Analysis and modeling for a turbulent convective plume

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Abstract

Asymptotic and scaling analyses are applied to develop a simple model for a turbulent convective plume at high Rayleigh number R in a high Prandtl number fluid layer whose lower boundary surface maintains steady local buoyant sources. The convective plume is assumed to be axisymmetric and conical and to have a radius b that is small compared with the depth of the layer. Under statistically steady conditions, to the leading order terms, the dependence of the mean components of axial velocity, radial velocity, pressure and temperature of the plume, as well as the vertical volume flux in the plume, on R and b is determined. The main results are in satisfactory agreement with available experimental observations. The results are equally applicable to the case where any finite number of such plumes are present under similar conditions, provided no interactions take place between the individual plumes.

1 Introduction

Plumes are buoyant jets, which arise due to steadily supplied buoyancy, and their buoyant motion is continuous between the source and any level in their domain [1]. There have been a number of theoretical, numerical and experimental studies of turbulent plumes and a collection of relevant references for such studies are given by Turner [1]. Morton et al. [2] developed, in particular, a simple model for plume maintained in a uniform or stably stratified ambient fluid, where the main assumptions were that the velocity and buoyancy force are constant across the plume and zero outside it, the rate of entrainment at the edge of the plume is proportional to some characteristic velocity at that height, the profiles of mean axial vertical velocity and mean

buoyancy force in horizontal sections are of similar form at all heights, and the largest local variations of density are small in comparison with some chosen reference density. However, the governing equations that they used to build their model for the steady point source of the plumes involve only differentiation with respect to the axial variable z. These authors also carried out laboratory experiments on maintained plumes and compared their theoretical results to those due to their experiments. The experimental results show a more or less conical shape for the maintained plumes.

Telford [3,4,5] and Warner and Telford [6] investigated convective plumes with particular interest in the atmospheric plumes applications, but again the radial r variations of the dependent variables are not taken into account. Telford [3] studied an isolated plume in still air, under the assumptions that the plume is uniform over all the horizontal section, turbulent energy dissipation is zero, turbulent energy spectrum is not taken into account, vertical mixing is negligible and wind shear is zero. Warner and Telford [6] studied the observations made from the ground and from aircraft and concluded that clear air thermals (suddenly released buoyant elements [1]) are continuing plumes, they rise through a neutral to slightly stably stratified environment, flow within and outside the plumes are turbulence, and mixing into and out of the plumes take place. They then found that the real plumes in the atmosphere have little resemblance to most existing models. Telford [4] extended his earlier theory [3] to provide a theory of rising and descending air in a field of plumes, where the effect of time dependent heating during day-night cycle was taken into account. Telford [4] ignored wind stress and assumed each plume is immersed in the turbulent downdraft which comprises the return flow. He suggested that his extended theory could lead to a prediction of the onset of a different form of convection resulting from instabilities in the convection layer. Telford [5] generalized his earlier dimensional type solutions [3,4] by non-dimensionalizing the original equations that provided such solutions.

More recently Offut [7] carried out experimental investigation of turbulent thermal convection in a horizontal layer of fluid with Prandtl number about 7.35 using particle image velocimetry and shadowgraph visualization. The Rayleigh number R was about 10^8 . In regard to the convective plumes, he obtained, in particular, the following results. The fluid layer was densely populated with viscous, mushroom-headed plumes. The plume geometry was characterized typically by a thin stem 2-3 m.m. wide, non-dimensional radius b of the plume's cross section was about 0.05 to 0.5 for a non-dimensional unity of the layer thickness, and a mushroom shaped cap, which grew in width as the plume ascended. The stems grew very little in width as the plumes rose, while the caps grew from several m.m. across at the level very close to the lower boundary of the layer to 3-4 c.m. near the top boundary of the layer. However, plumes' interactions or merging problem was difficult to be observed and is presently unresolved.

In the present paper we have developed a more realistic theoretical model for a maintained plume which can also be used for a field of such plumes under similar conditions. The model obeys the Navier-Stokes and continuity equations subjected to the Boussinesq approximation and under a number of realistic assumptions and conditions some of which have been used in the previous models described before. The full assumptions and conditions, underwhich such model is built, are given in the next section.

2 Formulation

We consider a thin horizontal layer of high viscous fluid and of thickness d above a heated horizontal surface. The gravity vector is assumed to be in the downward direction perpendicular to the direction of the surface and anti-parallel to the vertical z-axis. We assume that there are steady maintained plumes initiated at discrete locations on the surface. These plumes with steady point sources are assumed to have similar shapes and to be subjected to similar conditions. Since we will be concerned mainly with convection in any conical type shape plume [2], whose axis is assumed to be parallel to the z-axis, within the horizontal fluid layer, we consider the governing Navier-Stokes, continuity and temperature equations, under the so-called Boussinesq approximation [1], in a cylindrical coordinate whose axial direction is along the z-axis. We assume that the temperature difference across the layer is sufficiently high and the fluid in the layer, both inside and outside the plumes, is in turbulent motion. We shall consider mean time averaged axisymmetric convection in a conical plume, whose apex and axis coincides with the origin 0 and the z-axis of the coordinate system, respectively, and in the fluid layer in the asymptotic limit of strong buoyancy force, due to the vertical temperature gradient, and sufficiently large Prandtl number $P_r = \nu/K$. Here ν is the kinematic viscosity and K is the sum of eddy and molecular thermal diffusivity.

The non-dimensional form of the equations for the momentum, continuity and temperature for the statistically steady mean flow component of the fluid in the plume are then

$$\nabla^2 u = R(\nabla P + \theta \hat{z}),\tag{1.1}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1.2}$$

$$\nabla^2 \theta = \boldsymbol{u} \cdot \nabla \theta, \tag{1.3}$$

where $u = u\hat{r} + w\hat{z}$ is the mean velocity vector, u is the radial component of the mean velocity, \hat{r} is a unit vector in the direction of the radial r-axis, w is the axial component of u, \hat{z} is a unit vector in the direction of the axial z-axis, p is the mean pressure, θ is the mean temperature, $R = \beta \Delta T g d^3/(K\nu)$ is the Rayleigh number, β is the coefficient of thermal expansion, ΔT is the temperature difference imposed across the layer, and g is acceleration due to gravity.

The boundary conditions for the flow within the plume and outside are

$$u = \theta - 1 = 0$$
 at $z = 0$, (1.4)

$$\hat{z} \cdot \boldsymbol{u} = \theta = 0 \quad \text{at} \quad z = 1, \tag{1.5}$$

where it is assumed that the fluid layer is bounded above by a horizontal free surface at z=1.

It is seen from the governing equations (1.1)-(1.3) that, in contrast to the previous models, the radial rate of change of the dependent variables are taken into account. In the next section we shall apply asymptotic and scaling analyses for (1.1)-(1.5), in the asymptotic limit of sufficiently large R, to determine the strongly nonlinear statistically steady state and axisymmetric behavior of the mean flow in the layer and mainly mean turbulent convection features in each plume for sufficiently large P_r . Extension to arbitrary P_r for convection with non-axisymmetric behavior is planned to be done by the present author in near future. It should be noted that no analyses of the plumes' interactions or merging will be done in the present paper. However, mixing on the plume's boundary will be taken care of due to entrainment modeling [2] which will be described in the next section.

3 Analysis and results

Let us designate b(z) to be the radius of the horizontal cross section of a plume under consideration whose axis coincides with the z-axis. It is assumed that b is small (b << 1) [7]. We assume that the orders of magnitude of r and z are, respectively b and 1. Assuming |u| in the plumes is much larger than the corresponding one outside the plumes, then (1.1) implies that to the leading terms pressure field is unaffected by the flow velocity everywhere and θ is independent of r. The governing equations (1.1) and (1.3) then imply that $p = p_0(z), w = \tilde{w}_0(z)$ and $\theta = \theta_0(z)$ at most, where p_0, \tilde{w}_0 and θ_0 are the leading order terms for p, w and θ , respectively, and satisfy the following equations

$$\frac{dP_0}{dz} + \theta_0 = 0, (2.1)$$

$$\tilde{w}_0 \frac{d\theta_0}{dz} = \frac{d^2\theta_0}{dz^2}. (2.2)$$

Here \tilde{w}_0 is leading order term for the downward axial velocity for the flow outside the plumes. Let us now introduce a stream function $\psi(r,z)$ for the flow, so that

$$(u,w) = \left(-\frac{1}{r}\frac{\partial\psi}{\partial z}, \frac{1}{r}\frac{\partial\psi}{\partial r}\right). \tag{3}$$

Using (1.2) and (6), we have $\psi \sim bu \sim b^2 w$. Using these results, (1.3) yield

$$\frac{\partial}{\partial r} \left(r \frac{\partial \theta_1}{\partial r} \right) = \frac{\partial \psi}{\partial r} \frac{d\theta_0}{dz},\tag{4}$$

where $\theta_1(r,z)$ is the deviation of θ from θ_0 , and it is assumed that $\theta_1 \ll \theta_0$. Integrating (4) twice with respect to r and assuming mean flow solution with closed streamlines, we find

$$\theta_1 \sim \psi(b, z) \frac{d\theta_0}{dz} \ln r,$$
 (5.1)

where

$$2\pi\psi(b,z) = \int_0^b 2\pi r w dr \tag{5.2}$$

is the vertical volume flux in the plume.

Using (3), we have

$$u \sim -\frac{\partial \psi(b,z)}{\partial z} \left(\frac{1}{r}\right) \quad \text{as} \quad r \to b.$$
 (6)

Considering the radial component of (1.1), we have to the leading terms

$$\frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) = rR \frac{\partial P}{\partial r},\tag{7}$$

Near the wall of the plume, use (6) in (7) and then integrate the resulting equation to find

$$\Delta P \sim \frac{\partial \psi(b, z)}{\partial z} \left(\frac{1}{2b^2 R}\right),$$
 (8)

where ∇P represents the pressure near the wall of the plume.

Considering the axial component of (1.1), we have to the leading terms

$$\frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) = rR\theta_1. \tag{9}$$

Using (5.1) in (9) and integrating the resulting equation in r for w, we find

$$w \sim \left(\frac{b^2 R}{r}\right) \psi(b, z) \frac{d\theta_0}{dz} \left(\ln b - 1\right), \tag{10}$$

which holds near the wall of the plume. Using (6), (10) and the continuity constraint (1.2), we find

$$\frac{d\theta_0}{dz}Rb^4\ln b \sim 1. \tag{11}$$

from (6) and (10) near the wall of the plume, it is seen that the wall of the plume should be inclined with respect to z-axis as is consistently assumed.

Now denote u_0 to be the leading order term for u. The equations (2.1)-(2.2), derived from the axial component of (1.1) and from (1.3), together with the equation

$$\frac{\partial}{\partial r}(ru_0) = -r\frac{\partial w_0}{\partial z},\tag{12}$$

derived from (1.2), subjected to the boundary conditions

$$u_0 = w_0 = \theta_0 - 1 = 0 \quad \text{at} \quad z = 0,$$
 (13.1)

$$w_0 = \theta_0 = 0 \quad \text{at} \quad z = 1,$$
 (13.2)

derived from (1.4)-(1.5), can be solved in principle if the expression for w_0 , which is the leading order term for w in the plume, was known or if w_0 was given in terms of an equation involving any one or more of these

dependent variables. However, the expression for w_0 is not yet known and there is no additional equation for w_0 . The radial component of (1.1) is not of much help here since its leading form,

$$\frac{\partial}{\partial r} \left(r \frac{\partial u_0}{\partial r} \right) = rR \frac{\partial (P - P_0)}{\partial r},\tag{14}$$

involves the deviation of P from P_0 which serves as additional unknown. To determine w_0 , we proceed in the following way described in the next paragraph.

In order to determine the expression for w_0 on the boundary of the plume, we make use of (10)-(11) to the leading term to obtain

$$4w_0 = \psi(b, z)/b^2 \quad \text{as} \quad r \to b. \tag{15}$$

Thus, the expression for $w_0(b, z)$ will be known once the expression for $\psi(r, z)$ is evaluated at r = b. We use a Polhausen type method [8] suggested by Lighthill [9] to determine ψ . The boundary conditions to be satisfied by ψ are

$$\psi = 0 \quad \text{at} \quad r = 0, \tag{16.1}$$

$$\frac{\partial}{\partial z} \left(b^2 \frac{\partial \psi}{\partial r} \right) = 2b\alpha \frac{\partial \psi}{\partial r} \quad \text{at} \quad r = b, \tag{16.2}$$

where (16.2) relates the inflow velocity at the edge of the plume to the vertical velocity within the plume and involves an entrainment constant parameter α [1,2]. We now introduce a trial function for

$$(\theta - 1 + z) = (\theta_0 - 1 + z)T(S), \tag{17}$$

where S = r/b and T(S) is a polynomial in S. The boundary conditions

$$\frac{\partial \theta}{\partial r} = 0 \quad \text{at} \quad r = 0,$$
 (18.1)

$$\theta - \theta_0 = \frac{\partial \theta}{\partial r} = 0$$
 at $r = b$, (18.2)

chosen for θ , then leads to the following conditions for T(S)

$$\frac{dT}{dS} = 0 \quad \text{at} \quad S = 0, \tag{19.1}$$

$$T - 1 = \frac{dT}{dS} = 0$$
 at $S = 1$. (19.2)

A candidate for T(S) satisfying (19.1)-(19.2) is

$$T(S) = S^3 - \frac{3}{2}S_+^2 \frac{3}{2}. (20)$$

Using (3), (17) and (20) in (9), we find

$$\frac{1}{S}\frac{\partial}{\partial S}\left[S\frac{\partial}{\partial S}\left(\frac{1}{S}\frac{\partial\psi}{\partial S}\right)\right] = b^4R(\theta_0 - 1 + z)\left(S^3 - \frac{3}{2}S^2 + \frac{1}{2}\right). \tag{21}$$

Integrating (21) with respect to S three times and making use of (16.1)-(16.2) and the finiteness value of the vertical velocity at S = 0, we find

$$\psi = b^4 R(\theta_0 - 1 + z) \left(\frac{1}{175} S^7 - \frac{1}{64} S^6 + \frac{1}{32} S^4 + CS^2 \right), \tag{22.1}$$

where

$$C = \left[60(\theta_0 - 1 + z) \frac{db}{dz} + 114\alpha(\theta_0 - 1 + z) - 57b \left(\frac{d\theta_0}{dz} + 1 \right) \right] /$$

$$\left[1600b \left(\frac{d\theta_0}{dz} + 1 \right) - 3200\alpha(\theta_0 - 1 + z) \right].$$
(22.2)

Using (1.4)-(1.5), (2.1)-(2.2) and (22.1)-(22.2), we find the following results to the leading order terms:

$$2\pi\psi(b,z) = 2\pi b^4 R\phi \left[\frac{239}{11200} + \frac{\left(60\phi \frac{db}{dz} + 114\alpha\phi - 57\frac{d\phi}{dz}b\right)}{1600\left(\frac{d\phi}{dz}b - 2\alpha\phi\right)} \right],\tag{23.1}$$

$$P_0(z) = \int_0^z (\phi + 1 - z) dz,$$
 (23.2)

$$\frac{d^2\phi}{dz^2} = -2\pi\psi(b,z)N\left(\frac{d\phi}{dz} - 1\right),\tag{23.3}$$

$$\phi = 0 \quad \text{at} \quad z = 0, 1,$$
 (23.4)

where

$$\phi = \theta_0(z) - 1 + z,\tag{23.5}$$

N is the number density of all the plumes in the fluid layer and (23.1) represents total vertical volume flux in the plume.

The solution to (23.3)-(23.4) was determined numerically using a shooting type iterations, and then the vertical volume flux in the plume is determined from (23.1). The results for θ_0 and $2\pi\psi(b,z)$ are determined fro N=10, which is close to the observation in [7], and

$$b(z) = 0.01z + 0.02 \tag{24}$$

and for two different values of R. Talbe 1 presents these results for different z values across the fluid layer. It is seen from these results that the leading order temperature and the volume flux within the plume increase with R. At the bottom and top boundaries ϕ and $2\pi\psi$ are zero as (23.4)-(23.5) also indicate. Thus very close to the upper boundary the vertical volume flux decreases rapidly and the flow is mainly in the radial (horizontal) direction.

In relation of the present results to those available experimental results, it is of interest to refer to offut [7] observational and experimental results regarding the plumes in the Rayleigh-Benard type system. Offut [7] investigated experimentally turbulent thermal convection in a horizontal layer of water $(P_r = 7.35)$ of depth 10 cm and at $R = 10^8$. He observed the layer was densely populated with plumes. He characterized a hot plume geometry by a thin stem 2-3 mm wide and a mushroom shaped cap, which grew in width as the hot plume rose. Upon reaching the upper fourth of the layer, the plumes spread out horizontally and stagnated before diffusing away. Plumes moved upward with an initial vertical velocity that was a fraction of their terminal velocity. Plumes, interactions or merging was difficult to be observed with the shadowgraph technique employed. For $R = 10^8$, about 32 particular plumes were used to determine the mean cap vertical

speed which was found to be about 0.61 cm/s. In the present theoretical studies, we used the relation (11) and the numerical finding that $\left|\frac{d\theta_0}{dz}\right|$ is close to unity to find that dimensional value of b at $R=10^8$ is about 0.5 mm. This result is in satisfactory agreement with observation reported in [7] despite the fact that infinite Prandtl number and steady axisymmetric conditions are employed here. Next, using the present result presented in table 1 for $R=10^8$, we determined the dimensional value of the vertical velocity in the plume at z=0.9 to be about 0.39cm. This result is also in satisfactory agreement with the corresponding experimental one reported in [7].

It should be noted that the present asymptotic limit of large R taken in the analysis did not allow N to be too large, and the acceptable order of magnitude of \tilde{w}_0 implies that

$$N_{\text{max}} = O\left[\frac{1}{\int_0^1 \psi(b, z)dz}\right],\tag{25}$$

where N_{max} is the maximum allowed value of N. Hence the value of N chosen in the present study is reasonable.

4 Some conclusions

- (i). The radius of the plume is of the order of $(R \ln R)^{\frac{-1}{4}}$ and its value is in satisfactory agreement with the experimental observation [7].
- (ii). Both vertical velocity and temperature in the plume increase with R.
- (iii). The value of the vertical velocity in the plume is in satisfactory agreement with the experimental result [7].
- (iv). The vertical volume flux in the plume increases with R. It also increases with z up to a value of vertical level close to the upper boundary beyond which it decreases rapidly. This result is also in agreement with the experimental observation [7] that the plumes spread out horizontally upon reaching the upper fourth of the layer.

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z	$R = 10^7$		$R = 10^8$	
	$\theta_0(z)$	$2\pi\psi(b,z)$	$\theta_0(z)$	$2\pi\psi(b,z)$
0.0	1.0000	0.0000	1.0000	0.0000
0.1	0.9002	0.0001	0.9002	0.0006
0.2	0.8004	0.0004	0.8004	0.0028
0.3	0.7006	0.0012	0.7010	0.0076
0.4	0.6010	0.0024	0.6024	0.0200
0.5	0.5016	0.0044	0.5058	0.0536
0.6	0.4026	0.0082	0.4147	0.1473
0.7	0.3045	0.0156	0.3371	0.4317
0.8	0.2076	0.0299	0.2883	1.5438
0.9	0.1143	0.0582	0.2776	7.5108
1.0	0.0000	0.0000	0.0000	0.0000

Table 1: θ_0 and $2\pi\psi(b,z)$

versus z for $R = 10^7$ and 10^8 .

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