The effect of an undamped finite degree of freedom "fuzzy" substructure: numerical solutions and theoretical discussion

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Abstract

A single degree of freedom master oscillator attached to a simple undamped N degree of freedom "fuzzy substructure" is studied numerically and theoretically. Results are found to be in accord with the predictions of the Pierce-Sparrow-Russell theory at early times, in particular the master oscillation manifests an apparent damping. At later times, however, the damping ceases and the energy is returned from the fuzzy to the master. The precise manner in which the energy is returned, and the time taken to do this, depend on the details of the mass and frequency distribution within the fuzzy, and in particular on the distribution of spacings between the fuzzy resonances.

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In recent years the Structural Acoustics community has considered a proposal by Soize [1,2] that the effect of complex uncertain substructures, when attached to a simple master structure, may be represented by means of the impedance presented to the master structure. In the parlance of the field the substructure is termed a "fuzzy sub-structure." The word fuzzy is not used here in the sense of fuzzy sets and a better term might be uncertain or stochastic substructures. The idea has been amplified by recent work by Pierce, Sparrow and Russell[3,4]. In particular it has been emphasized that in the limit that the number of degrees of freedom in the substructure becomes infinite, it provides an effective damping, and a less interesting modification of the effective mass and stiffness. Ruckman[5], and Ruckman and Feit[6] have provided reviews and tutorials recently in this area.

The simplest model of such a system is shown in Figure 1. For such a system, in the limit that the number of subsystem degrees of freedom $N \rightarrow \infty$ (such that the fuzzy mass remains finite and of order M: $\Sigma \mu_\alpha = O(M)$), the question arises as to whether the substructure can be represented in a simple manner that is independent of the details of the substructure. Pierce *et al.*[3] have studied systems like this and emphasized that, in this limit, the impedance $Z(\omega)$ presented by the fuzzy to the Master has a dissipative real part that corresponds to an effective damping, and an reactive imaginary part which is of somewhat less interest. Inasmuch as the above system lacks any dissipative elements, the meaning of this damping is less than obvious. Certain contentions have arisen[7].

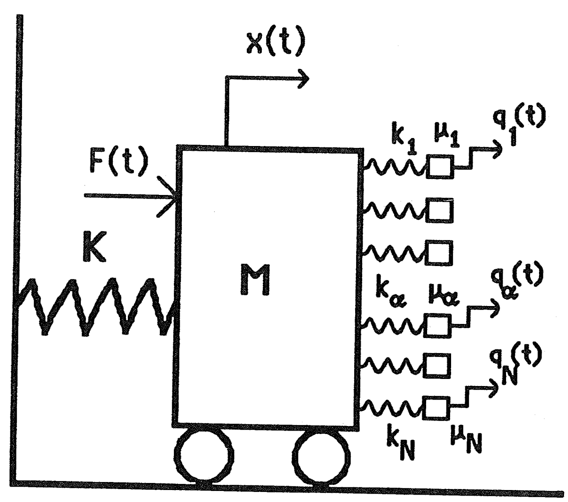

Figure 1. A master structure consisting of the mass M and stiffness K described by the coordinate x(t) is attached to a many degree of freedom fuzzy substructure consisting of masses μ_α and stiffnesses k_α described by coordinates $q_\alpha(t)$ and having isolated natural frequencies $\omega_\alpha = \sqrt{k_\alpha/\mu_\alpha}$.

The coupled Ordinary Differential Equations that describe the above system are

$$M\ddot{x} + Kx + \left(\sum_{\alpha}^{N} k_\alpha\right)x = \sum_{\alpha}^{N} k_\alpha q_\alpha + F(t)$$
$$m_\alpha \ddot{q}_\alpha + k_\alpha q_\alpha = k_\alpha x \tag{1}$$

These equations are often studied in the frequency domain. The Fourier transform is defined here by

$$\tilde{x}(\omega) \equiv \int_{-\infty}^{\infty} x(t)\exp\{-i\omega t\}\, dt \tag{2}$$

Causality (the vanishing of F and x for negative values of t) assures that their Fourier transforms are analytic in the lower half complex ω plane. $\omega \rightarrow \omega - \iota\varepsilon$ where ε is an infinitesimal positive quantity. In the frequency domain the coupled equations become

$$(K - M\omega^2)\tilde{x}(\omega) + \left(\sum_{\alpha} k_\alpha\right)\tilde{x}(\omega) = \sum_{\alpha} k_\alpha \tilde{q}_\alpha(\omega) + \tilde{F}(\omega)$$
$$(k_\alpha - \mu_\alpha\omega^2)\tilde{q}_\alpha(\omega) = k_\alpha \tilde{x}(\omega) \tag{3}$$

The later equation can be solved for q_α

$$\tilde{q}_\alpha(\omega) = \frac{\tilde{x}(\omega)}{1 - \omega^2/\omega_\alpha^2} \tag{4}$$

and the result substituted into the former equation to obtain

$$(K - M\omega^2)\tilde{x}(\omega) - i\,\omega\,\tilde{Z}(\omega)\,\tilde{x}(\omega) = \tilde{F}(\omega) \tag{5}$$

where the impedance Z presented to the master by the fuzzy is given by

$$i\omega\,\tilde{Z}(\omega) \equiv -\omega^2 \sum_{\alpha} \frac{\mu_\alpha}{[1 - (\omega - i\varepsilon)^2/\omega_\alpha^2]}. \tag{6}$$

The term in ε allows one to resolve the singularity.

This impedance is composed of a discrete sum of resonances. The real, or dissipative, part of the impedance is found by the invoking the usual identity from the theory of distributions

$$\lim_{\varepsilon\rightarrow 0^+} \frac{1}{z - i\varepsilon - z_o} = i\pi\,\delta(z - z_o) + P\frac{1}{z - z_o} \tag{7}$$

where P represents the Cauchy principle part. One obtains

$$\tilde{Z}(\omega) = i\omega\; P \sum_{\alpha=1}^{N} \frac{\mu_\alpha}{1 - \omega^2/\,\omega_\alpha^2} + \frac{\pi}{2}\sum_{\alpha=1}^{N} \mu_\alpha\,\omega_\alpha^2\,\delta(\omega - \omega_\alpha) \tag{8}$$

Thus the dissipative part of the impedance is infinite, but only on a set of measure zero consisting of the natural frequencies of the fuzzy substructure. It vanishes elsewhere.

If the number of degrees of freedom, N, of the fuzzy is large, but in such a fashion that the individual masses μ of the fuzzy are small, this sum, it is presumed[3,4], can be replaced with an integral

$$\tilde{Z}(\omega) = i\omega \int_0^\infty \frac{m(\Omega)}{1 - (\omega - i\varepsilon)^2 / \Omega^2} d\Omega \tag{9}$$

where $m(\Omega)$ is the smoothed spectral density of mass in the fuzzy substructure. Pierce[8] has discussed how one might estimate the appropriate $m(\Omega)$ for a real system.

Invoking the identity(7), one finds that Z becomes

$$\tilde{Z}(\omega) = \frac{\pi}{2} \omega^2 m(\omega) + i\omega P \int_0^\infty \frac{m(\Omega)}{1 - \omega^2/\Omega^2} d\Omega \tag{10}$$

which has a real part proportional to the spectral mass density in the fuzzy.

$$c_{\text{effective}}(\omega) = \frac{\pi}{2} \omega^2 m(\omega) \tag{11}$$

This is one of the central results of the current activity in this field [3,4,6,7]. The meaning of the dissipation represented by this term is not obvious.

The imaginary, or reactive, part of Z depends on the fuzzy parameters in a more complicated fashion than does the resistive part. It does, though, become simple in the low and high frequency limits:

$$\begin{aligned} \text{Im}\,\tilde{Z}(\omega) &\approx i\,\omega \int_0^\infty m(\Omega)\, d\Omega \quad (\omega \text{ small}) \\ &\approx (i/\omega) \int_0^\infty \Omega^2 m(\Omega)\, d\Omega \quad (\omega \text{ large}) \end{aligned} \tag{12}$$

The first of the above integrals can be recognized as the total mass in the fuzzy, the latter as the total stiffness.

The replacement of the sum with an integral is the most problematic of the steps taken here. Without that the impedance remains reactive almost everywhere. Indeed, for $N < \infty$, Z, eqn(8), is a highly discontinuous function of ω. Were it a smooth function of ω, the replacement of the sum with an integral would be less problematic. It may be that addition of a small amount of true dissipation, so that Z would in fact be smooth, would suffice to assure the legitimacy of the substitution. It is typically surmised that it is necessary to have a dissipation sufficient to assure modal overlap.

These arguments have so far not led to a consensus as to the meaning of the effective resistance $c_{\text{effective}}$, or to the possible errors incurred by applying the $N \to \infty$ limit to the case of finite N. It is to clarify these points that this present paper is addressed.

It is the conviction of this author that transient responses, studied in the time domain, will often illuminate issues that can remain obscure in the frequency domain. In the remainder of this paper, therefore, we present and discuss numerical solutions to the system described by (1). Theoretical arguments are advanced with which to understand them.

Numerical Method

Equations (1) are solved by central differences for the case of a unit impulsive force $F(t) = \delta(t)$.

$$M\{ x(t+\delta) - 2x(t) + x(t-\delta) \} / \delta^2 + K\{ x(t) \} = \sum_\alpha k_\alpha \{ q_\alpha(t) - x(t) \}$$

$$\mu_\alpha\{q_\alpha(t+\delta) - 2q_\alpha(t) + q_\alpha(t-\delta) \} / \delta^2 + k_\alpha\{q_\alpha(t) - x(t) \} = 0 \qquad (13)$$

$$q_\alpha(0) = q_\alpha(\delta) = 0$$

$$x(0) = 0;\; x(\delta) = \delta / M \qquad (14)$$

where δ is the chosen temporal step size. In the limit of vanishing step size, the difference equations (13) become equivalent to the differential equations(1). The difference equations allow calculation of x and q at time $t+\delta$ in terms of their values at two previous values of time, t and $t - \delta$. Iteration of the formula allows calculation of x and q at all subsequent times $t = n\,\delta$.

It is desirable to choose a value of step size that is sufficiently small that the computed response has the required accuracy, but not so small that one wastes computational time or incurs extra errors due to round-off. At finite, non infinitesimal, values of step size the solution of (13) deviates from the solution of (1). Accuracy at long integration times requires a correspondingly small step size. One of the virtues of the central difference method, however, is that a kind of qualitative accuracy may be assured by a moderately large step size even for long integration times. The requirement is that the step size δ be less than $2/\omega_{max}$ where ω_{max} is the largest natural frequency of the N +1 degree of freedom system. This assures that the numerical solution to (13) is stable. The effect of step size choice can be explored by comparing the formal solutions to (1) and (13):

The exact equations(1) have a solution that can be written in the following form:

$$\begin{Bmatrix} x \\ q \end{Bmatrix} = \mathrm{Re} \sum_{m=1}^{N+1} A_m \{ \phi^m \} \exp(i\,\omega_m t) \qquad (15)$$

where the $\{\phi^m\}$ are the N+1 component column vector normal modes of the N+1 degree of freedom system, and the ω_m are their natural frequencies. The A_m are the complex modal amplitudes, determined by the initial conditions.

The corresponding solution of the difference equations(13) is

$$\begin{Bmatrix} x \\ q \end{Bmatrix} = \mathrm{Re} \sum_{m=1}^{N+1} A'_m \{ \phi^m \} \exp(i\,\omega'_m t) \qquad (16)$$

The mode shapes $\{ \phi \}$ are identical to those of the exact system, but the modal amplitudes are slightly different:

$$A'_m = A_m + O(\delta). \qquad (17)$$

The natural frequencies are different also:

$$\omega_m' = 2 \arcsin (\omega_m \delta / 2) / \delta. \qquad (18)$$

Clearly as $\delta \to 0$ the results are identical. If, however, one wishes accuracy at late times T, one would require $(\omega_m' - \omega_m)T << 1$, or

$$\delta << (T \omega_m^3)^{-1/2} \qquad (19)$$

where m is a typical mode of interest.

This can, in some cases, be a prohibitive constraint on the step size. The easier condition $\delta < 2/\omega_{max}$ (which, according to (18), assures that none of the ω' are complex and that therefore the solution (16) does not grow exponentially in time) suffices to assure the qualitative accuracy that is usually all that we need. The error incurred by choice of a finite δ is therefore only a matter of some minor phase distortions that may be termed numerical dispersion. For a band limited process the error is only a matter of a slight rescaling of time.

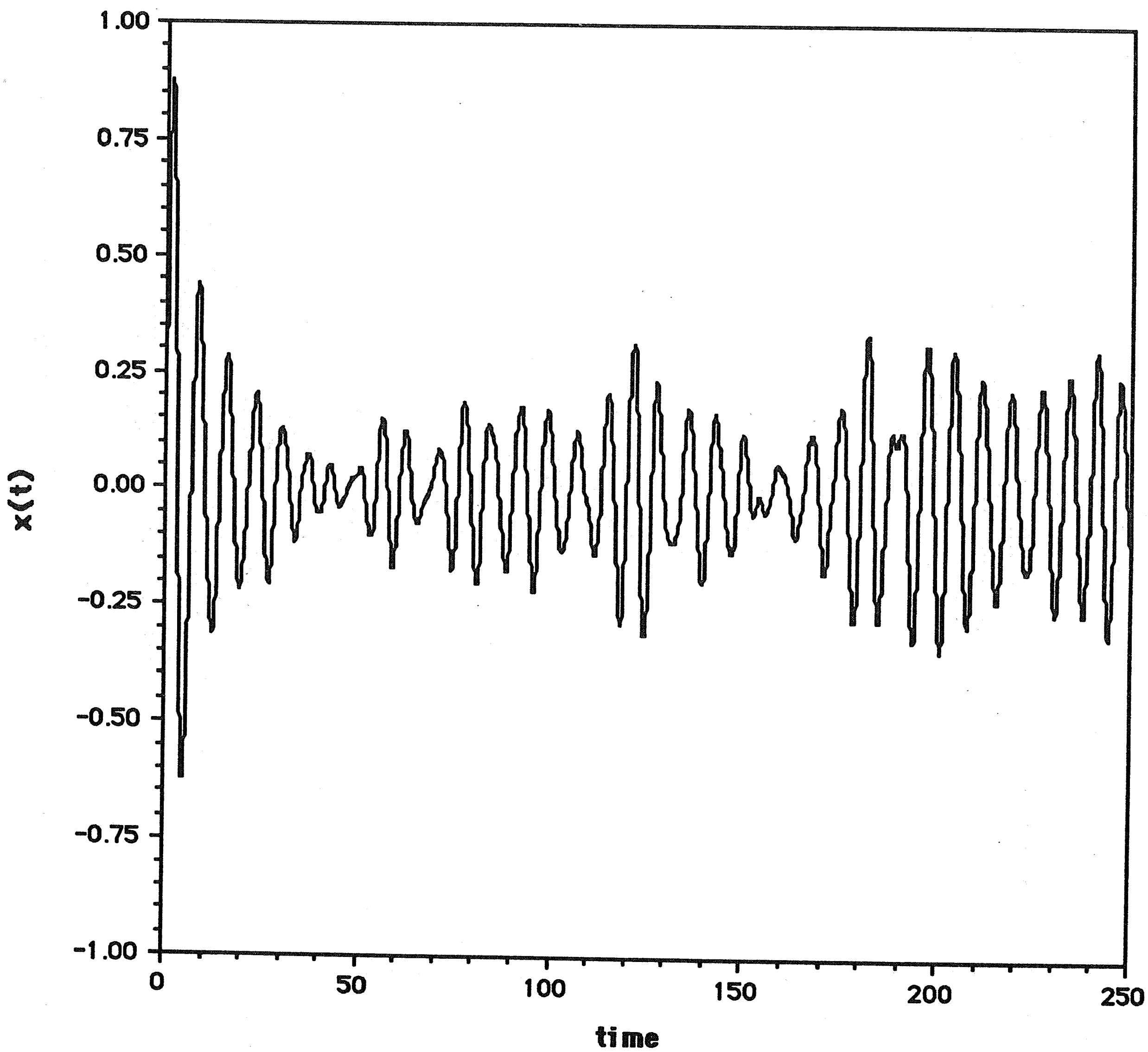

Figure 2a] The numerically determined response x(t) to a unit impulse. The substructure consisted of 100 equal masses $\mu = 0.003$ with natural frequencies chosen from an exponential distribution.

Solutions

Without loss of generality we choose the mass and stiffness of the master structure to be unity:

$$M = K = 1 \tag{20}$$

There remains a great deal of latitude in our choice for N, for the small masses μ_α and for their associated stiffnesses k_α.

The first case studied, shown in figures 2, corresponds to the choice N=100 and the choice $\mu_\alpha = \mu = 0.003$ for all α. Thus the total mass in the fuzzy is 0.3 times the mass of the master. The natural frequencies ω_α were taken randomly from an exponential probability density function $p(\omega_1, \omega_2, \ldots \omega_N) = \exp(-\omega_1 - \omega_2 - \ldots -\omega_N)$. The mean natural frequency in the substructure is therefore $< \omega > = 1$, the same as the natural frequency of the isolated

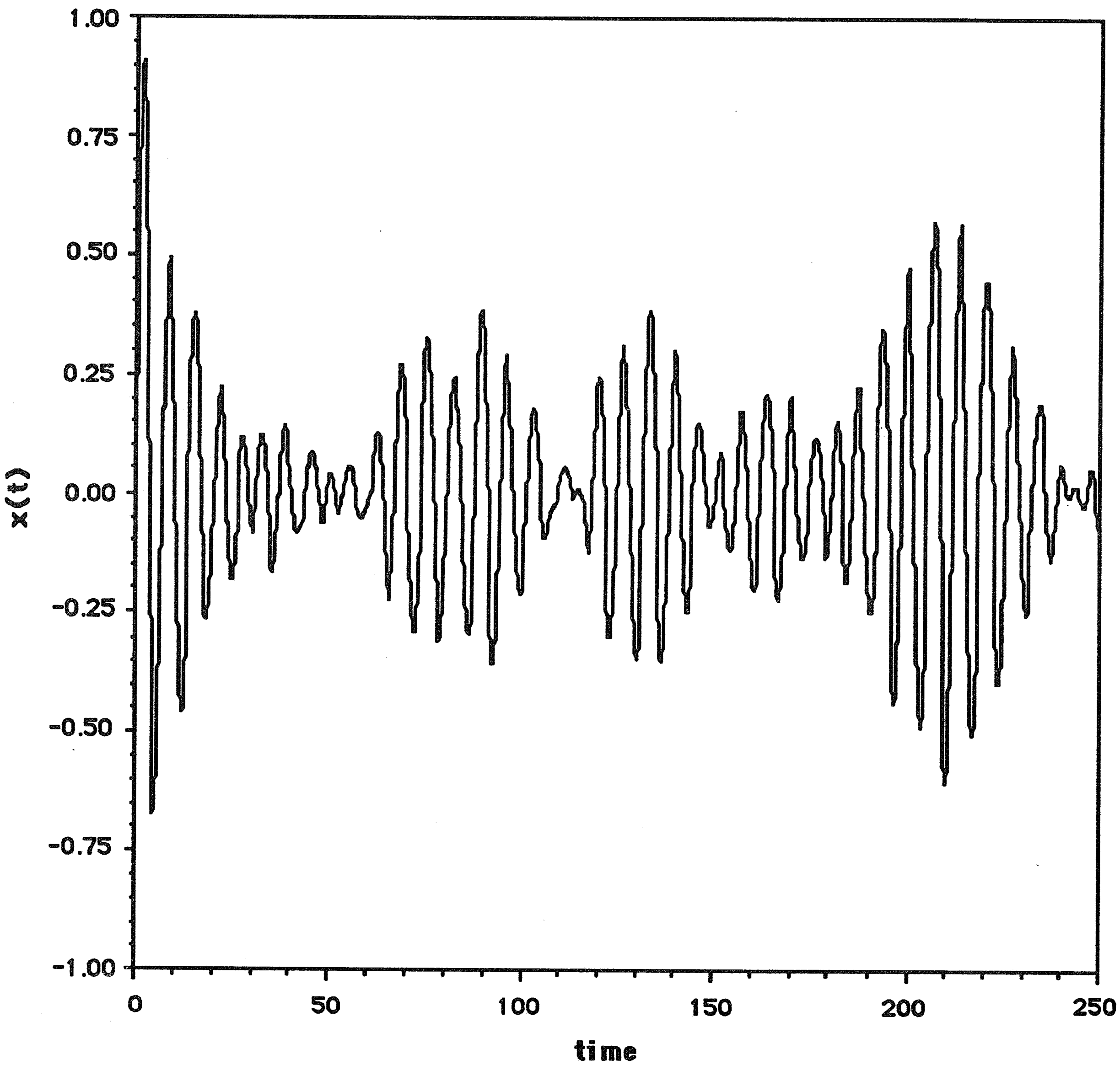

Figure 2b] A different set of randomly chosen frequencies in the fuzzy substructure, but with all other parameters unchanged. The response still shows the early time decay, but the random oscillations at late times are different.

master oscillator. Two realizations from the random ensemble were studied. For the case shown in figure 2a the largest natural frequency ω_α in the random collection was 4.93; for case b it was 3.34. These numbers provide estimates for ω_{max}. The step size δ was chosen equal to 0.1. The constraint $\delta\omega_{max} < 2$ was well satisfied. One may calculate the effective resistance (11) for this system, $c_{effective} = (\pi/2)\ \omega^2\ (0.3)\ e^{-\omega}$. Naive application of the Pierce-Sparrow-Russell theory [3,4] would predict a free vibration of the form

$$x(t) \sim \exp\{\, i\, \omega_{natural}\, t - c_{effective}\, t \,/\, 2M \,\} \qquad (21)$$

where $\omega_{natural}$ is approximately $\sqrt{K/M} = 1$, with any difference being ascribable to the reactive part of Z. $c_{effective}$ is evaluated at $\omega_{natural}$.

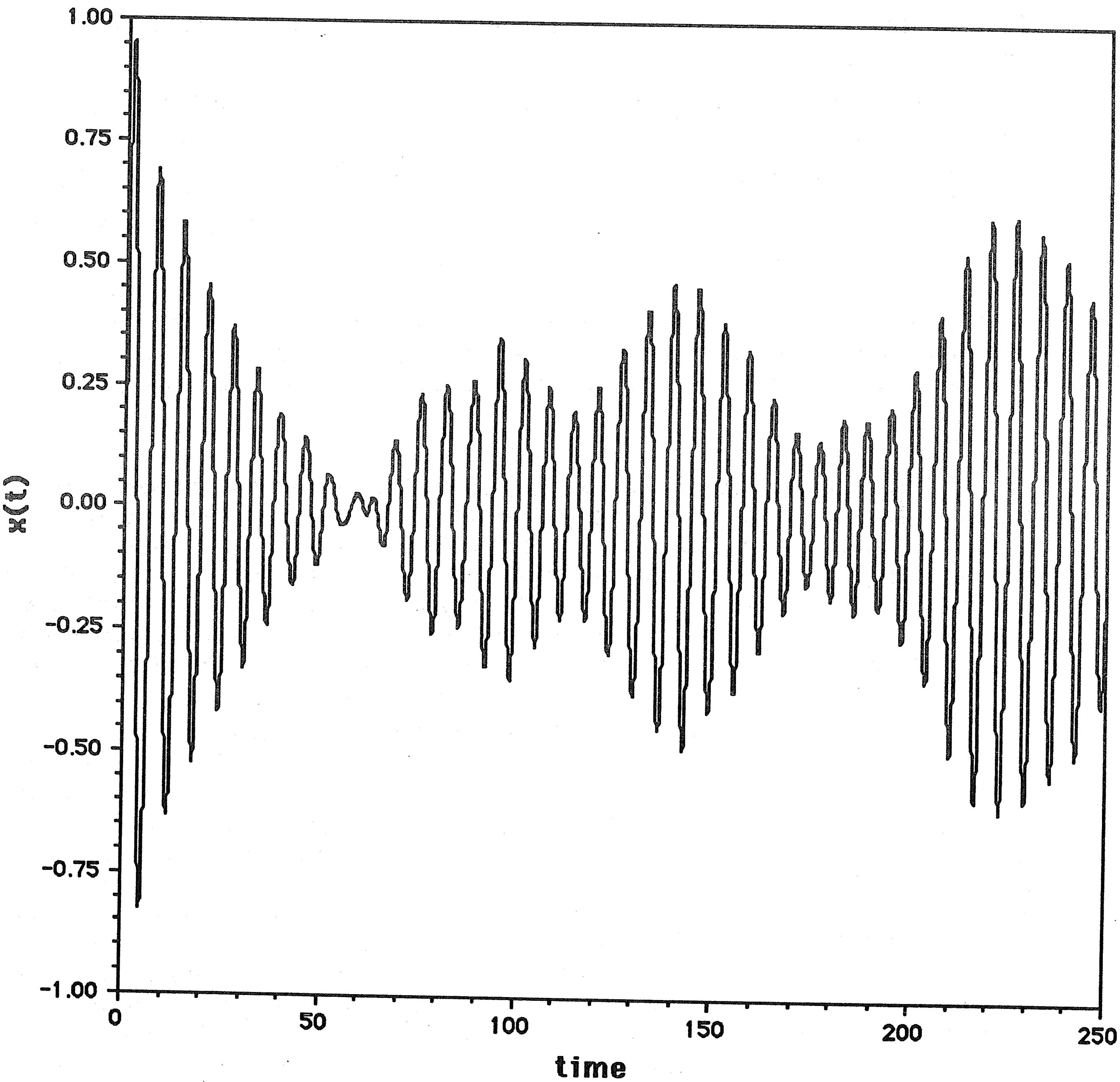

Figure 3] The response x(t) to a unit impulse for the case of a substructure consisting of 100 equal masses μ = 0.0015 with natural frequencies chosen from an exponential distribution.

In each figure the response at early times decays and oscillates at a frequency very nearly equal to that of the master oscillator in isolation. Based on the average period over the first few cycles one estimates that x(t) in figures 2 has a frequency f_{nat} = 0.138±0.005 and 0.148±0.005 respectively. These numbers may be compared to the frequency of the master oscillator in isolation, $f_{nat} = 1/2\pi = 0.159$. The difference is slight but apparently significant; it may be ascribed to the reactive part of the impedance contributed by the fuzzy. The figures also show, at early times, that the response decays at the rate predicted by the simple ($N \rightarrow \infty$) theory. The observed log decrements, based on the first three maxima of figures 2 are 0.55 and 0.49 respectively. The predicted log decrement is, evaluated at $\omega = 2\pi f_{nat}$, $c_{eff} / 2 M f_{nat}$ = 0.54.

At late times, however, x(t) exhibits an apparent randomness, with a spectral content centered near f_{nat}, and a spectral peak width and autocorrelation time consistent with the Quality factor $Q = M\omega/c_{eff}$. The late time amplitude is not insignificant. Even though N=100 might be considered large, it appears that the prediction (21) can be misleading.

Figure 3 shows the case of a smaller fuzzy structure mass, $\mu = 0.0015$, and a new realization of the set of random frequencies. This time ω_{max} was 5.81. The apparent natural frequency of free oscillation is now 0.151 ±0.005. The early time decay rate is now slower, with a log decrement of 0.25 ± .02. This is in accord with the prediction (21) of 0.27. The late time behavior has a longer autocorrelation time which may be associated with the lower decay rate. The amplitude at late times appears to be greater also.

Figure 4 shows the case N=1000, m = 0.0003, again with an exponential frequency distribution having $< \omega > = 1$. The prediction (11) for the effective damping is unmodified from the case studied in Figures 2. The predicted log decrement is again 0.54; the measured log decrement is 0.50. The late time random oscillation has substantially lower amplitude.

Discussion

These numerical studies have demonstrated that the substitution of the integral (9) for the sum(8) has correctly predicted the short time behavior, even for finite values of N. In particular it has correctly predicted the apparent dissipation. In view of the highly discontinuous nature of the function $Z(\omega)$ this might be viewed as surprising. The substitution has failed, however, to predict the cessation of damping at late times and the subsequent random oscillations. The failure at late times can be understood by simple arguments that appeal to the absence of any true dissipation and to the concept of equipartition. The short time accuracy of the prediction can be understood by appeal to another argument.

That the late time behavior is not quiescent, that the decay predicted by (21) does not continue indefinitely, could have been anticipated. The absence of any dissipative elements in the system implies that the total system energy (equal to $E_{total} = I^2/2M$ where I is the applied impulse, I = 1) must be a constant. That the master structure shares in this total energy is to be expected. If the master structure has a mean energy, < E > at late times, then its rms amplitude should be $\sqrt{E/K}$. Simple equipartition arguments (*e.g.* SEA[9]) suggest that the single degree of freedom master structure should have a mean energy < E > equal to 1/n+1 of the total energy $I^2/2M$. Here n should be the number of oscillators in the fuzzy substructure within the relevant frequency band. A rough guess for this number is the modal density in the substructure times the bandwidth of the deposited energy. The appropriate quantitative definition of that bandwidth is not obvious here. A good guess is the usual one of ω/Q, where Q is the quality factor; $Q = \omega M / c_{effective}$. The modal density can be expressed as $m(\omega)/ < \mu >$ where $< \mu >$ is the average mass of an oscillator at the frequency of interest. The prediction for n, therefore, is $n = \pi\omega^2 m(\omega)^2/2M<\mu>$.

These ideas imply a late time root mean square amplitude given by the expression:

$$\mathrm{rms} \sim \sqrt{\frac{E_{total}/K}{1+m(\omega)^2\omega^2\pi/2<\mu>M}} = I\sqrt{\frac{<\mu>}{K}}\sqrt{\frac{1}{2M<\mu>+m(\omega)^2\omega^2\pi}} \qquad (22)$$

which may be compared with the amplitudes observed in the figures. In the cases studied in Figures 2 the formula (22) predicts an rms level of 0.26; in the case studied in Figure 3, it predicts an rms level of 0.35; in the case studied in Figure 4 it predicts an rms level of 0.085. These quantities are in approximate agreement with the late times amplitudes in the figures.

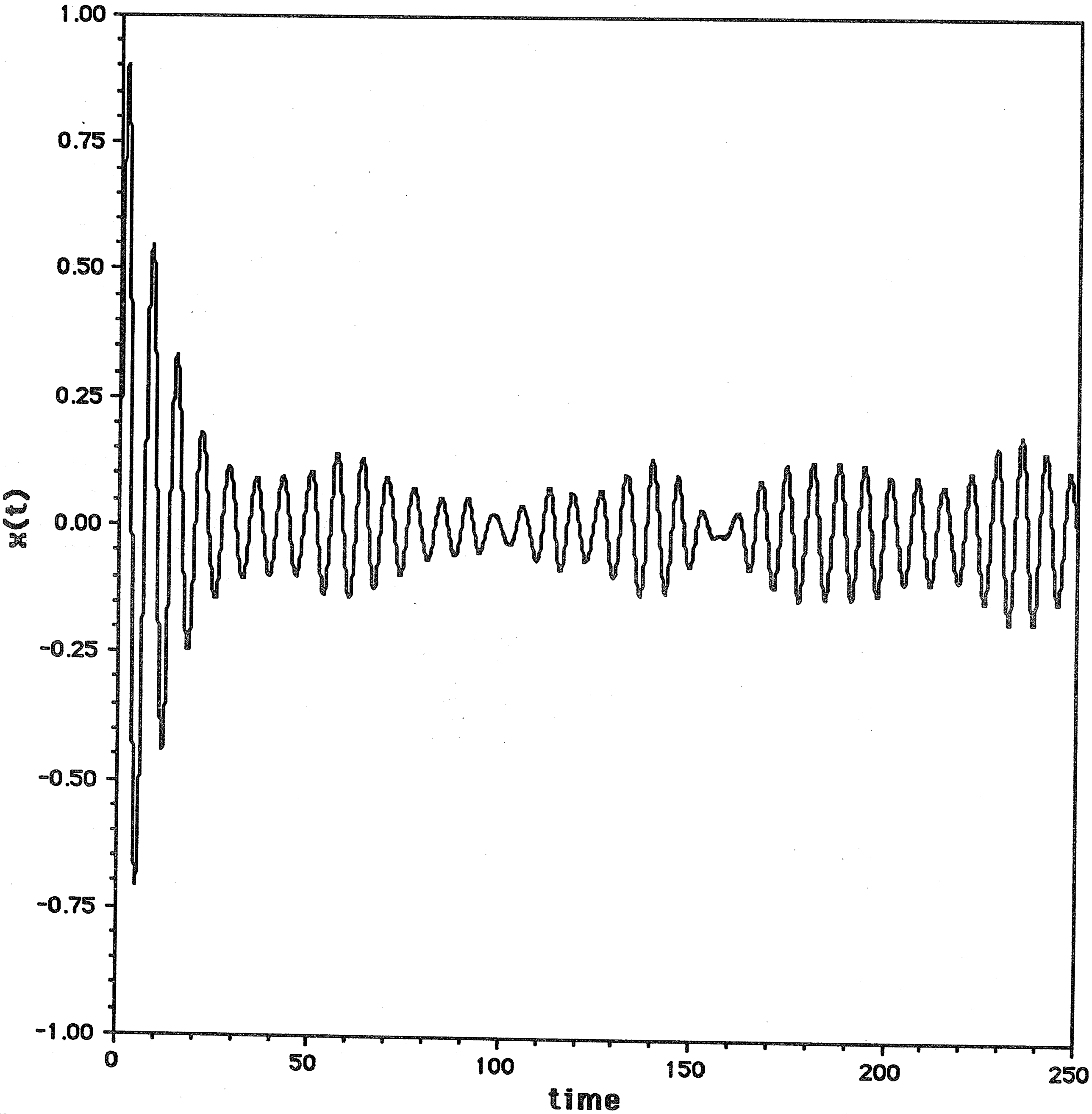

Figure 4] The case N=1000, μ = 0.0003. The early time decay rate is unchanged from the case described in figures 2, as predicted by the simple theory. The late time rms amplitude is less, in accord with equipartition.

The predicted levels are arguably slightly greater than the observed levels. The differences are not significant for the purposes of this paper, but they might tentatively be ascribed to an incorrect guess for the appropriate definition of bandwidth.

These arguments suggest that, as $N \to \infty$, the late time level falls to zero (albeit slowly, like $N^{-1/2}$), and that the prediction (21) that simple decay continues indefinitely ultimately becomes correct. On the other hand one also sees that expressions like (11) for the effective damping can be very misleading if N is not very large.

The prediction (11),(21) for the late time behavior was found to be incorrect at finite N. At short times, however, the prediction was accurate, even though N was finite and the actual impedances Z were highly discontinuous functions of frequency; the substitution of an integral for the sum is not well justified. The accuracy at short times may be understood by considering the effect of the introduction of damping. Consider the device of replacing $\omega \to \omega - i\eta$, where η is a non-infinitesimal quantity. This changes transient impulse responses by $x(t) \to x(t) \exp(-\eta\, t)$ At times t much less than $1/\eta$ the change in x(t) is negligible. The substitution $\omega \to \omega - i\eta$ has also made Z much smoother. If η is much greater than typical spacings between resonances in the fuzzy, then Z will lose its highly discontinuous nature and become smooth. The replacement of the sum with an integral would then be straightforward. For sufficiently early times one therefore expects the replacement of the sum with smooth integral to be accurate. The maximum value of time over which the simple theory should be accurate, "T," is therefore the inverse of the frequency bandwidth over which one must smooth the impedance $Z(\omega)$. If the resonances in the fuzzy are spaced by $\Delta\omega$, one would require only $T < 2\pi/\Delta\omega$. The mean modal spacing (at a value of $\omega = 1$) in the cases studied in Figures 2 and 4 was e/N, or $< \Delta\omega > = 0.027$ and 0.0027 respectively, for an estimate $T < 2\pi/\Delta\omega = 231$ and 2310 respectively. It is very clear that the estimate $T < 231$ grossly overestimates the domain over which the prediction (21) is accurate in Figures 2 and that the estimate $T < 2310$ does so even more severely in Figure 4. Thus this prediction for the range of accuracy of (21) is incorrect, and the speculation that a moderate value of mean modal overlap suffices to assure its accuracy is also incorrect. In the above examples, however, while there is an average spacing $< \Delta\omega >$, there are fluctuations also. For these cases the constraint on T must be more severe: damping must be sufficient to assure that even the more widely spaced resonances have significant overlap.

<u>Uniformly spaced resonances in the substructure</u>

The above argument suggests that the case of uniform resonance spacing could be of particular interest. Consider a fuzzy substructure consisting of N equally spaced ($\Delta\omega << 1$) natural frequencies $\omega_\alpha = \alpha\, \Delta\omega$, each associated with a different mass μ_α for $\alpha = 1,2,3 \ldots.$ N.

The impedance presented by this substructure is

$$\tilde{Z}(\omega) = i\,\omega \sum_{a} \mu_\alpha \;\; / \; [\, 1 - \omega^2/\omega_\alpha^2\,] \tag{23}$$

In the time domain this is

$$Z(t) = \frac{d}{dt}[\, \exp(-\eta t) \sum_{\alpha} m_\alpha \;\; \omega_\alpha^2\, H(t)\; \sin \omega_\alpha t \;\, / \;\, \omega_\alpha\,] \tag{24}$$

where H is the unit step function and the factor $\exp(-\eta t)$ represents the effect of the optional device $\omega \to \omega - i\eta$.

The governing equation for the master now reads, in the time domain,

$$M\ddot{x} + Kx - Z * \dot{x} =$$

$$M\ddot{x}(t) + Kx(t) - \int_0^t Z(t-\tau)\,\dot{x}(\tau)\,d\tau = F(t) \qquad (25)$$

where * denotes a convolution.

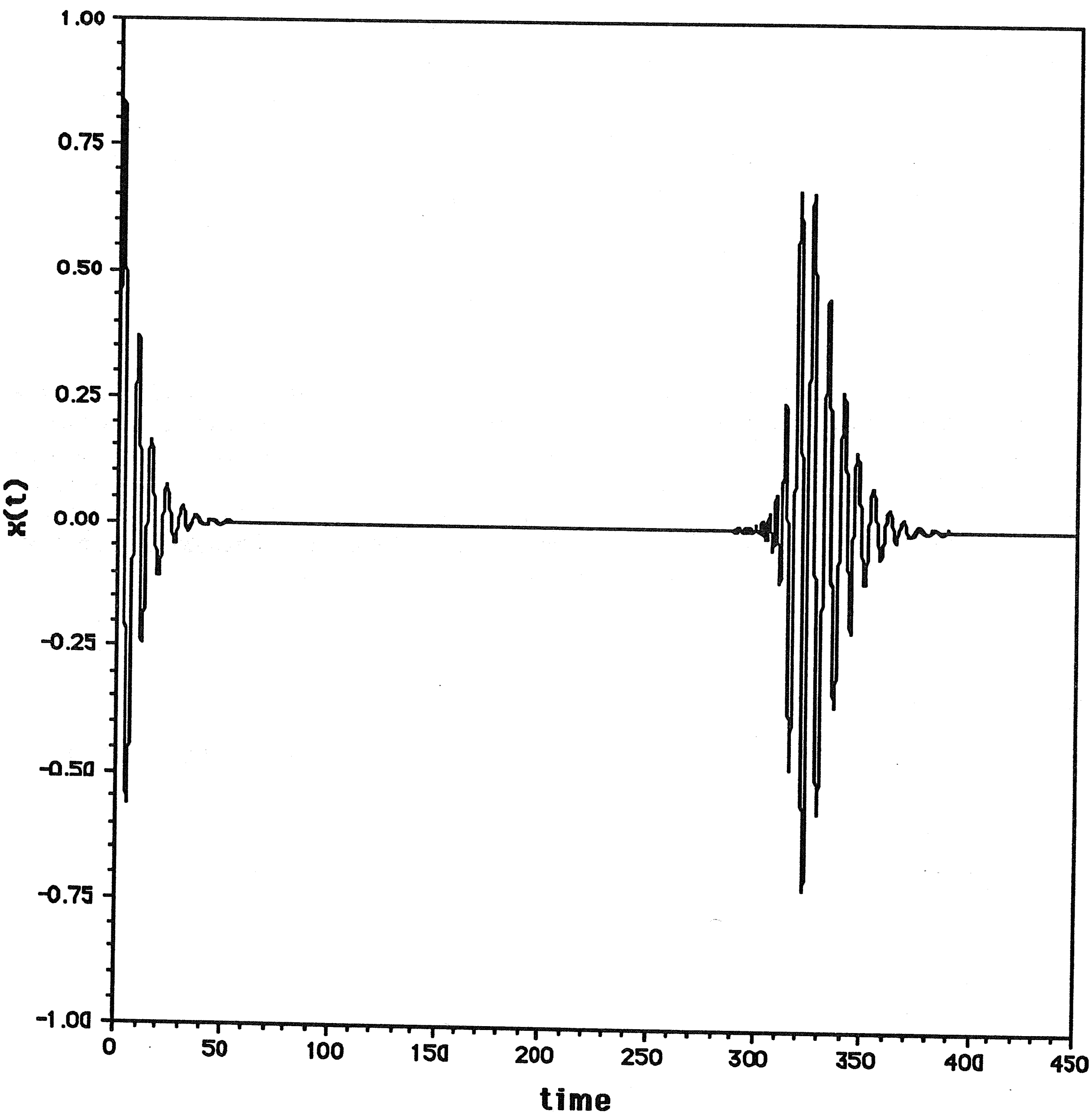

Figure 5] The case of N=300 equally spaced substructure resonances with an exponential distribution of mass. In accord with the simple theory, the substructure acts to damp the master structure at early times. But at time T_z =314 the substructure returns the energy it had originally absorbed.

If η is sufficiently greater than the spacing $\Delta\omega$, then the sum in (24) may be replaced by an integral in the usual fashion and the usual result obtained. Thus one expects, at early times, to see an effective damping $c_{effective}$. If, however, one is interested in the response at later times one must take $\eta = 0$. In this case the above expression for Z may be recognized to be a Fourier series (times a unit step function). Therefore Z(t) is periodic. Its period is $T_Z = 2\pi/\Delta\omega$. Thus the term in Z above acts at early times as if the system has damping, but at late times provides a force on the master structure proportional to the master structure velocity at earlier times, $t - n\,T_Z$ for all integer n. One comes to the remarkable conclusion that the master structure should exhibit damping at early times, but be re-excited by the substructure at a much later time. The moment of the first such re-excitation should be at $t = T_Z$.

Figure 5 shows the computed response x(t) of the master structure for the case outlined above, with the choice $\Delta\omega = 0.02$ and $\mu_\alpha = \mu \exp(-\omega_\alpha)$, with $\mu = 0.01$. Thus the total mass within the substructure is $\Sigma_\alpha \mu_\alpha \sim 0.48$. The temporal step size was chosen equal to 0.2. The sum over α was truncated at N=300 terms so that the highest frequency in the fuzzy substructure was 6.0. The early time damping is clear; it continues for a long time, unlike the cases (Figures 2-4) of randomly chosen values of fuzzy substructure natural frequencies. At late times, however, the substructure returns the energy it had originally absorbed from the master. The return time is, as predicted, $T_Z = 2\pi/\Delta\omega = 314$.[10]

Conclusions

Numerical demonstrations, and theoretical arguments, have been presented which show that the action of a finite degree of freedom undamped "fuzzy " substructure is, at early times, in accord with the predictions of the Pierce-Sparrow-Russell theory. At later times, however, the description in terms of a simple damping becomes incorrect; the energy is returned from the fuzzy to the master. The precise manner in which the energy is returned, and the time taken to do this, depend on the details of the mass and frequency distribution within the fuzzy, and in particular on the distribution of spacings between the fuzzy resonances.

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10] It is interesting to note that the higher frequencies return a little sooner than T_z. This may be ascribed to the numerical dispersion generated by the finite temporal step size.

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731	Acharya, A., and T. G. Shawki	A second-deformation-gradient theory of plasticity—*Journal of the Mechanics and Physics of Solids*, in press (1995)	Oct. 1993
732	Michaleris, P., D. A. Tortorelli, and C. A. Vidal	Tangent operators and design sensitivity formulations for transient nonlinear coupled problems with applications to elasto-plasticity—*International Journal for Numerical Methods in Engineering* **37**, 2471–2500 (1994)	Nov. 1993
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777	Cherukuri, H. P., T. G. Shawki, and M. El-Raheb	An accurate finite-difference scheme for elastic wave propagation in a circular disk	Nov. 1994
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779	Phillips, J. W., D. L. de Camara, M. D. Lockwood, and W. C. C. Grebner	Strength of silicone breast implants—*Plastic and Reconstructive Surgery*, in press (1995)	Jan. 1995
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783	Batchelor, G. K., and TAM faculty and students	A conversation with Professor George K. Batchelor	Feb. 1995
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790	Figa, J., and C. J. Lawrence	Linear stability analysis of a gravity-driven Newtonian coating flow on a planar incline	May 1995
791	Figa, J., and C. J. Lawrence	Linear stability analysis of a gravity-driven viscosity-stratified Newtonian coating flow on a planar incline	May 1995
792	Cherukuri, H. P., and T. G. Shawki	On shear band nucleation and the finite propagation speed of thermal disturbances	May 1995
793	Harris, J. G.	Modeling scanned acoustic imaging of defects at solid interfaces	May 1995
794	Sottos, N. R., J. M. Ockers, and M. J. Swindeman	Thermoelastic properties of plain weave composites for multilayer circuit board applications	May 1995
795	Aref, H., and M. A. Stremler	On the motion of three point vortices in a periodic strip	June 1995
796	Barenblatt, G. I., and N. Goldenfeld	Does fully-developed turbulence exist? Reynolds number independence versus asymptotic covariance—*Physics of Fluids* 7, 3078–3082 (1995)	June 1995
797	Aslam, T. D., J. B. Bdzil, and D. S. Stewart	Level set methods applied to modeling detonation shock dynamics	June 1995
798	Nimmagadda, P. B. R., and P. Sofronis	The effect of interface slip and diffusion on the creep strength of fiber and particulate composite materials—*Mechanics of Materials*, in press (1996)	July 1995
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801	Riahi, D. N.	Perturbation analysis and modeling for stratified turbulence	Aug. 1995
802	Thoroddsen, S. T.	Conditional sampling of dissipation in high Reynolds number turbulence	Aug. 1995
803	Riahi, D. N.	On the structure of an unsteady convecting mushy layer	Aug. 1995
804	Meleshko, V. V.	Equilibrium of an elastic rectangle: The Mathieu–Inglis–Pickett solution revisited	Aug. 1995
805	Jonnalagadda, K., G. E. Kline, and N. R. Sottos	Local displacements and load transfer in shape memory alloy composites	Aug. 1995
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809	Xin, Y.-B. and K. J. Hsia	Simulation of the brittle-ductile transition in silicon single crystals using dislocation mechanics	Oct. 1995
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