Blow-up in semilinear parabolic equations with weak diffusion

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Finite time blow-up in the semilinear reactive-diffusive parabolic equation $\phi_t = \mu\phi_{xx} + e^\phi$ is examined in the limit of weak diffusion $\mu \ll 1$, for a Cauchy initial value problem with $\phi(x, t = 0) = \phi_i(x)$ in which $\phi_i(x)$ possesses a smooth global maximum. An asymptotic description of the evolution of ϕ is obtained from the initial time through blow-up using singular perturbation techniques. Near blow-up, an exact self-similar focusing structure for ϕ, identical to that previously associated with non-diffusive thermal runaway, is shown to be appropriate. However, in an exponentially small layer close to the blow-up time, the focusing structure must be modified to ensure a uniformly valid solution. This modification uncovers the asymptotically self-similar focusing structure previously recognized for blow-up in equations of the form $\phi_t = \phi_{xx} + e^\phi$. In contrast to previous studies, however, the structure arises here as a natural consequence of removing the non-uniformity in the expansions which occurs exponentially close to blow-up when the effects of diffusion have to be reinstated. Identical weak-diffusion limit asymptotics can be applied to a variety of semilinear or quasilinear parabolic equations that exhibit finite time blow-up in order to reveal the associated focusing structure.

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1. Introduction

A number of reactive-diffusive processes which undergo rapid transient behaviour in short time intervals are modelled by semilinear or quasilinear parabolic equations which exhibit singular behaviour or blow-up in a finite time interval. Examples are the heterogeneous reaction-diffusion equation,

$$\phi_t = \phi_{xx} + e^\phi, \tag{1.1}$$

with an exponential rate law (Kassoy & Poland 1980, Kapila 1980), or alternatively,

$$\phi_t = \phi_{xx} + \phi^p, \quad p > 1, \tag{1.2}$$

with a power rate law, and the porous-medium reaction-diffusion equations,

$$\phi_t = (|\phi_x|^\sigma \phi_x)_x + e^u, \quad \sigma > 0, \tag{1.3}$$

(Budd, Dold & Galaktionov 1993), or

$$\phi_t = (\phi_x^\alpha \phi_x)_x + u^p, \quad \alpha > 0, \quad p > 1, \tag{1.4}$$

(Galaktionov & Posashkov 1991). A knowledge of the spatial structure of blow-up is a pre-requisite to understanding the physical dynamics associated with such systems which lead to the rapid transient behaviour. Moreover, this knowledge can be useful in developing very accurate numerical algorithms for computing blow-up profiles (Berger & Kohn 1988, Budd, Huang & Russell 1996). Determining both the nature and spatial structure of blow-up in many semilinear and quasilinear parabolic equations is in general a difficult task, and is a topic that has received much attention by many researchers, as described in Bebernes & Eberly (1989). In the present paper we explore the idea of using weak-diffusion limit asymptotics to derive analytical solutions to reaction-diffusion equations of the form (1.1) – (1.4), from which both the spatial structure and time of blow-up can be obtained formally. In particular, analytical solutions will be obtained for equation (1.1), although solutions for equations (1.2) – (1.4) follow along analogous lines.

Equation (1.1) describes the induction stage of a reactive-diffusive evolution in a combustible atmosphere with a one-step Arrhenius reaction in which diffusion and reaction time scales are initially comparable. Kapila (1980) and Kassoy & Poland (1980) independently proposed that the spatial structure of blow-up, or thermal runaway, could be described in terms of the exactly self-similar focusing variable

$$s = \frac{x}{\sqrt{t_I - t}}, \quad s \sim O(1) \text{ and fixed as } t \to t_I, \tag{1.5}$$

where

$$\phi(x,t) \sim -\ln(t_I - t) + G(s) + \cdots, \text{ as } t \to t_I, \tag{1.6}$$

with t_I the blow-up time and G an order one function of s. This choice of focusing variable corresponds to a situation where diffusion and reaction would have equal importance through thermal runaway. In a paper that exhibited remarkable insight and ingenuity, Dold (1985) suggested that the influence of diffusion should diminish relative to reaction through blow-up and proposed that the proper variable to describe the spatial structure of thermal runaway should have the alternate, asymptotically self-similar form,

$$\eta = \frac{x}{\sqrt{t_I - t}(\alpha + \ln(t_I - t))}, \; \eta \sim O(1) \text{ and fixed as } t \to t_I, \tag{1.7}$$

with α a constant. Extensive research followed, e.g., Bebernes & Troy (1987), Bebernes & Kassoy (1988) and Dold (1991), which established rigorously that the focusing variable (1.5) could not yield a self-consistent asymptotic description of blow-up in equations of the form (1.1) or (1.2). In this context and of relevance to the present analysis, Dold (1991) also investigated the possibility of nearly uniform solutions conforming to the variable (1.5) near to blow-up. In terms of the variable (1.7), Dold (1991) has additionally derived high-order solutions for the spatial structure of ϕ near blow-up in equation (1.1) using a coordinate-perturbation analysis. Finally, a rigorous justification that (1.7) is indeed the correct focusing variable for describing blow-up in equations of the form (1.1) and (1.2) was given by Bebernes & Bricher (1992), Filippas & Kohn (1992) and Herrero & Velazquez (1993) based on center manifold techniques.

All these studies, however, concentrate on regimes asymptotically close to blow-up, and to date no analytical solutions of equation (1.1) have been presented from which the focusing structure (1.7) can be recovered naturally from a formal asymptotic solution from the initial time. In this paper such an analysis is presented, which uses the limit of weak diffusion to obtain asymptotic descriptions of the evolution of $\phi(x,t)$ in (1.1) from the initial time through blow-up for a Cauchy initial value problem with $\phi(x,0) = \phi_i(x)$. We reiterate that the weak diffusion limit asymptotics applied here to (1.1) can be used for a variety of semilinear or quasilinear parabolic equations that exhibit finite time blow-up in order to reveal the focusing structure associated with the blow-up.

2. A reaction-diffusion equation with an exponential rate-law

Equation (1.1) describes the induction stage of a reactive-diffusive thermal explosion in which the temperature T of the mixture has the form,

$$T \sim 1 + \epsilon\phi, \tag{2.1}$$

where ϵ is the large inverse activation energy for the one-step Arrhenius reaction and ϕ satisfies the semilinear parabolic equation,

$$\phi_t = \mu\phi_{xx} + e^{\phi}. \tag{2.2}$$

The parameter μ is the ratio of the reaction time t_r to diffusion time t_d,

$$\mu = t_r/t_d \tag{2.3}$$

where for a given reference length scale l_r,

$$t_d = l_r^2/\kappa \tag{2.4}$$

where κ is the thermal diffusivity of the mixture. The reaction time t_r is defined by,

$$t_r = \frac{\epsilon}{Q} t_{ch}, \tag{2.5}$$

where t_{ch} is the standard chemical time of the mixture and Q the heat release factor. When diffusion and reaction time-scales are of comparable magnitude initially, $\mu = 1$. On the other hand, when the time-scale for diffusion is much longer than that associated with the reaction scale, as occurs for spatially slowly-varying initial disturbances, the role of diffusion is weaker than that of reaction initially and $\mu \ll 1$.

We consider Cauchy problems with initial data,

$$\phi(x,0) = \phi_i(x) \tag{2.6}$$

where $\phi_i(x)$ is assumed to have a smooth global maximum $\phi_i = 0$ at $x = 0$.

3. Asymptotic solution for negligible diffusion

When μ is vanishingly small, $\mu \to 0$, i.e. for diffusion times t_d significantly longer than reaction times t_r, equation (2.2) describes thermal runaway in an unconfined, constant pressure, non-diffusive system. The solution of (2.2) for $\mu = 0$, subject to (2.6) is

$$\phi = -\ln\left(e^{-\phi_i(x)} - t\right). \tag{3.1}$$

It is easily established that blow-up first occurs at $x = 0$ as $t \to 1$, with the spatial structure of blow-up given by

$$\phi = -\ln\tau + \ln(1 - \frac{1}{2}\left[\phi_i''(0)\right] s) + o(1) \tag{3.2}$$

(Dold 1988, Jackson, Kapila & Stewart 1989), where $\tau = 1 - t$ and s is the exact self-similar focusing variable

$$s = \frac{x^2}{\tau} \sim O(1) \text{ and fixed as } \tau \to 0. \tag{3.3}$$

Thus the spatial extent of the blow-up layer shrinks like $x = O(\sqrt{\tau})$ as $\tau \to 0$.

4. Asymptotic solution for weak diffusion

When diffusion is weak but finite the assumption $\mu \ll 1$ is appropriate. However, in order to ensure the validity of (2.2), the ordered limit

$$1 \gg \mu \gg \epsilon \tag{4.1}$$

is assumed. Anticipating that the time of blow-up t_I will be modified by the inclusion of the diffusive term, the transformation

$$\tau = t_I - t, \quad t_I \sim t_{I0} + \mu t_{I1} + \cdots, \tag{4.2}$$

is made. At this stage it is also convenient to introduce the transformation

$$\phi = -\ln\psi, \tag{4.3}$$

whereupon (2.2) becomes

$$\psi_\tau = \mu\left[-\psi_{,xx} + \frac{\psi_{,x}^2}{\psi}\right] + 1. \tag{4.4}$$

An expansion for ψ should now take the regular form

$$\psi(x, \tau; \mu) = \psi_0(x, \tau) + \mu\psi_1(x, \tau) + \cdots, \tag{4.5}$$

where ψ_0 and ψ_1 are subject to the initial conditions,

$$\psi_{i0}(x, t_{I0}) = e^{-\phi_i(x)}, \quad \psi_{i1}(x, t_{I0}) = -t_{I1}\psi_{i0,\tau}(x, t_{I0}). \tag{4.6}$$

(*a*) *Leading-order equation*

By substituting (4.5) into (4.4), it can be shown that ψ_0 satisfies the equation

$$\psi_{0,\tau} = 1. \tag{4.7}$$

Its solution, subject to the first of (4.6), is

$$\psi_0 = A(x) + \tau, \quad A(x) = e^{-\phi_i(x)} - t_{I0}. \tag{4.8}$$

(*b*) *First-order equation*

Similarly, ψ_1 can be shown to satisfy the equation,

$$\psi_{1,\tau} = -\psi_{0,xx} + \frac{\psi_{0,x}^2}{\psi_0}. \tag{4.9}$$

Its solution, subject to the second of (4.6), is

$$\psi_1 = A''(x)(t_{I0} - \tau) + A'^2(x)\left[\ln(A(x)+\tau) - \ln(A(x)+t_{10})\right] - t_{I1} \qquad (4.10)$$

(c) Analysis of outer layer blow-up structure

For a singularity to develop in ϕ in a finite time, it is clear that as both $\tau \to 0$ and $x \to 0$, $\psi \to 0$. Now as $x \to 0$,

$$e^{-\phi_i(x)} \sim 1 + ax^2 + bx^4, \quad a = -\frac{1}{2}\phi_i''(0), \quad b = -\frac{1}{24}\phi_i''''(0) + \frac{1}{8}\phi_i''^2(0), \qquad (4.11)$$

so that as $\tau \to 0$,

$$\psi_0 \sim 1 - t_{I0} + ax^2 + bx^4 + \tau, \qquad (4.12)$$

and

$$\psi_1 \sim (2a + 12bx^2)\left[1 - \tau\right] + 4a^2x^2\ln(ax^2+\tau) - t_{I1}. \qquad (4.13)$$

The requirement that $\psi \to 0$ thus implies

$$t_{I0} = 1, \quad t_{I1} = 2a. \qquad (4.14)$$

The form of equations (4.12) and (4.13) suggests, as before, the introduction of the exact self-similar focusing variable,

$$s = \frac{x}{\sqrt{\tau}}, \; s > 0 \text{ and fixed as } \tau \to 0. \qquad (4.15)$$

Then, ψ_0 (4.12) can be shown to have the behaviour

$$\psi_0 = \tau(1 + as^2) + O(\tau^2). \qquad (4.16)$$

Similarly, the expression (4.13) for ψ_1 can be shown to have the form,

$$\psi_1 = \tau\left[4a^2s^2\ln\tau + 4a^2s^2\ln(1+as^2) - 2a + 12bs^2\right] + O(\tau^2\ln\tau). \qquad (4.17)$$

Combining (4.16) and (4.17),

$$\psi \sim \tau\left(1 + as^2 + \mu\left[4a^2s^2\ln\tau + 4a^2s^2\ln(1+as^2) - 2a + 12bs^2\right]\right), \qquad (4.18)$$

and upon using (4.3), an expression for the behaviour of ϕ near blow-up can be obtained as,

$$\phi \sim -\ln\tau - \ln(1+as^2) - \frac{\mu}{1+as^2}\left(4a^2s^2\ln\tau + 4a^2s^2\ln(1+as^2) - 2a + 12bs^2\right) + \cdots. \qquad (4.19)$$

As $\tau \to 0$, at fixed s, this expression indicates a non-uniformity occurs in the expansions when

$$\ln\tau = O(\mu^{-1}), \qquad (4.20)$$

suggesting the presence of an exponentially small inner layer to account for the non-uniformity before the end of the induction zone is reached, which occurs when $\ln\tau = O(\epsilon^{-1})$. The presence of a non-uniformity in regions exponentially close to blow-up, but still within regions in which the induction zone expansion (2.1) is valid, has previously been found in problems associated with reactive-acoustic thermal runaway (Blythe & Crighton 1989, Short 1996), but to the authors' knowledge, has not be seen before in a problems associated with reactive-diffusive thermal-runaway. As observed below, it plays an critical role in modifying the nature of the spatial focusing of ϕ near blow-up.

5. Analysis of the inner layer blow-up structure

To analyse the structure of the exponentially small inner layer blow-up structure, it is appropriate to define a new time variable,

$$\sigma = -\mu \ln \tau, \quad (\tau = e^{-\sigma/\mu}) \tag{5.1}$$

In (s, σ) variables, equation (2.2) becomes

$$\phi_\sigma = -\frac{s\phi_s}{2\mu} + \phi_{ss} + \frac{1}{\mu} e^{\phi - \sigma/\mu}. \tag{5.2}$$

The form of the outer behaviour (4.19) suggests an expansion for ϕ in the form

$$\phi = \frac{\sigma}{\mu} + \Theta(s, \sigma; \mu), \tag{5.3}$$

where

$$\Theta(s, \sigma; \mu) = \Theta_0(s, \sigma) + \mu\Theta_1(s, \sigma) + \mu^2\Theta_2(s, \sigma) + \cdots. \tag{5.4}$$

The solution of the system (5.2)–(5.4) both must match with the outer layer behaviour (4.19) as $\sigma \to 0$, and satisfy conditions of spatial regularity at $s = 0$.

(a) *Leading-order behaviour*

By substituting (5.3) & (5.4) in (5.2) and collecting terms of $O(\mu^{-1})$, Θ_0 can be shown to satisfy the equation

$$\frac{s}{2}\Theta_{0,s} + 1 = e^{\Theta_0}. \tag{5.5}$$

Its solution is

$$\Theta_0 = -\ln\left(1 + A(\sigma)s^2\right), \tag{5.6}$$

where the function $A(\sigma)$ is as yet undetermined.

(b) *First-order behaviour*

Similarly, collecting terms at $O(1)$, Θ_1 can be shown to satisfy the equation

$$\frac{s}{2}\Theta_{1,s} - e^{\Theta_0}\Theta_1 = \Theta_{0,ss} - \Theta_{0,\sigma}, \tag{5.7}$$

whose solution is

$$\begin{aligned} \Theta_1 \;=\; & \frac{s^2 \ln s}{(1 + A(\sigma)s^2)}\left[8A^2(\sigma) + 2A'(\sigma)\right] + \frac{2A(\sigma)}{(1 + A(\sigma)s^2)} \\ & - \frac{4A^2(\sigma)s^2}{(1 + A(\sigma)s^2)} \ln(1 + A(\sigma)s^2) + \frac{B(\sigma)s^2}{(1 + A(\sigma)s^2)}. \end{aligned} \tag{5.8}$$

where the function $B(\sigma)$ is unknown. It is readily seen that the term in the square brackets generates a singular behaviour in the second derivative of Θ_1 as $s \to 0$. Thus the terms in the square brackets must be set to zero, so that

$$4A^2(\sigma) = -A'(\sigma) \tag{5.9}$$

generating an equation for $A(\sigma)$. The solution of (5.9) is,

$$A(\sigma) = \frac{1}{4\sigma + C} \tag{5.10}$$

for a constant C. At this stage, the equation for ϕ in the inner layer can be written as

$$\begin{aligned}\phi &= \frac{\sigma}{\mu} - \ln\left(1+A(\sigma)s^2\right) + \mu\left[\frac{2A(\sigma)}{(1+A(\sigma)s^2)} - \frac{4A^2(\sigma)s^2}{(1+A(\sigma)s^2)}\ln(1+A(\sigma)s^2)\right. \\ &\left. +\frac{B(\sigma)s^2}{(1+A(\sigma)s^2)}\right] + \mu^2\Theta_2(s,\sigma) + \cdots .\end{aligned} \tag{5.11}$$

Matching with (4.19) as $\sigma \to 0$ requires

$$C = \frac{1}{a}, \quad B(0) = -12b. \tag{5.12}$$

To complete the solution for ϕ at $O(\mu)$, the function $B(\sigma)$ can be determined by considering the second-order term Θ_2.

(*c*) *Second-order behaviour*

Collecting terms of $O(\mu)$ after substituting (5.3) and (5.4) in (5.2), the equation for Θ_2 is given by

$$\frac{s}{2}\Theta_{2,s} - e^{\Theta_0}\Theta_2 = \Theta_{1,ss} - \Theta_{1,\sigma} + \frac{\Theta_1^2}{2}e^{\Theta_0}. \tag{5.13}$$

Near $s=0$, the expression (5.8) for Θ_1 has the behaviour,

$$\Theta_1 = 2A(\sigma) + s^2\left(B(\sigma) - 2A^2(\sigma)\right) - s^4\left[2A^3(\sigma) + B(\sigma)A(\sigma)\right] + \cdots . \tag{5.14}$$

Requiring also that Θ_2 have the regular solution

$$\Theta_2 \sim \Theta_{20}(\sigma) + \Theta_{21}(\sigma)s^2 + \cdots , \tag{5.15}$$

as $s \to 0$, substituting (5.14) and (5.15) in (5.13) implies that the two conditions

$$\Theta_{20}(\sigma) = 2A^2(\sigma) + 2A'(\sigma) - 2B(\sigma) \tag{5.16}$$

and

$$B'(\sigma) + 8A(\sigma)B(\sigma) = -32A^3(\sigma) + 2A(\sigma)A'(\sigma), \tag{5.17}$$

must hold. Equation (5.17) determines $B(\sigma)$ with solution,

$$B(\sigma) = -\frac{10\ln(4\sigma + C)}{(4\sigma + C)^2} + \frac{D}{(4\sigma + C)^2}. \tag{5.18}$$

From the second of matching conditions (5.12),

$$D = -12bC^2 + 10\ln C. \tag{5.19}$$

To $O(\mu)$, the equation for ϕ can finally be written as,

$$\begin{aligned}\phi &= \frac{\sigma}{\mu} - \ln\left(1+A(\sigma)s^2\right) + \mu\left[\frac{2A(\sigma)}{(1+A(\sigma)s^2)} - \frac{4A^2(\sigma)s^2}{(1+A(\sigma)s^2)}\ln(1+A(\sigma)s^2)\right. \\ &\left. +\frac{\left[A^2(\sigma)\left(10\ln A(\sigma) + D\right)\right]s^2}{(1+A(\sigma)s^2)}\right] + \cdots .\end{aligned} \tag{5.20}$$

Equation (5.20) now indicates the presence of a new focusing structure η that is appropriate in the inner layer, defined by

$$\eta^2 = A(\sigma)s^2 = \frac{x^2}{(4\sigma + C)e^{-\sigma/\mu}} = \frac{x^2}{(-4\mu\ln\tau + C)\tau}, \qquad \eta > 0 \text{ and fixed for } \sigma = O(1),\ \tau \to 0. \tag{5.21}$$

But this is precisely the form of the asymptotically self-similar focusing variable (1.7) previously identified with blow-up in equations of the form (1.1). Here the variable is found to be the appropriate focusing variable in an exponentially small inner layer near to blow-up, which arises due to a requirement to remove a non-uniformity in the outer layer expansions obtained under the assumption of a weakly diffusing process. Near blow-up, the outer layer expansions are associated with an exactly self-similar focusing structure. The physical reason for the appearance of the modified focusing structure can be explained as follows. In the outer layer, diffusion plays a negligible role. As the spatial extent of the outer layer shrinks, like $x = \sqrt{\tau}$, a regime is reached exponentially close to the blow-up time where diffusive processes again become important and must be reinstated. The role of diffusion is to conduct heat away from $x = 0$, and thus broaden the spatial extent of the blow-up layer. The presence of diffusive processes is reflected in the form of the modified focusing variable (5.21), the constant C relating to the non-diffusive exactly self-similar focusing structure, but the 4σ term relating to the role played by diffusion in broadening the blow-up layer.

In terms of the variable η, (5.20) can be written in the form

$$\phi \sim \frac{\sigma}{\mu} - \ln\left(1+\eta^2\right) + \mu A(\sigma)\left[\frac{2}{(1+\eta^2)} - \frac{4\eta^2}{(1+\eta^2)}\ln(1+\eta^2) + \frac{[(10\ln A(\sigma) + D)]\,\eta^2}{(1+\eta^2)}\right] + \cdots. \tag{5.22}$$

As $\sigma \to \infty$, $A(\sigma) \sim 1/4\sigma$, and further increases in ϕ are governed by the structure

$$\phi = \frac{\sigma}{\mu} - \ln\left(1+\eta_1^2\right) + O\left[\mu\sigma^{-1}\ln\sigma^{-1}\right], \tag{5.23}$$

where

$$\eta_1^2 \sim \frac{x^2}{4\sigma e^{-\sigma/\mu}}, \eta_1 > 0 \text{ and fixed } \sigma \to \infty. \tag{5.24}$$

This form was previously obtained by Dold (1991), by anticipating (1.7) to be the appropriate structure, and considering regimes asymptotically close to blow-up. In contrast, it is recovered here by considering the limit of a weakly diffusively process. Apart from influencing the blow-up time t_I, the expression (5.23) is essentially independent of the initial conditions, but reflects the stronger role played by diffusion as the blow-up layer shrinks further.

6. Summary

Finite time blow-up in the semilinear reactive-diffusive parabolic equation $\phi_t = \mu\phi_{xx} + e^{\phi}$ has been examined as a singular perturbation problem using the limit of weak diffusion where $\mu \ll 1$. An outer zone, whose temporal extent is of order unity, describes the behaviour of ϕ from the initial to a time near blow-up. An exact self-similar focusing variable determines the spatial structure of blow-up. In an exponentially small layer close to the blow-up time, however, expansions for the outer layer become non-uniform. An examination of this inner layer reveals that diffusive processes, which conduct heat away from $x = 0$, have begun to play a significant role. The result is that the focusing structure must be modified to ensure a uniformly valid solution, and this modification uncovers the asymptotically self-similar focusing structure previously recognized for blow-up in equations of the form $\phi_t = \phi_{xx} + e^{\phi}$. In contrast to previous studies, however, here the focusing structure arises as a natural consequence of removing, from the asymptotic expansion, a nonuniformity that occurs under the assumption of an initially weak diffusion. Identical weak-diffusion limit asymptotics can be applied to a variety of other semilinear or quasilinear parabolic equations that exhibit finite time blow-up in order to reveal the associated focusing structures.

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List of Recent TAM Reports (cont'd)

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List of Recent TAM Reports (cont'd)

No.	Authors	Title	Date
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List of Recent TAM Reports (cont'd)

No.	*Authors*	*Title*	*Date*
851	Stewart, D. S., and J. Yao	The normal detonation shock velocity–curvature relationship for materials with non-ideal equation of state and multiple turning points	Apr. 1997
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861	Short, M.	A parabolic linear evolution equation for cellular detonation instability	Aug. 1997
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