

# OSCILLATORY CRACK GROWTH IN GLASS

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## Introduction

For those seeking to understand the behavior of cracks, the controlled quasi-static cracking observed in the experiments by Yuse and Sano (1,2) is a very interesting problem. By slowly lowering a glass plate through a heater and into cold water, they observed a crack whose tip stays ahead of the water line and whose path depends upon the dipping velocity (0.01-50 mm/s) and the temperature difference between the heater and water (50-300°C). Crack morphology was seen to change from straight to oscillatory to branching as the temperature difference was increased. Ronsin *et al* (3,4) added to the experimental knowledge with their work at low speeds and with multiple cracks. Many theoretical studies have been made (5-7), with that by Sasa *et al* (6) appearing to correlate best with the experimental data (2). On the numerical side, Bahr *et al* (8) used an incremental crack tip extension in a finite element formulation to study the onset of the oscillatory instability but were limited to very small amplitudes of oscillation. Therefore, they could not study the evolving crack morphology. The spring model of Hayakawa (9) was successful at reproducing many of the morphological features seen in experiments, but was unable to make any direct comparison to experiments quantitatively.

Clearly, many aspects of this interesting phenomenon are not well understood. In the present communication, we present some experimental observations as well as preliminary results of numerical simulations of rapid dipping using a cohesive element method (10).

## Experimental Observations

We performed initial experiments on ordinary window glass with a thickness of 1.85 mm. On one edge of the specimens, we generated one, two, three, or four pre-existing crack initiation sites by scratching with a tungsten carbide blade. Then, like in many other experiments, the specimens were heated to a higher temperature and dipped into a cold bath at speeds in the 5-10 mm/s range. We obtained the mosaic of crack growth patterns shown in Fig. 1.

The crack patterns in Fig. 1 demonstrate that, if the crack initiation sites are not uniformly spaced, the cracks tend to turn towards the direction such that they would be roughly equally spaced, as they grow longer. On the other hand, even if the initiation sites are equally spaced, the crack path may still become oscillatory when the temperature difference is sufficiently large as observed by others (4).

Another interesting phenomenon was observed in the specimen with three initial scratches. Looking at the picture in Fig. 1, one may think that the two side cracks propagated all the way down, whereas the center one stopped early. In fact, this is not true. More careful examination of the specimen surface revealed that the center crack was just as long as the two side cracks. We believe that the reason it cannot be seen in this picture is that the relative displacement between the faces of the center crack is smaller than the wavelength of light so that a light wave passes

through the crack without being reflected. Considering the fact that glass is an amorphous material with molecules randomly oriented, being able to generate a crack while keeping the relative displacement so small is surprising. It should be noted that this specimen is not the only one in which we observed such a phenomenon. We did see “coherent cracks” (or crack segments) in several other specimens.

More experiments were done on 1.0-mm thick glass microscope slides and at dipping speeds of 1-10 mm/s. Two results with single and repeated branching behavior are shown in Fig. 2. Specimen (a) also illustrates a case where the crack branches back onto itself forming a loop. We have seen many other interesting patterns and are refining the experimental setup to characterize these behaviors more precisely. This will include using thinner glass and/or a slower motor to make the thermal diffusion length ( $D/V$ ) greater than half the glass thickness such that the temperature field can more truly be considered two-dimensional.

This wavy cracking phenomenon is observed not only in glass, but also in single crystal silicon (11). The driving force for crack turning must be quite strong since silicon is an anisotropic material with well-defined cleavage planes.

### Numerical Formulation

The cohesive element formulation in (10, 12) provides a framework for modeling crack growth along arbitrary paths. The continuum is characterized by a volumetric constitutive law relating stress and strain, and a cohesive surface constitutive law relating tractions and displacement jumps across a set of cohesive surfaces that are dispersed throughout the material. A Lagrangian description and finite strain kinematics are employed. The amplitude of crack oscillation is not limited and the simulation can be done at the same length scale as the experiment. Furthermore, the method could be expanded to include thermal conductivity and anisotropic fracture strengths. The calculations are fully dynamic and are carried out within a two-dimensional plane strain framework. Features of the formulation that are different from (12) are given below.

A thermal wave passes through the specimen at a prescribed velocity. The elements are linear displacement triangles and are given a single temperature based upon the location of the element centroid with respect to the thermal wave. The temperature,  $\theta$ , in an element at time,  $t$ , with a centroid at  $x$  is given as (6)

$$\theta(x, t) = \theta_1 + [\theta_0 - \theta_1] \left[ 1 - \exp\left(-\frac{V(x - Vt)}{D}\right) \right] H(x - Vt) \quad (1)$$

where  $\theta_0$  is the heater temperature,  $\theta_1$  is the water temperature,  $V$  is the dipping velocity,  $D$  is the thermal diffusion coefficient, and  $H$  is the Heavyside step function. This is simply a step function for the velocities considered herein.

The stress caused by these temperature changes for the plane strain case is

$$\mathbf{S} = -\frac{E\alpha\Delta\theta}{1-2\nu} \mathbf{I} \quad (2)$$

where  $\mathbf{S}$  is the second Piola-Kirchhoff stress in the reference configuration,  $E$  is the Young's Modulus,  $\alpha$  is the coefficient of thermal expansion,  $\Delta\theta$  is the elemental temperature change over a single time step,  $\nu$  is the Poisson's ratio, and  $\mathbf{I}$  is the identity tensor. This thermal stress is added to the elastic stress before forming the global force vector.

The rectangular specimen of length  $L$  and width  $W$  is shown in Fig. 3. All boundaries are traction free. The elements in the center region are of constant size. Cohesive surfaces are found between every element. The precrack is usually centered in the width and it extends to the boundary of the constant mesh region. It is visible in Fig. 3 because a thermal wave has traveled eighty percent of the precrack length. Rather than setting the cohesive strengths to zero along the precrack as in (12), the cohesive strengths for the precrack are set to some fraction (typically 10%)

of the material cohesive strength. This provides a nucleation site for further cracking, while preventing interpenetration should the crack faces come together. Only cases with a single precrack have been investigated numerically to date, but multiple precracks could be introduced as desired.

The properties of glass used in this simulation were  $E=69$  GPa,  $\nu=0.2$ ,  $\alpha=7.2 \times 10^{-6}$  /°C,  $D=4.7 \times 10^{-7}$  m<sup>2</sup>/s,  $\rho=2500$  kg/m<sup>3</sup> for the mass density, and  $K_{Ic}=0.75$  MPa m<sup>1/2</sup> for the fracture toughness. The cohesive surface properties are based on the fracture toughness and the cohesive strength of the material, with the normal cohesive surface strength set to  $E/100$  (rather than  $E/30$ ) to give a larger stable time step.

### Numerical Results

An explicit time integration scheme is used so that the run time is inversely proportional to the dipping speed. The calculations were run on a 533 MHz DEC Alpha and to shorten the computing time an unrealistically high dipping speed of 1000 m/s was prescribed. In addition to moving to faster computers, other methods of improving the calculation speed are being investigated to allow simulations at the experimental velocities of 0.01-100 mm/s. Our purpose here is to demonstrate the abilities of this model and to discuss the interesting rapid quenching results obtained to date.

The Rayleigh wave speed for the given material properties is 3061 m/s. At the prescribed dipping speed of 1000 m/s, material inertia clearly affects the computed results. In particular, the fast dipping speed favors branching.

Long precracks were used to avoid unwanted influence of the boundary. A crack that oscillated before branching was observed for a 21-mm precrack in a 24x48-mm specimen with a temperature change of 420°C as shown in Fig. 4. The stress contours for the y-direction normal stress are shown in this and subsequent figures. The wavelength of the oscillations is approximately 0.13 W which compares well to the range 0.05-0.2 W reported by theory and experiment (2, 6, 7) as the asymptotic value for wavelength at high velocities at the onset of oscillating behavior. These oscillations make us hopeful of finding other oscillating regimes at lower dipping speeds.

For a 5-mm long edge crack centered along the short side of a 10x16-mm specimen only three behaviors were observed: no crack growth up to 160°C, branched cracks such as that shown at 220°C in Fig. 5, and repeated branching above approximately 300°C. In Fig. 5 the displacements are given a scale factor to make the crack more visible. Even for no sustained crack growth (not shown), the tendency seemed to be to branch immediately. The branched cracks continue to oscillate 180° out of phase as seen in Fig. 2(a) and by others (1, 2, 4). The space between the crack branches is 0.28 W while that in Fig. 2(a) and other branching (2) or two crack systems (4) is 0.33 W. Although locally restricted to the mesh angle, the global crack behavior with this type of mesh can approach any random path (12). Numerically, the branching we saw always had an included angle between branches of 90°. This angle was often, but not always, around 90° in our experiments as in Fig. 2(a) where it is 87°.

The tendency for these cracks to center themselves as seen experimentally in the first specimen of Fig. 1 is demonstrated numerically in Fig. 6. Here a 5-mm precrack was set 1 mm off center in a 10x16-mm specimen at 220°C. The crack oscillates towards the center before branching.

### Conclusions

Past experiments have dealt mostly with finding the straight-oscillatory transition as a function of the critical parameters such as temperature difference, velocity and crack spacing or specimen width. There are many more questions to be answered about the transient behavior, branching behavior, and interactions between multiple cracks. Also, questions about the cause of coherent cracks and the effect of anisotropy on the oscillatory crack behavior are worth investigating.

Calculations of thermal stress induced crack growth have been carried out for a single crack. The results exhibit both oscillatory behavior and crack branching. Because, for numerical reasons, the calculations are carried out at unrealistically high dipping speeds, branching tends to be favored more than in the experiments. It also remains to be seen whether the cohesive surface framework can predict the multiple crack morphologies seen in Fig. 1. However,

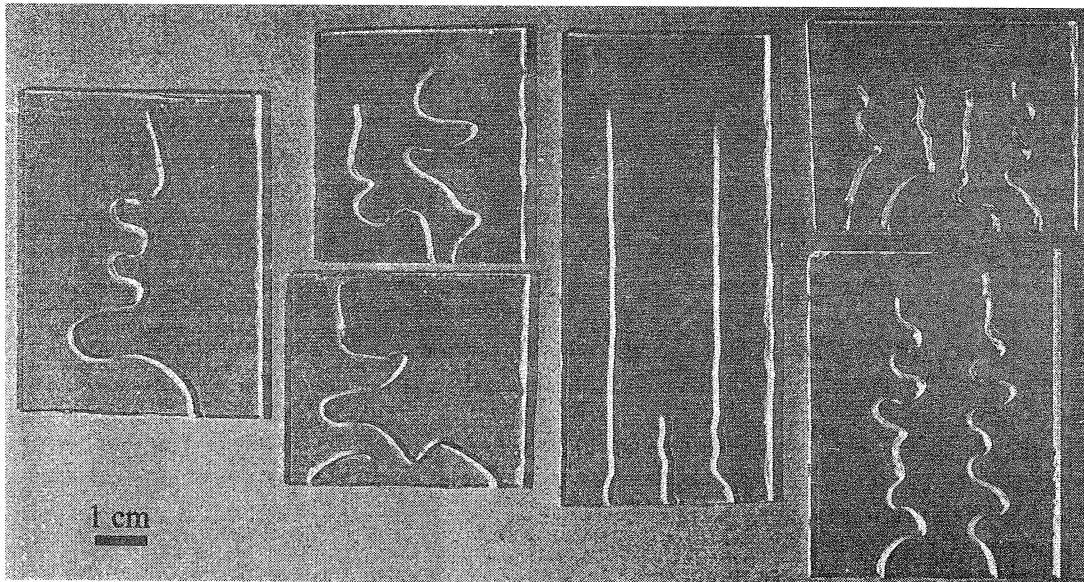
many features of the single crack behavior are reproduced by the computations, suggesting that a more quantitative agreement can be attained if computing strategies, currently being explored, are successful in permitting calculations with the slower dipping speeds of the experiments.

#### Acknowledgments

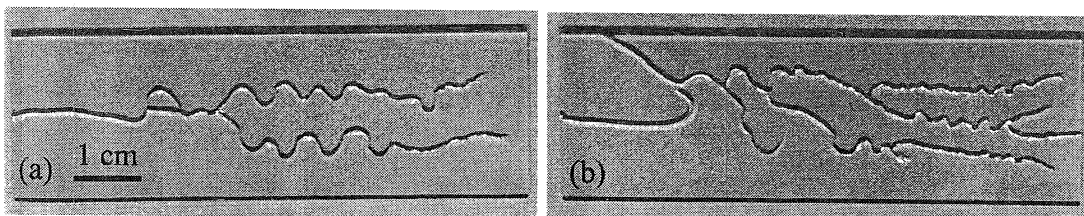
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#### References

1. A. Yuse and M. Sano, *Nature* 362, 329 (1993).
2. A. Yuse and M. Sano, *Physica D* 108, 365 (1997).
3. O. Ronsin, F. Heslot, and B. Perrin, *Physical Review Letters* 75, 2352 (1995).
4. O. Ronsin and B. Perrin, *Europhysics Letters* 38, 435 (1997).
5. M. Marder, *Physical Review E* 49, R51 (1994).
6. S. Sasa, K. Sekimoto, and H. Nakanishi, *Physical Review E* 50, R1733 (1994).
7. M. Adda-Bedia and Y. Pomeau, *Physical Review E* 52, 4105 (1995).
8. H.-A. Bahr, A. Gerbatsch, U. Bahr, and H.-J. Weiss, *Physical Review E* 52, 240 (1995).
9. Y. Hayakawa, *Physical Review E* 49, R1804 (1994).
10. A. Needleman, *J. Appl. Mech.* 54, 525 (1987).
11. B.D. Ferney and K.J. Hsia, unpublished research (1998).
12. X.-P. Xu and A. Needleman, *J. Mech. Phys. Solids* 42, 1397 (1994).



**Figure 1** Crack patterns formed when hot glass is dipped in cold water.



**Figure 2** Branching crack patterns.

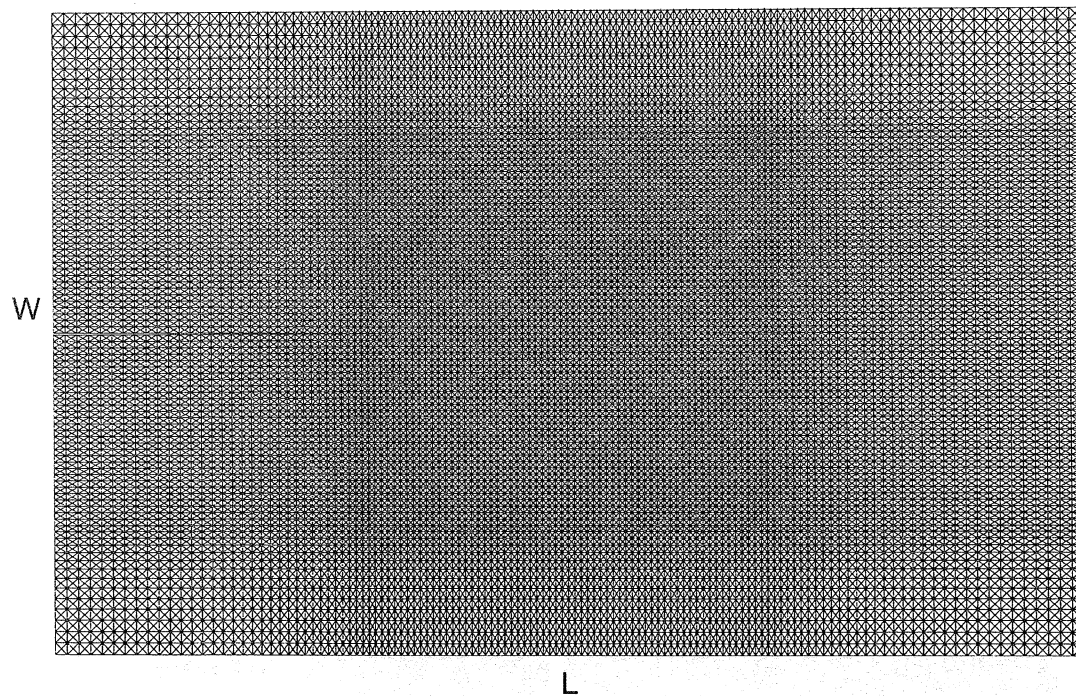


Figure 3 Finite element mesh with 11264 elements.

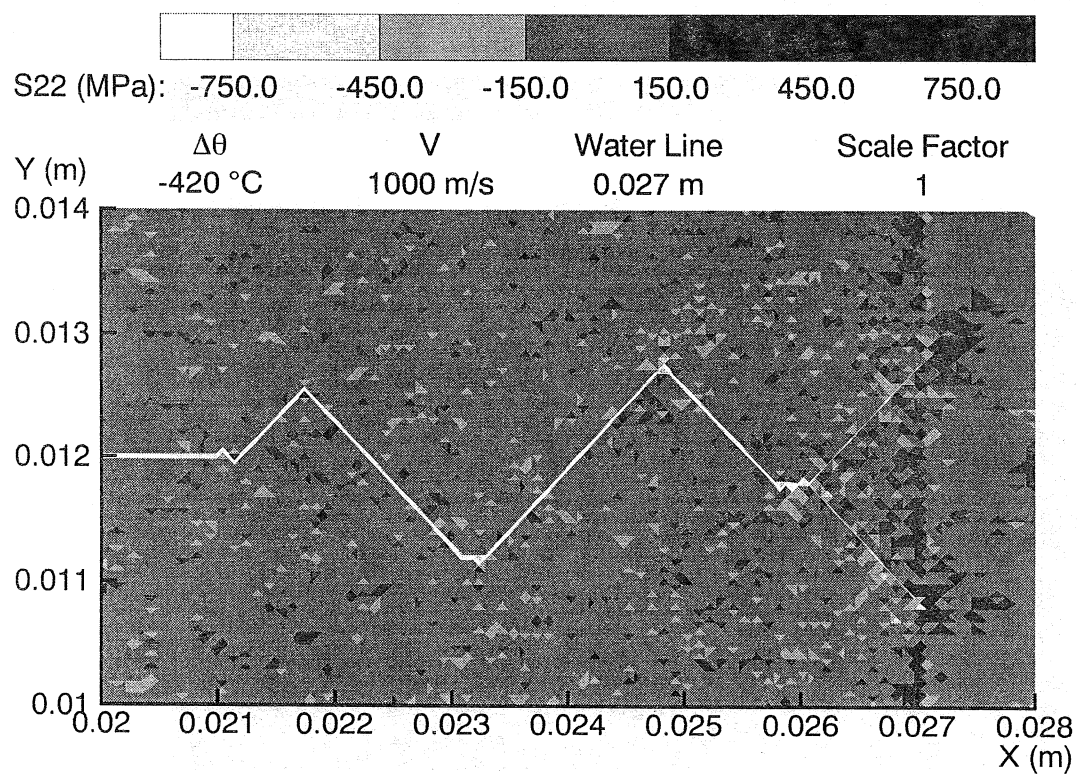


Figure 4 Oscillating and branching crack in a 24x48-mm specimen.



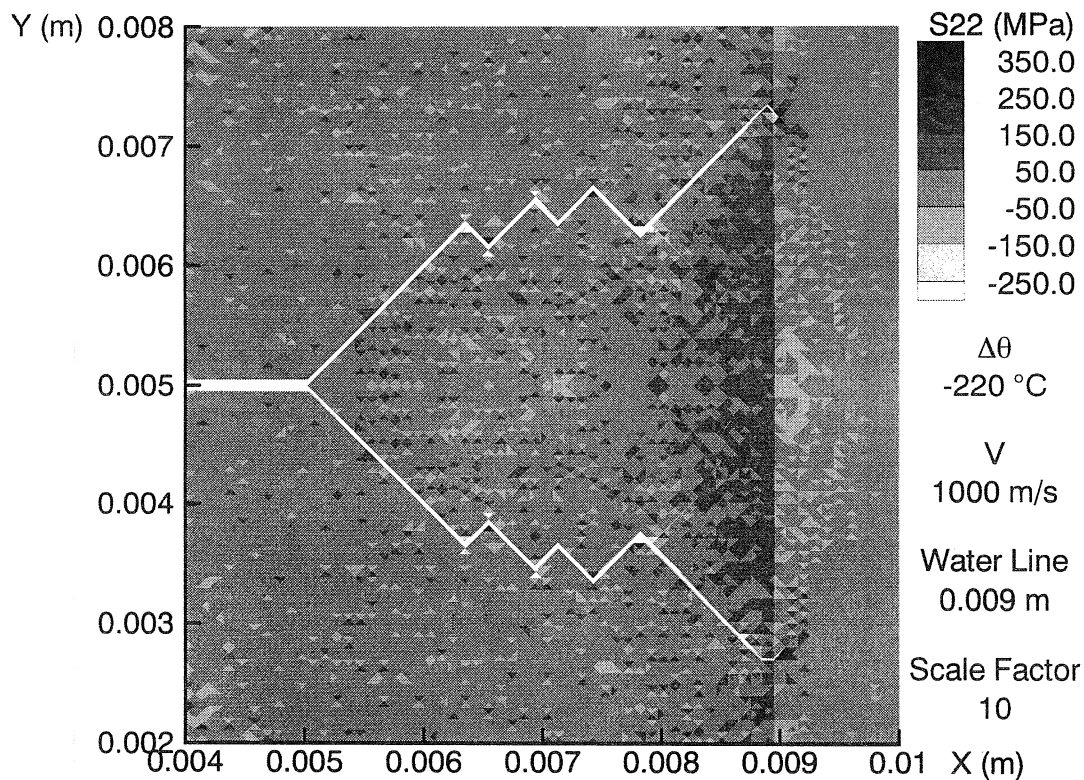


Figure 5 Branched crack with oscillations in a 10x16-mm specimen.

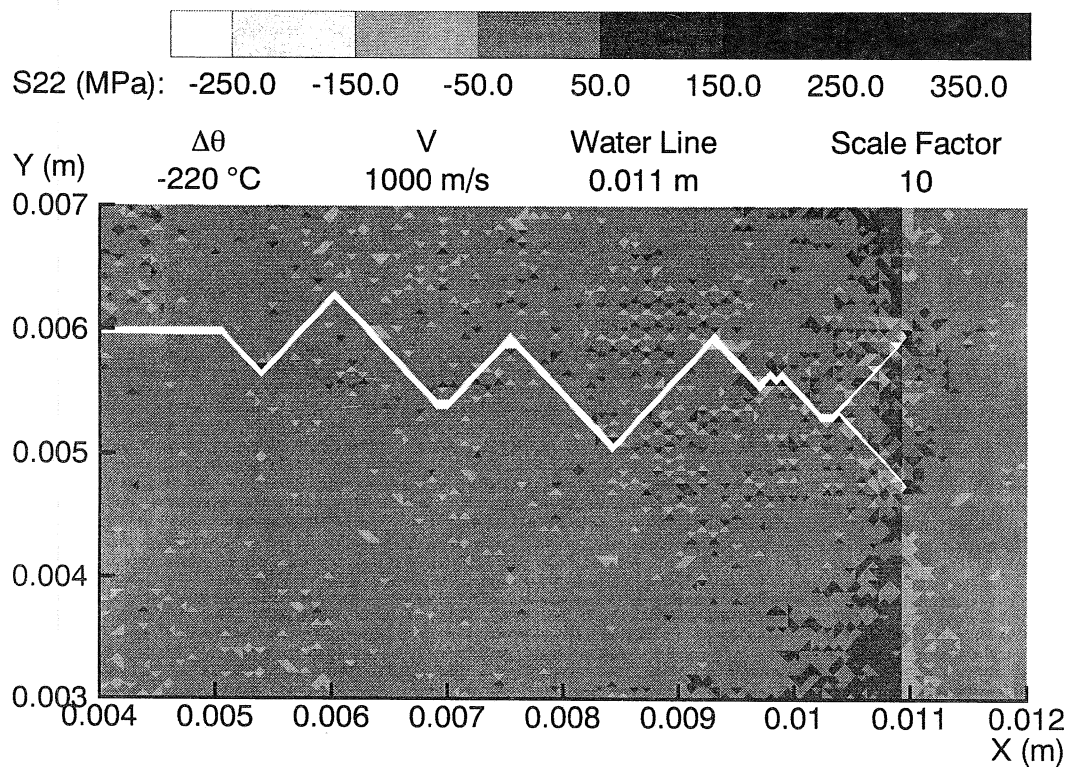


Figure 6 Oscillating crack in a 10x16-mm specimen with precrack offset vertically by 1 mm.







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896	Harris, J. G.	<i>Elastic waves</i> —Part of a book to be published by Cambridge University Press	Dec. 1998
897	Paris, A. J., and G. A. Costello	Cord composite cylindrical shells	Dec. 1998
898	Students in TAM 293–294	Thirty-fourth student symposium on engineering mechanics (May 1997), J. W. Phillips, coordinator: Selected senior projects by M. R. Bracki, A. K. Davis, J. A. (Myers) Hommemma, and P. D. Pattillo	Dec. 1998
899	Taha, A., and P. Sofronis	A micromechanics approach to the study of hydrogen transport and embrittlement	Jan. 1999
900	Ferney, B. D., and K. J. Hsia	The influence of multiple slip systems on the brittle–ductile transition in silicon	Feb. 1999
901	Fried, E., and A. Q. Shen	Supplemental relations at a phase interface across which the velocity and temperature jump	Mar. 1999
902	Paris, A. J., and G. A. Costello	Cord composite cylindrical shells: Multiple layers of cords at various angles to the shell axis	April 1999
903	Ferney, B. D., M. R. DeVary, K. J. Hsia, and A. Needleman	Oscillatory crack growth in glass	April 1999
904	Fried, E., and S. Sellers	Microforces and the theory of solute transport	April 1999