

Mean and Mean Square Responses of a Prototypical Master/Fuzzy Structure

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Abstract

Analytical estimates are made for mean and mean square transient responses of a single degree of freedom master oscillator attached to an undamped N degree of freedom random substructure. The ensemble averaged response is found to be, asymptotically for a large number of substructural degrees of freedom, in accord with the predictions of the Pierce-Sparrow-Russell theory. In particular the mean response of the master oscillator manifests an apparent damping. Corrections to this behavior are also predicted, but are found to lie well below the root mean square level of the fluctuations. The corrections are therefore, for practical purposes, unimportant. The ensemble average of the square of the response is also investigated and found to take a value at later times that is in accord with simple equipartition arguments advanced in an earlier paper. The power spectral density of the late time behavior is shown to be proportional to the square of the power spectrum of the early time response, and thus to be sharper than that of the early time behavior. Numerical simulations are presented which agree with the analytical predictions at moderate times, but also show an enhanced backscatter that develops over longer time scales.

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Introduction

In recent years the Structural Acoustics community has considered a proposal by Soize [1,2] that the effect of complex uncertain substructures, when attached to a simple master structure, may be represented by means of the impedance presented to the master structure. In the parlance of the field the substructure is termed a "fuzzy sub-structure." The idea has been amplified in recent work by Pierce, Sparrow and Russell[3,4] and by Strasberg and Feit[5]. Ruckman[6], and Ruckman and Feit[7] have provided reviews and tutorials recently in this area.

The simplest model of such a system is shown in Figure 1. In the limit that the number of subsystem degrees of freedom N is large (such that the fuzzy mass remains finite and of order M : $\sum \mu_\alpha = O(M)$), the question arises as to whether the effect of the substructure can be represented in a simple manner that is independent of the details of the substructure. Pierce *et al.*[3,4] and Strasberg and Feit[5] have studied systems like this and emphasized that, in this limit, the impedance presented by the fuzzy to the Master has a dissipative real part that corresponds to an effective damping, and an reactive imaginary part which is of somewhat less interest. Strasberg and Feit[5] emphasize, however, that their analysis is restricted to the case in which there is sufficient damping in the substructure to warrant certain approximations. In this case the conclusion is clearly sound - the master oscillator manifests an apparent damping that is independent of the precise degree and mechanism of damping in the substructure. If however, the substructure has insufficient damping to justify the approximations, such a conclusion is more problematic.

Nevertheless Weaver[8] recently argued, and showed numerically, that the conclusion is correct even if the substructure lacks damping, *if* one confines attention to early times. He furthermore showed that at later times the master structure oscillates in a random fashion with a mean square amplitude that appears to be in accord with simple SEA-like equipartition arguments. It is the intention of this paper to explore these questions analytically. In the following sections a procedure (related to certain methods in quantum field theory used in recent years for studying the electronics of disordered structures) is introduced for calculating response moments. Mean and mean square responses are then evaluated. Mean responses are

calculated to be different from, but practically indistinguishable from, the responses predicted by others [3,4,5] Calculations of mean square responses are then presented and found to be in accord with the equipartition arguments. In the last section the results are compared to numerical simulations.

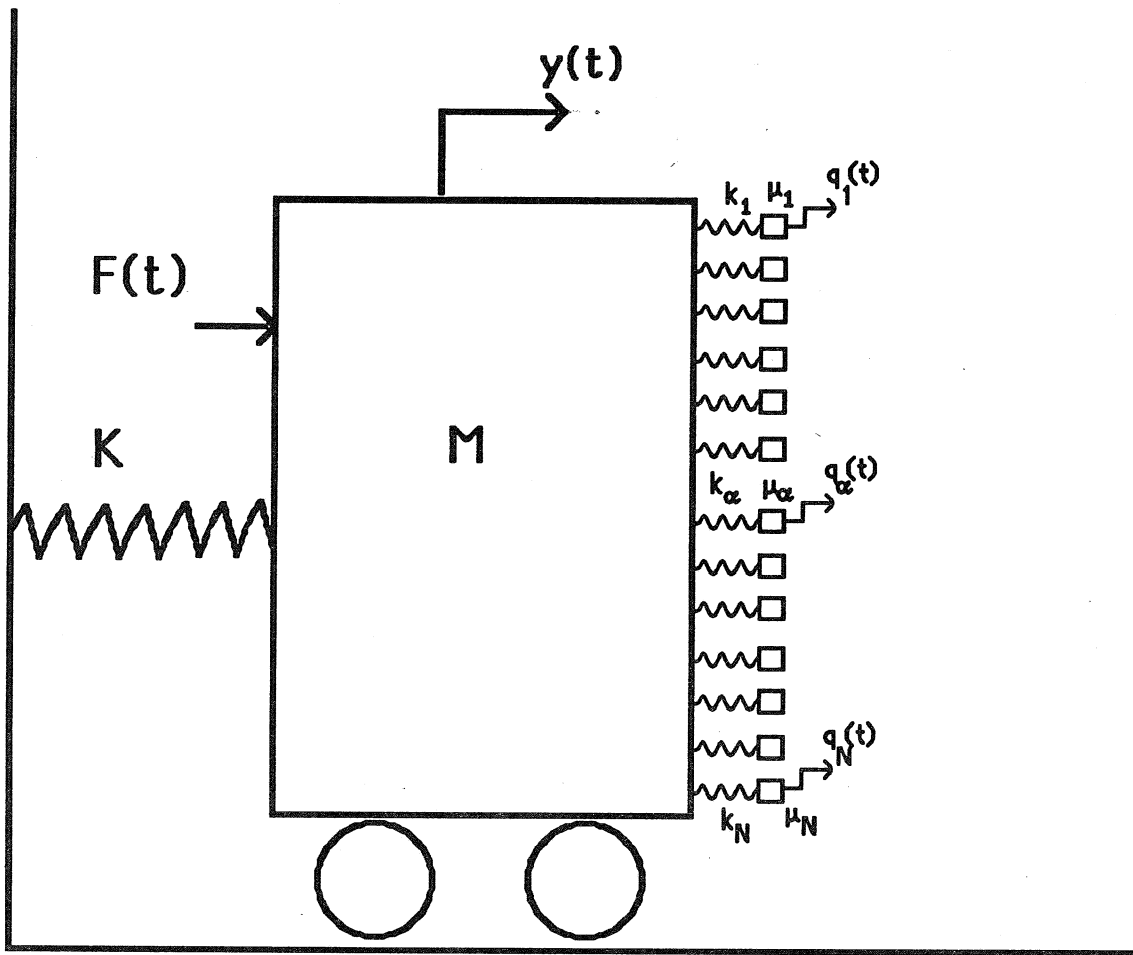


Figure 1. A master structure consisting of the mass M and stiffness K described by the coordinate $y(t)$ is attached to a many degree of freedom fuzzy substructure consisting of masses μ_α and stiffnesses k_α described by coordinates $q_\alpha(t)$ and having isolated natural frequencies $\omega_\alpha = \sqrt{k_\alpha/\mu_\alpha}$

Theory. preliminaries

The coupled Ordinary Differential Equations that describe the system of Figure [1] are

$$M \ddot{y} + K y + \left(\sum_{\alpha}^N k_{\alpha} \right) y = \sum_{\alpha}^N k_{\alpha} q_{\alpha} + F(t) \quad (1)$$

$$\mu_{\alpha} \ddot{q}_{\alpha} + k_{\alpha} q_{\alpha} = k_{\alpha} y$$

The Fourier Transform is defined here by

$$\tilde{y}(\omega) \equiv \int_{-\infty}^{\infty} y(t) \exp\{-i\omega t\} dt \quad (2)$$

in such a way that causality (the vanishing of F and x for negative values of t) assures that the Fourier transforms are analytic in the lower half complex ω plane. $\omega \rightarrow \omega - i\varepsilon$, where ε is an infinitesimal positive quantity. In the frequency domain the coupled equations become

$$(K - M\omega^2) \tilde{y}(\omega) + \left(\sum_{\alpha} k_{\alpha} \right) \tilde{y}(\omega) = \sum_{\alpha} k_{\alpha} \tilde{q}_{\alpha}(\omega) + \tilde{F}(\omega) \quad (3)$$

$$(k_{\alpha} - \mu_{\alpha}\omega^2) \tilde{q}_{\alpha}(\omega) = k_{\alpha} \tilde{y}(\omega)$$

The later equation can be solved for q_{α}

$$\tilde{q}_{\alpha}(\omega) = \frac{\tilde{y}(\omega)}{1 - \omega^2/\omega_{\alpha}^2} \quad (4)$$

where ω_{α}^2 is defined as k_{α}/μ_{α} . The result is substituted into the former equation to obtain

$$(K - M\omega^2) \tilde{y}(\omega) - \tilde{V}(\omega) \tilde{y}(\omega) = \tilde{F}(\omega) \quad (5a)$$

where the "scattering potential" V presented to the master by the fuzzy is given by

$$\tilde{V}(\omega) \equiv \sum_{\alpha} \tilde{v}_{\alpha}(\omega) \equiv \omega^2 \sum_{\alpha} \frac{\mu_{\alpha}}{[1 - (\omega - i\varepsilon)^2/\omega_{\alpha}^2]} \quad (6)$$

The term in ε allows one to resolve the singularity.

The impulse response is the solution, $y = G$, in the case that $F(t) = \delta(t)$,

$$(K - M\omega^2)\tilde{G}(\omega) - \tilde{V}(\omega)\tilde{G}(\omega) = 1 \quad (5b)$$

If the number of degrees of freedom, N , of the fuzzy is large, but in such a fashion that the individual masses μ of the fuzzy are small, the sum, (6) it has been presumed[3,4,5], can be replaced with an integral

$$\tilde{V}(\omega) = \omega^2 \int_0^\infty \frac{m(\Omega)}{1 - (\omega - i\varepsilon)^2 / \Omega^2} d\Omega \quad (7)$$

where $m(\Omega)$ is the smoothed spectral density of mass in the fuzzy substructure. If the substructure has sufficient damping that the resonances in (6) overlap (this can be achieved formally by letting ε be finite) then the substitution of the integral (7) for the sum (6) is justified.[5]

Invoking the usual identity from the theory of distributions, or by doing the above integral by residues, one finds that V is

$$\tilde{V}(\omega) = -i\frac{\pi}{2}\omega^3 m(\omega) + \omega^2 P \int_0^\infty \frac{m(\Omega)}{1 - \omega^2/\Omega^2} d\Omega \quad (8)$$

where 'P' indicates a Cauchy principle part. This has an imaginary part representing an apparent damping of the master structure that is proportional to the spectral mass density in the fuzzy.

$$c_{\text{effective}}(\omega) = \frac{\pi}{2} \omega^2 m(\omega) \quad (9)$$

This is one of the central results of the current activity in this field. This quantity represents the loss of energy from the master to the multi-degree of freedom substructure.

The replacement of the sum with an integral is the most questionable of the steps taken here. Indeed, for $N < \infty$, V , eqn(6), is a highly discontinuous function of ω . Were it a smooth function of ω , the replacement of the sum with an integral would be less problematic. As emphasized by Strasberg and Feit[5] the introduction of damping sufficient to provide for modal overlap in the substructure would make it smooth. Weaver[8] emphasized that, *if* interest is confined to early times, then introduction of damping will not change responses. He concluded that the above value (9) for effective damping applies at early times even in undamped systems.

If, however, interest is also directed towards later times, and in undamped systems with finite N , there is at present no theory for the response. Such a theory is presented here.

Mean Responses

For purposes of definiteness a particular distribution for the masses and stiffnesses within the substructure is considered here. All stiffnesses k_α are chosen equal, at a value of order $1/N$; $k_\alpha = k = \kappa K/N$, with κ of order unity and dimensionless. Values of κ that are somewhat less than unity are probably the most relevant in practice, as one usually wishes to consider a master structure that is only moderately perturbed by the substructure. Such values of κ will assure that the loss tangent associated with the effective damping is less than unity. The masses μ_α are taken from the associated substructure frequencies ω_α ($\mu_\alpha = k_\alpha / \omega_\alpha^2$) and these frequencies are chosen from a distribution

$$p(\omega_1, \omega_2, \omega_3, \dots, \omega_N) = \prod_{\alpha} p(\omega_{\alpha}); \quad p(\omega_{\alpha}) = \frac{\omega_o / \pi}{\omega_o^2 + \omega_{\alpha}^2}; \quad (-\infty < \omega_{\alpha} < \infty) \quad (10)$$

This corresponds to a smoothed modal density within the substructure of

$$\rho(\omega) = \frac{2N}{\pi} \frac{\omega_o}{\omega_o^2 + \omega^2} \quad (0 < \omega < \infty) \quad (11)$$

and a smoothed spectral mass density

$$m(\omega) = \kappa \frac{K\omega_o}{\pi\omega^2} \frac{2}{\omega_o^2 + \omega^2} \quad (0 < \omega < \infty) \quad (12)$$

The distribution has been chosen so as to allow easy analytic evaluation of certain integrals. It is not expected that the choice will affect the conclusions of this paper. This distribution also has the peculiar property of corresponding to a finite total stiffness κK , but a total mass with infinite expectation: $\langle \sum \mu_\alpha \rangle = \infty$. It could be modified so as to have a finite value for both quantities, at a cost of a slight additional complexity.

Ensemble average of a quantity $f(\omega_1, \omega_2, \dots, \omega_N)$ is indicated by brackets $\langle f \rangle$, and is evaluated by the multiple integral:

$$\langle f \rangle = \prod_{\alpha} \int_{-\infty}^{\infty} p(\omega_{\alpha}) d\omega_{\alpha} f(\omega_1, \omega_2, \dots, \omega_N) \quad (13)$$

The mean impulse response is given, in the frequency domain, by the average of the solution of eqn(5b).

$$\langle G(\omega) \rangle = \left\langle \frac{1}{-M\omega^2 + K - V(\omega)} \right\rangle \quad (14)$$

The Pierce-Sparrow-Russell (PSR) [3,4] expression for the response is given by replacing V with its average:

$$G^{PSR}(\omega) = \frac{1}{-M\omega^2 + K - \langle V(\omega) \rangle} \quad (15)$$

But $\langle V \rangle = \langle \Sigma v \rangle$ is

$$\begin{aligned} \langle V(\omega) \rangle &= \sum_{\alpha=1}^N \int_{-\infty}^{\infty} d\omega_{\alpha} p(\omega_{\alpha}) \frac{k_{\alpha}\omega^2}{\omega_{\alpha}^2 - \omega^2} = \kappa \omega^2 K \int_{-\infty}^{\infty} dx p(x) / [x^2 - (\omega - i\epsilon)^2] \\ &= \kappa \omega^2 K \int_{-\infty}^{\infty} dx \frac{\omega_0/\pi}{x^2 + \omega_0^2} \frac{1}{x^2 - (\omega - i\epsilon)^2} \end{aligned} \quad (16)$$

This integral may be done by analytic continuation and evaluation of residues. There are simple poles in the lower half x -plane, at $x = \omega - i\epsilon$ and at $x = -i\omega_0$. The result is

$$\langle \tilde{V}(\omega) \rangle = -\kappa \omega^2 K \left[\frac{1}{\omega^2 + \omega_0^2} + i \frac{\omega_0/\omega}{\omega^2 + \omega_0^2} \right] \quad (17)$$

Thus the PSR impulse response function is given by

$$G^{PSR}(\omega) = 1 / [K - M\omega^2 + i\kappa\omega\omega_0 K / (\omega^2 + \omega_0^2) + \kappa\omega^2 K / (\omega^2 + \omega_0^2)] = \frac{1}{K A(\omega)} \quad (18)$$

which serves also to define the quantity $A(\omega)$.

For values of κ that are small enough that the substructure only slightly modifies the behavior of the master, G^{PSR} may be approximated by its behavior in the vicinity of the dominating poles near $\omega = \sqrt{K/M}$:

$$G^{PSR}(\omega) \approx \frac{1}{K} \frac{1}{1 - \omega^2/\omega_{PSR}^2 + i \operatorname{sgn}(\omega) \alpha_{PSR} \frac{2M}{K}} \quad (19)$$

where

$$\omega_{\text{PSR}} \equiv \sqrt{\frac{K}{M + \frac{\kappa K}{(K/M + \omega_o^2)}}} \approx \sqrt{\frac{K}{M}} \quad (20)$$

$$\alpha_{\text{PSR}} \equiv \frac{\kappa \omega_o \omega_{\text{PSR}}^2}{2(\omega_{\text{PSR}}^2 + \omega_o^2)} = [c_{\text{eff}}(\omega_{\text{PSR}})/2M] \frac{\omega_{\text{PSR}}^2}{K/M} \approx c_{\text{eff}}(\omega_{\text{PSR}})/2M$$

corresponding to a damped simple harmonic oscillator with a frequency ω_{PSR} and a decay rate $\sim \exp(-\alpha_{\text{PSR}} t)$. It has been, and will continue to be, occasionally convenient to disregard the small difference (at moderate values of κ) between ω_{PSR} and $\sqrt{K/M}$.

Notwithstanding the attractiveness and simplicity of the Pierce *et al.*[3,4] and Strasberg and Feit[5] arguments, it may also be desirable to attempt a derivation which does not depend on a limit of infinite N or on an assumption of sufficient damping in the substructure. An appeal to well developed formal methods for mean solutions of stochastic differential equations is therefore indicated. There is a substantial literature treating responses of disordered linear systems. Amongst the more popular methods are those sometimes termed multiple scattering methods in which the response (5b) is expanded in a Born-Neumann series of powers of the perturbing potential V . [9] The infinite series so obtained is averaged and the result re-summed. This author is not comfortable with the notion of a power series in an unbounded quantity. He nevertheless found that the multiple scattering method did yield reasonable answers for mean responses $\langle G \rangle$. The author was, however, unable to apply the methods successfully for the case of $\langle G^2 \rangle$. In lieu of the multiple scattering method therefore a different procedure is followed here. The method is related to certain path integral procedures used in quantum field theory in recent years for studying the electronics of disordered structures[see, *e.g.* 10].

The exact, nonstochastic, response is given formally by eqn (5b); which is difficult to average as it stands. It is also given by the following expression:

$$K G(\omega) = -i \int_0^\infty dy \exp\{ iy(1 - (\omega - i\epsilon)^2/\omega_b^2) - iy \sum_\alpha v_\alpha/K \} \quad (21)$$

where the quantity ω_b is the frequency of the bare master; $\omega_b^2 = K/M$. The integral converges by virtue of the positive quantity ϵ [11]. This expression may be averaged. We exchange the order of integrations and obtain

$$\begin{aligned}
 K \langle G(\omega) \rangle &= -i \int_0^{\infty} dy \exp\{iy(1 - \omega_{-i\epsilon}^2 / \omega_b^2)\} \prod_{\alpha=1}^N \int_{-\infty}^{\infty} d\omega_{\alpha} p(\omega_{\alpha}) \exp\{-iyv_{\alpha}/K\} \\
 &= -i \int_0^{\infty} dy \exp\{iy(1 - \omega_{-i\epsilon}^2 / \omega_b^2)\} X^N
 \end{aligned} \tag{22}$$

where

$$X \equiv \langle \exp\{-iy v_{\alpha}/K\} \rangle = \int_{-\infty}^{\infty} dx \frac{\omega_0}{\pi} \frac{1}{x^2 + \omega_0^2} \exp\left\{-iy \frac{\kappa}{N} \frac{(\omega - i\epsilon)^2}{x^2 - (\omega - i\epsilon)^2}\right\} \tag{23}$$

X may be evaluated exactly by analytic continuation. There is, in the lower half x -plane, a simple pole at $x = -i\omega$, and an essential singularity at $x = \omega - i\epsilon$. We write X as the sum of the contributions from each singularity: $X = X_1 + X_2$

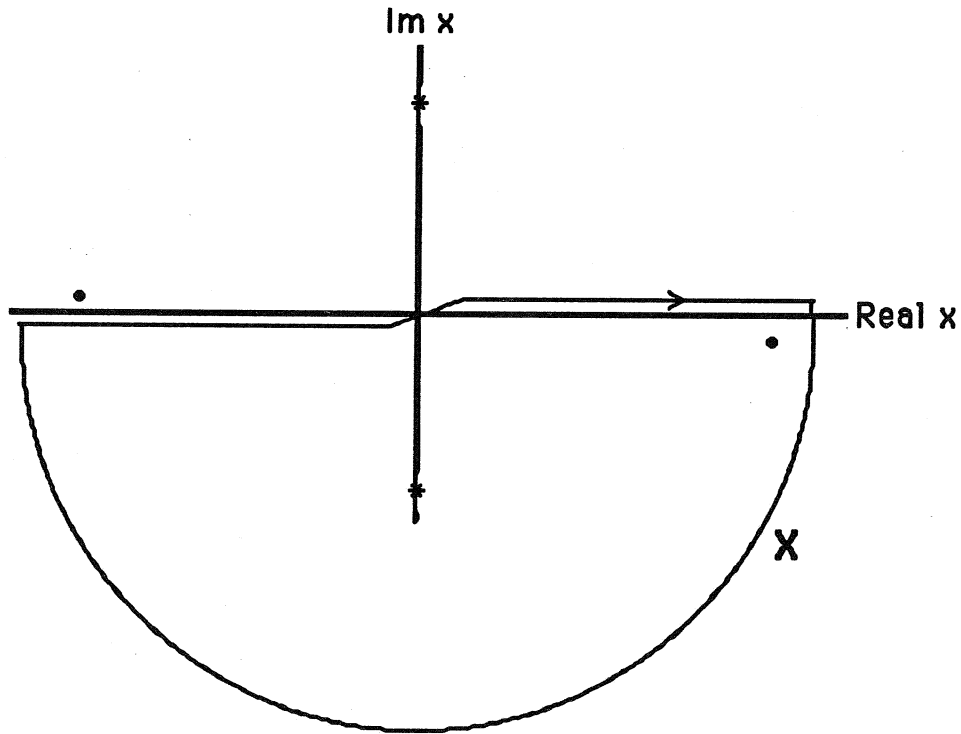


Figure 2a] The contour for the evaluation of the integral of eqn(23)

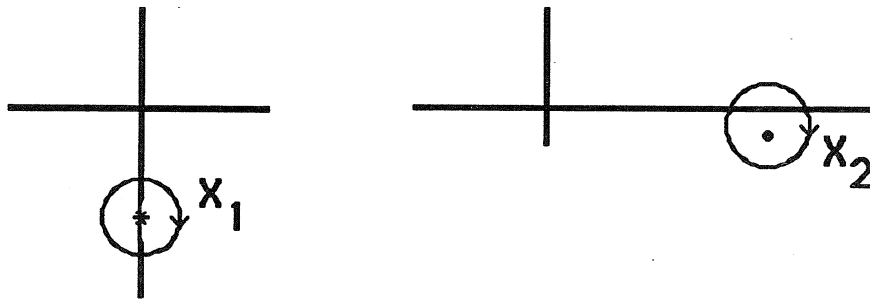


Figure 2b] The contours for the evaluation of the contributions of each singularity.

The contribution from the simple pole is

$$X_1 = \exp\left\{ iy \frac{\kappa}{N} \frac{\omega^2}{\omega^2 + \omega_o^2} \right\} \quad (24)$$

and the contribution from the essential singularity is

$$X_2 = - \frac{\kappa y \omega \omega_o}{N (\omega^2 + \omega_o^2)} \quad (25)$$

It is the Nth power of X which is needed, so, as $X^N = \exp\{ N \ln X \}$, we construct an asymptotic estimate for $\ln X$:

$$\ln X = \frac{\kappa \xi}{N} (i - \omega_o / \omega) + \frac{\kappa^2 \xi^2}{2N^2} (2i \omega_o / \omega - \omega_o^2 / \omega^2) + \dots \quad (26)$$

where the quantity ξ is defined by

$$\xi = y \omega^2 / (\omega^2 + \omega_o^2). \quad (27)$$

This allows an asymptotic estimate for X^N :

$$X^N = \exp\{N \ln X\} \approx \exp\{ \kappa \xi (i - \omega_o / \omega) \} \exp\left\{ \frac{\kappa^2 \xi^2}{2N} (2i \omega_o / \omega - \omega_o^2 / \omega^2) + \dots \right\} \quad (28)$$

and a corresponding estimate for $\langle G \rangle$. If the second factor is ignored as being, for large N, essentially unity, it is found that

$$\begin{aligned} K \langle G \rangle &= -i \int_0^\infty dy \exp\left\{ iy(1 - \omega_{-ie}^2 / \omega_o^2) + i y \kappa \omega^2 / (\omega^2 + \omega_o^2) - y \kappa \omega \omega_o / (\omega^2 + \omega_o^2) \right\} \\ &= -i \int_0^\infty dy \exp\{ i y A(\omega) \} \\ &= 1 / A(\omega) \end{aligned}$$

or (29)

$$\langle G \rangle = G^{PSR}. \quad (30)$$

The mean impulse response is, to leading order at large N, equal to the PSR impulse response. This is one of the chief results of this section. The first correction may be obtained by retaining the second factor in equation(28).

$$K \langle \tilde{G}(\omega) \rangle = -i \int_0^\infty dy \exp\{ i y A(\omega) \} \exp\{ - y^2 B(\omega) / N \} \quad (31)$$

where

$$B(\omega) = \frac{\kappa^2 \omega^4}{2(\omega^2 + \omega_o^2)^2} [\omega_o^2 / \omega^2 - 2 i \omega_o / \omega] \quad (32)$$

The second factor has a significant effect on the integrand only for y greater than or of the order of $(N/|B|)^{1/2}$. For such large values of y , however, the first factor has a magnitude of the order of $\exp\{-(2N)^{1/2}\} \ll 1$. One concludes that these large values of y are unimportant in the evaluation of the integration (31); the second factor may be ignored.

At late times, however, the inverse Fourier transform reconstruction of $\langle G(t) \rangle$ can be very sensitive to slight errors in $\langle G(\omega) \rangle$. Some indication of the form taken by $\langle G(t) \rangle$ at finite N and late time may be found by assuming that $\omega \approx \omega_b$ is the important part of the inverse Fourier transform, thus

$$K \langle \tilde{G}(\omega) \rangle = -i \int_0^\infty dy \exp\{2iy(\omega_b - \omega)/\omega_b + iy\kappa\omega_b^2/(\omega_b^2 + \omega_o^2) - \kappa\omega_b\omega_o/(\omega_b^2 + \omega_o^2) - B(\omega_b)y^2/N\} \quad (33)$$

plus its complex conjugate. The inverse Fourier transform to the time domain is then obtained, from inspection, after making the change of variables $y = \omega_b t / 2$:

$$K \langle G(t) \rangle = \frac{\omega_b}{2i} \exp\{i(\omega_b + \frac{\kappa\omega_b^3}{2(\omega_b^2 + \omega_o^2)})t - \frac{\kappa\omega_o\omega_b^2}{(\omega_b^2 + \omega_o^2)}t\} * \exp\{-t^2\omega_b^2 B(\omega_b)/4N\} + \text{c.c.} \quad (34)$$

The effect of the correction is to multiply the right hand side of equation (30) by a very slowly decaying Gaussian. The Gaussian correction is important only for times of the order or greater than $(2/\omega_b)(N/B(\omega_b))^{1/2}$. At these times G^{PSR} has decayed by a factor of $\exp\{-(2N)^{1/2}\}$ and is very small anyway. As will be seen in the next section, the fluctuating part $\langle G^2(t) \rangle^{1/2}$ dominates the low amplitude of the mean response $\langle G(t) \rangle$ at these late times. These Gaussian corrections to the PSR response are therefore arguably irrelevant and would in any case be very hard to detect.

Mean Square Responses

It is useful to also consider the square of the response

$$E(t) = G^2(t) \quad (35)$$

and in particular, its Fourier Transform:

$$\begin{aligned} \tilde{E}(\Omega) &\equiv \int_0^\infty G^2(t) \exp\{-i(\Omega - i\delta)t\} dt = \int_{-\infty}^\infty G^2(t) \exp\{-i(\Omega - i\delta)t\} dt = \\ &= \frac{1}{4\pi^2} \int_{-\infty}^\infty \exp\{-i\Omega t - \delta t\} \int_{-\infty}^\infty \tilde{G}(\omega) e^{i\omega t} d\omega \int_{-\infty}^\infty \tilde{G}^*(\omega') e^{-i\omega' t} d\omega' dt = \\ &= \frac{1}{2\pi} \int_{-\infty}^\infty \left[\tilde{G}\left(\omega + \frac{\Omega - i\delta}{2} - i\epsilon\right) \tilde{G}^*\left(\omega - \frac{\Omega - i\delta}{2} + i\epsilon\right) \right] d\omega \end{aligned} \quad (36)$$

A small positive quantity δ that assures the analyticity of the transform has been introduced above. It will now be suppressed until it is needed. We also define a filtered, smoothed, $E(t)$

$$E'(t) \equiv \frac{1}{2\pi} \int_{-\infty}^\infty \tilde{E}(\Omega) \exp\{i\Omega t\} W(\Omega) d\Omega \quad (37)$$

where the filter function W is constructed to cut off the high frequencies in a smooth fashion (this will aid in the evaluation of integrals later)

$$W(\Omega) \equiv \frac{\Omega_0^2}{\Omega_0^2 + \Omega^2} \quad (38)$$

and Ω_0 is some characteristic time scale, long compared to $1/\omega_b$, over which one is willing to smooth. With these definitions one finds that

$$\langle E'(t) \rangle = \frac{1}{4\pi^2 K^2} \int_{-\infty}^\infty \int_{-\infty}^\infty d\omega H(\omega, \Omega) \exp(i\Omega t) W(\Omega) d\Omega \quad (39)$$

where the key quantity of interest H has been defined by

$$H(\omega, \Omega) \equiv \langle K^2 \tilde{G}\left(\omega + \frac{\Omega}{2} - i\epsilon\right) \tilde{G}^*\left(\omega + \frac{\Omega}{2} + i\epsilon\right) \rangle \quad (40)$$

and it is understood that the "outer frequency" Ω is much less than the "inner" or carrier frequency ω . It is for H that we now construct a field theoretic description.

As before G may be represented by the expression (21). We append a similar representation for G^* and take an average:

$$H = \int_0^\infty \int_0^\infty dy dy' \exp\{iy(1 - (\omega + \frac{\alpha}{2} - i\epsilon)^2 / \omega_b^2) - iy'(1 - (\omega - \frac{\alpha}{2} + i\epsilon)^2 / \omega_b^2)\} * \prod_{\alpha=1}^N \int_{-\infty}^\infty d\omega_\alpha p(\omega_\alpha) \exp\{-iyv_\alpha / K + iy'v_\alpha^* / K\} \quad (41)$$

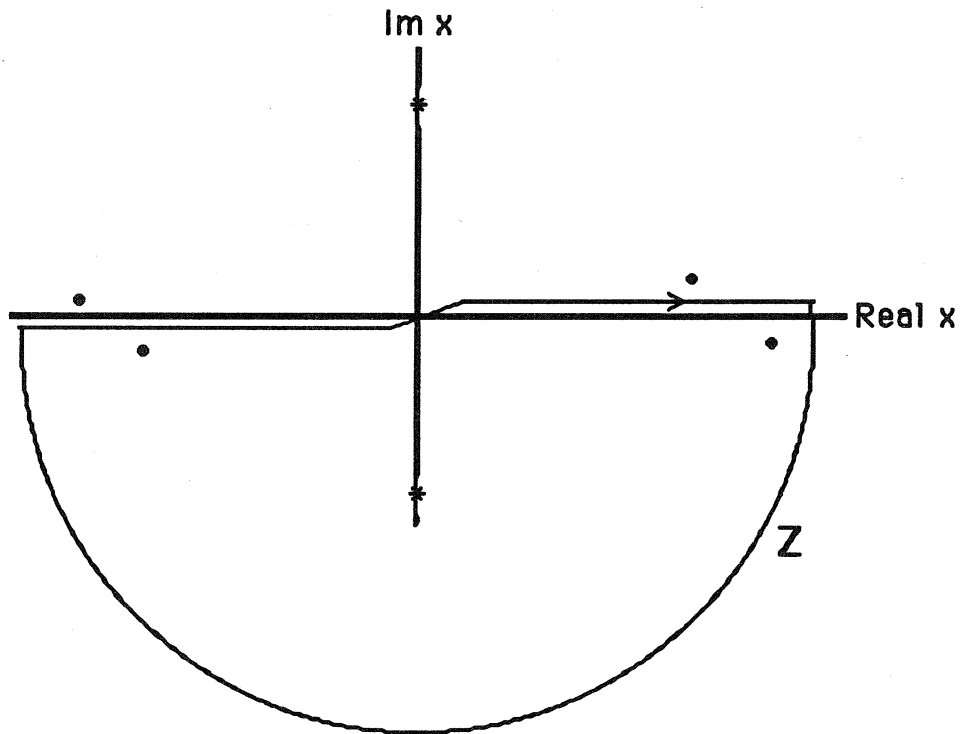


Figure 3a] The contour for the evaluation of the integral of eqn(42)

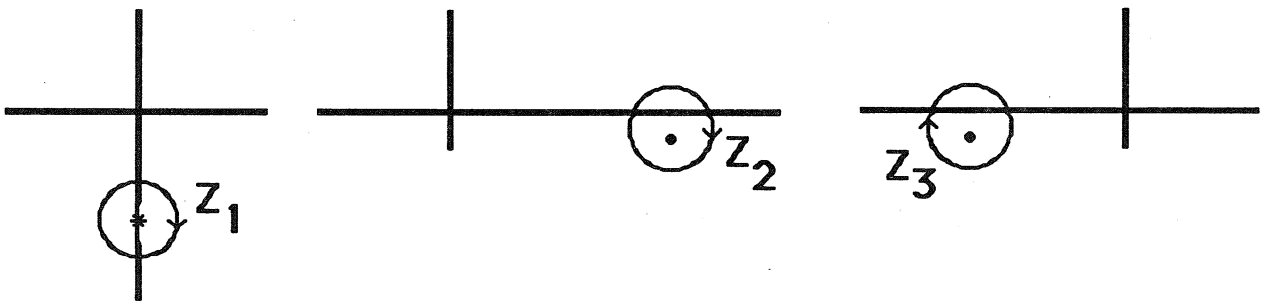


Figure 3b] The contours for the evaluation of each contribution to Z .

where v and v^* are defined by equation (6) but now with the implicit understanding of the respective substitutions $\omega \rightarrow \omega + \Omega/2 - i\varepsilon$ and $\omega \rightarrow \omega - \Omega/2 + i\varepsilon$.

The latter factor consists of N identical factors of Z , where Z is

$$Z \equiv \frac{\omega_0}{\pi} \int_{-\infty}^{\infty} dx \frac{1}{x^2 + \omega_0^2} \exp \left\{ -iy \frac{\kappa}{N} \frac{\omega^2}{x^2 - (\omega + \Omega/2 - i\varepsilon)^2} + iy' \frac{\kappa}{N} \frac{\omega^2}{x^2 - (\omega - \Omega/2 + i\varepsilon)^2} \right\} \quad (42)$$

This integrand, when analytically extended, has a simple poles at $x = \pm i\omega_0$ and essential singularities at $x = \pm(\omega + \Omega/2 - i\varepsilon)$ and $x = \pm(\omega - \Omega/2 + i\varepsilon)$. We close the contour in the lower half plane, as in Figure 3a] and represent Z exactly as the sum of three contributions, one from the contour around each singularity in the lower half plane. $Z = Z_1 + Z_2 + Z_3$. Z_1 is the contribution from the simple pole at $x = -i\omega_0$.

$$Z_1 = \exp \left\{ i \frac{\kappa}{N} \omega^2 \left[\frac{y}{\omega_0^2 + (\omega + \Omega/2)^2} - \frac{y'}{\omega_0^2 + (\omega - \Omega/2)^2} \right] \right\} \quad (43)$$

$$Z_1 \approx \exp \left\{ i \frac{\kappa}{N} \omega^2 \frac{y - y'}{\omega_0^2 + \omega^2} \right\}$$

Z_2 is the contribution from the singularity at $x = \omega + \Omega/2 - i\varepsilon$.

$$Z_2 = -\frac{\omega \omega_0}{(\omega^2 + \omega_0^2)} \frac{\kappa}{N} y \exp \left\{ iy' \frac{\kappa}{N} \frac{\omega}{2\Omega} \right\} \quad (44)$$

Similarly,

$$Z_3 = -\frac{\omega \omega_0}{(\omega^2 + \omega_0^2)} \frac{\kappa}{N} y' \exp \left\{ iy \frac{\kappa}{N} \frac{\omega}{2\Omega} \right\} \quad (45)$$

To leading order $\ln Z$ is then given by

$$\ln Z = \frac{1}{N} \left\{ \frac{i\kappa\omega^2}{\omega^2 + \omega_0^2} (y - y') - \frac{\kappa\omega\omega_0}{\omega^2 + \omega_0^2} [y e^{iy'\kappa\omega/2N\Omega} + y' e^{iy\kappa\omega/2N\Omega}] \right\} + \dots \quad (46)$$

so Z^N is given, to leading order, by

$$Z^N = \exp \left\{ \frac{i\kappa\omega^2}{\omega^2 + \omega_0^2} (y - y') - \frac{\kappa\omega\omega_0}{\omega^2 + \omega_0^2} [y e^{iy'\kappa\omega/2N\Omega} + y' e^{iy\kappa\omega/2N\Omega}] \right\} \quad (47)$$

and H is, to this leading order,

$$H = \int_0^\infty \int_0^\infty dy dy' \exp \left\{ iy \left(1 - \frac{(\omega + \Omega/2 - i\epsilon)^2}{\omega_b^2} + \kappa \frac{\omega^2}{\omega^2 + \omega_o^2} \right) - iy' \left(1 - \frac{(\omega - \Omega/2 + i\epsilon)^2}{\omega_b^2} + \kappa \frac{\omega^2}{\omega^2 + \omega_o^2} \right) \right\} * \\ \exp \left\{ -\frac{\kappa \omega \omega_o}{\omega^2 + \omega_o^2} [y e^{iy' \omega / 2N\Omega} + y' e^{iy \omega / 2N\Omega}] \right\} \quad (48)$$

If the product $N\Omega$ were treated as asymptotically large (While N is large, Ω is arbitrarily small, depending on the time scale one wishes to probe, so the treating of $N\Omega$ as large implies that Ω is in some sense not small, or that interest is in early times.) then the integrals over y and y' would decouple.

$$H_{\Omega \gg \kappa \omega / N} = \int_0^\infty dy \exp \left\{ iy \left(1 - \omega^2 / \omega_b^2 + \kappa \frac{\omega^2}{\omega^2 + \omega_o^2} + i \frac{\kappa \omega \omega_o}{\omega^2 + \omega_o^2} \right) \right\} * \\ \int_0^\infty dy' \exp \left\{ -iy' \left(1 - \omega^2 / \omega_b^2 + \kappa \frac{\omega^2}{\omega^2 + \omega_o^2} - i \frac{\kappa \omega \omega_o}{\omega^2 + \omega_o^2} \right) \right\} \quad (49) \\ = 1 / |A(\omega)|^2$$

One concludes that at early times the mean square response is the square of the mean. Not surprisingly, fluctuations vanish at early times.

The chief interest is, however, with later times, for which the approximation $N\Omega$ large is not justified. In order to most easily evaluate $E'(t)$ at late times we define an intermediate quantity

$$\mathcal{E}(\omega, t) \equiv \int_{-\infty}^{\infty} H(\omega, \Omega) W(\Omega) \exp(i\Omega t) d\Omega \quad (50)$$

and note

$$E'(t) = \frac{1}{4\pi^2 K^2} \int_{-\infty}^{\infty} d\omega \mathcal{E}(\omega, t) \quad (51)$$

By exchanging the order of the Ω and y and y' integrations one constructs an expression for $\mathcal{E}(\omega, t)$

$$\mathcal{E}(\omega, t) \equiv \int_0^\infty \int_0^\infty dy dy' \exp \left\{ i(y - y') \left(1 - \omega^2 / \omega_b^2 + \kappa \frac{\omega^2}{\omega^2 + \omega_o^2} \right) \right\} * e(y, y', t) \quad (52)$$

where

$$e(y, y', t) \equiv \int_{-\infty}^{\infty} \frac{\Omega_o^2}{\Omega^2 + \Omega_o^2} \exp \left\{ i\Omega t - i\Omega(y + y')\omega / \omega_b^2 \right\} \exp \left\{ -Dye^{i\Omega y / N\Omega} - Dy'e^{i\Omega y' / N\Omega} \right\} d\Omega \quad (53)$$

and where D and F are defined by

$$D = \frac{\kappa \omega \omega_0}{\omega^2 + \omega_0^2}; \quad F = \kappa \omega / 2 \quad (54)$$

We also recall that $\Omega = \Omega - i\delta$. Thus when the integral (53) over Ω is performed by evaluating residues, we must recognize that there are simple poles at $\Omega = \pm i\Omega_0$ and that there is an essential singularity at $\Omega = i\delta \sim 0$.

The contour must be closed in the lower half Ω plane if $t < (y+y')\omega/\omega_b^2$. In this case one has only contributions from the pole at $\Omega = -i\Omega_0$. But for the very late times of chief interest, the integral over y and y' will then be confined to only the largest values of $y+y'$, and the corresponding contribution to \mathcal{E} unimportant compared to the contribution from smaller values of $y+y'$.

Thus, in the large t limit, the integral over Ω must be closed in the upper half plane. There are contributions from the simple pole at $i\Omega_0$ and from the essential singularity at $\Omega = i\delta$:

$$e = e_1 + e_2 \quad (55)$$

where e_1 is the contribution from the simple pole:

$$e_1 = \pi \Omega_0 \exp\{-\Omega_0(t - (y+y')\omega/\omega_b^2)\} \exp\{-Dy e^{Fy/N\Omega_0} - Dy' e^{Fy'/N\Omega_0}\} \quad (56)$$

It is exponentially unimportant at large times. Thus e is dominated by e_2 , the contribution from the essential singularity at $\Omega = i\delta$.

$$e \approx e_2 = 4\pi \frac{DF y y'}{N} \exp\{-D(y+y')\} \quad (57)$$

If this expression is substituted into (52), one obtains, for late time,

$$\begin{aligned} \mathcal{E}(\omega, \infty) &= \frac{2\pi}{N} \int_0^\infty \int_0^\infty dy dy' yy' \frac{\kappa^2 \omega^2 \omega_0}{\omega^2 + \omega_0^2} \exp\{i(y-y')(1 - \omega^2/\omega_b^2 + \kappa \frac{\omega^2}{\omega^2 + \omega_0^2})\} \exp\{-(y+y') \frac{\kappa \omega \omega_0}{\omega^2 + \omega_0^2}\} \\ &= \frac{2\pi}{N} \frac{\kappa^2 \omega^2 \omega_0}{\omega^2 + \omega_0^2} \int_0^\infty \int_0^\infty dy dy' yy' \exp\{iyA(\omega) - iy'A^*(\omega)\} = \frac{2\pi}{N} \frac{\kappa^2 \omega^2 \omega_0}{\omega^2 + \omega_0^2} \frac{1}{|A(\omega)|^4} \end{aligned} \quad (58)$$

This is presumably the power spectral density of the response at late times. It is noteworthy that it is centered at the same place (ω_{PSR}) as is the mean response G^{PSR} . It is also noteworthy that it is proportional to the fourth power of $\langle G \rangle$, and so is more strongly peaked. The random process which is $G(t)$ at late times is narrower band than is the mean impulse response $\langle G(t) \rangle$.

The late time mean square level is given by an integral (51) of the above expression. If that integral is approximated by the dominant contribution from the vicinity of the double poles in $|A|^{-4}$ near $\omega = \pm\omega_{PSR}$, then

$$\begin{aligned}
 E'(\infty) &\approx \frac{1}{2\pi K^2 N} \frac{\kappa^2 \omega_{PSR}^2 \omega_o}{\omega_{PSR}^2 + \omega_o^2} \int_{-\infty}^{\infty} d\omega \left| 1 - \omega^2 / (\omega_{PSR} + i \alpha_{PSR})^2 \right|^{-4} \\
 &= \frac{1}{2\pi K^2 N} \frac{\kappa^2 \omega_{PSR}^2 \omega_o}{\omega_{PSR}^2 + \omega_o^2} * \\
 &\int_{-\infty}^{\infty} d\omega \left[1 - \frac{\omega}{\omega_{PSR} + i \alpha_{PSR}} \right]^{-2} \left[1 + \frac{\omega}{\omega_{PSR} + i \alpha_{PSR}} \right]^{-2} \left[1 - \frac{\omega}{\omega_{PSR} - i \alpha_{PSR}} \right]^{-2} \left[1 + \frac{\omega}{\omega_{PSR} - i \alpha_{PSR}} \right]^{-2}
 \end{aligned} \tag{59}$$

On closing the contour in the upper half plane it is found that the first and fourth of the above factors contribute residues from the double poles at $\omega = \omega_{PSR} + i \alpha_{PSR}$ and $\omega = -\omega_{PSR} + i \alpha_{PSR}$ respectively. In the convenient limit in which we assume that $\alpha_{PSR} \ll \omega_{PSR}$, each double pole gives a contribution of $(\pi/32) \omega_{PSR}^4 \alpha_{PSR}^{-3}$ (plus neglected terms of order α^{-2}). One concludes

$$\begin{aligned}
 E'(\infty) &= \frac{1}{32 K^2 N} \frac{\kappa^2 \omega_{PSR}^6 \omega_o}{\omega_{PSR}^2 + \omega_o^2} \alpha_{PSR}^{-3} = \frac{1}{8 K^2 N} \omega_{PSR}^2 \frac{\omega_{PSR}^2 + \omega_o^2}{\omega_o} \alpha_{PSR}^{-1} \\
 &= \frac{1}{4\pi} \frac{\omega_{PSR}^2}{K^2} \frac{1}{\alpha_{PSR} \rho(\omega_{PSR})}
 \end{aligned} \tag{60}$$

This is the main result of this section. It may be interpreted as consistent with an SEA-like equipartition argument. The amount of energy originally deposited by the unit impulse is $1/2M$. The mean amount of strain energy stored in the spring K at late times is $KE(\infty)/2$, also equal to the mean amount of kinetic energy in the master oscillator at late times. The ratio of the mean amount of total energy at late times to that originally deposited is thus

$$\begin{aligned}
 \frac{\text{Energy in late time fluctuations}}{\text{Energy originally deposited}} &= 2MKE(\infty) = \frac{1}{2\pi} \frac{M}{K} \frac{\omega_{PSR}^2}{\alpha_{PSR} \rho(\omega_{PSR})} \\
 &\approx \frac{1}{2\pi \alpha_{PSR} \rho(\omega_{PSR})}
 \end{aligned} \tag{61}$$

It is seen that the master oscillator has, at late times, a share of the originally deposited energy, as if it were shared equally amongst the substructural modes within a band of width $2\pi \alpha_{PSR}$. This was anticipated in an argument advanced earlier [8] based on equipartition notions from Statistical Energy Analysis.

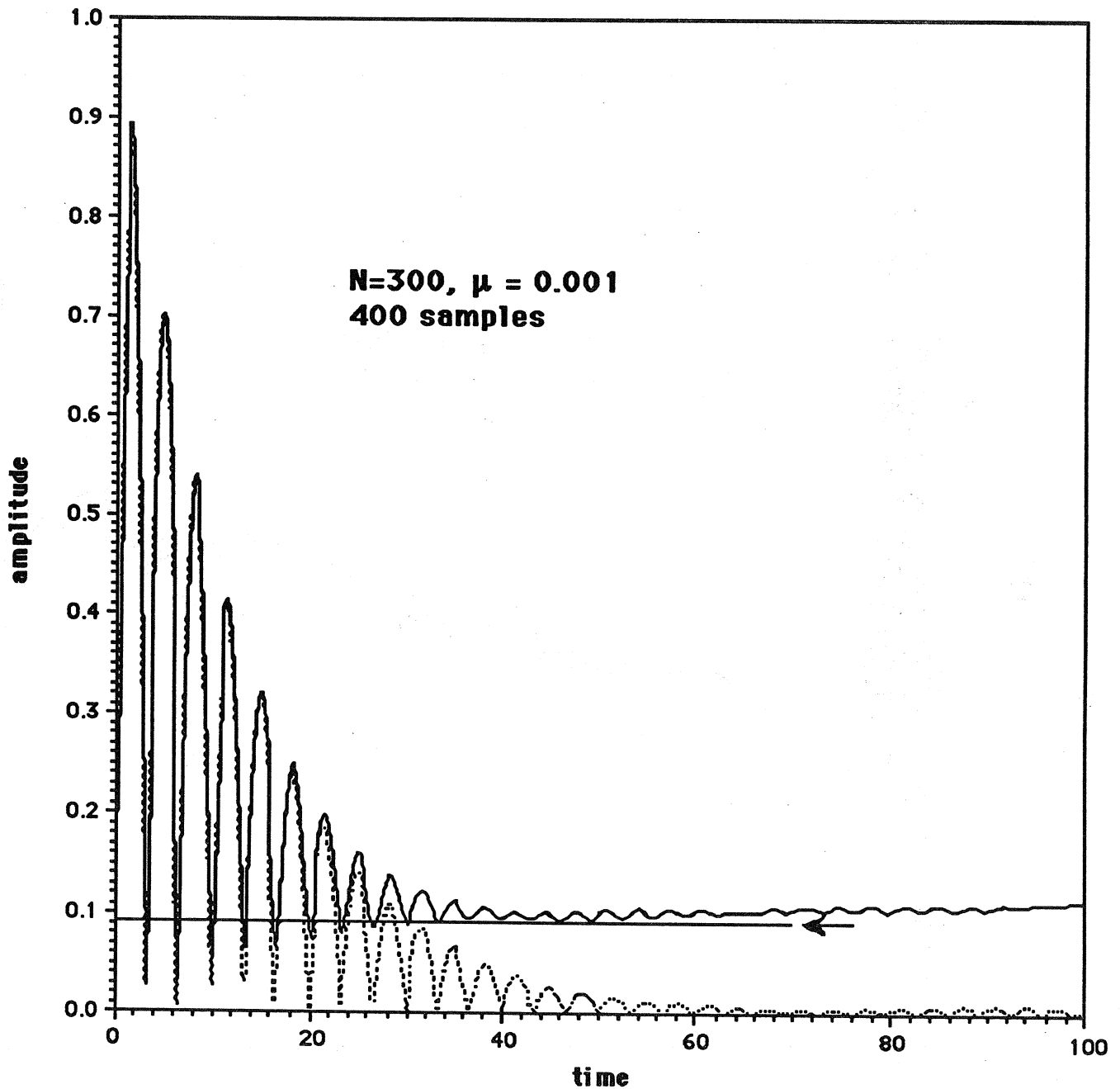


Figure 4] Plots of numerically calculated absolute value of the mean (dotted line), and root mean square (solid line), response of the master/fuzzy system of reference[8]. The horizontal line and arrow is the prediction of equation (60).

Numerical Simulations

In order to corroborate these results a numerical integration of the transient dynamics was undertaken. Many (400) equivalent samples were integrated. Ensemble averages of the solutions and square roots of ensemble averages of their squares are plotted below. The statistics used in the earlier [8] numerical investigation were retained; the masses of the oscillators in the substructure were fixed, and their frequencies ω_α were taken from an

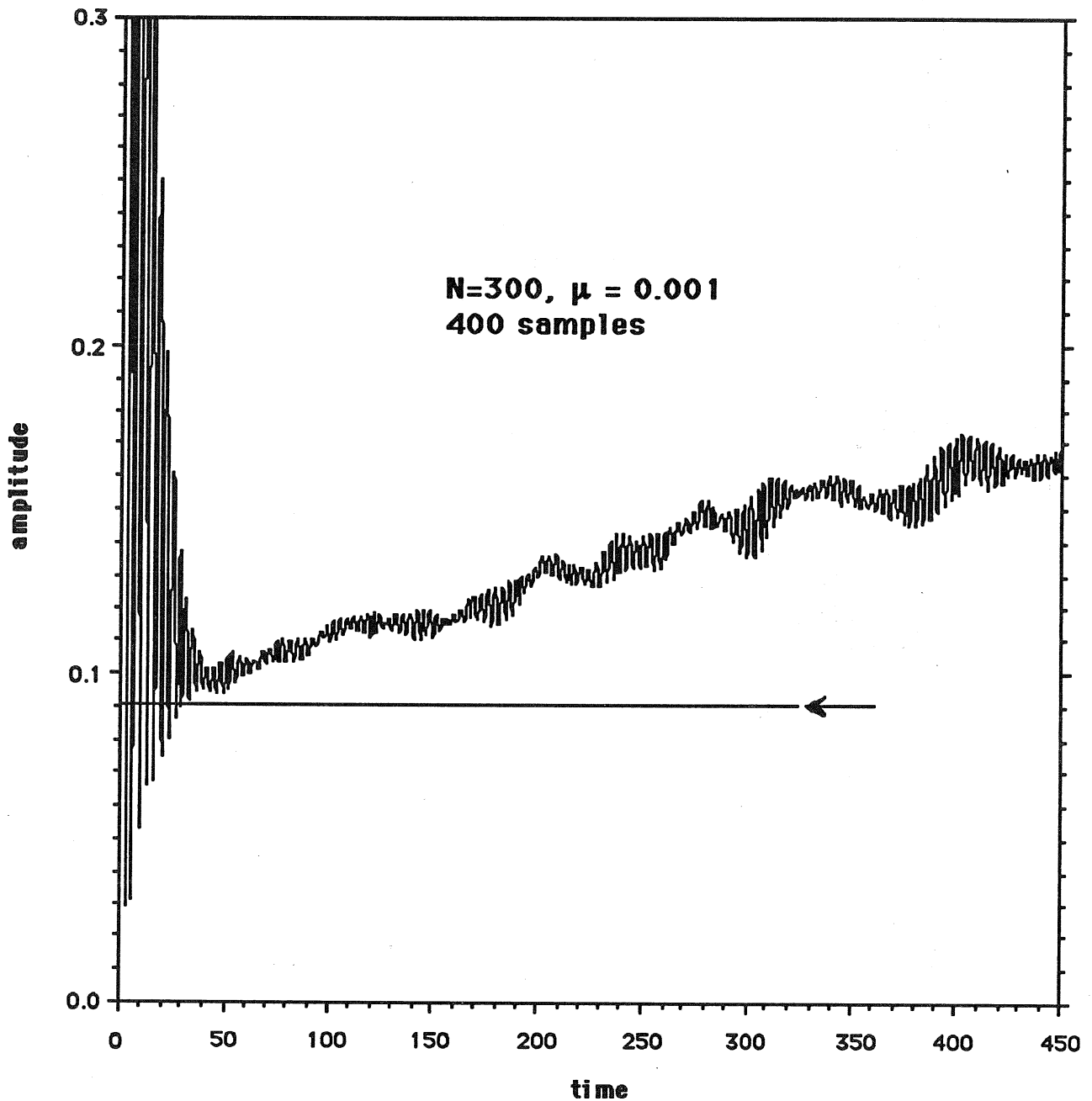


Figure 5] The root mean square response of the system of figure 4 is studied over longer times. The discrepancy between theory and simulation is seen to grow.

exponential distribution: $p(\omega_\alpha) = \exp(-\omega_\alpha)$. M and K were chosen to be unity. In figure 4] the absolute value of the mean response, and the root mean square response, are plotted versus time for the case of $N=300$ substructural degrees of freedom, each with a small mass $\mu = 0.001$. As predicted, equation (49), there is little difference at early times, reflecting the small variance and weak fluctuations. At late times, however, the mean response has decayed to a level which is essentially zero. It is non-zero presumably only because of the finite number of

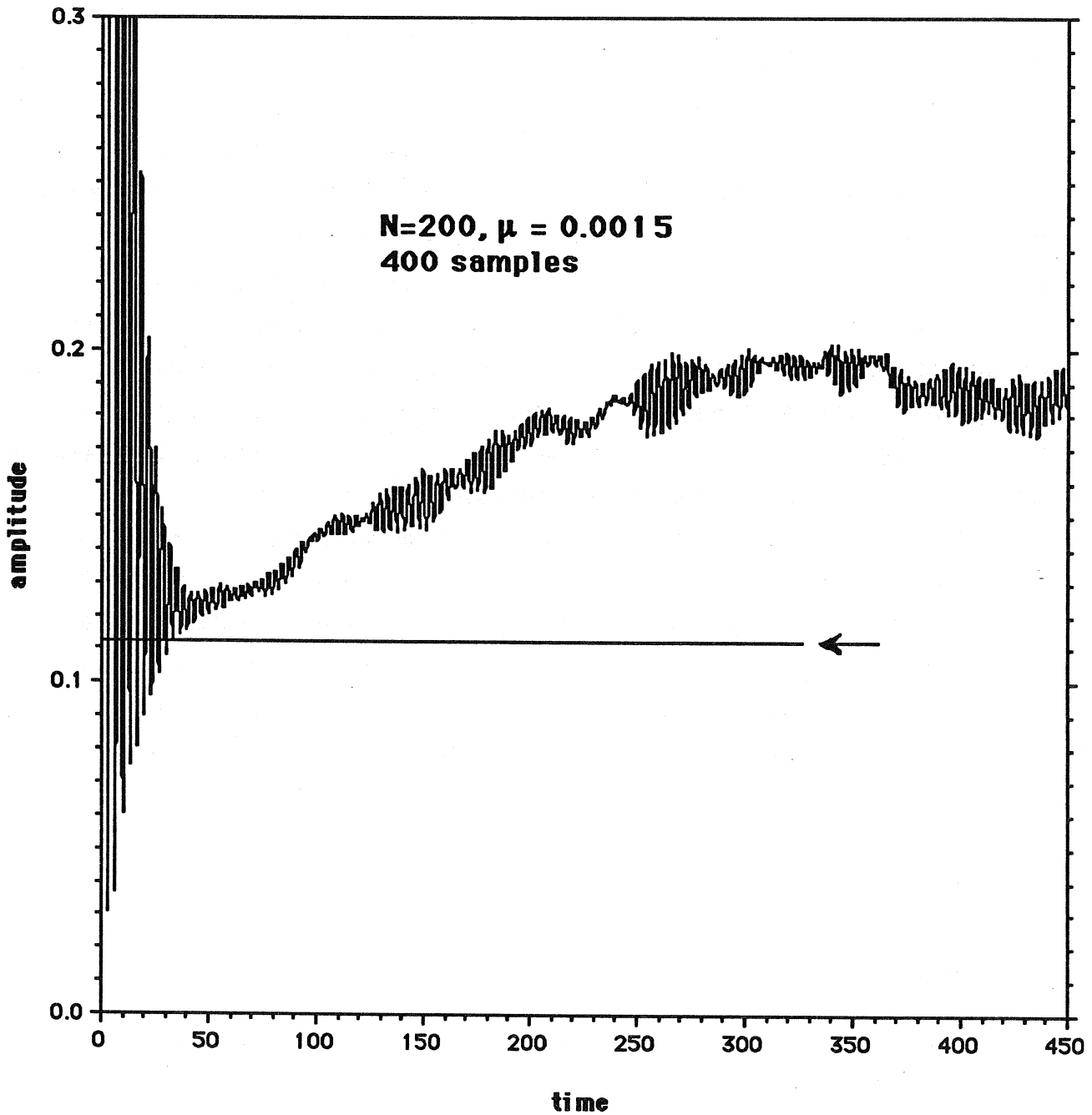


Figure 6] A different system is studied, in which it can be seen that the discrepancy between theory and measurement ceases its growth, and does so at a time of order $\pi\rho$.

samples, 400, averaged over. In contrast, the root mean square response achieves an approximately constant value slightly higher than that predicted by the square root of equation(60) and indicated in the plot by the horizontal line and arrow. The agreement with theory is fairly good. The small difference is, though, intriguing. One is at first tempted to ascribe the slight difference to having had only a finite number of samples to average over, and to speculate that the difference would vanish as this number was increased. Alternatively one might ascribe the discrepancy to the neglect, here, of the slight difference between ω_{PSR} and $\sqrt{K/M}$, or to neglect of corrections of order α/ω . Such speculations are not confirmed by further study. Indeed, if the system studied in Figure 4 is studied over longer times (Figure 5) one sees that the difference grows with time.

The discrepancy does not grow indefinitely. Figure 6] shows the case of $N=200$, $\mu=0.0015$, again with 400 samples taken from the ensemble. These parameters correspond to the same value of c_{eff} , but to a smaller modal density ρ , so the predicted root mean square level is greater. It is seen in figure 6 that the actual root mean square level saturates at a value roughly 50% higher than predicted. (There are some residual fluctuations so this number is only approximate.) The saturation takes place on a time scale comparable to a time that would be required to resolve the modes, $\pi\rho = 231$. These features, the final 50% discrepancy, and the time scale being of order $\pi\rho$, were observed over a range of system parameters with independent variations of N and μ . It is hypothesized here that these values are general.

This behavior is strongly reminiscent of the enhanced backscatter effect described recently in a room-acoustics context[12]. It was shown there that reverberant structural acoustic systems will show an enhancement of their energy densities at the position of the transient source, and that this enhancement grows in time by 50%, over a time scale of the order of $\pi\rho$. That the analytical calculation presented above has failed to predict the enhancement is not surprising; as shown in [11] the effect depends on eigenfrequency correlations and can only be derived by means of random matrix theory or related methods. The present analytical derivation has however correctly described the level of backscatter at times short compared to $\pi\rho$.

Conclusions

An analytical theory has been presented for the calculation of mean and mean square responses of an undamped master/fuzzy system. Ensemble average responses are found to agree, to high practical accuracy, with the simple predictions of Pierce *et al.* Calculated mean square responses agree with SEA-like arguments and with numerical simulations at moderately

late times but disagree on very late times scales comparable to the modal density. It is as yet unclear what analytical methods might be used to predict these late time enhancements. It also remains to apply these methods to the cases of multi-degree of freedom master structures, to nonlocal substructures, and to continuous systems.

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806	Nimmagadda, P. B. R., and P. Sofronis	On the calculation of the matrix-reinforcement interface diffusion coefficient in composite materials at high temperatures— <i>Acta Metallurgica et Materialia</i> , in press (1996)	Aug. 1995
807	Carlson, D. E., and D. A. Tortorelli	On hyperelasticity with internal constraints— <i>Journal of Elasticity</i> 42, 91-98 (1996)	Aug. 1995
808	Sayre, T. L., and D. N. Riahi	Oscillatory instabilities of the liquid and mushy layers during solidification of alloys under rotational constraint— <i>Acta Mechanica</i> , in press (1996)	Sept. 1995
809	Xin, Y.-B., and K. J. Hsia	Simulation of the brittle-ductile transition in silicon single crystals using dislocation mechanics	Oct. 1995
810	Ulysse, P., and R. E. Johnson	A plane-strain upper-bound analysis of unsymmetrical single-hole and multi-hole extrusion processes	Oct. 1995
811	Fried, E.	Continua described by a microstructural field— <i>Zeitschrift für angewandte Mathematik und Physik</i> , in press (1996)	Nov. 1995
812	Mittal, R., and S. Balachandar	Autogeneration of three-dimensional vortical structures in the near wake of a circular cylinder	Nov. 1995
813	Segev, R., E. Fried, and G. de Botton	Force theory for multiphase bodies— <i>Journal of Geometry and Physics</i> , in press (1996)	Dec. 1995
814	Weaver, R. L.	The effect of an undamped finite-degree-of-freedom "fuzzy" substructure: Numerical solutions and theoretical discussion	Jan. 1996
815	Haber, R. B., C. S. Jog, and M. P. Bensøe	A new approach to variable-topology shape design using a constraint on perimeter— <i>Structural Optimization</i> 11, 1-12 (1996)	Feb. 1996
816	Xu, Z.-Q., and K. J. Hsia	A numerical solution of a surface crack under cyclic hydraulic pressure loading	Mar. 1996
817	Adrian, R. J.	Bibliography of particle velocimetry using imaging methods: 1917-1995— <i>Produced and distributed in cooperation with TSI, Inc., St. Paul, Minn.</i>	Mar. 1996
818	Fried, E., and G. Grach	An order-parameter based theory as a regularization of a sharp-interface theory for solid-solid phase transitions— <i>Archive for Rational Mechanics and Analysis</i> , in press (1996)	Mar. 1996
819	Vonderwell, M. P., and D. N. Riahi	Resonant instability mode triads in the compressible boundary-layer flow over a swept wing	Mar. 1996
820	Short, M., and D. S. Stewart	Low-frequency two-dimensional linear instability of plane detonation	Mar. 1996
821	Casagrande, A., and P. Sofronis	On the scaling laws for the consolidation of nanocrystalline powder compacts	Apr. 1996
822	Xu, S., and D. S. Stewart	Deflagration-to-detonation transition in porous energetic materials: A comparative model study	Apr. 1996
823	Weaver, R. L.	Mean and mean-square responses of a prototypical master/fuzzy structure	Apr. 1996