EFFECTS OF ROUGHNESS ON NONLINEAR STATIONARY VORTICES IN ROTATING DISK FLOWS

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ABSTRACT

Primary stability of rotating disk boundary layer flow over a rough surface for stationary modes was investigated by using the weakly nonlinear theory where the Reynolds number R is close to its critical value R_c as determined by linear theory. Both the single mode case, where the wave vector $\underset{\sim}{K}$ equals its critical $\underset{\sim}{K}_c$ at the onset of stationary primary stability, and the bi-modal case, where the wave vectors $\underset{\sim}{K}_n (n=1,2)$ are close to $\underset{\sim}{K}_c$ for the primary stability of the flow, are considered. The analysis leads to stable solutions for particular roughness forms and magnitude, and particular wave vectors $\tilde{\underset{\sim}{K}}_n (n=1,2)$ of the surface roughness.

INTRODUCTION

Recently Vonderwell and Riahi [1] investigated the problem of primary stability of rotating disk boundary flow for stationary modes using the weakly non-linear theory for the Reynolds number R close to its critical value R_c as determined earlier by linear theory [2]. They derived a Landau type equation for the disturbance amplitude A and found that for $R < R_c$ subcritical instability occurs, while $|A|$ becomes very large and the solution breaks down in a finite time for $R > R_c$. The problem treated by Vonderwell and Riahi [1] was for flow over a smooth disk where roughness effects were absent. In the present study, we extended [1] to the case of flow over rough surface. In contrast to the results reported in [1], we find that roughness effects can lead to single mode or bi-modal steady stable solutions, for either $R > R_c$ or $R < R_c$, with R smaller than the corresponding one in the smooth surface case [1].

The results presented in this paper and described briefly above are the first reported theoretical ones on the effects of rough surface upon a shear flow. Theoretical studies, prior to the present one, are largely absent despite the fact that roughness effects are known both experimentally and technologically to be quite significant affecting the instabilities and the transition path to turbulence [3,4]. The importance of roughness effects in technological applications is numerous. For example, the influence of aircraft manufacturing inhomogeneities on three-dimensional flows and on the cross flow vortices, resulted from the instability of such flows, can be quite important.

The development of the present investigation was facilitated by the prior author's work on the effects of surface corrugation upon thermal convection [5] which resulted, in particular, to the theoretical development of two distinct regimes, corresponding to the cases of $\delta = \varepsilon^3$ and $0(\varepsilon^2) < \delta < 0(\varepsilon)$. Here δ represents a small parameter describing the order of magnitude of surface corrugation, and ε is the small amplitude of the primary disturbances superimposed on the basic steady state. It turns out that in the present problem we also have two distinct regimes of analysis of the types detected in [5].

Although the specific problem of rotating disk flow is considered for the illustrative purpose of the present method of approach to demonstrate the significance of the presence of certain families of surface roughness shapes, the qualitative results presented in this paper remain unchanged for nonlinear development of primary stationary disturbances in any wall bounded shear flow, provided such disturbances are the most critical ones for the Reynolds number near its critical value.

FORMULATION

Consider the problem of an infinite horizontal rough plane rotating about a vertical axis with angular velocity Ω. It is convenient to use cylindrical coordinates r^*, θ, z^*, with $z^*=0$ being the averaged location of the plane of the disk and assume that an incompressible fluid flow lies in the region $z^*>0$. Let the over-bar variables, $\bar{p}$, $\bar{u}$, $\bar{v}$, $\bar{w}$, denote the steady basic states for pressure and velocity in the radial (r^*), azimuthal (θ) and axial (z^*) directions, respectively, in the rotating coordinate system. The well-known Von Karman's exact solution of the Navier-Stokes and continuity equations [6] is obtained by setting

$$\bar{u}=r^*\Omega F(z),\ \ \bar{v}=r^*\Omega G(z),\ \ \bar{w}=(\nu\,\Omega)^{1/2}H(z),\ \ \bar{p}=\varphi\nu\underline{p}(z), \tag{1}$$

where $z=z^*(\Omega/\nu)^{1/2}$, ν is kinematic viscosity and φ is a constant (reference) density. The functions F, G, H, and $\underline{\mathrm{P}}$ satisfy a well known system [6] whose solution is easily determined [6].

To study the stability of the basic state (1) with respect to small disturbances, we first non-dimensionalize the governing Navier-Stokes and continuity equations by using $(\nu/\Omega)^{1/2}$ as the reference length, $r_e^*\Omega$ as the reference velocity, and $\varphi\, r_e^{*2}\,\Omega^2$ as the reference pressure, where r_e^* denotes the radial location near which the analysis is to be made. The disturbance variables u, v, w, and p superimposed on the base flow variables (1) are used to satisfy the Navier-Stokes and continuity equations in cylindrical coordinates. The resulting equations for the disturbance variables, as functions of non-dimensional space and time variables r, θ, z, and t, can be simplified by applying the usual quasi-parallel flow approximation [2]. We are interested in the stability near a radial location such that the Reynold's number $R = r_e^*(\Omega/\nu)^{1/2}$, equals its minimum value, R_c, below which no stationary mode is amplified based on the linear theory [2]. The full non-dimensional form of the equations are given in [1] and will not be repeated here. The non-dimensional boundary conditions for the present problem are that

$$\underset{\sim}{u} = 0 \text{ at } z = \infty, \tag{2}$$

where $\underset{\sim}{u}$ is the disturbance velocity vector, and the total velocity vector (sum of base flow and disturbance velocity vectors) equals one at $z = \delta h(r, \theta)$, where δ is a small (constant) parameter $(\delta << 1)$ describing the magnitude of the lower boundary roughness and $h(r, \theta)$ is surface roughness shape function which is assumed to be of the single or bi-modal form

$$h(r, \theta) = \sum_{n=1}^{2} \tilde{A}_n \tilde{E}_n + C.C., \tag{3a}$$

where

$$\tilde{E}_n = \exp\left(i \underset{\sim}{\tilde{K}_n} \cdot \underset{\sim}{X} \right), \tag{3b}$$

$\tilde{A}_n$ is a constant amplitude, i is the pure imaginary number $(\sqrt{-1})$, $\underset{\sim}{\tilde{K}_n} = (\tilde{\alpha}_n, R_c\tilde{\beta}_n)$ is the horizontal wave number vector for the boundary roughness, $\tilde{\alpha}_n$ is the radial component of $\underset{\sim}{\tilde{K}}$, $R_c\tilde{\beta}_n$ is the azimuthal component of $\underset{\sim}{K_n}$, $\underset{\sim}{X} = (r, \theta)$ and $C.C.$ indicates the complex conjugate of the entire preceding expression. Applying Taylor-series expansion about $z = 0$ for the total velocity vector, we find the following boundary conditions at the lower boundary

$$u = -\delta h\left[F'(0) + \frac{\partial u}{\partial z} \right] \text{ at } z = 0, \tag{4a}$$

$$v = -\delta h\left[G'(0) + \frac{\partial v}{\partial z}\right] \text{ at } z = 0, \tag{4b}$$

$$w = -\delta h\frac{\partial w}{\partial z} \text{ at } z = 0, \tag{4c}$$

where only the lowest order terms of order δ, which is needed here, are retained in (4), a prime denotes a derivative with respect to z and

$$F'(0) = 0.51023, \quad G'(0) = -0.61592 \tag{4d}$$

[7].

ANALYSIS FOR CASE $\delta = \varepsilon^3$

This case corresponds to the critical regime where $R \approx R_c$ and $\delta = \varepsilon^3$ [5], where ε here is a small perturbation parameter defined by

$$\varepsilon^2 = (R - R_0)/R_2, \tag{5}$$

R_2 is a real constant and R_0 is the value of R on the neutral stability curve close to R_c as determined by Malik [2] based on the stationary modes of linear theory. We also assume that R_0 does not deviate much from R_c $\left(R_0 - R_c \sim 0(\varepsilon^2) \text{ or less}\right)$ and define the slowly varying time t_s

$$t_s \equiv \varepsilon^2 t \tag{6}$$

[1]. Applying the weakly nonlinear theory [8], we pose expansions in powers of ε like

$$\begin{pmatrix} \tilde{u} \\ P \end{pmatrix} = \sum_{n=1} \varepsilon^n \begin{pmatrix} \tilde{u}_n \\ P_n \end{pmatrix} \tag{7}$$

Using (5)-(7) in the governing system for disturbances, we find to the lowest order in ε the linear system for $\underset{\sim}{u}_1$ and P_1 quantities. This system admits solution of the form

$$\begin{pmatrix} \underset{\sim}{u}_1 \\ P_1 \end{pmatrix} = \sum_{n=-2}^{2} \begin{bmatrix} \underset{\sim}{u}_{1n}(z) \\ P_{1n}(z) \end{bmatrix} A_n E_n, \tag{8a}$$

where

$$E_n = \exp\left(i \underset{\sim}{K}_n \cdot \underset{\sim}{X} \right), \tag{8b}$$

the amplitude function $A_n(t_s)$ satisfies the condition

$$A_{-n} = A_n^* \tag{8c}$$

a 'star' indicates complex conjugate, $\underset{\sim}{K}_n = (\alpha_n,\ R_c\beta_n)$ is the horizontal wave vector of the disturbance, α_n is the radial component of $\underset{\sim}{K}_n$, $R_c\beta_n$ is the azimuthal component of $\underset{\sim}{K}_n$, and the coefficient functions $\underset{\sim}{u}_{1n}$ and P_{1n} satisfy a system of the type given by (6) in [1], provided $(\alpha,\ \beta)$ in that system is replaced by $(\alpha_n,\ \beta_n)$.

It should be noted that the solution of the form (8a), which consists of two modes plus their complex conjugates, is considered here since the results for the neutral stability curve reported in [2] indicated two distinct neutral modes for given $R = R_0$ close to R_c. For $R_0 = R_c$, these two modes merge into the critical mode at the onset of primary instability of the stationary modes.

The linear system for $\underset{\sim}{u}_{1n}$ and P_{1n} represents a linear eigenvalue problem which was solved in [1] using a collocation method to find the following results for the minimum value of $R_0 = R_c$ for the neutral stability of the stationary modes and the associated critical values of $\alpha_n = \alpha_c$ and $\beta_n = \beta_c$

$$R_c = 275.36, \quad \alpha_c = 0.38318, \quad \beta_c = 0.078232 \tag{9}$$

The solution to the adjoint linear system is of the form

$$\begin{pmatrix} \hat{\underset{\sim}{u}}_1 \\ \hat{P}_1 \end{pmatrix} = \sum_{n=-2}^{2} \begin{bmatrix} \hat{\underset{\sim}{u}}_{1n}(z) \\ \hat{P}_{1n}(z) \end{bmatrix} A_n E_n, \tag{10}$$

where the coefficient functions $\hat{\underset{\sim}{u}}_{1n}$ and $\hat{P}_{1n}$ satisfy a system of the type given by (11) in [1], provided (α, β) in that system is replaced by (α_n, β_n).

Later to form the so-called solvability condition for the higher order system [8], we need to consider particular solution $\left(\underset{\sim}{u}_{an}, P_{an}\right)$ of the adjoint linear system which can be isolated from (10) in the form

$$\begin{pmatrix} \underset{\sim}{u}_{an} \\ P_{an} \end{pmatrix} = \begin{bmatrix} \hat{\underset{\sim}{u}}_{1n}(z) \\ \hat{P}_{1n}(z) \end{bmatrix} E_n. \tag{11}$$

The solution to order ε^2 system can be written in the form

$$\begin{pmatrix} \underset{\sim}{u}_2 \\ P_2 \end{pmatrix} = \sum_{l,\,p=-2}^{2} \begin{bmatrix} \underset{\sim}{u}_{2lp}(z) \\ P_{2lp}(z) \end{bmatrix} A_l A_p E_l E_p, \tag{12}$$

where the coefficient functions $\underset{\sim}{u}_{2lp}$ and P_{2lp} satisfy a system of the form given by (7) in [1], provided $(2\alpha, 2\beta)$ in the homogeneous version of the system is replaced by $\left[(\alpha_l + \alpha_p),\ (\beta_l + \beta_p)\right]$, while (α, β) in the non-homogeneous part is replaced by (α_p, β_p).

We now form the solvability condition for order ε^2 system. Take dot product of the momentum equation, in the order ε^2, with $\underset{\sim}{u}_{an}$, multiply the continuity equation, in the order ε^2, by P_{an}, sum the equations, integrate in z from $z = 0$ to $z = \infty$, average in the $(r,\ \theta)$ plane, apply integration by parts and use the zero boundary conditions for both $\underset{\sim}{u}_2$ and $\underset{\sim}{u}_{an}$. We then find $0 = 0$ so that such condition always holds for the order ε^2 system.

In the order ε^3, the boundary conditions at $z=0$ are no longer zero, in general, as can be seen from (4). The solvability condition for this order system then yield the following system of equations for $A_n(n=1,2)$:

$$\frac{\partial A_n}{\partial t_s} = R_2 a_n A_n + b_n \sum_m \tilde{A}_m \langle E_n^* \tilde{E}_m \rangle +$$

$$\sum_{m,\,l,\,p} \hat{C}_{mnlp} A_m A_l A_p \langle E_n^* E_m E_l E_p \rangle, \tag{13}$$

where the expressions for the coefficients a_n, b_n and $\hat{C}_{mnlp}$ are given by (A1)-(A3) in the appendix. Here an angular bracket $\langle\ \rangle$ denotes an average over the fluid layer.

The integral expression $\langle E_n^* E_m E_l E_p \rangle$ in (13) is non-zero only if

I) $p=n$, $l=-m$	II) $l=n$, $p=-m$, $p\neq n$	III) $m=n$, $p=-l$, $p\neq \pm n$

Using these conditions, (13) is simplified to the form

$$\frac{\partial A_n}{\partial t_s} = R_2 a_n A_n + b_n \sum_m \tilde{A}_m \langle E_n^* \tilde{E}_m \rangle +$$

$$C_n |A_n|^2 A_n + \sum_{\substack{m \\ m\neq \pm n}} e_{mn} |A_m|^2 A_n,\ (n=-2,\ldots,2), \tag{14a}$$

where

$$C_n = \left(\hat{C}_{nn,-1n,n} + \hat{C}_{-n,nnn} + \hat{C}_{nnn,-n}\right), \tag{14b}$$

$$e_{mn} = \left(\hat{C}_{mn,-m,n} + \hat{C}_{mnn,-m} + \hat{C}_{nnm,-m}\right). \tag{14c}$$

To distinguish the physically realizable solution(s) among all the solutions of (14a), the stability of $A_m(m=-2,...,2)$ with respect to disturbances $B_m(t_s)$ are investigated. The system of equations for the time dependent disturbances is given by

$$\left(\frac{\partial}{\partial t_s} - R_2 a_n\right) B_n = C_n\left(A_n^2 B_n^* + 2|A_n|^2 B_n\right) + \sum_{\substack{m \\ m\neq \pm n}} e_{mn}$$

$$\left(|A_m|^2 B_n + A_m A_n B_m^* + A_n A_m^* B_m\right), \quad (15)$$

where B_m also satisfies condition of the form (8c).

It is clear from (14a) and (15) that the surface roughness affects the solutions directly as source term in (14a), while the surface roughness affects the disturbances indirectly through the solutions. It is also seen from (14a) that the surface roughness can affect the solutions only if there is at one m for which $\tilde{\underset{\sim}{K}}_m = \underset{\sim}{K}_n$, otherwise the second term in the right-hand-side of (14a) vanishes and surface roughness becomes insignificant.

Before considering (14)-(15) for significant surface roughness effects, it is instructive to analyse these equations for the cases where none of the wave vectors $\tilde{\underset{\sim}{K}}_m$ of the surface roughness equals $\underset{\sim}{K}_n$. In this situation the second term in the right-hand-side of (14a) vanishes and the most preferred solution, which corresponds to the smallest R, is that for which $R_0 = R_c$. Hence, $A_n = A$ for all n, where A is the amplitude of the critical mode, corresponding to $R_0 = R_c$, and (14a)-(15) reduce to

$$\frac{\partial A}{\partial t_s} = R_2 a A + c|A|^2 A, \quad (16a)$$

$$\left(\frac{\partial}{\partial t_s} - R_2 a\right) B = c\left(A^2 B^* + 2|A|^2 B\right), \quad (16b)$$

where B is the disturbance amplitude and

$$a = (2.8993 - 2.5127i)\left(10^{-5}\right), \quad c = 0.0976 + 5.1792i \tag{17}$$

[1]. Using (16), it can be shown that, due to positive real part c_r of c, the steady solution always leads to positive growth rate for B and, thus, the steady solution is unstable. For time dependent solution, it is found that, due to $c_r > 0$, $|A|$ increases indefinitely with respect to t_s leading to solution breakdown of the present weakly nonlinear equation. On the other hand, it is known from solution of the Landau equation [8] that steady stable solution is possible if c_r were negative and $a_r R_2$ were positive. We shall demonstrate below that, with the aid of particular surface roughness, it is possible to have steady stable solution for the present problem.

Consider first the case of single critical-mode, where $R_0 = R_c$ and $\underset{\sim}{K}_n = \underset{\sim}{K}_c$. Here $\underset{\sim}{K}_c$ is the critical wave vector $(\alpha_c,\ R_c B_c)$. For significant surface roughness, where $\tilde{\underset{\sim}{K}}_n = \underset{\sim}{K}_c$, (14a) is reduced to the form

$$\frac{\partial A_1}{\partial t_s} = R_2 a A_1 + c|A_1|^2 A_1 + b\tilde{A}_1, \tag{18}$$

where $\tilde{A}_1$ is the amplitude of the surface roughness and b is value of b_n at the critical mode. We are going to select particular $\tilde{A}_1$ which leads to steady stable A_1.

The form of $\tilde{A}_1$ which leads to steady stable A_1 is given by

$$b\tilde{A}_1 = \left[\left(a_r|R_2| + |\hat{a}_2| + ib_2\right) + \left(-c_r - |\hat{a}_1| + ib_1\right)\left(a_r|R_2| + a_r R_2 + |\hat{a}_2| / |\hat{a}_1|\right]\cdot$$

$$\cdot\left[\left(a_r|R_2| + a_r R_2 + |\hat{a}_2|\right)/|\hat{a}_1|\right]^{1/2}\cdot(\cos\ \gamma + i\ \sin\ \gamma), \tag{19a}$$

where $|\hat{a}_1|$, $|\hat{a}_2|$, b_1 and γ are arbitrary constants (real),

$$b_2 = -a_i R_2 - (c_i + b_1)\left(a_r|R_2| + a_r R_2 + |\hat{a}_2|\right)/|\hat{a}_1|, \tag{19b}$$

and a_i and c_i are the imaginary components of a and c, respectively. The steady solution for (18) subjected to (19) is then

$$A_1 = \left[\left(a_r|R_2| + a_r R_2 + |\hat{a}_2|\right)/|\hat{a}_1|\right]^{1/2}(\cos\ \gamma + i\ \sin\ \gamma) \tag{20}$$

The solution (20) subjected to the surface roughness amplitude of the general form (19) is also found to be stable with respect to infinitesimal disturbances, provided

$$R_2 < -2C_r|\hat{a}_2|/\left(a_r|\hat{a}_1|\right). \tag{21}$$

These disturbances are found to be only stationary, which satisfy equation of the form (166). It is seen from (19-(21) that there are infinite number of possible forms of the surface roughness amplitudes which lead to steady stable solutions which are subcritical by the requirement (21).

It should be noted that in order to arrive at the form (19) for $\tilde{A}_1$, which led to the solution (20) for A_1, we first assumed that $b\tilde{A}_1$ is the sum of a term proportional to A_1 and a term proportional to $A_1|A_1|^2$. Then, we used such form for $\tilde{A}_1$ in (18) and applied conditions for prediction of steady stable solutions, where the resulting coefficient of A_1 in (18) should have positive real part and the resulting coefficient of $A_1|A_1|^2$ in (18) should have negative real part. This led to (20) which was used in the assumed form of $\tilde{A}_1$ to find (19).

We now consider the case of non-critical bi-modal flow, where R_0 is slightly larger than R_c and $\underset{\sim}{K}_n \neq \underset{\sim}{K}_c$. For significant surface roughness, where $\tilde{\underset{\sim}{K}}_n = \underset{\sim}{K}_n (n = 1,2)$, say (14a) is reduced to the following two equations

$$\frac{dA_1}{dt_s} = \left(R_2 a_1 + \tilde{e}_{21}|A_2|^2\right)A_1 + c_1|A_1|^2 A_1 + b_1\tilde{A}_1, \tag{22a}$$

$$\frac{dA_2}{dt_s} = \left(R_2 a_2 + \tilde{e}_{12}|A_1|^2\right)A_2 + c_2|A_2|^2 A_2 + b_2\tilde{A}_2, \tag{22b}$$

where

$$\tilde{e}_{21} \equiv e_{21} + e_{-2,1}, \; \tilde{e}_{12} = e_{12} + e_{-1,2}. \tag{22c}$$

For $\tilde{A}_1 = \tilde{A}_2$, roughness effect is absent in (22), $R_0 > R_c$ and such flow is not preferred as compared to critical mode case [8]. For the case where either $\tilde{A}_1$ or $\tilde{A}_2$ is zero, then (22a) or (22b), respectively, implies indefinite increase of $|A_1|$ or $|A_2|$ with time since c_1 and c_2 has positive real part. Consequently, such case also is not preferred. Thus we consider the case where both $\tilde{A}_1$ and $\tilde{A}_2$ are assumed non-zero. Following the procedure described before for the single critical mode case, we select $\tilde{A}_1$ and $\tilde{A}_2$ appropriately such that the resulting coefficients of A_1 and A_2 in (22a) and (22b) have positive real parts and the resulting coefficients of $A_1|A_1|^2$ and $A_2|A_2|^2$ in (22a) and (22b) have negative real parts. This procedure then leads to steady solutions for $A_i (i=1,2)$ which are stable for sufficiently large $|R_2|$ $(\mathrm{R}_2 < 0)$. Again, as in the case of single critical mode, there are many different possible forms of the surface roughness amplitudes which can lead to steady stable bi-modal flow solutions which are also subcritical.

Before considering another major regime of interest in the next section, where a double expansions in powers of δ and ε for the dependent variables and for R is required, it is of interest here to consider briefly other particular $\delta = \varepsilon^n$ cases for $n \neq 3$ with n a positive integer. For $n > 3$, the magnitude of the roughness elements is so small that all the main results follow, up to order ε^3, without consideration of the surface roughness. For $n = 1$, the trivial result that the roughness controls the linear solution follows. For $n = 2$, one needs to define ε and t_s like

$$\varepsilon^2 R_2 + \varepsilon R_1 + (R - R_0) = 0, \quad t_s = \varepsilon t. \tag{23}$$

In the order ε^2, the preference of the solution, in the form of either single critical mode or non-critical bi-modal, can be followed by the effect of surface roughness as in the case discussed in this section. The steady solutions for $A_i (i = 1,2)$ are then determined in terms of $\tilde{A}_i$ and R_1, and they are also stable with respect to infinitesimal disturbance for sufficiently large $|R_1|$ and $\varepsilon R_1 < 0$. The solvability condition in the order ε^3 then determines R_1, R_2 and A_i in terms of $\tilde{A}_i$, and, of course, the preferred solutions are those stable ones which correspond to the smallest value of R.

ANALYSIS FOR CASE $0(\varepsilon^2) < \delta < 0(\varepsilon)$

This case belongs to the critical regime where $R \approx R_c$ and $0(\varepsilon^n) < \delta < 0(\varepsilon^{(n-1)})$ for $n = 2$. Here ε is the amplitude of the primary stationary modes. We consider double expansions in powers of δ and ε for the dependent variables and for R

$$\begin{pmatrix} \underset{\sim}{u} \\ P \\ R \end{pmatrix} = \sum_{m,n} \varepsilon^m \delta^n \begin{pmatrix} \underset{\sim}{u}_{mn} \\ P_{mn} \\ R_{mn} \end{pmatrix}, \quad \underset{\sim}{u}_{00} = P_{00} = 0, \tag{24}$$

and we define the slowly varying time t_s in the form

$$t_s = \delta t \tag{25}$$

Using (24)-(25) in the governing disturbance system, we find to the order ε the linear system for $\underset{\sim}{u}_{10}$ and P_{10} quantities. This system admits solution of the form (8), provided R_0, $\left(\underset{\sim}{u}_1, P_1\right)$ and $\left(\underset{\sim}{u}_{1n}, P_{1n}\right)$ are replaced, respectively, by R_{00}, $\left(\underset{\sim}{u}_{10}, P_{10}\right)$ and $\left(\underset{\sim}{u}_{10n}, P_{10n}\right)$.

The results (9) then follow from such system. The solution to the adjoint linear system is of the form (10), provided $\left(\hat{\underset{\sim}{u}}_1, \hat{P}_1\right)$ and $\left(\hat{\underset{\sim}{u}}_{1n}, \hat{P}_{1n}\right)$ are replaced, respectively, by $\left(\hat{\underset{\sim}{u}}_{10}, \hat{P}_{10}\right)$

and $\left(\hat{\underset{\sim}{u}}_{10n},\hat{P}_{10n}\right)$. Particular solution of the adjoint linear system is of the form (11), provided $\left(\hat{\underset{\sim}{u}}_{1n},\hat{P}_{1n}\right)$ is replaced by $\left(\hat{\underset{\sim}{u}}_{10n},\hat{P}_{10n}\right)$.

In order δ, the governing equations and the upper boundary conditions are of the same form to the corresponding ones in the order ε, provided $\left(\underset{\sim}{u}_{10},P_{10}\right)$ is replaced by $\left(\underset{\sim}{u}_{01},P_{01}\right)$. The lower boundary conditions are, however, of the order δ version of (4), provided $\underset{\sim}{u}$ is replaced by $\underset{\sim}{u}_{01}$. This system admits solution of the form

$$\begin{pmatrix}\underset{\sim}{u}_{01}\\ P_{01}\end{pmatrix}=\sum_{n=-2}^{2}\begin{bmatrix}\underset{\sim}{u}_{01n}(z)\\ P_{01n}(z)\end{bmatrix}\tilde{A}_n\tilde{E}_n, \tag{26}$$

where the equations and the upper boundary conditions for the coefficient functions $\left(\underset{\sim}{u}_{01n},P_{01n}\right)$ are of the form to the corresponding ones for $\left(\underset{\sim}{u}_{10n},P_{10n}\right)$ described before. However the lower boundary conditions for these coefficients are now

$$u_{01n}(z)=-F'(0),\ v_{01n}(z)=-G'(0),\ w_{01n}(z)=0 \text{ at } z=0. \tag{27}$$

In the order ε^2, the solvability condition leads to the result $R_{10}=0$. The leading possible non-zero term $R_{01}\delta$, in the expansion (24) for R, beyond R_{00} appears in the solvability condition for the $\varepsilon\delta$-order system. This condition can be reduced to

$$\frac{dA_n}{dt_s}=R_{01}a_{n1}A_n+\sum_{l,p}\hat{C}_{nlp}\tilde{A}_lA_p\left\langle E_n^*\tilde{E}_lE_p\right\rangle,(n=-2,...,2), \tag{28}$$

where the expressions for the coefficients a_{n1} and $\hat{C}_{nlp}$ are given by (A4)-(A5) in the appendix.

The integral expression $\left\langle E_n^*\tilde{E}_lE_p\right\rangle$ in (28) is non-zero only if

$$\underset{\sim}{K}_n = \underset{\sim}{\tilde{K}}_l + \underset{\sim}{K}_p \tag{29}$$

for at least some l and p.

Before considering (28)-(29) for significant surface roughness effects, it is of interest to note that for the cases where none of the surface roughness wave vectors $\underset{\sim}{\tilde{K}}_l$ satisfy (29). In this situation the second term in the right-hand-side of (28) vanishes. Referring to (A1), (A4) and the results presented in the previous section, it is apparent that the real part of a_{n1} is positive, and the solution A_n to the reduced equation (28) increases indefinitely with t_s, unless R_{01}. Hence $R_{01} = 0$ and A_n becomes constant. Therefore, non-trivial and significant result can follow only if $\underset{\sim}{\tilde{K}}_l$ satisfy (29) for at least some l.

Consider first the case of single critical-mode, where $R_{00} = R_c$ and $\underset{\sim}{K}_n = \underset{\sim}{K}_c$. For significant surface roughness, where (29) holds, we have

$$\underset{\sim}{\tilde{K}}_l = 2\underset{\sim}{K}_c, \tag{30}$$

for, say $l = 1$. Using (30) in (29) (for $n = -1,1$), we find

$$\frac{dA_n}{dt_s} = R_{01}a_{n1}A_n + \hat{C}_{nn,-n}\tilde{A}_n A_{-n}, \quad (n = -1,1) \tag{31}$$

The form of $\tilde{A}_n$ which leads to steady stable A_n is found to be

$$\hat{C}_{nn,-n}\tilde{A}_n = -R_{01}a_{n1}, \tag{32a}$$

provided

$$R_{01}a_{n1r} < 0,$$

where a_{n1r} denotes the real part of a_{n1}. Since $a_{n1r} > 0$, we have

$$R_{01} < 0. \tag{32b}$$

Using the property (8c) and (32) in (31), we find that A_n is a real constant, and it turns out to be stable with respect to infinitesimal disturbances. Again as in the corresponding result presented in the previous section, in order to arrive at (32), we had to choose $\tilde{A}_n$ according to (32a) which suppresses the time dependence part in (31) and leads to stable solution for A_n if (32b) holds.

The flow solution described above is preferred and since

$$\delta > 0, \tag{33}$$

it leads to subcritical steady stable solution $(\delta R_{01} < 0)$.

Next, consider the case of non-critical bi-modal flow, where R_{00} is slightly larger than R_c and $\underset{\sim}{K}_n \neq \underset{\sim}{K}_c$. For significant surface roughness, there are two cases of general interest. For the first case, the surface roughness wave vectors satisfy the conditions

$$\tilde{\underset{\sim}{K}}_n = 2\underset{\sim}{K}_n, \quad (n = 1,2). \tag{34}$$

Using (29) and (34) in (28), we find equations of the form (31) which now hold for $n = -2,...,2$. Again the form of $\tilde{A}_n$ which leads to steady stable A_n is found to be of the form (32a) which now hold for $n = -2,...,2$, provided

$$R_{01} a_{n1r} < 0 \text{ and } R_{01}\delta < 0. \tag{35}$$

These results again imply subcritical steady stable solution for the particular non-critical bi-modal flow satisfying the conditions (34).

For the second case of non-critical bi-modal flow, the surface roughness wave vectors satisfy the conditions, say, of the form

$$\tilde{K}_1 = 2K_1, \quad \tilde{K}_2 = K_1 + K_{-2}, \tag{36}$$

where the wave vectors $\tilde{K}_2$, $\tilde{K}_{-2}$, K_1, K_{-1}, K_{-2} and K_2 form a six-sided polygon. Using (36) in (28), we find

$$\left(\frac{d}{dt_s} - R_{01}a_{11}\right)A_1 = \hat{C}_{12,-2}\tilde{A}_2 A_{-2} + \hat{C}_{11,-1}\tilde{A}_1 A_{-1}, \tag{37a}$$

$$\left(\frac{d}{dt_s} - R_{01}a_{-1,1}\right)A_{-1} = \hat{C}_{-1,-2,2}\tilde{A}_{-2} A_2 + \hat{C}_{-1,-1,1}\tilde{A}_{-1} A_1, \tag{37b}$$

$$\left(\frac{d}{dt_s} - R_{01}a_{21}\right)A_2 = \hat{C}_{2,-2,-1}\tilde{A}_{-2} A_{-1}, \tag{37c}$$

$$\left(\frac{d}{dt_s} - R_{01}a_{-2,1}\right)A_{-2} = \hat{C}_{-2,2,1}\tilde{A}_2 A_1 . \tag{37d}$$

There can exist steady stable and real solutions A_n for (37), provided

$$\tilde{A}_{-2}C_{2,-2,-1} = -R_{01}a_{21}, \tag{38a}$$

$$\tilde{A}_1\hat{C}_{11,-1} + R_{01}a_{11} = -\tilde{A}_2\hat{C}_{12,-2}, \tag{38b}$$

real part of

$$\left(\tilde{A}_1\hat{C}_{11,-1} + R_{01}a_{11} + R_{01}a_{21}\right) < 0, \tag{38c}$$

and such solutions are preferred if

$$R_{01}\delta < 0. \tag{39}$$

Finally, for completeness, we consider here briefly other particular $0(\varepsilon^n) < \delta < 0(\varepsilon^{(n-1)})$ cases for $n \neq 2$ with n a positive integer. For $n = 1$, the trivial result that roughness controls the linear solution follows. For $n \geq 3$, the $\varepsilon^2 R_{20}$ term in the series expansion (24) for R dominates all the other terms containing factor $\delta^m (m > 0)$ and thus implying negligible roughness effect on the amplitude solution A_l.

SOME CONCLUDING REMARKS

In this paper we evaluated the effect of small roughness elements which are found to have significant influence, under certain conditions on their magnitude, amplitudes and wave vectors, on the selection of particular flow development resulted from the primary instability of the stationary modes in rotating disk flows. In particular, we found that there are certain families of the surface roughness shapes with particular wave vectors and amplitudes which can lead to the preference of steady stable single critical-mode vortex or non-critical bi-modal vortices.

An important feature not considered in the present study is that of consideration of wave packet disturbances. In the absence of any roughness effects, Riahi and Vonderwell [9] investigated weakly nonlinear development of wave packet disturbances in a rotating disk flow and found that the solution of the amplitude equation can increase indefinitely with time at the center of the wave packet disturbances. In the presence of some particular forms of surface roughness, it is anticipated that steady stable flow can be predicted as the results of some preliminary investigation by the present author indicates but the completed full results will be reported elsewhere. Some roles of surface roughness was detected experimentally by Wilkinson and Malik [10] who found that the stationary disturbances can originate from isolated roughness sites on the rotating disk and several vortex pattern emerges only when the different wave packets have spread and filled the entire disk circumference.

A related important problem investigated presently by the present author is the determination of the surface roughness effects on the nonlinear development of travelling wave disturbances which can be preferred as in the case of Tollmien-Schlichting waves development in a channel flow. Similar to the rotating disk flow, it is known that in the absence of roughness no steady stable solution can be predicted based on the weakly nonlinear theory [11]. Hence, consideration of particular surface roughness may have significant effects on the flows in such cases as well.

References

1. M.P. Vonderwell and D.N. Riahi Weakly non-linear devlopment of rotating disk flow, *International Journal of Engineering Science,* **31**, 549-558 (1993).
2. M.R. Malik The neutral curve for stationary disturbances in rotating-disk flow, *Journal of Fluid Mechanics,* **164**, 275-287 (1986).
3. M.V. Morkovin Understanding Transition to Turbulence in Shear Layers, Defence Technical Information Center, U.S.A. (1983).
4. I.A. Waitz and S.R. Wilkinson Rotating disk transition due to isolated roughness with intense accoustic irradiation, preceed. AIAA/ASME/SIAM/APS First National Fluid Dynamics Congress, 1281-1288 (1988).
5. D.N. Riahi Preferred pattern of convection in a porous layer with a spatially non-uniform boundary temperature, *Journal of Fluid Mechanics,* **246**, 529-543 (1993).
6. H. Schlichting Boundary Layer Theory, seventh edition, McGraw-Hill Book Co. (1979).
7. F.M. White Viscous Fluid Flow, second edition, McGraw-Hill Book Co. (1991).
8. P.G. Drazin and W.H. Reid Hydrodynamic Stability, Cambridge University Press, Cambridge, U.K. (1981).
9. D.N. Riahi and M.P. Vonderwell Wave packet development of rotating disk flow, proceedings of third Pan American Congress of Applied Mechanics (Sao Paulo, Brazil, Jan 1993) 331-334 (1993).
10. S.P. Wilkinson and M.R. Malik Stability experiment in the flow over a rotating disk, *AIAA Journal* , **23**, 588-595 (1983).
11. K. Stewartson and J.T. Stuart A nonlinear instability theory for a wave system in plane Poiseuille flow, *Journal of Fluid Mechanics*, **48**, 525-545 (1971).

Appendix

The expressions for the coefficients a_n, b_n and $\hat{C}_{mnlp}$ introduced in (13) are given below

$$\left.\begin{aligned}
&a_n = -\langle[\hat{u}^*_{1n}(L_{1n}u_{1n} - 2i\beta_n v_{1n}/R_c) + \hat{v}^*_{1n}(L_{1n}v_{1n} + 2i\beta_n u_{1n}/R_c) \\
&+\hat{w}^*_{1n}(L_{1n} + 1/R_c^2)w_{1n}]\rangle/(R_0^2 d_n), \\
&\text{where} \\
&L_{1n} \equiv \frac{d^2}{dz^2} - (\alpha_n^2 + \beta_n^2 + 1/R_c^2) + i\alpha_n/R_c, \\
&d_n \equiv \langle(\hat{u}^*_{1n}u_{1n} + \hat{v}^*_{1n}v_{1n} + \hat{w}^*_{1n}w_{1n})\rangle,
\end{aligned}\right\} \tag{A1}$$

$$b_n = -\left[F'(0)\frac{d\hat{u}^*_{1n}}{dz}(0) + G'(0)\frac{d\hat{v}^*_{1n}(0)}{dz}\right]\Big/(R_0 d_n), \tag{A2}$$

$$\left.\begin{aligned}
&\hat{C}_{mnlp} = -\langle\hat{u}^*_{1n}[i(\alpha_l + \alpha_p)u_{1m}u_{2lp} + i\alpha_m u_{1m}u_{2lp} + \\
&i(\beta_l + \beta_p)v_{1m}u_{2lp} + i\beta_m u_{1m}v_{2lp} 2v_{1m}v_{2lp}/R_c + w_{1m}\cdot \\
&\frac{d}{dz}u_{2lp} + \frac{du_{1m}}{dz}\cdot w_{2lp}] + \hat{v}^*_{1n}[i(\alpha_l + \alpha_p)u_{1m}v_{2lp} + i\alpha_m \\
&v_{1m}u_{2lp} + i(\beta_l + \beta_p)v_{1m}v_{2lp} + i\beta_m v_{1m}v_{2lp} - (u_{1m}v_{2lp} + \\
&v_{1m}u_{2lp})/R_c + w_{1m}\frac{dv_{2lp}}{dz} + w_{2lp}\frac{dv_{1m}}{dz}] + \hat{w}^*_{1n}[i(\alpha_l + \alpha_p)\cdot \\
&u_{1m}w_{2lp} + i\alpha_m w_{1m}u_{2lp} + i(\beta_l + \beta_p)v_{1m}w_{2lp} + i\beta_m w_{1m}v_{2lp} \\
&+w_{1m}\frac{dw_{2lp}}{dz} + \frac{dw_{1m}}{dz}w_{2lp}]\rangle/d_n
\end{aligned}\right\} \tag{A3}$$

The expressions for the coefficients a_{n1} and $\hat{C}_{nlp}$ introduced in (28) are given below

$$\left.\begin{aligned}
&a_{n1} = -\langle\hat{u}^*_{10n}(L_{1n}u_{10n} - 2i\beta_n v_{10n}/R_c) + \hat{v}^*_{10n}(L_{1n}v_{10n} + 2i\beta_n u_{10n}/R_c) \\
&+\hat{w}^*_{10n}(L_{1n} + 1/R_c^2)w_{10n}\rangle/[(R_{00})^2 d_{n1}], \\
&\text{where } L_{1n} \text{ is given in (A1)}, \\
&\mathrm{d}_{n1} = \langle(\hat{u}^*_{10n}u_{10n} + \hat{v}^*_{10n}v_{10n} + \hat{w}^*_{10n}w_{10n})\rangle,
\end{aligned}\right\} \tag{A4}$$

$$\hat{C}_{nlp} = -\left\langle \left[\frac{d\hat{u}^*_{10n}(0)}{dz} \frac{du_{10p}(0)}{dz} + \frac{d\hat{v}^*_{10n}}{dz} \frac{dv_{10p}(0)}{dz} + \frac{d\hat{w}^*_{10n}(0)}{dz} \frac{dw_{10p}(0)}{dz} \right] \right.$$
$$/R_{00} + \Big[\hat{u}^*_{10n} \big(i\tilde{\alpha}_l u_{10p} u_{01l} + i\alpha_p u_{01l} u_{10p} + i\tilde{\beta}_l v_{10p} u_{01l} + i\beta_p v_{01l} u_{10p} -$$
$$2 v_{01l} v_{10p}/R_c + w_{01l} + w_{01l} \frac{du_{10p}}{dz} + w_{10p} \frac{du_{01l}}{dz} \big) + \hat{v}^*_{10n} \big(i\tilde{\alpha}_l u_{10p} v_{01l} +$$
$$i\alpha_p u_{01l} v_{10p} + i\tilde{\beta}_l v_{10p} v_{01l} + i\beta_p v_{01l} v_{10p} - u_{10p} v_{01l}/R_c - u_{01l} v_{10p}/R_c + \qquad \text{(A5)}$$
$$w_{10p} \frac{dv_{01l}}{dz} + w_{01l} \frac{dv_{10p}}{dz} \big) + \hat{w}^*_{10n} \big(i\tilde{\alpha}_l u_{10p} w_{01l} + i\alpha_p u_{01l} w_{10p} + i\tilde{\beta}_l v_{10p}$$
$$\left. w_{01l} + i\beta_p v_{01l} w_{10p} + w_{10p} \frac{dw_{01l}}{dz} + w_{01l} \frac{dw_{10p}}{dz} \big) \Big] \right\rangle \Big/ d_{n1}$$

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