Effects of Centrifugal and Coriolis Forces on Chimney Convection During Alloy Solidification

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Abstract

Nonlinear natural convection in cylindrical chimneys within a mushy layer during solidification of a binary alloy is investigated under a high gravity environment where the rotation axis is inclined at an angle γ to the high gravity vector. These chimneys produce freckles in the final form of the solidified materials. Freckles are imperfections that reduce the quality of the solidified materials. Asymptotic and scaling analyses are applied to weakly non-axisymmetric convection in the chimneys. Three important non-dimensional parameters are the solutal Rayleigh number R, the centrifugal acceleration parameter A and the Coriolis parameter T. It is found that, for some moderate values of the rotation rate and $\sin\gamma \neq 0$, axial convection in the chimneys decreases rapidly with increasing A above some azimuthally dependent axial level. This level can also depend on the radii of the chimneys and on γ and T. The axial convection also decreases with increasing T in certain range which depends on γ, T and R. The Coriolis effect is destabilizing for T

outside this range of values, but it is less destabilizing for $\gamma = \gamma_0 > 180^\circ$ as compared with those cases with inclined angle $\left(\gamma_0 - 180^\circ\right)$.

1. Introduction

Recently Riahi and Sayre [1] investigated nonlinear natural convection in a mushy layer of a solidification system under a high gravity condition where the rotation axis is inclined at an angle γ to the high gravity vector. They considered only axisymmetric convection and did not include the effect of Coriolis force. They applied asymptotic and scaling analyses to convection within the mushy layer and in cylindrical chimneys and found that, for some particular moderate range of rotation rate, where $A \sim R$, vertical velocity in the chimneys reduces significantly in magnitude. Here R is the solutal Rayleigh number in the mushy zone and A is the gradient acceleration parameter in the mushy layer due to deviation of the centrifugal acceleration from its average value. The chimneys referred to above were characterized by having zero solid fraction, and they were assumed to be in vertical direction parallel to the high gravity vector which is the resultant vector of those due to normal gravity and averaged centrifugal acceleration term. These channels subsequently become locations of severe compositional non-homogeneities which in their final form are called freckles. Freckles are imperfections that interrupt the uniformity of the solidified materials causing areas of mechanical weakness.

Sample and Hellawell [3,4] did Nh_4Cl alloy experiment in a cylindrical mold with a chilled bottom surface where solidification was induced. They employed a rotation and tilting technique to change the orientation of gravity relative to the axis of the cylinder. They found that for slow and steady rotation of the mold about the vertical axis, the channel formation and development was more or less the same as in the case of zero rotation. But, for slow and steady rotation of a tilted mold the number of channels was reduced significantly and, under some conditions, was completely eliminated. Other experimental results reported by Kou et al. [5] on rotated remelted ingot indicated that for too large rotational speed, segregates formed along a ring between the axis and the outer edge of the ingot.

The main purpose of the present study was to investigate theoretically the effect of inclined rotation on chimney convection using analyses of the type due to Riahi and Sayre [1] at large R but for realistic cases taking into account the Coriolis force effects and the non-axisymmetric behavior of convection that were ignored in Riahi and Sayre [1]. Such extension can be important for the flow regime under consideration where R is assumed to be large so that convection is likely to be three-dimensional and there is no justification to ignore the Coriolis effects.

The motivations for the present study have been those discussed above as well as to determine realistic results, under no un-justifiable assumption, which can be compared with the available experimental results [3-5] and could be used, in a useful way, to eliminate or at least reduce the production of undesirable freckles effects. The preliminary studies by Riahi and Sayre [1] indicated that such goal may be achieved by applying rotational constraint upon convection in the chimneys. If axial convection can be decreased with increasing the rotation rate, then it can be expected that the chimney formation in the mushy layer can be halted or atleast can be reduced since these chimneys initially grow due to thermal plumes that develop by convective flow in the mushy layer. Suppression of convection in the mushy layer is ideal because it can provide favorable condition for production of uniform crystal which is quite desirable for crystal growers.

2. Governing system

We consider a thin mushy layer adjacent to solidifying surface of a binary alloy melt and of thickness h. The binary alloy melt of some constant composition C_0 and temperature T_∞ is solidified at a rate V_0, with the eutectic temperature T_e at the position $z = 0$ held fixed in a frame moving with the solidification velocity in z—direction, where the z-axis is assumed to be anti-parallel to the high gravity vector to be described below. The physical model at normal gravity is based on the assumptions of the type considered by Worster [6], and we refer the reader to this reference for details regarding such assumptions. The extension of such model in high gravity is based on assumptions of the type considered by Arnold et al. [2] for solidification in a centrifuge, and we refer the reader to this reference for details regarding high gravity considerations. We assume that the solidifying system is placed in a centrifuge rotating at some constant angular velocity Ω about the centrifuge axis which makes an angle γ with respect to z—axis. The centrifuge axis is assumed to be anti-parallel to the earth gravity vector.

Next, we consider the equations for momentum, continuity, heat and solute, based on the Boussinesq approximation [2,6], in the moving coordinate system whose origin 0 is centered on the solid-mush interface. The governing system of these equations for the solidifying system rotating with the centrifuge basket [2] and translating with the solidification front at speed V_0 is non-dimentionalized by using the procedure of the type described in Worster [6] and in Riahi and Sayre [1] and will not be repeated here. In the momentum equations the centrifugal acceleration term is split into an average term, which is superimposed on the gravity term, and a so-called gradient acceleration term [2]. The non-dimensional parameter representing the modified gravity term can then become significantly larger than the corresponding one due to earth's gravity alone for significant rotation rate. Following [6-8], we consider the mushy layer as a porous medium where Darcy's law holds. However, the original equations in a liquid layer hold for the flow of melt inside each chimney since the solid fraction ϕ is assumed to be zero there.

Since we will be concerned mainly with the convection in any cylindrical chimney, whose axis is assumed to be parallel to the z-axis, within the mushy layer [6], we consider the governing equations in a cylindrical coordinate whose axial direction is along the z-axis. We shall consider particular non-axisymmetic convection in a cylindrical chimney, whose axis coincides with the z-axis, and in the mushy layer in the asymptotic limit of strong buoyancy force due to the solute concentration, negligible thermal buoyancy and sufficiently large Prandtl number ν/K and Lewis number K/D. Here ν is the kinematic viscosity, and K and D are thermal and solute diffusivity, respectively. The non-dimensional form of the equations for the momentum, continuity, temperature and solute concentration for the steady flow of melt in the chimney are

$$0 = -HR\left(\Delta P + S\hat{Z}\right) + \nabla^2 \underset{\sim}{u} + HT\underset{\sim}{u} \times \hat{\Omega} + HAS\underset{\sim}{G}, \tag{1a}$$

$$\nabla \cdot \underset{\sim}{u} = 0, \tag{1b}$$

$$\left(-\frac{\partial}{\partial z} + \underset{\sim}{u} \cdot \nabla\right)\theta = \nabla^2 \theta, \tag{1c}$$

$$\left(-\frac{\partial}{\partial z} + \underset{\sim}{u} \cdot \nabla\right)S = 0, \tag{1d}$$

where $\underset{\sim}{u} = u\hat{r} + v\hat{\xi} + w\hat{z}$ is the velocity vector, u is the radial component of the velocity, $\hat{r}$ is a unit vector in the direction of the radial r—axis, v is the aximuthal component of the velocity, $\hat{\xi}$ is a unit vector in the direction of the aximuthal ξ—axis, w is the axial component of $\underset{\sim}{u}$, $\hat{z}$ is a unit vector in the direction of the axial z—axis, P is the pressure, S is the solute concentration, θ is the temperature, $R = \beta \Delta C N_g K^2 / \left(V_0^3 \nu H\right)$ is the solutal Rayleigh number, β is the expansion coefficient for solute, $\Delta C \equiv C_0 - C_e$, C_e is the eutectic concentration of the alloy, $N_g = \left(g^2 + \Omega^4 R_0^2\right)^{1/2}$ is the acceleration due to high gravity, $N = 1$ corresponds to normal gravity case while $N > 1$ indicates level of high gravity, g is acceleration due to gravity, R_0 is the perpendicular distance from the center of gravity of the centrifuge basket to the rotation axis [2], $H = K^2 / \left(V_0^2 \Pi_0\right)$ is a non-dimensional parameter $\left(H >> 1 \quad [6]\right)$ representing ratio of R in the liquid zone inside the chimney to that in the mushy zone, Π_0 is a reference value of the permeability $\Pi(\phi)$ of the porous medium, $T = 2\Omega K^2/\left(V_0^2 \nu H\right)$ is the Coriolis parameter (square root of the Taylor number), $A = \beta\Delta C\Omega^2 K^3/\left(V_0^4 \nu H\right)$ is the gradient acceleration parameter, $\hat{\Omega}$ is a unit vector along the rotation axis defined by

$$\hat{\Omega} = \cos\gamma\hat{z} + \sin\gamma\left(\cos\xi\hat{r} - \sin\xi\hat{\xi}\right), \tag{2a}$$

ξ is the azimuthal variable, $\underset{\sim}{G}$ is a position vector defined by

$$\begin{aligned} \underset{\sim}{G} = &\left(r\cos^2\xi\cos^2\gamma + r\sin^2\xi - z\sin\gamma\cos\gamma\cos\xi\right)\hat{r} + \\ &\left(z\sin\gamma\cos\gamma\sin\xi + r\sin\xi\cos\xi - r\sin\xi\cos\xi\cos^2\gamma\right)\hat{\xi} + \\ &\left(z\sin^2\gamma - r\sin\gamma\cos\gamma\cos\xi\right)\hat{z}, \end{aligned} \tag{2b}$$

r is the radial variable, and z is the axial variable. The steady non-dimensional form of the equations for momentum, continuity, temperature and solute concentration in the mushy zone are

$$\underset{\sim}{u}/\Pi = -R\left(\nabla P + S\hat{Z}\right) + T\underset{\sim}{u}\times\hat{\Omega} + AS\underset{\sim}{G}, \tag{3a}$$

$$\nabla\cdot\underset{\sim}{u} = 0, \tag{3b}$$

$$\left(-\frac{\partial}{\partial z} + \underset{\sim}{u}\cdot\nabla\right)\theta = \nabla^2\theta + S_t\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z}\right)\phi, \tag{3c}$$

$$\frac{\partial}{\partial z}\left[(1-\phi)(C_r - S)\right] + \underset{\sim}{u}\cdot\nabla S = 0, \tag{3d}$$

where $S_t = L/(C\Delta T)$ is the Stefan number, C is the specific heat per unit volume, L is the latent heat of solidification per unit volume, $\Delta T \equiv T_L - T_e$, T_L is the local liquidus temperature, $C_r = \left(C_s - C_0\right)/\Delta C$ is a concentration ratio, and C_s is the composition of the solid phase forming the dendrites. The boundary conditions are given in Worster [6] and will not be repeated here. Analysis in Worster [6] indicates that $\theta = S$ in the mushy zone which is also valid here.

In the next section we shall apply asymptotic and scaling analyses for (1)-(3), in the asymptotic limit of sufficiently large R, to determine the strongly nonlinear steady state and weakly non-axisymmetic behavior of convection in the mushy zone and mainly convection features in the chimney for sufficiently large P_r. Extension to arbitrary P_r for either steady or unsteady convection cases with fully non-axisymmetric behavior is planned to be done by the present author in near future.

3. Analysis

Let us designate a to be the radius of a chimney under consideration whose axis coincides with the z-axis. It is assumed that a is small $(a << 1)$. We assume that the orders of magnitude of r, ξ and z are, respectively, a, 1 and 1. Assuming $|\underset{\sim}{u}| \sim 1$ in the mushy zone, then (3a) implies that to the leading terms pressure field is unaffected by the flow velocity and $\theta = S$ is independent of r and ξ. The equations (3c)-(3d) then imply that $w - w_0(z)$, $\phi = \phi_0(z)$ and $\theta = \theta_0(z)$ at most, where w_0, ϕ_0 and θ_0 are the leading order terms for w, ϕ and θ, respectively. It is also assumed that $C_r >> \theta$ in the mushy zone [6]. Due to a comparison between the two non-axial terms in (1b) or (3b), we find that a weak non-axisymmetric flow case is based on the conditions that

$$v \leq 0(u), \quad \frac{\partial F}{\partial \xi} << F, \tag{4}$$

where F can be any dependent variable.

Using (4) in (1b) implies that, to the leading terms, a stream function $\psi(r, \xi, z)$ for the flow in the chimney can be introduced, so that

$$(u, w) = \left(-\frac{1}{r}\frac{\partial \psi}{\partial z}, \frac{1}{r}\frac{\partial \psi}{\partial r} \right). \tag{5}$$

Considering the flow in the chimney described by (1a)-(1d) and assuming $S \sim 1$ and $w >> 1$ [6], (1a) then implies that

$$w \sim HRa^2, u \sim HRa^3, \psi \sim HRa^4. \tag{6}$$

Now, θ_0 is the leading order temperature solution in the mushy zone outside the chimney and its surrounding boundary layer [6]. Designate $\theta_1(r, \xi, z)$ to be the deviation of θ from θ_0. From (1b) and the condition.

$$1/a^2 << HR << 1/a^4, \tag{7}$$

we find $\theta_1 << 1$. Using these results, (1a)-(1d) in the chimney are simplified and, in particular, (1c) reduces to

$$\frac{\partial}{\partial r}\left(r\frac{\partial \theta_1}{\partial r}\right) = \frac{\partial \psi}{\partial r}\theta_0', \tag{8}$$

where prime indicates derivative with respect to z.

Integrating (8) in r from $r = 0$ to $r = a$, averaging in ξ and following [6], we find

$$\theta_1 \sim \psi_a \theta_0' \ln r, \tag{9a}$$

where

$$2\pi\psi_a = \int_0^a 2\pi r w dr, \tag{9b}$$

is the vertical flux in the chimney.

Using (5), we have

$$u \sim -\psi_0'/r \qquad \text{as} \quad r \to a. \tag{10}$$

From (3a), we have

$$u \sim -R\frac{\partial P}{\partial r} + TF_1 + A\theta_0 G_1, \tag{11a}$$

$$V \sim -R\frac{1}{r}\frac{\partial P}{\partial \xi} + TF_2 + A\theta_0 G_2, \tag{11b}$$

where it is assumed that $\Pi = 0(1)$ and

$$F_1\hat{r} + F_2\hat{\xi} + F_3\hat{z} \equiv \underset{\sim}{u} \times \hat{\Omega}, \quad G_1\hat{r} + G_2\hat{\xi} + G_3\hat{z} \equiv \underset{\sim}{G}. \tag{11c}$$

Using (10) in (11a), we find

$$\Delta_r P \sim (\psi'_a/R)\ln a + (T/R)\int F_1 dr + \theta_0(A/R)\int G_1 dr, \tag{12}$$

where $\Delta_r P$ represents the radial pressure difference near the chimney. Using (9a) and (3a), we find that

$$w \sim -R\psi_a \ln a[1-(A/R)G_3] + TF_3, \tag{13}$$

holds near the wall of the chimney.

Using (3d) and the condition $C_r >> 1$, we find

$$C_r \frac{\partial \varphi}{\partial z} \sim \underset{\sim}{u} \cdot \nabla S. \tag{14}$$

Within the mushy zone outside the chimney, (14) and (4) imply

$$C_r \frac{\partial \phi}{\partial z} \sim u\frac{\partial \theta_1}{\partial r} + w\theta'_0. \tag{15}$$

For $u << 1$, (15) reduces to

$$C_r \frac{\partial \phi}{\partial z} \sim w\theta'_0. \tag{16}$$

Near the wall of the chimney, $\theta'_0 \sim 1$, $C_r >> 1$ and $\phi C_r \sim 1$ [6], which together with (16) imply that

$$w \sim 1. \tag{17}$$

Using (6), (10) and (17) in (13), we find

$$HR^2 a^4 \ln a[1-(A/R)G_3] \sim 1 + TF_3. \tag{18}$$

Using (13) and (17) in (9a), we obtain

$$\theta - \theta_0 \sim (1+TF_3)/(R-AG_3). \tag{19}$$

Using (6), (9a) and (18), we find

$$u\frac{\partial \theta_1}{\partial r} \sim (RH)^{1/2}\left[\frac{1+TF_3}{\ln a(R-AG_3)}\right]^{3/2}. \tag{20}$$

Thus $u\frac{\partial \theta_1}{\partial r}$ term in (15) is negligible if the right-hand-side in (20) is small. For

$$RH\left(1+TF_3\right)^3\rangle\rangle\left[1na\left(R-AG_3\right)\right]^3, \tag{21}$$

all the three terms in (15) must balance and, thus,

$$w \sim \left(RHa^3\right)^2. \tag{22}$$

Using (6), (13) and (22), we find

$$H \sim \left(1na/a^2\right)\left[1-AG_3/R\right]+TF_3/\left(R^2a^6H\right). \tag{23}$$

Using (6) and (23) in (12), we obtain

$$\Delta_r P \sim (a\ln a)^2(1-AG_3/R)+TF_3 1_n\, a/\left(R^2a^2H\right)$$

$$+(T/R)\int F_1 dr+\theta_0(A/R)\int G_1 dr. \tag{24}$$

Using (18) and (23) in (19), we find

$$\theta-\theta_0 \sim HR(1na)^3\left(1-AG_3/R\right)^2/\left[H-TF_3/\left(R^2a^6H\right)\right]^2. \tag{25}$$

Vertical and horizontal advection of solute balance here in this regime. Using (3c) and (22)-(23), we find

$$\frac{\partial\phi}{\partial z} \sim \left[1na\left(R-AG_3\right)\right]^3/\left[RH\left(1-TF_3/R^2a^6H^2\right)^3\right]. \tag{26}$$

Using (21)-(22) and (26), we find that the right-hand-side in (26) is small based on the condition (21) if

$$TF_3 << w. \tag{27}$$

However, if (27) does not hold it can not be concluded that the right-hand-side of (26) is small based on (21).

Now, the wall of the chimney can be defined by

$$\phi\left[a(z,\xi),\xi,z\right]=0 \quad \text{at} \quad r=a(z,\xi). \tag{28}$$

Taking derivative with respect to z of (28) implies

$$a'/a \sim \frac{\partial\phi}{\partial z} << 1, \tag{29}$$

provided that (21) and (27) hold. This result indicates that, based on scalings of the type (23)-(25) and the conditions (21) and (27), the wall of the chimney can be in the axial direction to leading order terms.

Applying the scalings of the type (23)-(25), using a Polhausen type method suggested by Lighthill [9] and following [6], we find that the total volume flux $2\pi\psi a$ in the chimney, due to upward flow [6], is given by

$$2\pi\psi_a = 2\pi RHa^4(1+\theta_0)\left[\lambda_1\left(1-\frac{A}{R}z\sin^2\gamma\right)+\lambda_2 a\sin 2\gamma\cos\xi\right]$$

$$\text{for} \quad THa^3\sin\gamma(\sin\xi + a\cos\xi) << 1 \qquad (30a)$$

and

$$2\pi\psi_a = \frac{2\pi aR}{T(\sin\xi + a\cos\xi)\sin\gamma}\left\{\lambda_3[\int_0^z(1+\theta_0)dz-(A/R)\sin^2\gamma\right.$$

$$\left.\int_0^z(1+\theta_0)zdz]+\lambda_4\cdot(a\,A/2R)\sin 2\gamma\cos\xi\int_0^z(1+\theta_0)dz\right\}$$

$$\text{for} \quad THa^3\sin\gamma(\sin\xi + a\cos\xi) >> 1, \qquad (30b)$$

where $\lambda_i (i = 1,2,3,4)$ are order one parameters which can depend, in general, on $a, \gamma, (A/R)$ and z. It should be noted that for $\sin\xi \le 0(a\cos\xi)$, the equality sign in (30b) should be replaced by the symbol '~' since in this situation v is at most of order u which was then related to ψ through (5) in order to arrive at (30b).

To satisfy the mass conservation, downward flow through the mushy zone $w_0(z)$ must be equal to the total upflow through all the chimneys per unit horizontal area. Thus

$$w_0 = -2\pi\psi_a N, \qquad (31)$$

where N is the number density of the chimneys.

4. Results and discussion

In order to present the main results in their simplest explicit form, we shall consider the results for two particular cases, corresponding to $\xi = 90^\circ$ and 0°, which also illustrate the dependence of the results on the non-axisymmetric (azimuthal) variable.

4.2 Case for $\xi = 90^\circ$

In this case the results of the analysis presented in the previous section are simplified and, in particular, (9)-(10), (13) and (30) yield

$$u \sim \psi_a / a, \quad \theta_1 \sim \psi_a \ln a, \quad w \sim \psi_a \left[R \ln a \left(1 - \frac{A}{R} z \sin^2 \gamma \right) + \frac{T}{a} \sin \gamma \right], \tag{32}$$

$$\psi_a \sim \begin{cases} RHa^4 \left(1 - \frac{A}{R} z \sin^2 \gamma \right) & \text{for} \quad THa^3 \sin \gamma << 1, \\ aR\lambda_3 \left(b_1 - \frac{A}{R} \sin^2 \gamma \, b_2 \right) \Big/ T \sin \gamma & \text{for} \quad THa^3 \sin \gamma >> 1, \end{cases} \tag{33}$$

where b_1 and b_2 represent the first two integrals in (30b). These integrals are functions of z and of order one quantities with $0 < b_2 < b_1$. For non-inclined rotation, $\gamma = 0$ and the results are seen to be independent of the rotational constraint to the leading order terms and, in particular, the axial convection and the volume flux in the chimney increases with R. For inclined rotation $(\sin \gamma \neq 0)$ and for the case $THa^3 \sin \gamma << 1$, the flow velocity and the volume flux in the chimney can decrease with increasing A. For some particular moderate rotation limit, where $Az \sin^2 \gamma \to R$ for some critical value $z = z_c$, the leading order terms for the flow velocity and the volume flux approach zero, and the next leading order terms will determine the order of magnitude of these quantities, which turn out to be much less than those for weak rotation case. Here by weak rotation, we mean case where $Az \sin^2 \gamma << R$ and $T \sin \gamma <<$ minimum $\left[a, 1/\left(Ha^3 \right) \right]$. However, for $z > z_c$, (33) indicates that ψ_a has opposite sign to that for $z < z_c$. Furthermore, in such limiting situation, a smaller z_c corresponds to a larger $A\sin^2\gamma/R$ and vice versa a larger z_c corresponds to a smaller $A\sin^2\gamma/R$. For given R and γ, these results clearly indicate the stabilization of convection in the chimney by the rotational constraint due to the gradient acceleration effect. These results also indicate some possible double-cell formation and circulation within the chimney due to the effect of a moderate rotation rate for gradient acceleration effect. The condition (27), which reduces to the form

$$T\sin\gamma << aR1na\left(1-\frac{A}{R}z\sin^2\gamma\right), \qquad (34)$$

for $\xi = 90^\circ$, may not be satisfied for z close enough to z_c, so that it can be concluded that the wall of the chimney may not be vertical. This result can reinforce the possibility of double-cell formation and circulation within the chimney in such situation. Although gradient acceleration effect can be stabilizing for the inclined rotation and $THa^3\sin\gamma << 1$, the Coriolis effect is destabilizing in the sense that the axial convection increases with T.

For inclined rotation and for the case $THa^3\sin\gamma >> 1$, then (33) implies that the flow behavior described above can be predicted about a different critical z-level. Here the Coriolis effect is stabilizing in the sense that ψ_a and the flow velocity decrease with increasing T, while the gradient acceleration effect is stabilizing for $b_1 - b_2\frac{A}{R}\sin^2\gamma \geq 0$ and destabilizing otherwise.

The results presented above generally are in agreement with the experimental finding of Sample and Hellawell [3,4] referred to in the introduction that inclined rotation with some moderate range of values of the rotation rate can have beneficial effect in the sense that convection can be reduced significantly. Here by a moderate Ω we mean about which $A\sin^2\gamma = 0(R)$. For $\gamma = 30^\circ$ and $P_r = 100$, it corresponds to Ω about $6\,rad/\text{sec}$ and with a corresponding acceleration due to high g of about $N_g = 10g$ [1] in a typical centrifuge [2]. Weak rotation then corresponds to rotation rates about which

$A\sin^2\gamma << R$. For $\gamma = 30^\circ$ and $P_r = 100$, it can corresponds to Ω about $0.1 rad/\sec$ and with N_g hardly beyond $1g$ [1].

For strong rotation, $A\sin^2\gamma >> R$, we find from (32)-(33) that the order of magnitudes of the velocity components and the volume flux in the chimney generally increase with $\frac{A}{R}\sin^2\gamma$. For $THa^3 \sin\gamma << 1$, the gradient acceleration and Coriolis effects are both destabilizing. For $THa^3 \sin\gamma >> 1$, the gradient acceleration effect is destabilizing, while the Coriolis effect is stabilizing. However, for sufficiently large Ω the gradient acceleration effect dominates over the Coriolis effect. Hence, rotational effects are destabilizing for sufficiently strong rotation and can lead quickly to channel formation in the mushy zone resulting segregates in the solidified material. These results generally agree with the experimental finding due to Kou et al. [5].

It can be seen from (32)-(33) that the results for the case $\gamma = \gamma_0 \left(\gamma_0 < 180^0\right)$ corresponds to larger order of magnitude for the axial convection in the chimney as compared to the corresponding case for $\gamma = \gamma_0 + 180^\circ$. Also sign of ψ_a for the case $\gamma = \gamma_0$ appears to be opposite to that for $\gamma = \gamma_0 + 180^\circ$ case. These two cases correspond to counter-clockwise or clockwise sense of rotation about the earth's gravity vector.

It is of interest, particularly in relation to the results presented in the previous paragraph, to note that there have been experimental [10] and computational studies [11] whose results generally are in agreement with those described in the previous paragraph of the present study. Experimental studies of the flow of molten tin a horizontal boat under centrifugation were conducted by Ma et al. [10]. Two cases corresponding to two different rotation senses of the centrifuge were considered. The Coriolis effect showed different influences on the flow stability depending on the rotation sense of the centrifuge. Numerical investigation for melt flow in a two-dimensional rectangular boat influenced by both centrifugal acceleration and the Coriolis force was carried out by Tao et al [11]. For sufficiently small Ω, the results were the same as those for zero rotation case. For Ω beyond some critical value, the results became different from those for zero rotation case. If centrifuge rotated counter clockwise, it was found that the strength of the convection in the melt was enhanced by the Coriolis force. However, if the centrifuge rotated clockwise, it was found that the convection effect was reduced by the Coriolis force.

4.2 Case for $\xi = 0°$

In this case also the results of the analysis presented in the previous section are simplified and, in particular, (9)-(10), (13) and (30) yield

$$u \sim \psi_a/a, \quad \theta_1 \sim \psi_a \ln a, \quad w \sim \psi_a\left[R\ln a\left(1-\frac{A}{R}z\sin^2\gamma+\frac{aA}{2R}\sin 2\gamma\right)+T\sin\gamma\right], \quad (35)$$

$$\psi_a \sim \begin{cases} RHa^4\left[\lambda_1\left(1-\frac{A}{R}z\sin^2\gamma\right)+\lambda_2 a\sin 2\gamma\right] & \text{for } THa^4\sin\gamma \ll 1, \\ R\left[\lambda_3\left(b_1-\frac{A}{R}\sin^2\gamma\, b_2\right)+\lambda_4\frac{aA}{2R}\sin 2\gamma\, b_1\right]\Big/(T\sin\gamma) & \text{for } THa^4\sin\gamma \gg 1. \end{cases} \quad (36)$$

comparing (35)-(36) to (32)-(33), we find that the Coriolis effect for $\xi = 0°$ is represented less strongly (of order aT) as compared to one for $\xi = 90°$ (of order T). For non-inclined rotation $(\gamma = 0)$, the results are again independent of rotational effect to the leading order terms. For inclined rotation $(\sin\gamma \neq 0)$ and for the case $THa^4\sin\gamma \ll 1$, the flow velocity and the volume flux in the chimney can decrease with increasing A. For some moderate rotation limit, where $Az\sin^2\gamma \to R\left(1+\frac{\lambda_2}{\lambda_1}a\sin 2\gamma\right)$ for some critical value $z = z_c$, the leading order terms for the flow velocity and the volume flux approach zero, and the next leading order terms will determine the order of magnitude of these quantities, which turn out to be much less than those for weak rotation case. Here by weak rotation, we mean case where $Az\sin^2\gamma \ll R$ and $T\sin\gamma \ll \text{minimum}\left[1, 1/\left(Ha^4\right)\right]$. It is seen that the conditions for z_c, moderate rotation and weak rotation for $\xi = 0^\circ$ are generally different from the corresponding ones for $\xi = 90^\circ$. Hence, the results are generally depend on the azimuthal location in the chimney. For $z > z_c$, (36) indicates that ψ_a has opposite sign to that for $z < z_c$ and the same qualitative results regarding stabilization of convection and possible double-cell formation and circulation within the chimney that were described in sub-section 4.1 for the $\xi = 90^\circ$ case follow here as well. The condition (27), which reduces to the form

$$T \sin\gamma << R \ln a\left(1 - \frac{A}{R} z \sin^2\gamma + \frac{aA}{2R}\sin 2\gamma\right), \tag{37}$$

for $\xi = 0^\circ$, may not be satisfied for $z \to z_c$, so that the wall of the chimney may not also be vertical at $\xi = 0^\circ$. Although gradient acceleration effect can be stabilizing for the inclined rotation of moderate level and $THa^4 \sin\gamma << 1$, the Coriolis effect is destabilizing since axial convection increases with T.

For inclined rotation and for the case $THa^4 \sin\gamma >> 1$, then (36) implies that the flow behavior described above can be predicted about a different critical z-level. Here the Coriolis effect is stabilizing, while the gradient acceleration effect is stabilizing for

$$b_1\left[1 + \left(\frac{\lambda_4}{\lambda_3}\right)\left(\frac{aA}{2R}\right)\sin 2\gamma\right] - b_2\left(\frac{A}{R}\right)\sin^2\gamma \geq 0 \tag{38}$$

and destabilizing otherwise.

For strong rotation, $A \sin^2\gamma >> R$, we find from (35)-(36) that the order of magnitudes of the velocity components and the volume flux generally increase with $\frac{A}{R}\sin^2\gamma$. For $THa^4 \sin\gamma << 1$, the gradient acceleration and Coriolis effects are both destabilizing. For $THa^4 \sin\gamma >> 1$, the gradient acceleration effect is destabilizing, while the Coriolis effect is stabilizing. However, for sufficiently large Ω rotational constraint is destabilizing since gradient acceleration dominates over Coriolis force. Again, the same qualitative results as in the case $\xi = 90^\circ$ for the reversal of the centrifuge sense of rotation follow here as well

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