Measurements of coupled Rayleigh wave propagation in an elastic plate

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**Abstract** 

At frequencies where the thickness of an elastic plate is more than a wavelength thick, the propagation of the two lowest Rayleigh-Lamb modes in an elastic plate can be viewed as the propagation of a Rayleigh surface wave over two weakly coupled, surfacewave waveguides. That is, a Rayleigh wave launched on one surface gradually transfers to the other and then back. It does so in a length we call the beatlength. Measurements of the beatlength for brass plates are reported as a function of frequency and thickness. This phenomenon is readily excited and persists over a wide range of thicknesses and frequencies.

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1

## INTRODUCTION

The two lowest waveguide modes of an elastic plate can be described as the propagation of a Rayleigh surface wave that starts on one surface, but that gradually transfers to the other surface. It then transfers back to the surface from which it started, the whole cycle taking place over a distance we call the beatlength. Provided the product of wave number and thickness is sufficiently large, one can thus view a plate as two weakly coupled, surface-wave waveguides. The objective of this letter is to report measurements of the beatlength in brass plates. This may be one of the first detailed experimental assessments of this phenomenon, though its possibility has been noted by Auld¹, Viktorov² and Brekhovskikh and Goncharov³. Moreover, Li and Thompson⁴ made similar measurements, but did not widely report them.

The longer service life of structures such as pipelines means that they must be monitored more thoroughly, and over a longer period of time, for damage. Using coupled surface waves may be one way to inspect the inner, and therefore not easily accessible, surface of a pipe from its outer surface. Moreover, if the damage were a small surface-breaking fatigue crack, then a surface wave would readily detect the crack because the surface wave would strike the crack broadside, or if the damage were corrosion, then a surface wave would be more severely attenuated by the patch of corrosion at the surface than a bulk wave. The present measurements indicate that the coupled surface waves are readily excited and detected so that they can be used for such nondestructive testing.

The basic idea is presented graphically. Figure 1 shows the dispersion relation for the two lowest, Rayleigh-Lamb modes. The lower curve is that for the lowest antisymmetric mode, while the upper curve is for the lowest symmetric mode. The vertical axis is the normalized angular frequency ( $\omega$  multiplied by  $(h/c_t)$ , where h is one half the thickness of the plate and  $c_t$  is the transverse wave speed). The horizontal axis is the normalized wave number ( $\beta$  multiplied by h). The short dashed line indicates the straight line formed by  $\omega h/c_t$  plotted against  $\beta_r h$ , where  $\beta_r$  is the Rayleigh wave number. The slope is  $(c_r/c_t)$ , where  $c_r$  is the Rayleigh wave speed. In the neighborhood of the intersection points of the long dashed horizontal line with the dispersion curves, the x

particle displacements look roughly as sketched in Fig. 2. Figure 2(a) sketches the symmetric mode, Fig. 2(b) the antisymmetric mode and Fig. 2(c) their algebraic sum. If the symmetric and antisymmetric modes are both excited in phase, then the sum approximates a Rayleigh wave on the upper surface. This is indicated by the solid line in Fig. 2(c). However, each mode propagates with a slightly different wave number,  $\beta_s$  for the symmetric mode and  $\beta_a$  for the antisymmetric mode. After a distance L/2, the two modes move  $\pi$  out of phase. Adding the two modes together at this location approximates a Rayleigh wave on the lower surface. This is indicated by the dashed line in Fig. 2(c). After propagating an additional distance L/2, the modes move back into phase (more accurately  $2\pi$  out of phase) and their sum again approximates a Rayleigh wave, now on the upper surface. In this sense the propagation of the two modes can be viewed as a Rayleigh surface wave coupling from one surface-wave waveguide to another. Figure 1, by means of the vertical dashed lines, indicates the difference  $2\varepsilon$  between the wave numbers  $\beta_s$  and  $\beta_a$ . The difference between  $\beta_s$  and  $\beta_r$  is almost equal to that between  $\beta_r$ and  $\beta_a$ . The beatlength L of the coupled waves is that distance over which the two modes move out of phase by  $2\pi$ , that is,

$$(\beta_a - \beta_s)L = 2\pi \tag{1}$$

or

$$L/h = \pi/h\varepsilon \tag{2}$$

Brekhovskikh and Goncharov<sup>3</sup> give an estimate of  $\varepsilon$  for large  $\beta h$ . However, we use the exact, Rayleigh-Lamb dispersion relation<sup>5</sup> to calculate  $\varepsilon$  and L/h.

## I MEASUREMENTS

The longitudinal and shear wave speeds in brass are experimentally determined using standard pulse-echo techniques with a Panametrics 5800 Pulser/Receiver. The longitudinal wave speed is found using a 15 MHz Panametrics V3619 immersion transducer. The shear wave speed is found using a 20 MHz Panametrics V222-BA-RM normal incidence shear-wave contact transducer.

The view, from above, of the experimental arrangement is sketched in Fig. 3. A focused 3 MHz Panametrics V3680 transducer directs a focused ultrasonic beam at the surface of a 2.38 mm thick brass plate at the Rayleigh angle  $\theta_r$ .

The sound is coupled to the plate by a jacket of water that surrounds the sound beam, as shown in Fig. 3. The plate is otherwise in air, so that no substantial fluid-loading occurs.

The Rayleigh angle is experimentally determined as follows. The normal beam axis to the brass plate is found by adjusting the focused transducer to an angle where the reflection, measured using the pulse-echo mode, is maximum. Then the transducer is rotated to  $\theta_r$ , which angle is calculated (see Section III) from the measured wave speeds. The rotation is computer controlled by a positioning system that has a linear accuracy of about 5  $\mu m$  and a rotational accuracy of about 0.02°.

The beatlength is experimentally determined as follows. With the incident beam at the Rayleigh angle and the focus on the metal plate, the transmitter is moved parallel to the plate surface, toward the broadband receiver, a Deci Model SE 1025-H308 surface contact transducer. The transmitter is moved in 200  $\mu m$  intervals, and at each interval, a 1024-point, 10 MHz data record of the temporal, received sinusoidal signal is recorded by the receiver. The RMS value for each position is calculated and recorded.

The transmitter is moved a distance slightly greater than 3 times the estimated beatlength. This corresponds to collecting 150 RMS values. The RMS values are Fourier transformed using a 4096 FFT. The power spectrum is calculated to determine the peak corresponding to the beatlength.

## II RESULTS

The measured longitudinal wave speed in brass is 4536 m/s. The measured shear wave speed in brass is 2215 m/s. Their ratio is 2.048.

Figure 4 shows a typical measurement (case 4 of Table 1). Figure 4(a) shows the amplitude modulated signal from a 100 mm scan, that is, the transmitter was moved 100 mm, in 200  $\mu m$  intervals, toward the receiver. Figure 4(b) shows the spatial power spectrum of the signal from Fig 4(a). The vertical line marks the theoretically predicted spatial beat frequency calculated using the exact, Rayleigh-Lamb dispersion relation<sup>5</sup>.

Figure 5 plots L/h against  $\omega h/c_i$ . The error bars represent  $\pm 10\%$  of a theoretically predicted value. The transmitter's operating frequency f is varied to produce the 7 different  $\omega h/c_i$  cases.

Table 1 summarizes the results of the experiment conducted on brass. The wave number  $\beta$  must be greater than  $\varepsilon$  to produce a spatially modulated signal. Thus  $\beta/\varepsilon$  is tabulated. The theoretical value for L/h is calculated from the exact dispersion relations. The experimentally determined spatial beat frequency, beatlength and normalized beatlength are tabulated in the last three columns.

## III DISCUSSION

The Rayleigh angle,  $\theta_r$  is defined as

$$\theta_r = \sin^{-1}(c/c_r) \tag{3}$$

where c is the speed of sound in water. A very good approximation for  $c_r$  is

$$c_r/c_t = (0.87 + 1.12\nu)/(1 + \nu)$$
 (4)

where

$$V = \left[ \left( c_t / c_t \right)^2 - 2 \right] / 2 \left[ \left( c_t / c_t \right)^2 - 1 \right]$$
 (5)

From the measured values of  $c_t$  and  $c_t$ ,  $c_r = 2069$  m/s and  $\theta_r = 47.58^\circ$ . This is the angle used in these measurements.

Figure 4 (b) suggests that the beat phenomenon is robust in the spatial frequency domain, and that the measured spatial frequency is in agreement with theory (see Table 1). Figure 5 indicates that the measured values of the spatial beat frequency lie close to or within  $\pm 10\%$  of the theoretically predicted values. Brekovskikh and Goncharov<sup>3</sup> suggest that  $\beta h$  must be large for the coupling phenomena to be clearly observed and their estimate of  $\varepsilon$  assumes this. However, our values of  $\beta h$  ranged from 2 to 5 and yet we observe the coupling phenomena without difficulty. Moreover, the beatlengths indicate that the use of the coupled waves to access an inner surface is realistic. None of the beatlengths is so great that the signals would become too severely attenuated, with distance, to carry information from the far surface.

Initially, we had been concerned that, in addition to the two lowest modes, we might excite higher modes and as a consequence some power would be carried by them and lost to the coupling phenomenon. This appears not to be the case suggesting that when we are incident at the Rayleigh angle we strongly excite only the two lowest modes.

## **ACKNOWLEDGMENTS**

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- <sup>3</sup> L. Brekhovskikh and V. Goncharov, *Mechanics of Continua and Wave Dynamics* (Springer, New York, 1985), pp. 75-81. Note that the first of Eqs (5.16) has a sign error. The plus sign between the two terms should be replaced by a minus sign.
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Figure captions Measurements of coupled Rayleigh wave propagation in an elastic plate by Ti, O'Brien and Harris

Figure 1. The dispersion relation for the two lowest modes of an elastic plate. The upper curve is the symmetric mode and the lower the antisymmetric mode. The beatlength  $L = \pi/\varepsilon$ . Figure 2 indicates the particle displacement in the neighborhood of the intersections of the long dashed horizontal line with the dispersion curves.

Figure 2. A sketch of the x particle displacements corresponding to the neighborhoods of the intersection of the long dashed horizontal line in Fig. 1 with each dispersion curve. The solid lines indicate that the modes are in phase, while the dashed lines indicate that they are  $\pi$  out of phase. (a) symmetric mode; (b) antisymmetric mode; (c) algebraic sum of the two modes.

Figure 3. A sketch of the experimental arrangement viewed from above. The central axis of the focused transducer makes an angle  $\theta_r$  with the vertical. The sound is coupled to the plate by means of a water-filled jacket. The plate is otherwise loaded only by the surrounding air. The focal point is placed at the plate surface and moved toward the stationary receiving transducer.

Figure 4. Representative data, case 4 of Table I. (a) The spatially amplitude-modulated, received signal plotted against position along the plate's surface (500 RMS data points, 100 mm scan distance). (b) The power spectrum of the signal from (a) plotted against the spatial frequency (1/distance). The vertical line indicates the theoretically estimated, spatial beat frequency.

Figure 5. A plot of  $\omega h/c_i$  against L/h. The data points indicated by  $\square$  are the measured results. The data points indicated by  $\spadesuit$  are the theoretically predicted results. A  $\pm 10\%$  error bar is attached to the theoretically estimated results.

Table captions Measurements of coupled Rayleigh wave propagation in an elastic plate by Ti, O'Brien and Harris

Table 1. The measured and calculated values of the important parameters. The temporal frequency f is the transducer operating frequency, f is the spatial frequency, f is the beatlength and f is the half-thickness of the brass plate. Note that case 4 is illustrated in Fig. 4.

Fig. 1 Ti, et al.

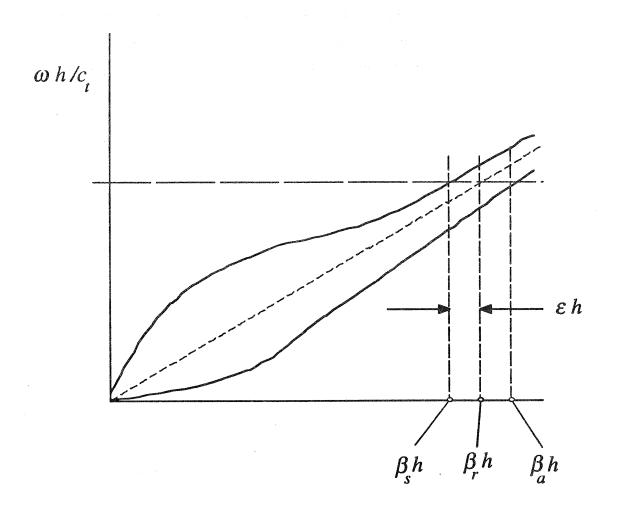


Fig.2 Ti, et al.

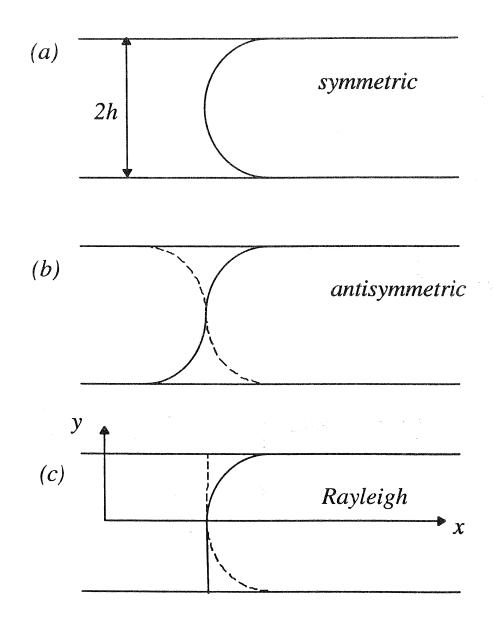


Fig. 3 Ti, et al.

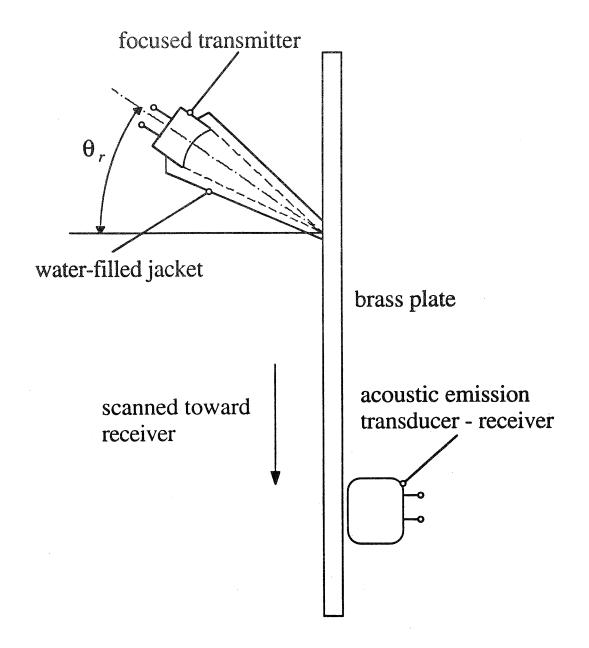


Fig. 4. Ti, et al.

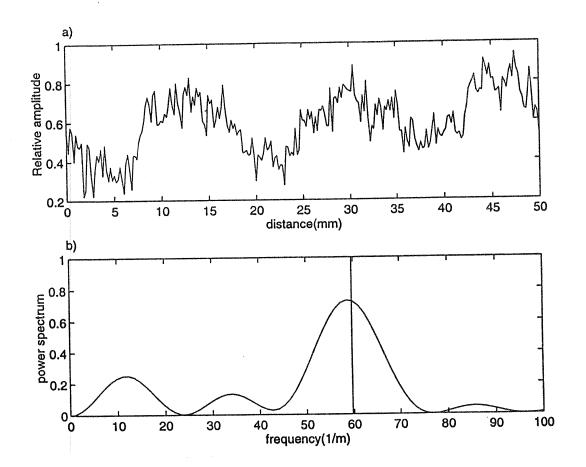


Fig. 5. Ti, et al.

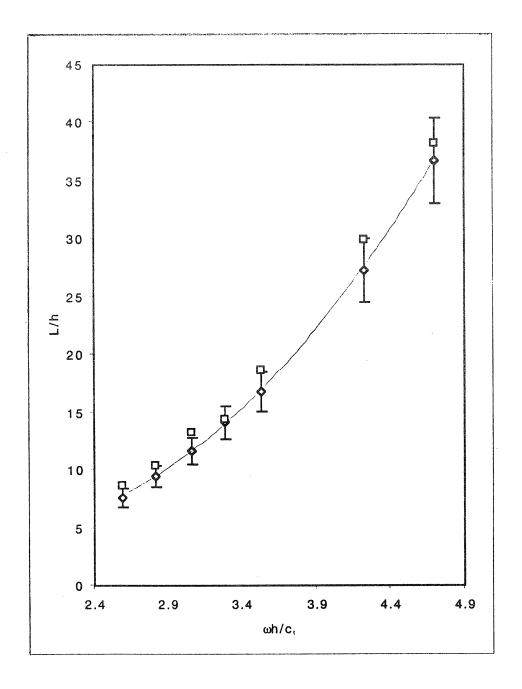


Table 1. Ti, et al.

				theory			experiment		
Case	f(MHz)	ωh/c,	β/ε	f <sub>a</sub> (m <sup>-1</sup> )	L (mm)	L/h	f <sub>a</sub> (m <sup>-1</sup> )	(mm)	L/h
1	0.733	2.59	6.88	109.8	9.1	7.6	97.7	10.2	8.6
2	0.800	2.82	7.64	88.9	11.3	9.4	80.6	12.4	10.4
3	0.866	3.06	13.01	72.3	13.8	11.6	63.5	15.7	13.2
4	0.933	3.29	17.14	59.7	16.7	14.1	58.6	17.1	14.4
5	1.00	3.53	22.11	50.0	20.0	16.8	45.2	22.1	18.6
6	1.20	4.23	43.73	30.8	32.5	27.3	28.1	35.6	29.9
7	1.33	4.70	66.03	22.9	43.7	36.7	22.0	45.5	38.2

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827	Riahi, D. N.	Nonlinear instabilities of shear flows over rough walls	June 1996
828	Weaver, R. L.	Multiple scattering theory for a plate with sprung masses: Mean and mean-square responses	July 1996
829	Moser, R. D., M. M. Rogers, and D. W. Ewing	Self-similarity of time-evolving plane wakes	July 1996
830	Lufrano, J. M., and P. Sofronis	Enhanced hydrogen concentrations ahead of rounded notches and cracks—Competition between plastic strain and hydrostatic constraint	July 1996
831	Riahi, D. N.	Effects of surface corrugation on primary instability modes in wall-bounded shear flows	Aug. 1996
832	Bechel, V. T., and N. R. Sottos	Measuring debond length in the fiber pushout test	Aug. 1996
833	Riahi, D. N.	Effect of centrifugal and Coriolis forces on chimney convection during alloy solidification	Sept. 1996
834	Cermelli, P., and E. Fried	The influence of inertia on configurational forces in a deformable solid— <i>Proceedings of the Royal Society of London A</i> , in press (1996)	Oct. 1996
835	Riahi, D. N.	On the stability of shear flows with combined temporal and spatial imperfections	Oct. 1996
836	Carranza, F. L., B. Fang, and R. B. Haber	An adaptive space–time finite element model for oxidation-driven fracture	Nov. 1996
837	Carranza, F. L., B. Fang, and R. B. Haber	A moving cohesive interface model for fracture in creeping materials	Nov. 1996
838	Balachandar, S., R. Mittal, and F. M. Najjar	Properties of the mean wake recirculation region in two-dimensional bluff body wakes	Dec. 1996
839	Ti, B. W., W. D. O'Brien, Jr., and J. G. Harris	Measurements of coupled Rayleigh wave propagation in an elastic plate	Dec. 1996