Contribution to the issue of *Wave Motion* honouring Gerry Wickham

DIFFRACTION BY A SLIT IN AN INFINITE POROUS BARRIER

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Abstract

The diffraction of an acoustic wave by a slit in an infinite, plane, porous barrier is investigated. The barrier is modeled as a rigid material filled with narrow pores, normal to the plane of the barrier, that provide sound damping. However, the barrier is thin enough that sound transmission takes place. An approximate boundary condition is derived that models both these effects. The source point is assumed far from the slit so that the incident spherical wave is locally plane. The slit is wide and the barrier thin, both with respect to wavelength. The principal purpose of the barrier is to reduce the reflected and transmitted sound so that we assume that the flow resistance of the pores is large. The diffracted field is calculated using integral transforms, the Wiener-Hopf technique and asymptotic methods. While a formal solution to the complete problem is given, only the diffracted wavefield is studied, and that only in the farfield of the slit. The diffracted field is the sum of the wavefields produced by the two edges of the slit and an interaction wavefield. The dependence on the barrier parameters of the power removed from the reflected wavefield by the diffraction at the slit is exhibited.

1. Introduction

An effective method of noise reduction is to use sound absorbent barriers in heavily built up areas [1,2]. In most calculations with such a barrier, no sound is assumed to be transmitted through it. However, many barriers are not sufficiently thick to completely prevent sound transmission. The aim of this work is to calculate the scattered wavefield excited by a spherical wave incident to a slit in a barrier exhibiting both absorption and transmission. The source is assumed to be sufficiently far from the slit that its wavefront is locally plane. Throughout we assume that the field is harmonic in time. In this paper we give a formal solution to the complete problem and demonstrate that, in the limit of a rigid barrier, the solution reduces to that calculated by the geometrical theory of diffraction. The asymptotic analysis of the resulting integrals is only carried far enough to permit the calculation of the diffracted wavefields far from the slit as well as the power removed from the reflected wavefield by interference with the diffracted one. We anticipate extending the analysis of these integrals, so that expressions for the wavefield in the slit and close to the barrier can be obtained, and have therefore given more details of the solution than is necessary to calculate only the farfield results.

Scattering from a slit or strip is a well-studied problem in diffraction theory. Asvestas and Kleinman [3, pp. 181-239] summarize and review much of the work done on it. Jones [4, pp. 602-607] and Noble [5, pp. 196-207] discusses diffraction from a slit or strip using the Wiener-Hopf method. We follow their approach very closely. To calculate the diffracted wavefield from the interaction between the edges we assume that the slit is large, with respect to wavelength, and asymptotically approximate several integrals using this assumption. Karp and Keller [6] calculate this interaction term for diffraction from a slit in a perfectly rigid barrier using the geometrical theory of diffraction (this theory also assumes that the slit is large with respect to wavelength). Their work is a limiting case for ours and we show that, in this limit, the power removed from the reflected wavefield by interference with the diffracted one, that we calculate, agrees with theirs. Lastly, the same overall approach used here has been taken by Asghar [7] in his study of scattering from an absorbing strip in a moving fluid.

Rawlins [8], continuing his earlier work on diffraction from an absorbing barrier [9], presented a model of an acoustically penetrable but absorbing half plane barrier, and calculated the diffraction from its edge. He used a boundary condition, having two parameters, that mixes the pressure and its normal derivative at each side of the barrier. The boundary condition produces discontinuities across the barrier in both the pressure and its normal derivative. The magnitudes of the discontinuities are set by the two parameters. They are chosen to give approximately the same reflection and transmission coefficients as those found for the case of a plane wave incident to a thin layer, whose governing equation is a scalar wave equation. Adopting the same form of boundary condition here, we identify the parameters in a different way. Using a simple theory of porous materials described in Morse and Ingard [10, pp. 252-256], our model assumes the barrier is made from a rigid material that is riddled with small pores that are approximately normal to the plane of the barrier. No particle velocity in the barrier parallel to its plane is permitted (a kinematic constraint). We take limited account of the compressibility of the gas in the pores. However, the gas in each pore behaves primarily as an incompressible cylinder, driven back and forth by the harmonic wavefield, but opposed by the frictional force generated at the pore walls (the flow resistance). The barrier is thin enough (with respect to wavelength) that sound is communicated from one side to the other by the motion of the numerous incompressible cylinders. The model is accurate provided $h\Phi/\rho c = O(1)$, where h is half the thickness, Φ the flow resistance and ρc the specific acoustic impedance of the surrounding gas.

There have been other attempts do derive approximate boundary conditions that model thin layers, though, unlike the one discussed here, they have not involved a kinematic constraint. Bovik [11] derives approximate boundary conditions for thin fluid and elastic layers in a differential form, using Taylor expansions as the basis of the approximation procedure. Wickham[12] takes a different approach and reduces the approximate boundary condition to an integral formulation that avoids the need to approximate the boundary conditions pointwise, but imposes instead a condition averaged over the boundary. Though our approach lies somewhat mid-way between the two, we end with a differential form because the boundary conditions are locally reacting. The gas in each pore responds only to the wavefield in its immediate neighborhood.

The final results are presented in the form of the power removed from the reflected wavefield by interference with the diffracted one. To make this calculation we adopt an argument given by Newton[13, pp. 18-20]. Normalized with respect to the reflected intensity times twice the width of the slit, this gives a measure of the effectiveness of the barrier, with the slit, at reducing sound transmission. This term is a function of the slit width and the properties of the barrier.

2. Formulation

We consider the diffraction of an acoustic wave excited by a point source located at (x_0, y_0, z_0) or (r_0, θ_0, z_0) by a slit in the plane $y = 0$ of width $2a$, $-a \le x \le a$. We shall also ask that $0 < \theta_0 \le \pi/2$. The geometry is shown in the Fig. 1. Throughout, the time harmonic factor $e^{-i\omega t}$ is understood. We shall work with the velocity potential σ, where the particle velocity $\mathbf{u}$ is given by $\mathbf{u} = -\nabla\sigma$. The total velocity potential σ_t satisfies

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right)\sigma_t = -\,\delta(x - x_0)\delta(y - y_0)\delta(z - z_0) \qquad (1)$$

where

$$k = k_1 + ik_2 \qquad (2)$$

is the wavenumber. The wavenumber k is assumed to have a small positive imaginary part whenever this is needed to ensure the convergence (regularity) of the Fourier transform integrals defined subsequently. The term k_2 is otherwise set to zero. The boundary conditions satisfied by σ_t on $(-\infty < x \le -a) \cup (a \le x < \infty)$, $y = 0^\pm$ are

$$\pm\frac{\partial}{\partial y}\sigma_t(x, 0^\pm, z) + ik\alpha\sigma_t(x, 0^\pm, z) + ik\beta\sigma_t(x, 0^\mp, z) = 0 \qquad (3)$$

We shall refer to this as the Rawlins [8] boundary condition. The parameters α and β will be identified shortly. The $0^\pm$ means that the field term is to be evaluated as $y \to 0$

through positive or negative values of y. The boundary conditions on $-a < x < a, y = 0^{\pm}$ are

$$\sigma_t(x, 0^+, z) = \sigma_t(x, 0^-, z) \tag{4}$$

and

$$\frac{\partial}{\partial y}\sigma_t(x, 0^+, z) = \frac{\partial}{\partial y}\sigma_t(x, 0^-, z) \tag{5}$$

In addition, we insist that σ_t satisfy the edge condition as $x \to -a^+$, a^-,

$$\sigma_t(x, 0, z) = O(1) \tag{6}$$

and

$$\frac{\partial}{\partial y}\sigma_t(x, 0, z) = O(x^{-1/2}) \tag{7}$$

The plus sign indicates a limit taken from the left and the minus sign one taken from the right.

It is useful to split the total field σ_t in two ways. To discuss the boundary condition we write

$$\sigma_t = \sigma_i + \sigma_s \tag{8}$$

where σ_i is the incident wave and σ_s is the scattered wavefield. We insist that σ_s represent an outward radiating wavefield. However, to discuss the diffraction problem, it is more useful to write σ_t as

$$\sigma_t = \begin{cases} \sigma_i + \sigma_r + \sigma, & y \geq 0^+ \\ \sigma \quad , & y \leq 0^- \end{cases} \tag{9}$$

where σ_i is again the incident wave, σ_r is the wave reflected from a *perfectly rigid* barrier and σ is the scattered wavefield. It is comprised of the diffracted wave, a correction to the reflected wave and a transmitted wave.

3. The Boundary Condition

Figure 2 shows a porous barrier of thickness $2h$ extending to infinity in the $\pm x$ directions. No slit is present. The space is divided into three regions. The regions V^+ and V^- are those above and below the barrier and are occupied by a gas having density ρ and sound speed c. The region V_0 is that occupied by the porous barrier. Following a formulation that is identical to that given in Section I.B of Harris *et al.* [14], the velocity potential σ_s scattered from this barrier is represented by

$$\sigma_s(\mathbf{x}) = -\int_S [\sigma_g(\mathbf{x}',\mathbf{x})\nabla\sigma_t(\mathbf{x}') - \sigma_t(\mathbf{x}')\nabla'\sigma_g(\mathbf{x}',\mathbf{x})]\cdot\hat{\mathbf{n}}\,dS(\mathbf{x}'), \quad \mathbf{x} \in V^+ \cup V^- \tag{10}$$

where σ_t is the total potential given by Eq. (8) and σ_g is the three-dimensional, free-space Green's function. The surface S is comprised of the upper and lower surfaces of the barrier, $\hat{\mathbf{n}}$ is a unit normal vector pointing out of the barrier and ∇' indicates that the gradient is taken with respect to the argument $\mathbf{x}'$. The vector $\mathbf{x}$ indicates the observation point and lies outside the barrier, while the vector $\mathbf{x}'$ indicates the source point and lies on the surface S.

Asking that the unit normal $\hat{\mathbf{n}}$ now point only in the positive y direction, we define the discontinuities

$$[\nabla\sigma_t \cdot \hat{\mathbf{n}}] = \nabla\sigma_t(x,h,z)\cdot\hat{\mathbf{n}} - \nabla\sigma_t(x,-h,z)\cdot\hat{\mathbf{n}} \tag{11}$$

and

$$[\sigma_t] = \sigma_t(x,h,z) - \sigma_t(x,-h,z) \tag{12}$$

These are the sources of the scattered sound as can be seen by noting that, provided the discontinuities in Eqs. (11) and (12) are no larger than $O(1)$, then the integral Eq. (10) can be approximated to $O(kh)$ by evaluating the Green's terms at $y'=0$. This leaves us with

$$\sigma_s(\mathbf{x}) = -\iint_S \left\{\sigma_g(x',0,z',\mathbf{x})[\nabla\sigma_t\cdot\hat{\mathbf{n}}] - [\sigma_t]\nabla'\sigma_g\cdot\hat{\mathbf{n}}\right\}dx'dz' + O(kh) \tag{13}$$

where $\mathbf{x}$ lies outside the volume enclosed by S. Note that we have approximated a function that we know and whose length scale is set by the wavenumber k and not by the wavenumber of the porous material. It is therefore the discontinuities, Eqs. (11) and (12), that Eq. (3) must mimic.

Returning to the Rawlins boundary condition, we note that if we take the limit $kh \to 0^{\pm}$ of the following

$$[\nabla\sigma_t\cdot\hat{\mathbf{n}}] = -ik(\alpha+\beta)[\sigma_t(x,h,z)+\sigma_t(x,-h,z)] \tag{14}$$

and

$$[\sigma_t] = -[ik(\alpha-\beta)]^{-1}[\nabla\sigma_t(x,h,z)\cdot\hat{\mathbf{n}} + \nabla\sigma_t(x,-h,z)\cdot\hat{\mathbf{n}}] \tag{15}$$

then, by adding and subtracting Eqs. (14) and (15), we recover Eq. (3). Accordingly, by estimating the discontinuities, Eqs. (11) and (12), we may use Eqs. (14) and (15) to determine the parameters α and β.

Adapting a simple theory of porous materials given in Morse and Ingard [10, pp. 252-256], the equations governing the acoustical behavior of the porous barrier are

$$i\omega\kappa_p\Omega p = du_2/dy \tag{16}$$

$$dp/dy = i\omega\rho_p\left[1+\left(i\Phi/\rho_p\omega\right)\right]u_2 \tag{17}$$

The particle velocity in the barrier u_2 is restricted to be in the normal direction *only,* the particle velocity in the tangential direction must be zero, and the acoustic pressure in the barrier is p. The parameters of the model are κ_p the compressibility of the gas in the pores, Ω the porosity or fraction of the volume occupied by the pores and hence by the gas, ρ_p the effective density of the gas in the pores and Φ the flow resistance. This last parameter determines the effective sound absorbing properties of the barrier. At the boundaries of the barrier the pressure and normal components of the particle velocity are continuous. No condition is placed on the tangential particle velocity components immediately outside the barrier. Integrating Eqs. (16) and (17), noting that p and u_2 are the total fields in the barrier and using the boundary conditions at the barrier walls gives

$$[\nabla\sigma_t \cdot \hat{\mathbf{n}}] = -\omega^2 \rho \kappa_p \Omega (-i\omega\rho)^{-1} \int_{-h}^{h} p dy \tag{18}$$

and

$$[\sigma_t] = i\omega\rho_p[1 + (i\Phi/\rho_p\omega)](-i\omega\rho)^{-1} \int_{-h}^{h} u_2\, dy \tag{19}$$

The barrier is both thin and absorbing. We wish to capture both these features. Defining $\kappa_e = \kappa_p\Omega$, $\rho_e = \rho_p[1 + (i\Phi/\rho_p\omega)]$ and $c_e = (\rho_e\kappa_e)^{-1/2}$, the effective wavenumber in the barrier is $k_e = \omega/c_e$. We assume that p and u_2 vary slowly enough through the barrier to be approximated accurately by the first two terms of a Taylor series in the scaled thickness variable $k_e h(y/h)$. This assumes that the flow resistance is not so strong as to cause the wavefield in the barrier to very rapidly decay We are therefore able to relate Eqs. (14) and (15) to the porous barrier model by noting that

$$\frac{1}{(-i\omega\rho)2h} \int_{-h}^{h} p dy = [\sigma_t(x,h,z) + \sigma_t(x,-h,z)]/2 + \mathrm{O}(k_e h)^2 \tag{20}$$

and

$$\frac{-1}{2h} \int_{-h}^{h} u_2\, dy = [\nabla\sigma_t(x,h,z) \cdot \hat{\mathbf{n}} + \nabla\sigma_t(x,-h,z) \cdot \hat{\mathbf{n}}]/2 + \mathrm{O}(k_e h)^2 \tag{21}$$

Assuming that $(k_e h)^2$ is small, we find that

$$\alpha + \beta = -i\rho c^2 \kappa_p \Omega kh \tag{22}$$

and

$$\alpha - \beta = i\rho / kh\, \rho_p [1 + (i\Phi/\rho_p\omega)] \tag{23}$$

Note that only $(\alpha - \beta)$ contains the flow resistance term.

To estimate the sizes of these terms assume that κ_p and ρ_p are equal to the compressibility κ and density ρ of the surrounding gas, so that $\kappa_p\rho_p c^2 = 1$. This is not quite the case because ρ_p can be larger than ρ, and κ_p can be the isothermal compressibility rather than the adiabatic compressibility κ. Nevertheless, if the barrier is to absorb the incident sound then $\Phi/\rho\omega$ must be moderately large. Morse and Ingard [10, pp. 252-256] suggest a value as high as 10 at 1000 Hz. We are therefore left with the following estimates

$$\alpha + \beta = -i\Omega kh \tag{24}$$

and

$$(\alpha - \beta)^{-1} = kh\,\Phi/\rho\omega \tag{25}$$

For kh small $(\alpha + \beta)$ is small because $\Omega < 1$, but $(\alpha - \beta)^{-1}$ need not be because, for effective sound absorption, $\Phi/\rho\omega > 1$. Moreover, $|k_e h| = kh(\Omega\Phi/\rho\omega)^{1/2}$. Examining the approximation in Eqs. (20) and (21), we note that, provided $kh\Phi/\rho\omega = \mathrm{O}(1)$ or equivalently $h\Phi/\rho c = \mathrm{O}(1)$, then the error leading to the approximate equivalence between Eqs. (14) and(15), and Eqs. (20) and (21) is $\mathrm{O}(kh)$ throughout. As we continue with the calculation we shall find that some terms are proportional to $(\alpha + \beta)$ and can be dropped, while others contain $(\alpha - \beta)$ or $(\alpha - \beta)^{-1}$ and cannot. We could just set $(\alpha + \beta)$ to zero at this point, but, by carrying it through the calculation the different roles of the barrier thickness and absorption become clearer. Moreover, though we are assuming that $(\alpha - \beta)$ is not small, it can be set to zero to recover the case of a rigid barrier.

The reflection R and transmission T coefficients for the velocity potential using the boundary condition Eq. (3) are given in Rawlins [8] Eq. (38). Neglecting the $(\alpha + \beta)$, they are

$$R(\theta) = \frac{\sin\theta}{[\sin\theta + (\alpha - \beta)]} \tag{26}$$

and

$$T(\theta) = \frac{-2\beta}{[\sin\theta + (\alpha - \beta)]} \tag{27}$$

Note that $\alpha \approx -\beta$ and thus $-2\beta \approx (\alpha - \beta)$. The parameter β clearly controls transmission. For normal incidence, using the previous estimates $T(\pi/2)$ is approximately $-(\rho c/2h\Phi)$ so that the barrier allows weak transmission of sound. The coefficients have no poles on the real θ axis $(0 < \theta < \pi)$.

4. The Wiener-Hopf Problem

We now proceed with the calculation of the diffraction by the slit. The Fourier transform over z and its inverse are defined, respectively, as

$$\psi_t(x, y, \mu) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \sigma_t(x, y, z) e^{-i\mu z} dz \tag{28}$$

and

$$\sigma_t(x, y, z) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \psi_t(x, y, \mu) e^{i\mu z} d\mu \tag{29}$$

with identical definitions for the other potentials σ_i, σ_r and σ. The problem now becomes

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2 \right) \begin{matrix} \psi_i(x, y, \mu) \\ \psi_r(x, y, \mu) \end{matrix} = \frac{e^{-i\mu z_0}}{(2\pi)^{1/2}} \delta(x - x_0) \delta(y \mp y_0), \tag{30}$$

and

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2 \right) \psi(x, y, \mu) = 0, \tag{31}$$

where

$$\gamma = \left(k^2 - \mu^2\right)^{1/2}, \quad \operatorname{Im} \gamma > 0 \tag{32}$$

The boundary conditions at $y = 0$ are

$$\frac{\partial(\psi_i + \psi_r)}{\partial y} = 0 \tag{33}$$

for $(-\infty < x < \infty)$

$$\left(\frac{\partial}{\partial y} + ik\alpha \right) \psi(x, 0^+, \mu) + ik\alpha [\psi_i(x, 0, \mu) + \psi_r(x, 0, \mu)] + ik\beta \psi(x, 0^-, \mu) = 0 \tag{34}$$

$$\left(\frac{\partial}{\partial y} - ik\alpha \right) \psi(x, 0^-, \mu) - ik\beta [\psi_i(x, 0, \mu) + \psi_r(x, 0, \mu) + \psi(x, 0^+, \mu)] = 0 \tag{35}$$

for $(-\infty < x \le -a) \cup (a \le x < \infty)$ and

$$\psi(x, 0^+, \mu) - \psi(x, 0^-, \mu) = -[\psi_i(x, 0, \mu) + \psi_r(x, 0, \mu)] \tag{36}$$

$$\frac{\partial}{\partial y} \psi(x, 0^+, \mu) - \frac{\partial}{\partial y} \psi(x, 0^-, \mu) = 0 \tag{37}$$

for $(-a < x < a)$.

The solution to Eq. (30), giving the incident wave ψ_i, is

$$\psi_i(x, y, \mu) = -\frac{e^{-i\mu z_0}}{(2\pi)^{1/2}4i} H_0^{(1)}(\gamma|\mathbf{r}-\mathbf{r}_0|) \tag{38}$$

where $|\mathbf{r}-\mathbf{r}_0| = \left[(x-x_0)^2 + (y-y_0)^2\right]^{1/2}$. The reflected wave ψ_r has the same form with the source point replaced by its reflected image source $(x_0, -y_0, z_0)$. As indicted in the introduction, we are interested in a situation where the source point is far from the slit. Accordingly, we may use the asymptotic approximation to the Hankel function, assuming that $|\gamma r_0| \to \infty$, to obtain

$$\psi_i = b(\mu)e^{-i\gamma(x\cos\theta_0 + y\sin\theta_0)} \tag{39}$$

and

$$\psi_r = b(\mu)e^{-i\gamma(x\cos\theta_0 - y\sin\theta_0)} \tag{40}$$

where $x_0 = r_0\cos\theta_0$ and $y_0 = r_0\sin\theta_0$ ($0 < \theta_0 \le \pi/2$), and $x = r\cos\theta$ and $|y| = r\sin\theta$ ($0 < \theta < \pi$). The possibility that γ is near 0 can always be avoided. The term $b(\mu)$ is given by

$$b(\mu) = -\frac{e^{-i\mu z_0}}{(2\pi)^{1/2}4i}\left(\frac{2}{\pi\gamma r_0}\right)^{1/2} e^{i(\gamma r_0 - \pi/4)}. \tag{41}$$

Note that by asking that $\operatorname{Im}\gamma > 0$, we have succeeded only in causing the incident and reflected disturbance to be damped in the negative x direction.

We next define the Fourier transform pair

$$\overline{\psi}(v, y, \mu) = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} \psi(x, y, \mu)e^{ivx}dx \tag{42}$$

and

$$\psi(x, y, \mu) = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} \overline{\psi}(v, y, \mu)e^{-ivx}dv \tag{43}$$

with identical definitions for the other wavefield terms. Note the different sign convention in the exponential terms from that used in Eqs. (28) and (29). We split $\overline{\psi}(v, y, \mu)$ as

$$\overline{\psi}(v, y, \mu) = \overline{\psi}_+(v, y, \mu)e^{iva} + \overline{\psi}_-(v, y, \mu)e^{-iva} + \overline{\psi}_1(v, y, \mu), \tag{44}$$

where

$$\overline{\psi}_\pm(v, y, \mu) = \frac{1}{(2\pi)^{1/2}}\int_{a,-\infty}^{\infty,-a} \psi(x, y, \mu)e^{iv(x\mp a)}dx \tag{45}$$

and

$$\overline{\psi}_1(\nu, y, \mu) = \frac{1}{(2\pi)^{1/2}} \int_{-a}^{a} \psi(x, y, \mu) e^{i\nu x} dx \tag{46}$$

In Eq. (45) the first (reading from left to right) set of limits accompany the plus sign and the second the minus sign.

In calculating the partial transforms, Eq. (45), of ψ_i and ψ_r, Eqs. (39) and (40), care needs to be taken as $x \to \infty$. Accordingly, we shall assume that ψ_i and ψ_r are multiplied by $\mathrm{H}(x-a)e^{-\varepsilon(x-a)}$ for $x > 0$ and by $\mathrm{H}(x+a)e^{\varepsilon(x+a)}$ for $x < 0$. Later we shall let $\varepsilon \to 0$. This device allows us to sort out the regions of analyticity. Because ψ_r is that for a rigid rather than a porous barrier, the wavefield ψ will contain a transmitted term that behaves as $e^{-i\gamma(x\cos\theta_0)}e^{-\varepsilon(x-a)}$ for $x > a$, $e^{-i\gamma(x\cos\theta_0)}$for $-a < x < a$ and $e^{-i\gamma(x\cos\theta_0)}e^{\varepsilon(x+a)}$ for $x < -a$. This fact will dominate the regions of analyticity. The term $\overline{\psi}_+(\nu, y, \mu)$ is regular for $\mathrm{Im}\,\nu > [\mathrm{Im}(\gamma\cos\theta_0) - i\varepsilon]$ and $\overline{\psi}_-(\nu, y, \mu)$ for $\mathrm{Im}\,\nu < [\mathrm{Im}(\gamma\cos\theta_0) + i\varepsilon]$. The function $\overline{\psi}_1(\nu, y, \mu)$ is an integral function. We shall end with two Wiener-Hopf problems, one with the common region $\mathrm{Im}(\gamma\cos\theta_0 - i\varepsilon) < \mathrm{Im}\,\nu < \mathrm{Im}(\gamma\cos\theta_0)$ and one with the common region $\mathrm{Im}(\gamma\cos\theta_0) < \mathrm{Im}\,\nu < \mathrm{Im}(\gamma\cos\theta_0 + i\varepsilon)$.

Taking the Fourier transform over x of Eq. (31) and solving the resulting differential equation, so that the radiation condition is satisfied, gives

$$\overline{\psi}(\nu, y, \mu) = \begin{cases} A_1(\nu)e^{-\overline{\gamma}y} & y \geq 0^+ \\ A_2(\nu)e^{\overline{\gamma}y} & y \leq 0^- \end{cases} \tag{47}$$

where

$$\overline{\gamma} = (\nu^2 - \gamma^2)^{1/2}, \quad \mathrm{Re}\,\overline{\gamma} > 0 \tag{48}$$

Transforming boundary conditions Eqs. (34) to (37), and using Eqs. (39) to (41) we get

$$\frac{d\overline{\psi}_-}{dy}(\nu, 0^\pm, \mu) \pm ik[\alpha\overline{\psi}_-(\nu, 0^\pm, \mu) + \beta\overline{\psi}_-(\nu, 0^\mp, \mu)] \pm \frac{2k\frac{\alpha}{\beta}e^{i\gamma\cos\theta_0 a}b(\mu)}{(2\pi)^{1/2}[\nu - (\gamma\cos\theta_0 + i\varepsilon)]} = 0 \tag{49}$$

$$\frac{d\overline{\psi}_+}{dy}(\nu, 0^\pm, \mu) \pm ik[\alpha\overline{\psi}_+(\nu, 0^\pm, \mu) + \beta\overline{\psi}_+(\nu, 0^\mp, \mu)] \mp \frac{2k\frac{\alpha}{\beta}e^{-i\gamma\cos\theta_0 a}b(\mu)}{(2\pi)^{1/2}[\nu - (\gamma\cos\theta_0 - i\varepsilon)]} = 0 \tag{50}$$

$$\overline{\psi}_1(\nu, 0^+, \mu) - \overline{\psi}_1(\nu, 0^-, \mu) = 2iG(\nu)b(\mu), \tag{51}$$

$$\frac{d\overline{\psi}_1}{dy}\left(\nu, 0^+, \mu\right) = \frac{d\overline{\psi}_1}{dy}\left(\nu, 0^-, \mu\right), \tag{52}$$

In Eqs. (49) and (50) the term α goes with the upper sign and β with the lower sign. The term $G(\nu)$ is given by

$$G(\nu) = \frac{e^{i(\nu-\gamma\cos\theta_0)a} - e^{-i(\nu-\gamma\cos\theta_0)a}}{(2\pi)^{1/2}(\nu - \gamma\cos\theta_0)} \tag{53}$$

From Eq. (47) and using the boundary conditions (49) to (52) , we eliminate $d\overline{\psi}_+/dy$ and $d\overline{\psi}_-/dy$ to get

$$\begin{aligned} &e^{i\nu a}\overline{\eta}_+(\nu, 0, \mu)\left[\overline{\gamma} - ik(\alpha - \beta)\right] + \frac{d\overline{\psi}_1}{dy}(\nu, 0, \mu) + e^{-i\nu a}\overline{\eta}_-(\nu, 0, \mu)\left[\overline{\gamma} - ik(\alpha - \beta)\right] \\ &+ i\overline{\gamma}G(\nu)b(\mu) + \frac{kb(\mu)(\alpha - \beta)e^{i(\nu-\gamma\cos\theta_0)a}}{(2\pi)^{1/2}\left[\nu - (\gamma\cos\theta_0 - i\varepsilon)\right]} - \frac{kb(\mu)(\alpha - \beta)e^{-i(\nu-\gamma\cos\theta_0)a}}{(2\pi)^{1/2}\left[\nu - (\gamma\cos\theta_0 + i\varepsilon)\right]} = 0 \end{aligned} \tag{54}$$

where

$$2\overline{\eta}_\pm(\nu, 0, \mu) = \overline{\psi}_\pm\left(\nu, 0^+, \mu\right) - \overline{\psi}_\pm\left(\nu, 0^-, \mu\right) \tag{55}$$

Equation (54) is the Wiener-Hopf functional equation discussed by Noble [5, pp. 196-202]. Note how $(\alpha - \beta)$ enters this equation.

5. The Diffracted Wavefield

The unknown functions $\overline{\eta}_+(\nu, 0, \mu)$ and $\overline{\eta}_-(\nu, 0, \mu)$ have been determined by using the procedure discussed by Noble [5, pp. 196-202]. Several steps in the procedure are given in Appendix A. Terms multiplied by $(\alpha + \beta)$ are $O(kh)$ and are dropped, but terms containing $(\alpha - \beta)$ (that appear in $L(\nu)$ and $L_\pm(\nu)$) need not be small and are retained. Moreover, the procedure includes asymptotically evaluating the integrals appearing in Eqs. (A15) and (A17) for large ξa, where ξ scales with k. That is, we have taken ka to be large. With these approximations the functions are given by

$$\overline{\eta}_\pm(\nu, 0, \mu) = -\frac{ib(\mu)}{(2\pi)^{1/2}S_\pm(\nu)}\left[G_{1,2}(\pm\nu) + T(\pm\nu)C_{1,2}(\gamma)\right], \tag{56}$$

where the subscript 1 accompanies the upper sign and the subscript 2 the lower sign. The $C_{1,2}(\gamma)$ are

$$C_{1,2}(\gamma) = \left[S_+(\gamma)G_{2,1}(\gamma) + T(\gamma)G_{1,2}(\gamma)\right]\left[S_+^2(\gamma) - T^2(\gamma)\right]^{-1} \tag{57}$$

The $G_{1,2}(\nu)$ are

$$G_{1,2}(\nu) = P_{1,2}(\nu)e^{\mp i\gamma\cos\theta_0 a} - R_{1,2}(\nu)e^{\pm i\gamma\cos\theta_0 a} \tag{58}$$

where

$$P_{1,2}(\nu)=\frac{S_+(\nu)-S_\pm(\gamma\cos\theta_0)}{(\nu\mp\gamma\cos\theta_0)}-\frac{ik(\alpha-\beta)}{S_\mp(\gamma\cos\theta_0\mp i\varepsilon)[\nu\mp(\gamma\cos\theta_0\mp i\varepsilon)]} \tag{59}$$

and

$$R_{1,2}(\nu)=\frac{E_0\gamma^{1/2}\{W_0[-i(\gamma\pm\gamma\cos\theta_0)2a]-W_0[-i(\gamma+\nu)2a]\}}{2\pi i\,L_+(\gamma)(\nu\mp\gamma\cos\theta_0)} \tag{60}$$

The first subscript accompanies the upper sign and the second the lower sign . The $T(\nu)$ is

$$T(\nu)=\frac{E_0\gamma^{1/2}W_0[-i(\gamma+\nu)2a]}{2\pi i\,L_+(\gamma)} \tag{61}$$

where

$$E_0=2e^{i\pi/2}\frac{e^{i2\gamma a}}{(2\gamma a)^{1/2}} \tag{62}$$

The definition of $W_0(z)$ needed in this paper is

$$W_0(-iy)=\pi^{1/2}\{1+\pi^{1/2}e^{-iy}(-iy)^{1/2}\operatorname{erfc}[(-iy)^{1/2}]\} \tag{63}$$

where y is real and positive (for our work), and $\operatorname{erfc}(z)$ is the complimentary error function. It is closely related to the Fresnel integral.

6. Farfield Asymptotic Approximations to the Diffracted Wavefield

Substitution of Eqs. (56) and (57) in Eq. (A8) yields

$$A_{1,2}(\nu)=-\operatorname{sgn}(y)\frac{ib(\mu)}{(2\pi)^{1/2}}\left\{\frac{e^{i\nu a}}{S_+(\nu)}[G_1(\nu)+C_1(\gamma)T(\nu)]+\frac{e^{-i\nu a}}{S_-(\nu)}[G_2(-\nu)+C_2(\gamma)T(-\nu)]\right\} + i\operatorname{sgn}(y)G(\nu)b(\mu) \tag{64}$$

where the first subscript corresponds to $y>0$ and the second to $y<0$, and thus the wavefield $\psi(x,y,\mu)$ is calculated. We divide ψ as $\psi=\psi_1(x,y,\mu)+\psi_2(x,y,\mu)$. Each part is given by

$$\psi_1(x,y,\mu)=\operatorname{sgn}(y)\frac{ib(\mu)}{2\pi}\int_{-\infty}^{\infty}d\nu\,e^{-i\nu x}e^{-\bar{\gamma}|y|}\left\{\frac{S_+(\gamma\cos\theta_0)e^{i(\nu-\gamma\cos\theta_0)a}}{S_+(\nu)(\nu-\gamma\cos\theta_0)}-\frac{S_-(\gamma\cos\theta_0)e^{-i(\nu-\gamma\cos\theta_0)a}}{S_-(\nu)(\nu-\gamma\cos\theta_0)}\right.$$
$$\left.+\frac{ik(\alpha-\beta)e^{i(\nu-\gamma\cos\theta_0)a}}{S_+(\nu)S_-(\gamma\cos\theta_0-i\varepsilon)[\nu-(\gamma\cos\theta_0-i\varepsilon)]}-\frac{ik(\alpha-\beta)e^{-i(\nu-\gamma\cos\theta_0)a}}{S_-(\nu)S_+(\gamma\cos\theta_0+i\varepsilon)[\nu-(\gamma\cos\theta_0+i\varepsilon)]}\right\} \tag{65}$$

and

$$\psi_2(x,y,\mu) = \mathrm{sgn}(y)\frac{ib(\mu)}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} e^{-\bar{\gamma}|y|}\left\{\frac{e^{i\nu a}}{S_+(\nu)}\left[R_1(\nu)e^{i\gamma\cos\theta_0 a} - C_1(\gamma)T(\nu)\right]\right.$$
$$\left. + \frac{e^{-i\nu a}}{S_-(\nu)}\left[R_2(-\nu)e^{-i\gamma\cos\theta_0 a} - C_2(\gamma)T(-\nu)\right]\right\} \tag{66}$$

The first term $\psi_1(x, y, \mu)$ represents the field diffracted by the edges at $x = \pm a$, plus the geometrical wavefield not included earlier. Note that there is one pole above the contour and a second below it. These terms are the transmitted wavefield. Once these pole contributions are captured we can let $\varepsilon \to 0$. The second term $\psi_2(x, y, \mu)$ gives the interaction of one edge with the other.

The integrals appearing in Eqs. (65) and (66) can be evaluated asymptotically by using the method of steepest descents. Harris [15] shows that, beyond the Fresnel distance, $k(2a)^2/2\pi$, the exponential phase terms in the braces need not be considered and only the exponential with x and $|y|$ needs to be considered in making the steepest descents calculation. In other words we evaluate the diffracted wavefield at points sufficiently distant from the slit that it has evolved into a cylindrical wavefield (a spheroidal wavefield after the inversion in μ) with a radiation pattern. For that, we put $x = r\cos\theta$ and $|y| = r\sin\theta$, with $0 < \theta < \pi$, and deform the contour with the Sommerfeld transformation $\nu = -\gamma\cos(\tau)$. Hence, for large γr, the diffracted wavefields are

$$\psi_1(x,y,\mu) = \mathrm{sgn}(y)\frac{i\sin\theta\, b(\mu)}{(2\pi\gamma r)^{1/2}}F_1(-\gamma\cos\theta)e^{i(\gamma r - \pi/4)} \tag{67}$$

and

$$\psi_2(x,y,\mu) = \mathrm{sgn}(y)\frac{i\gamma\sin\theta\, b(\mu)}{(2\pi\gamma r)^{1/2}}F_2(-\gamma\cos\theta)e^{i(\gamma r - \pi/4)} \tag{68}$$

The radiation patterns are given by

$$F_1(-\gamma\cos\theta) = -\left\{\frac{S_+(\gamma\cos\theta_0)e^{-i\gamma(\cos\theta+\cos\theta_0)a}}{S_+(-\gamma\cos\theta)(\cos\theta+\cos\theta_0)} - \frac{S_-(\gamma\cos\theta_0)e^{i\gamma(\cos\theta+\cos\theta_0)a}}{S_-(-\gamma\cos\theta)(\cos\theta+\cos\theta_0)}\right.$$
$$\left. + \frac{ik(\alpha-\beta)e^{-i\gamma(\cos\theta+\cos\theta_0)a}}{S_+(-\gamma\cos\theta)S_-(\gamma\cos\theta_0)(\cos\theta+\cos\theta_0)} - \frac{ik(\alpha-\beta)e^{-i\gamma(\cos\theta+\cos\theta_0)a}}{S_-(-\gamma\cos\theta)S_+(\gamma\cos\theta_0)(\cos\theta+\cos\theta_0)}\right\} \tag{69}$$

and

$$F_2(-\gamma\cos\theta)=\left[R_1(-\gamma\cos\theta)e^{-i\gamma\cos\theta_0 a}-C_1(\gamma)T(-\gamma\cos\theta)\right]\left[\frac{e^{-i\gamma\cos\theta a}}{S_+(-\gamma\cos\theta)}\right]$$
$$+\left[R_2(\gamma\cos\theta)e^{i\gamma\cos\theta_0 a}-C_2(\gamma)T(\gamma\cos\theta)\right]\left[\frac{e^{i\gamma\cos\theta a}}{S_+(\gamma\cos\theta)}\right] \quad (70)$$

Next we take the inverse transform over μ using Eqs. (67) and (68) in Eq. (29).

$$\sigma_{d1}(x,y,z)=\mathrm{sgn}(y)\frac{i\sin\theta}{8\pi^2(rr_0)^{1/2}}\int_{-\infty}^{\infty}F_1(-\gamma\cos\theta)\frac{e^{i[\gamma(r+r_0)+\mu(z-z_0)]}}{\gamma}d\mu \quad (71)$$

$$\sigma_{d2}(x,y,z)=\mathrm{sgn}(y)\frac{i\sin\theta}{8\pi^2(rr_0)^{1/2}}\int_{-\infty}^{\infty}F_2(-\gamma\cos\theta)e^{i[\gamma(r+r_0)+\mu(z-z_0)]}d\mu \quad (72)$$

Introduce $r+r_0=r_{12}\sin\phi_{12}$ and $(z-z_0)=r_{12}\cos\phi_{12}$, with $0<\phi_{12}<\pi$. Using the transformation $\mu=k\cos(\tau)$, Eqs. (74) and (75) are approximated as

$$\sigma_{d1}(x,y,z)=\mathrm{sgn}(y)\frac{i\sin\theta}{4\pi(2\pi krr_0r_{12})^{1/2}}F_1(-k\cos\theta\sin\phi_{12})e^{i(kr_{12}-\pi/4)} \quad (73)$$

and

$$\sigma_{d2}(x,y,z)=\mathrm{sgn}(y)\frac{ik\sin\theta\sin\phi_{12}}{4\pi(2\pi krr_0r_{12})^{1/2}}F_2(-k\cos\theta\sin\phi_{12})e^{i(kr_{12}-\pi/4)} \quad (74)$$

where $F_{1,2}(-k\cos\theta\sin\phi_{12})$ are given by Eqs. (69) and (70), respectively.

7. Discussion

We are concerned with understanding how successfully the barrier reduces the sound transmission despite the presence of the slit. Moreover, we want to understand how the absorption of the barrier makes its presence felt. To do so we imagine that source lies on the positive y axis far from the slit and that the reflected sound is measured at a point on the y axis, also far from the slit. We take $b(\mu)=1$ and $\mu=0$, so that $\gamma=k$, in Eqs. (67) and (68). Moreover we set $\theta_0=\pi/2$. The power both carried by the reflected wavefield and by wavefield diffracted from the slit is then calculated in the farfield. The term resulting from their interference is then extracted. This term is the power removed from the reflected wavefield by that scattered by and transmitted through the slit, and by that absorbed by the barrier. It is then normalized by dividing by the reflected intensity times *twice* the width of the slit. This quantity is given by

$$\frac{\Gamma(ka,\alpha-\beta)}{4a}=\frac{1}{2ka}\mathrm{Im}\{[F_1(0)+kF_2(0)][1+(\alpha-\beta)]\} \quad (75)$$

The term $F_1(0)$ is given by

$$F_1(0) = \frac{2ika}{[1+(\alpha-\beta)]} - \left\{1 + \left[\frac{2(\alpha-\beta)}{1+(\alpha-\beta)}\right]^{1/2}\right\} \tag{76}$$

and $F_2(0)$ is given by

$$kF_2(0) = -\frac{e^{i(2ka-\pi/4)}}{(2\pi)^{1/2}(2ka)^{3/2}} \frac{1}{[L_+(k)]^2[1+(\alpha-\beta)]} \tag{77}$$

The interesting behavior is largely confined to the second term in Eq. (75). Setting

$$F(ka,\alpha-\beta) = \frac{1}{2ka}\mathrm{Im}\{kF_2(0)[1+(\alpha-\beta)]\} \tag{78}$$

Fig. 3 shows a plot of F against ka, for values of $\alpha-\beta$ from 0 to 1. The increasing effect of the absorption is apparent. The form of $kF_2(0)$ suggests that the interaction between the edges is affected more strongly by the properties of the barrier than are singly diffracted rays. However, because $2ka$ is large in our approximation, the interaction term is always small.

Note that the case $(\alpha-\beta)=0$ corresponds to a rigid barrier. In this case our expression for $\Gamma(ka,\alpha-\beta)/4a$ corresponds to the transmission cross-section given by Karp and Keller [6, Eq.(16)], namely,

$$\frac{\Gamma(ka,\alpha-\beta)}{4a} = 1 - \frac{\sin(2ka-\pi/4)}{(2\pi)^{1/2}(2ka)^{5/2}} \tag{79}$$

It is of interest to note how the parameters $(\alpha\pm\beta)$ enter the calculation. The parameter $(\alpha+\beta)$ represents essentially the thickness of the barrier and appears in the calculation separated from the other terms, while $(\alpha-\beta)$ represents the absorption of the barrier and is intimately included in the calculation through its role in the terms $L_\pm$ and L. We believe that the Rawlins boundary condition more adequately represents the mechanical response of a thin absorbing barrier than would a boundary condition with $\beta=0$.

While we have not explored our expressions in any completeness, we conjecture that they are more accurate than those calculated using the geometrical theory of diffraction and hence that they permit us to approximate the wavefields both near the slit and near the barrier itself.

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Appendix A The Solution to the Wiener-Hopf Problem

To solve Eq. (54), we make the following factorizations

$$\bar{\gamma} = K_+(v)K_-(v) = \left[e^{-i\pi/4}(v+\gamma)^{1/2}\right]\left[e^{-i\pi/4}(\gamma - v)^{1/2}\right], \tag{A1}$$

and

$$L(v) = \left[1 - ik(\alpha - \beta)/\bar{\gamma}\right] = L_+(v)L_-(v) \ , \tag{A2}$$

where $L_+(v)$ and $K_+(v)$ are regular for $\operatorname{Im} v > -\operatorname{Im}\gamma$, and $L_-(v)$ and $K_-(v)$ are regular for $\operatorname{Im} v < \operatorname{Im}\gamma$. Rawlins gives the exact factorization of Eq. (A2) in both [8] and [9]. The $L_\pm(0)$ are given by

$$L_+(0) = L_-(0) = \left[1 + (\alpha - \beta)k/\gamma\right]^{1/2} \tag{A3}$$

and

$$L_+(\gamma) = L_-(-\gamma) = \left(\frac{1+\cos\chi}{2}\right)^{1/2} \exp\left(\frac{-1}{2\pi}\int_{-\chi}^{\chi}\frac{u}{\sin u}du\right) \tag{A4}$$

where $\sin\chi = -(\alpha - \beta)(k/\gamma)$.

Using Eqs. (A1) and (A2), we rewrite Eq. (54) as

$$\begin{aligned} & e^{iva}\overline{\eta}_+(v,0,\mu) + \left[S_+(v)S_-(v)\right]^{-1}\frac{d\overline{\psi}_1}{dy}(v,0,\mu) + e^{-iva}\overline{\eta}_-(v,0,\mu) + iG(v)b(\mu) \\ & -\frac{kb(\mu)(\alpha-\beta)e^{ia(v-\gamma\cos\theta_0)}}{(2\pi)^{1/2}\left[S_+(v)S_-(v)\right]}\left[\frac{1}{(v-\gamma\cos\theta_0)} - \frac{1}{\left[v-(\cos\theta_0 - i\varepsilon)\right]}\right] \\ & +\frac{kb(\mu)(\alpha-\beta)e^{-ia(v-\gamma\cos\theta_0)}}{(2\pi)^{1/2}\left[S_+(v)S_-(v)\right]}\left[\frac{1}{(v-\gamma\cos\theta_0)} - \frac{1}{\left[v-(\cos\theta_0 + i\varepsilon)\right]}\right] = 0 \end{aligned} \tag{A5}$$

where.

$$S_\pm(v) = K_\pm(v)L_\pm(v) \tag{A6}$$

With the help of Eqs. (44), (47), and (49) to (52), the unknown functions $A_1(v)$ and $A_2(v)$ are given by

$$\begin{aligned} \pm 2A_{1,2}(v) = {} & e^{iva}\left[\overline{\psi}_+(v,0^+,\mu) - \overline{\psi}_+(v,0^-,\mu)\right] + e^{-iva}\left[\overline{\psi}_-(v,0^+,\mu) - \overline{\psi}_-(v,0^-,\mu)\right] + i2G(v)b(\mu) \\ & \pm\frac{ik(\alpha+\beta)}{\bar{\gamma}}\Big\{e^{iva}\left[\overline{\psi}_+(v,0^+,\mu) + \overline{\psi}_+(v,0^-,\mu)\right] + e^{-iva}\left[\overline{\psi}_-(v,0^+,\mu) + \overline{\psi}_-(v,0^-,\mu)\right] \\ & + \frac{i2b(\mu)e^{ia(v-\gamma\cos\theta_0)}}{\left[v-(\gamma\cos\theta_0 - i\varepsilon)\right]} - \frac{i2b(\mu)e^{-ia(v-\gamma\cos\theta_0)}}{\left[v-(\gamma\cos\theta_0 + i\varepsilon)\right]}\Big\} \end{aligned} \tag{A7}$$

The plus sign is used with the subscript 1 and the minus sign with the subscript 2. As we indicated in our discussion of the boundary conditions, terms multiplied by $(\alpha+\beta)$ are $O(kh)$ (after the inverse transforms are taken) and are dropped, but terms containing $(\alpha-\beta)$, that appear in $L_{\pm}(\nu)$, need not be small and are retained. Thus, using this approximation, Eq. (A7) becomes

$$A_1(\nu) = -A_2(\nu) = e^{i\nu a}\,\overline{\eta}_+(\nu,0,\mu) + e^{-i\nu a}\,\overline{\eta}_-(\nu,0,\mu) + iG(\nu)b(\mu) \tag{A8}$$

By multiplying Eq. (A5) by $S_+(\nu)e^{-i\nu a}$ and using the general decomposition theorem, we obtain

$$\begin{aligned}
&S_+(\nu)\overline{\eta}_+(\nu,0,\mu) + \frac{ib(\mu)e^{-i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)}\left[S_+(\nu)-S_+(\gamma\cos\theta_0)\right] + U_+(\nu) + V_+(\nu) \\
&+\frac{kb(\mu)(\alpha-\beta)e^{-i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}S_-(\gamma\cos\theta_0-i\varepsilon)\left[\nu-(\gamma\cos\theta_0-i\varepsilon)\right]} = -\frac{e^{-i\nu a}}{S_-(\nu)}\frac{d\overline{\Psi}_1}{dy}(\nu,0,\mu) \\
&-\frac{ib(\mu)e^{-i\gamma\cos\theta_0 a}S_+(\gamma\cos\theta_0)}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)} - U_-(\nu) - V_-(\nu) + \frac{ib(\mu)e^{-i(2\nu-\gamma\cos\theta_0)a}S_+(\gamma\cos\theta_0)}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)} \\
&+\frac{kb(\mu)(\alpha-\beta)e^{-i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}S_-(\nu)(\nu-\gamma\cos\theta_0)} - \frac{kb(\mu)(\alpha-\beta)e^{-i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}\left[\nu-(\gamma\cos\theta_0-i\varepsilon)\right]}\left[\frac{1}{S_-(\nu)}-\frac{1}{S_-(\gamma\cos\theta_0-i\varepsilon)}\right] \\
&-\frac{kb(\mu)(\alpha-\beta)e^{-i(2\nu-\gamma\cos\theta_0)a}}{(2\pi)^{1/2}S_-(\nu)}\left[\frac{1}{(\nu-\gamma\cos\theta_0)}-\frac{1}{\left[\nu-(\gamma\cos\theta_0+i\varepsilon)\right]}\right]
\end{aligned} \tag{A9}$$

The functions $U_{\pm}(\nu)$ and $V_{\pm}(\nu)$ are the decomposition [17] of

$$U(\nu) = S_+(\nu)\overline{\eta}_-(\nu,0,\mu)e^{-i2\nu a} \tag{A10}$$

and

$$V(\nu) = \frac{-ib(\mu)e^{-i(2\nu-\gamma\cos\theta_0)a}}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)}\left[S_+(\nu)-S_+(\gamma\cos\theta_0)\right] \tag{A11}$$

Similarly, multiplying Eq. (A5) by $S_-(\nu)e^{i\nu a}$, we obtain

$$
\begin{aligned}
&S_-(\nu)\bar{\eta}_-(\nu,0,\mu)-\frac{ib(\mu)e^{i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)}\left[S_-(\nu)-S_-(\gamma\cos\theta_0)\right]+P_-(\nu)-Q_-(\nu)\\
&-\frac{kb(\mu)(\alpha-\beta)e^{i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}S_+(\gamma\cos\theta_0+i\varepsilon)\left[\nu-(\gamma\cos\theta_0+i\varepsilon)\right]}=-\frac{e^{i\nu a}}{S_+(\nu)}\frac{d\bar{\Psi}_1}{dy}(\nu,0,\mu)\\
&+\frac{ib(\mu)e^{i\gamma\cos\theta_0 a}S_-(\gamma\cos\theta_0)}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)}-P_+(\nu)+Q_+(\nu)-\frac{ib(\mu)e^{i(2\nu-\gamma\cos\theta_0)a}S_-(\gamma\cos\theta_0)}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)}\\
&-\frac{kb(\mu)(\alpha-\beta)e^{i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}S_+(\nu)(\nu-\gamma\cos\theta_0)}+\frac{kb(\mu)(\alpha-\beta)e^{i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}\left[\nu-(\gamma\cos\theta_0+i\varepsilon)\right]}\left[\frac{1}{S_+(\nu)}-\frac{1}{S_+(\gamma\cos\theta_0+i\varepsilon)}\right]\\
&+\frac{kb(\mu)(\alpha-\beta)e^{i(2\nu-\gamma\cos\theta_0)a}}{(2\pi)^{1/2}S_+(\nu)}\left[\frac{1}{(\nu-\gamma\cos\theta_0)}-\frac{1}{\left[\nu-(\gamma\cos\theta_0-i\varepsilon)\right]}\right]
\end{aligned}
\tag{A12}
$$

The functions $P_\pm(\nu)$ and $Q_\pm(\nu)$ are the decomposition of

$$
P(\nu)=S_-(\nu)\bar{\eta}_+(\nu,0,\mu)e^{i2\nu a} \tag{A13}
$$

and

$$
Q(\nu)=\frac{-ib(\mu)e^{i(2\nu-\gamma\cos\theta_0)a}}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)}\left[S_-(\nu)-S_-(\gamma\cos\theta_0)\right] \tag{A14}
$$

Let $\bar{f}_1(\nu)$ define a function equal to both sides of Eq. (A9). The left hand side is regular for $\operatorname{Im}\nu>\operatorname{Im}(\gamma\cos\theta_0-i\varepsilon)$ and the right hand side is regular for $\operatorname{Im}\nu<\operatorname{Im}(\gamma\cos\theta_0)$. Therefore, by analytic continuation, the definition of $\bar{f}_1(\nu)$ can be extended throughout the complex ν plane. The form of $\bar{f}_1(\nu)$ is ascertained by examining the asymptotic behavior of the terms in Eq. (A9) as $|\nu|\to\infty$. We note from Rawlins [9] that $|L_\pm(\nu)|=O(1)$ as $|\nu|\to\infty$ and, with the help of the edge conditions, Eqs. (6) and (7), we find that $d\bar{\Psi}_1/dy$ is of $O\left(|\nu|^{-1/2}\right)$ as $|\nu|\to\infty$. Using the extended form of Liouville's theorem, it can be seen that $\bar{f}_1(\nu)$ can only be a constant equal to zero. Hence, from Eq. (A9), we obtain

$$
\begin{aligned}
&S_+(\nu)\bar{\eta}_+^*(\nu,0,\mu)+\frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic}\frac{K_+(\xi)\bar{\eta}_-^*(\xi,0,\mu)e^{-i2\xi a}}{L_-(\xi)(\xi-\nu)}d\xi\\
&-\frac{ib(\mu)e^{-i\gamma\cos\theta_0 a}S_+(\gamma\cos\theta_0)}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)}+\frac{kb(\mu)(\alpha-\beta)e^{-i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}S_-(\gamma\cos\theta_0-i\varepsilon)\left[\nu-(\gamma\cos\theta_0-i\varepsilon)\right]}=0
\end{aligned}
\tag{A15}
$$

where

$$
\bar{\eta}_\pm^*(\nu,0,\mu)=\bar{\eta}_\pm(\nu,0,\mu)\pm\frac{ie^{\mp i\gamma\cos\theta_0 a}b(\mu)}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)} \tag{A16}
$$

Similarly, from the equality of both sides of Eq. (A12 in the strip $\mathrm{Im}(\gamma\cos\theta_0) < \nu < \mathrm{Im}(\gamma\cos\theta_0 + i\varepsilon)$, we have

$$S_-(\nu)\overline{\eta}_-^*(\nu,0,\mu) - \frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id}\frac{K_-(\xi)\overline{\eta}_+^*(\xi,0,\mu)e^{i2\xi a}}{L_+(\xi)(\xi-\nu)}d\xi$$
$$+\frac{ib(\mu)e^{i\gamma\cos\theta_0 a}S_-(\gamma\cos\theta_0)}{(2\pi)^{1/2}(\nu-\gamma\cos\theta_0)} - \frac{kb(\mu)(\alpha-\beta)e^{i\gamma\cos\theta_0 a}}{(2\pi)^{1/2}S_+(\gamma\cos\theta_0+i\varepsilon)[\nu-(\gamma\cos\theta_0+i\varepsilon)]} = 0 \quad \text{(A17)}$$

The contours of the integrals in Eqs. (A15) and (A17) are such that $c < \mathrm{Im}(\gamma\cos\theta_0 - i\varepsilon)$ and $d > \mathrm{Im}(\gamma\cos\theta_0 + i\varepsilon)$. These integrals must be asymptotically approximated, as indicated in Noble [5, pp. 199-202]. Thus we arive at Eqs. (56) through (63).

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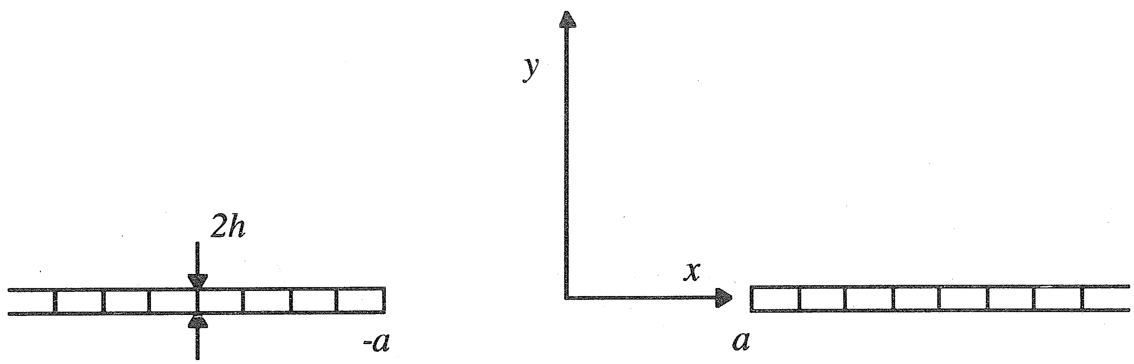

Figure 1 The geometry of the problem

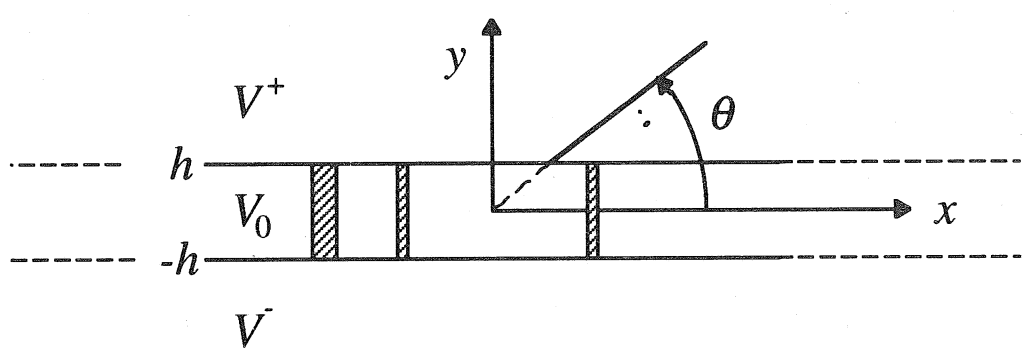

Figure 2 The geometry of the barrier. The hatched regions are intended to suggest the presence of pores in an otherwise rigid material. In practice the pores are unlikely to be so regular.

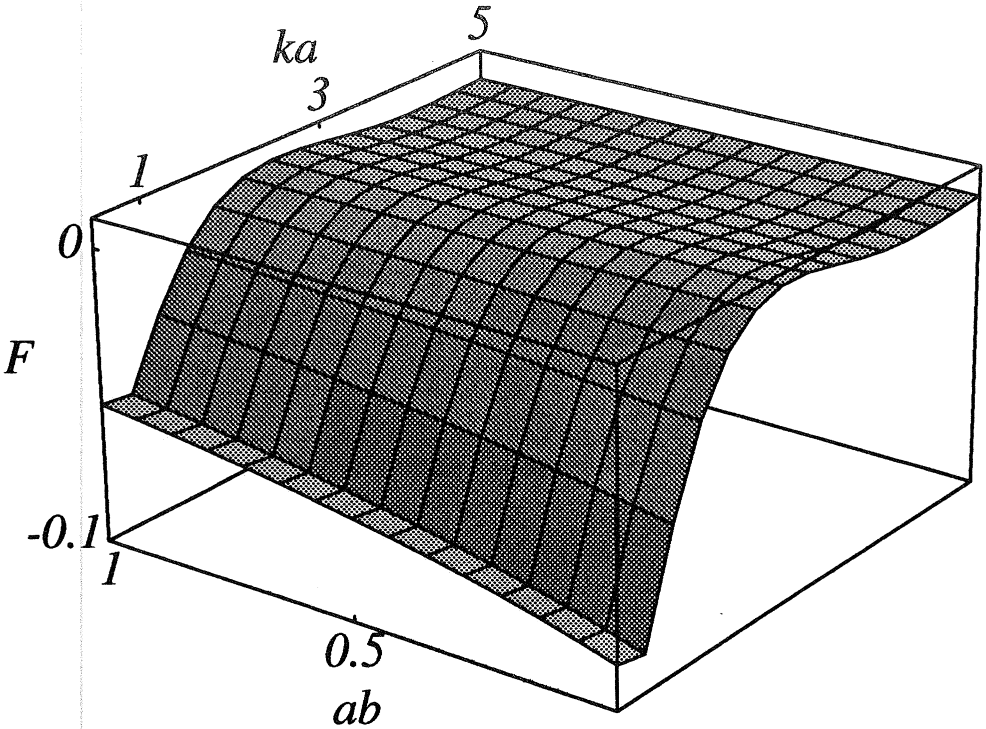

Figure 3 A three-dimensional graph of $F(ka, \alpha - \beta)$ against ka, for values between 0.5 and 10, and against $(\alpha - \beta)$ for values from 0 to 1.0. The $(\alpha - \beta)$ axis is labeled ab.

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