Direct resonance analysis and modeling for a turbulent boundary layer over a corrugated surface

by

D. N. Riahi

Department of Theoretical and Applied Mechanics

216 Talbot Laboratory, 104 S. Wright Street

University of Illinois

Urbana, IL 61801

Abstract

Effects of surface corrugation on turbulent flow in a boundary layer are studied using a model based on the direct resonance theory. The induced mean flow due to weakly nonlinear waves, superimposed on the mean and fluctuating components of turbulence, is determined. The mean turbulent flow is affected by the surface corrugation throughout the boundary layer. The corrugated surface generates higher harmonics and affects the streamwise vortices generated by the waves superimposed on turbulence whose mean flow includes secondary induced mean flow component due to the corrugation.

Introduction

The problem investigated in this paper is that of the effects of surface corrugation on a turbulent boundary layer flow over such surface which is determined by making use of the so–called direct resonance theory [1,2]. Gustavsson [1] was the first who proposed the idea of direct resonance where frequency and wave number of the Orr–Sommerfeld equation [3] coincide with the corresponding ones for the vertical vorticity equation in the linear regime. Benney and Gustavsson [2] developed a nonlinear theory for the direct resonance which results in evolution equations different from those of conventional nonlinear stability theory [3]. Jang *et al.* [4] provided a possible theoretical explanation for the appearance of streamwise vortices in a turbulent boundary layer flow over a smooth and flat surface by making use of the direct resonance concept in a weakly nonlinear perturbation analysis around the mean flow velocity.

The present paper extends the work by Jang *et al.* [4] to the case of a turbulent boundary layer flow over a corrugated surface. We have found some interesting results which demonstrate the significance of surface corrugation effects upon turbulence and its structure. In particular, we found that the turbulent mean flow can be modified by the corrugations, the spanwise scale of the streamwise vortices can be increased by the corrugations, and the vortices due to corrugations can dominate over those unaffected by the corrugations for sufficiently large time regardless of the order of magnitude of the surface corrugation.

The present investigation and that due Jang *et al.* [4] are based on weakly nonlinear analysis of wave components, centered about the direct resonance mode, of turbulent flows. As was shown by Jang *et al.* [4], who took into account the wave mode at resonance whose wave numbers and frequency were close to those associated with the most intense waves measured by Morrison and Kronauer [5] near the viscous sub-layer boundary, the low-speed streaks accompanying the streamwise vortices contained in the induced mean flow due to the resonant mode have spanwise spacing comparable to that of experimental finding.

The materials given in this paper are organized as follows. Governing system of equations and the boundary conditions appropriate for direct resonance in a turbulent boundary layer over corrugated surface and the corresponding assumptions are given in the next section followed by the analysis and results, where weakly nonlinear theory is used to derive the induced mean flow, and the associated vortical structures are analysed and the results are presented. The last section provides some concluding results followed by an appendix which contains the governing system and some subsequent equations and boundary conditions.

Governing system

The governing system of equations is briefly described here. For details see Jang *et al.* [4]. We consider an incompressible turbulent flow over a corrugated surface of infinite extent. It is assumed that the order of magnitude of the corrugations about $x_2 = 0$, designated by δ, is small $(\delta << 1)$. It is convenient to use a cartesian system of coordinates x_1, x_2, x_3, with $x_2 = 0$ being the average location of the surface. Following Reynolds and Hassain [6], we split the dependent variables for velocity and pressure field into mean motion, designated by overbar quantitites, wave-like motion, designated by lower-case quantities, and turbulent fluctuation, designated by prime quantitites. The equations for the wave-like motion (A.1) and (A.2), given in the appendix, are obtained by subtracting the mean parts of the Navier-Stokes and continuity equations from the phase averaged Navier-Stokes and continuity equations. An angular bracket is used to denote phase average, where average over a large ensemble of points having the same phase with respect to a reference oscillator is taken [6].

The assumptions for the analysis of (A.1)-(A.2) [4] are the following : i) The effects of the nonlinear terms in u_i are important only intermittently. ii) The streamwise vortices and the low-speed streaks are weakly nonlinear phenomena. iii) The effects of the last term in (A.1) are not important. See [4] for details regarding these assumptions and the corresponding justifications.

We now non-dimensionalize (A.1)-(A.2) by using $\delta^*, \bar{u}_\infty, \delta^*/\bar{u}_\infty$ and $\varphi\bar{u}_\infty^2$ as scales for length, velocity, time and pressure, respectively. Here δ^* is the boundary-layer displacement thickness and $\bar{u}_\infty$ is the free-stream velocity. Making use of the above assumptions, the non-dimensional form of $(A.1)-(A.2)$ can then be written as equations (A.3)-(A.7), given in the appendix, after the pressure term is eliminated. In these equations $R = \delta^* \bar{u}_\infty / v$ is the Reynolds number, ε is a small parameter representing amplitude of the waves, η is the transverse component of vorticity, $\underset{\sim}{u} = (u, v, w)$ is the non-dimensional velocity vector, and $x, y,$ and z are taken to denote non-dimensional streamwise, transverse and spanwise coordinates respectively. In addition, the mean flow is assumed to be parallel and $\underset{\sim}{\bar{u}} = (\bar{u}(y), 0, 0)$ is assumed to be known for the turbulent boundary layer [4].

The boundary conditions for the velocity $\underset{\sim}{u}$ are

$$\varepsilon \underset{\sim}{u} = -\sum_{m=1} \frac{(\delta h)^m}{m!} \frac{\partial^m}{\partial y^m} \left(\underset{\sim}{\bar{u}} + \varepsilon \underset{\sim}{u} \right) \quad \text{at} \quad y = 0, \tag{1a}$$

$$\underset{\sim}{u} = 0 \quad \text{at} \quad y = \infty, \tag{1b}$$

where $h(x, z)$ is surface corrugation shape function which is, in general, a function of the streamwise and spanwise variables. We have assumed that the boundary conditions $\underset{\sim}{u_t}$ and

$\underset{\sim}{u}_b$ for the phase averaged of the total flow velocity vector at the upper and lower boundaries, respectively, are prescribed constants. The conditions (1) are oabtained by the consideration that the mean flow velocity vector assumes the value $\underset{\sim}{u}_b = 0$ and $\underset{\sim}{u}_t = \bar{u}_\infty \hat{i}$ *at* $y = 0$ and $y = \infty$, respectively, since the corrugations on the lower boundary introduce simply surface wave perturbations which contribute to the wave disturbance system only. Here $\hat{i}$ is a unit vector along x—axis. The terms in the right-hand-side of (1a) arise simply by the contributions of the higher order terms in Taylor-series expansion about $y = 0$ of the phase averaged of the total velocity vector which are due to the boundary corrugations.

We shall assume that the surface corrugation shape function $h(x, z)$ is represented by

$$h(x, z) = \sum_{n=-\tilde{N}}^{\tilde{N}} \left[\tilde{A}_n \cos\left(\tilde{\beta}_n z\right) + \tilde{B}_n \sin\left(\tilde{\beta}_n z\right)\right] \exp\left(i\tilde{\alpha}_n x\right), \tag{2}$$

where $i = \sqrt{-1}$, $\tilde{N}$ is a positive integer and the streamwise wave numbers $\tilde{\alpha}_n$ and the spanwise wave numbers $\tilde{\beta}_n$ of the surface corrugation satisfy the properties

$$\tilde{\alpha}_{-n} = -\tilde{\alpha}_n, \quad \tilde{\beta}_{-n} = -\tilde{\beta}_n. \tag{3}$$

The amplitude coefficients $\tilde{A}_n$ and $\tilde{B}_n$ satisfy the conditions

$$\tilde{A}_n^* = \tilde{A}_{-n}, \quad \tilde{B}_n^* = -\tilde{B}_{-n}, \tag{4}$$

where 'asterisk' indicates complex conjugate. The conditions (3)-(4) ensure that the expression (2) for h is real.

We now follow [2, 4] to rescale the governing system (A.3)-(A.7) and (1) since, as was shown by Benney and Gustavsson [2], the following rescaling of the dependent variables should be made, provided the condition for the direct resonance is satisfied which we assume to be the case:

$$v = V, \quad (u, w, \eta) = \varepsilon^{-1/2}(U, W, T). \tag{5}$$

Using (5) in (A.3)-(A.7) and (1), the rescaled form of the governing system is found which is given by (A.8)-(A.13) in the appendix.

In the first part of the next section, we shall first analyse the system (A.8)-(A.13) for a particular form of $h(x, z)$ and then present and discuss the results. The case for general form of $h(x, z)$ will be discussed briefly in the second part of the next section.

Analysis and results

First we consider a particular simple case of the corrugation shape function $h(x, z)$ with

$$\tilde{N} = 1, \quad \tilde{B}_n = 0, \quad \tilde{A}_n = \text{a constant}, \quad \tilde{\alpha}_1 = \delta\hat{\alpha}_1, \quad \hat{\alpha}_1 \leq 0(1), \tag{6a}$$

and for

$$\varepsilon = \delta^2, \tag{6b}$$

since the effects of the surface corrugation on various results can be demonstrated well in a simple and clear manner in this case. Later we will discuss more general cases corresponding to other order of magnitudes for δ and other forms of $h(x, z)$.

As can be seen from (A.12), the case (6b) can correspond to an appropriate minimum order of magnitude for ε, for a given δ, or, equivalently, a maximum order of magnitude for δ, for a given ε, and it was found that the induced mean flow will be affected by the surface corrugation in this case only if the streamwise wave number $\tilde{\alpha}_1$ of the corrugation is zero or $\tilde{\alpha}_1 \leq 0(\delta)$.

Using (2) and (6) in (A.8)-(A.13), suggest expansion for the dependent variables in powers of δ

$$(U, V, W, T) = (U_0, V_0, W_0, T_0) + \delta(U_1, V_1, W_1, T_1) + \cdots \quad (7)$$

Using (7) in (A.8)-(A.13) yield systems to various orders in δ. To zero*th* order in δ, (A.9) yields a linear homogeneous equation for T_0 which is subjected to a non-homogeneous boundary condition at $y = 0$ and zero boundary condition at $y = \infty$. The solution for T_0 can be written in the form

$$T_0 = A_0 \eta_0(y) \sin(\beta_0 z) \exp[i(\alpha_0 x - \omega_0 t)] +$$

$$\tilde{A}_1 \eta_1(y) \sin(\tilde{\beta}_1 z) \exp(i \tilde{\alpha}_1 x) + C.C., \quad (8)$$

where $C.C.$ indicates complex conjugate. Here the coefficient A_0 is an amplitude (a constant), and α_0, β_0 and ω_o are the streamwise wave number, spanwise wave number and complex frequency at which direct resonance is made. Their values are [4]

$$\alpha_0 = 0.0093, \quad \beta_0 = 0.035, \quad \omega_0 = 0.090 - 0.037i. \quad (9)$$

According to Jang *et al.* [4] studies the values of these quantities given in (9) are relatively independent of R at least in the range $1000 \le R \le 15000$ examined by Jang *et al.* [4]. The y—dependent coefficients $\eta_0(y)$ and $\eta_1(y)$ introduced in (8) satisfy the systems (A.14)-(A.15) given in the appendix.

Using (8) in the zero*th* order systems in δ for U and W, derived from (A.10)-(A.13), we find the following results:

$$U_0 = -A_0 \beta_0 \eta_0(y) \cos(\beta_0 z) \exp[i(\alpha_0 x - \omega_0 t)] / (\alpha_0^2 + \beta_0^2) - \tilde{A}_1 \eta_1(y) \cdot$$

$$\cdot \cos(\tilde{\beta}_1 z) \exp(i \tilde{\alpha}_1 x) / \tilde{\beta}_1 + C.C., \quad (10)$$

$$W_0 = i\alpha_0 A_0 \eta_0(y)\sin(\beta_0 z)\exp[i(\alpha_0 x - \omega_0 t)]/(\alpha_0^2 + \beta_0^2) + C.C. \tag{11}$$

To zero*th* order in δ, (A.8) and (A.12)-(A.13) yield the following result

$$V_0 = V_{0m}(y,t)\cos(2\beta_0 z) + h.h. + C.C., \tag{12}$$

where *h.h.* refers to higher harmonics generated due to nonlinearities in (A.8) and to surface corrugation shape function h. The explicit forms of these higher harmonics are not needed here and are not given in this paper. The coefficient function V_{0m} introduced in (12) is the same as the corresponding one given in [4], and we refer the reader to this reference for details regarding V_{0m} and its calculated expression. Jang *et al.* [4] referred to terms in (7), which were independent of x and non–oscillatory in time, as induced mean flow terms. They were interested to determine the mean secondary flow due to the weakly nonlinear contribution of the direct resonance wave on the original mean flow. They identified only vertical component of the induced mean flow, in their examination of their zero*th* order system in $\sqrt{\varepsilon}$, as the first term in the right-hand-side of (12), despite the fact of their result that V_{0m} is a slowly decaying function in time and, consequently, such term becomes insignificant for sufficiently large time. However, if we identify the induced mean flow terms as those which are time averaged or time independent terms, then clearly the second terms in the right-hand-sides of (8) and (10) are the induced mean flow terms in the vertical vorticity and the streamwise velocity components, respectively.

In the present paper we identify the second terms in the right-hand-sides of (8) and (10) and the first term in the right-hand-side of (12) as the mean secondary flow in the vertical vorticity, streamwise velocity and transverse velocity components, respectively. In order to identify the mean secondary flow in the spanwise velocity component, we proceed to examine the δ—order system derived from (A.8)-(A.13). The system for T_1 yield

$$T_1 = T_{1m}(y,t)\sin(2\beta_0 z) + \left|\tilde{A}_1\right|^2 \eta_2(y)\sin\left(2\tilde{\beta}_1 z\right)$$

$$-i\tilde{\alpha}_1 \tilde{A}_1 \eta_3(y)\sin\left(\tilde{\beta}_1 z\right)\exp\left(i\tilde{\alpha}_1 x\right) + h.h. + C.C., \tag{13}$$

where the coefficient function $T_{1m}(y,t)$ is a slowly decaying function of time, whose expression is calculated in [4] and the reader is referred to [4] for details, the coefficient function $\eta_2(y)$ satisfies (A.16) given in the appendix and the coefficient function $\eta_3(y)$ satisfies (A.17) in the appendix. Again, terms *h.h.* in (13) referred to higher harmonics due to the nonlinearities and the surface corrugations and their explicit expressions are not needed here.

The systems for U_1 and W_1 yield

$$U_1 = -T_{1m}(y,t)\cos(2\beta_0 z)/(2\beta_0) -$$

$$\left|\tilde{A}_1\right|^2 \eta_2(y)\cos^2\left(\tilde{\beta}_1 z\right)\Big/\tilde{\beta}_1 + h.h. + C.C., \tag{14}$$

$$W_1 = -\frac{\partial V_{0m}}{\partial y}\sin(2\beta_0 z)/(2\beta_0) + h.h. + C.C., \tag{15}$$

and it is found that such solution is possible only if

$$\hat{\alpha}_1 \leq 0(\delta). \tag{16}$$

The result (16) indicates that the streamwise rate of change of the surface corrugation should, indeed, be quite small in order for the corrugations to be effective in the present direct resonance model.

The results (8) and (10)-(15) indicate that both nonlinearities and surface corrugations can produce induced secondary mean flow. However, for sufficiently large t only the induced mean flow due to corrugations remain significant. The resultant mean flow is then a function of y and z but may depend very weakly on x. Clearly, surface corrugation affects mainly the streamwise component of the mean flow and the associated vertical vorticity. In addition, it is of interest to note that the analysis of the δ^n—order systems, that we have carried out so far for $n=0$ and $n=1$ indicated that maximum order of magnitude of $\hat{\alpha}_1$ is δ^n. Although we have not been able to prove this rigorously for any arbitrary n, we speculate that such result may hold for any n which implies that, in order for the corrugation to be effective, the function h may have to be independent of x.

Based on the results discussed above, it is clear that the projection of the streamlines for the induced mean flow on the (y, z) plane is independent of the surface corrugation effects and is essentially the same as the one reported in [4]. It shows the counter–rotating streamwise vortex structure due to the induced flow and with spanwise wavelength which compares favourably with the experimental value.

In relation to the problem of regeneration of streamwise vortices, such as the one studied by Hamilton *et al.* [7] for turbulent Couette flow with flat boundaries, it should be noted that after sufficiently large t the turbulent mean flow of the present work is modified mainly by the surface corrugation and can be represented by $\bar{u} + U_0\delta$ to the leading terms. If one then employes such a resultant mean flow in the original formulation of the present paper and carry out a weakly nonlinear analysis, similar to that presented in this section, one finds, in particular, that the streamwise vortices, due to the new induced mean flow components in the transverse and spanwise directions, are, indeed, affected by the surface corrugations. In particular, the spanwise wavelength of these vortices can depend strongly on the structure of the surface corrugation.

Let us now discuss briefly other cases with more general δ and h. If $\varepsilon^{n/2} = \delta$ for any finite positive integer n, other than $n=1$ considered in this paper, then perturbation analysis, based on a single-series expansion for the dependent variables in powers of $\delta^{1/n}$, can be carried out similar to that for $n=1$ considered in this paper. However, results due to the corrugation effects will show up first in the analysis of the δ^{n-1}—order system and will be qualitatively similar to those reported in this paper. For the cases where $\varepsilon^{(n+1)/2} << \delta << \varepsilon^{n/2}$, a double–series expansion in powers of $\sqrt{\varepsilon}$ and δ is required for the dependent variables instead of (7). The results of the analysis for the $\varepsilon^{n/2}$—order systems will be independent of the surface corrugation effects, while those due to $\delta^m \varepsilon^{n/2}$—order systems, m a positive integer and n a non–negative integer, will depend on the corrugations and essentially the same types of qualitative results as those presented in this section will follow. More general form of the surface corrugation shape function can also be considered for finite $\tilde{N}$, other than $\tilde{N} = 1$, and the corresponding results are expected to be more general with inclusion of the effects of, for example, many spanwise ware numbers and the amplitude coefficients of the corrugation. But the essential qualitative results regarding the influence of the corrugation on various flow features will be similar to those presented in this paper and, thus, need not separate presentation here.

Some Conclusions

i) Mean secondary flow of turbulent boundary layer can be affected by the surface corrugation.

ii) The streamwise vortex structure of the induced mean flow can be affected by the surface corrugation.

iii) The inner and outer flow zones of turbulent boundary layer can be affected by the surface corrugation.

iv) Secondary harmonics can be generated by the surface corrugation which can interact with each other or with other harmonics through nonlinearities to form higher harmonics or initiate sources to generate new induced mean flow.

v) Direct resonance model admits only streamwise independent or weakly streamwise dependent surface corrugation.

Appendix

The dimensional form of the governing equations for the wave-like motion are given below

$$\left(\frac{\partial}{\partial t}+\bar{u}_j\nabla_j-\nu\nabla_j^2\right)u_i+u_j\nabla_j\,\bar{u}_i=-\frac{1}{\varphi}\nabla_i\,P+$$

$$\nabla_j\left(\overline{u_i u_j}-u_i u_j\right)-\nabla_j\left[\left\langle u_i' u_j'\right\rangle-\overline{u_i' u_j'}\right], \qquad \text{(A.1)}$$

$$\nabla_i\,u_i=0. \qquad \text{(A.2)}$$

Here p is the pressure, u_i is the velocity component in the x_i direction $(i=1,2,3)$, φ is the fluid density and ν is the kinematic viscosity.

The non-dimensional form of the equations for the wave-like motion are given below

$$\left[\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\nabla^2-\frac{d^2\bar{u}}{dy^2}\frac{\partial}{\partial x}-\frac{1}{R}\nabla^4\right]v=$$

$$\varepsilon\left[-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)S_2+\frac{\partial^2 S_1}{\partial x\partial y}+\frac{\partial^2 S_3}{\partial z\partial y}\right], \qquad \text{(A.3)}$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}-\frac{1}{R}\nabla^2\right)\eta+\frac{d\bar{u}}{dy}\frac{\partial v}{\partial z}=\varepsilon\left(\frac{\partial S_3}{\partial x}-\frac{\partial S_1}{\partial z}\right), \qquad \text{(A.4)}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=-\frac{\partial v}{\partial y}, \qquad \text{(A.5)}$$

$$\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=\eta, \qquad \text{(A.6)}$$

where

$$
\left.\begin{aligned}
S_1 &= \frac{\partial}{\partial x}\left(uu-\overline{uu}\right)+\frac{\partial}{\partial y}\left(vu-\overline{vu}\right)+\frac{\partial}{\partial z}\left(wu-\overline{wu}\right),\\
S_2 &= \frac{\partial}{\partial x}\left(uv-\overline{uv}\right)+\frac{\partial}{\partial y}\left(vv-\overline{vv}\right)+\frac{\partial}{\partial z}\left(wv-\overline{wv}\right),\\
S_3 &= \frac{\partial}{\partial x}\left(uw-\overline{uw}\right)+\frac{\partial}{\partial y}\left(vw-\overline{vw}\right)+\frac{\partial}{\partial z}\left(ww-\overline{ww}\right).
\end{aligned}\right\} \qquad \text{(A.7)}
$$

The rescaled form of the governing system (A.3)-(A.7) and (1) is given below

$$
\left[\left\{\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\nabla^2-\frac{d^2\bar{u}}{dy^2}\frac{\partial}{\partial x}-\frac{1}{R}\nabla^4\right\}V-\frac{\partial^3}{\partial x^2 dy}\left(UU-\overline{UU}\right)\right.
$$

$$
\left.-2\frac{\partial^3}{\partial x\partial z\partial y}\left(UW-\overline{UW}\right)-\frac{\partial^3}{\partial z^2\partial y}\left(WW-\overline{WW}\right)\right]+
$$

$$
\sqrt{\varepsilon}\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial y^2}\right)\left\{\frac{\partial}{\partial x}\left(UV-\overline{UV}\right)+\frac{\partial}{\partial z}\left(WV-\overline{WV}\right)\right\}\right]+
$$

$$
\varepsilon\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\frac{\partial}{\partial y}\left(VV-\overline{VV}\right)\right]=0, \qquad \text{(A.8)}
$$

$$
\left[\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)-\frac{1}{R}\nabla^2\right]T+\sqrt{\varepsilon}\left[\frac{d\bar{u}}{dy}\frac{\partial V}{\partial z}+\frac{\partial^2}{\partial x\partial z}\left\{\left(UU-\overline{UU}\right)\right.\right.
$$

$$
\left.\left.-\left(WW-\overline{WW}\right)\right\}+\left(\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial x^2}\right)\left(UW-\overline{UW}\right)\right]+
$$

$$
\varepsilon\left[\frac{\partial^2}{\partial y\partial z}\left(UV-\overline{UV}\right)-\frac{\partial^2}{\partial x\partial y}\left(VW-\overline{VW}\right)\right]=0, \qquad \text{(A.9)}
$$

$$
\frac{\partial U}{\partial x}+\frac{\partial W}{\partial z}+\sqrt{\varepsilon}\frac{\partial V}{\partial y}=0, \qquad \text{(A.10)}
$$

$$
T=\frac{\partial U}{\partial z}-\frac{\partial W}{\partial x}, \qquad \text{(A.11)}
$$

$$\left.\begin{aligned}
&\left(\sqrt{\varepsilon}U, V, W\right) = -\sum_{m=1} \frac{(\delta h)^m}{m!} \frac{\partial^m}{\partial y^m}\left[\left(\bar{u} + \sqrt{\varepsilon}U\right), V, W\right] \\
&V\bar{\varepsilon}T = -\sum_{m=1} \frac{(\delta h)^m}{m!} \frac{\partial^m}{\partial y^m}\left\{\frac{m}{h}\left[\frac{\partial h}{\partial z}\right.\right. \\
&\left.\left.\left(\bar{u} + \sqrt{\varepsilon}U\right) - \sqrt{\varepsilon}\frac{\partial h}{\partial x}W\right] + \sqrt{\varepsilon}T\right\} \quad \text{at} \quad y = 0,
\end{aligned}\right\} \tag{A.12}$$

$$U = V = W = T = 0 \quad \text{at} \quad y = \infty, \tag{A.13}$$

where additional boundary conditions given in (A.12) and (A.13) for T are derived from (1) and (5).

The y—dependent coefficients $\eta_0(y)$ and $\eta_1(y)$ given in (8) satisfy the following systems:

$$\left.\begin{aligned}
&\left[i(\alpha_0\bar{u} - \omega_0) - \frac{1}{R}\left(\frac{d^2}{dy^2} - \alpha_0^2 - \beta_0^2\right)\right]\eta_0 = 0, \\
&\eta_0(0) = \eta_0(\infty) = 0,
\end{aligned}\right\} \tag{A.14}$$

$$\left.\begin{aligned}
&\left[i\tilde{\alpha}_1\bar{u} - \frac{1}{R}\left(\frac{d^2}{dy^2} - \tilde{\alpha}_1^2 - \tilde{\beta}_1^2\right)\right]\eta_1 = 0, \\
&\eta_1(0) = \tilde{\beta}_1\frac{d\bar{u}(0)}{dy}, \quad \eta_1(\infty) = 0.
\end{aligned}\right\} \tag{A.15}$$

The y—dependent coefficient $\eta_2(y)$ given in (13) satisfies the following system:

$$\left.\begin{aligned}
&\left(4\tilde{\beta}_1^2 - \frac{d^2}{dy^2}\right)\eta_2 = 0, \\
&\eta_2 = -\tilde{\beta}_1\frac{d^2\bar{u}}{dy^2} - \frac{d\eta_1}{dy} \quad \text{at} \quad y = 0, \quad \eta_2 = 0 \quad \text{at} \quad y = \infty.
\end{aligned}\right\} \tag{A.16}$$

The y—dependent coefficient $\eta_3(y)$ given in (13) satisfies the following system:

$$\left.\begin{aligned}&\left(\frac{d^2}{dy^2}-\tilde{\beta}_1^2\right)\eta_3=-R\bar{u}\,\eta_1(y),\\&\eta_3=0\quad\text{at}\quad y=0,\infty.\end{aligned}\right\}\qquad\text{(A.17)}$$

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