Effects of a vertical magnetic field on chimney convection in a mushy layer

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Abstract

Nonlinear compositional convection in cyclindrical chimneys within a mushy layer during alloy solidification is investigated under an externally imposed magnetic field in the vertical direction. These chimneys produce freckles in the final form of the solidified materials. Freckles are imperfections that reduce the quality of the solidified materials. The undesirable effects of the chimneys can be reduced by reducing the effects of convection within the chimneys. Asymptotic and scaling examinations are done to weakly nonaxisymmetric convection in the chimneys. The effects of the externally imposed magnetic field is represented by the Chandrasekhar number Q. It is found that for sufficiently large values of Q, convection and the volume flux in the chimneys decrease with increasing Q. Conditions on Q and other parameters are determined under which the walls of the chimneys can or cannot be in the vertical direction.

1 Introduction

Convection effects during alloy solidification are known to be important. The convective flow affects the solid-liquid content within thin mushy layer adjacent to the solid-liquid interface and influences the flow pattern and the critical conditions for the generation of flow instabilities within the liquid zone of the solidification system. It is important to reduce the undesirable effects of convection as much as possible for the solidified system and also develop controlling procedures to prevent formation of localized channels, also called chimneys, within the mushy layer for such system, since it is known that chimney convection can lead to serious defects and imperfections in the solidified materials. These chimneys are vertical channels of negligible solid fraction and they eventually become locations of severe compositional nonhomogenities which in their final form are called freckles. Freckles are imperfections that unterrupt the uniformity of the solidified materials causing areas of mechanical weakness.

Worster [1, 2] developed and analysed the governing equations for a mushy layer in the asymptotic limit of large solutal Rayleigh number. He proposed a model in which there is downward flow everywhere in the mushy zone, except in and near localized chimneys. The chimneys support upward convective flow driven by compositional buoyancy. Chimney convection is governed by the Navier-Stokes, continuity, temperature and solute equations under Boussinesq approximation. Using asymptotic and scaling analyses, Worster [2] derived simple solutions, based on the governing equations for the flow within and outside the chimneys, to determine the structure of the mushy layer for strong natural convection. These results were determined under the restriction of infinite Prandtl number. Riahi [3, 4, 5] extended the model developed by Worster [2] to more general flow cases and under both normal [4] and high gravity environments [3, 5].

The magnetohydrodynamic effects on convection during solidification are of interest to the crystal growing community. In industrial crystal growth processes it is desirable to impose certain external constraints, such as rotation and/or magnetic field(s), in an optimized manner, upon the solidification system in order to suppress or at least reduce the undesirable effects of convection which can lead to micro defect density in the crystal and, thus, reduce the quality of the produced crystal. The possible beneficial effects of rotation has already been explored by Riahi [3, 5] for the solidification system. It was found that the external constraint of rotation can reduce the undesirable effects of convection in the mushy zone and in the chimneys, provided the axis of rotation is inclined with respect to the so-called high gravity axis, which is supposed to be perpendicular to the average location of the free surface of melt, and, furthermore, the values of the Coriolis parameter T and the centrifugal acceleration parameter A should lie in certain domain in (T, A)-space. However, this later condition may be difficult to be met in practice. Thus, it is quite desirable to explore the possibility of beneficial effects of a magnetic field in the solidification system. The present paper under takes such investigation by taking into account the effects of a vertical magnetic field on chimney convection in a mushy layer. We have found interesting results and, in particular, predicted certain domains for the magnetic field parameter Q (Chandrasekhar number) under which the stabilizing effects of the field can be effective in reducing the strength of the chimney convection.

2 Basic equations

The formulation for the solidification system used here follows most closely the approach of Worster [1, 2] in the absence of magnetohydrodynamic effects. The mushy region is treated thermodynamically as a single continuum phase. The temperature and the composition of the interstitial liquid are assumed to be uniform over length scales of the interdentric spacing. No expansion or contraction upon changes of phase is assumed, the solute diffusion D_s in the solid phase is ignored, and the volume changes upon change of phase is also ignored. The reader is referred to Worster

[2] for supporting arguments and justification regarding these and other assumptions and modeling formulation of the present solidification system.

Within the mushy zone we take into account continuity equation in its non-divergence form for the volume flux of the interdendric fluid. The mushy layer is considered as a porous medium, and Darcy's law is used to describe the equations for the fluid flow within the mushy zone and outside the chimneys. The permeability Π of the mushy layer is, in general, a function of the liquid fraction $(1-\phi)$, where ϕ is the volume fraction of solid dendrites or simply solid fraction. However, an explicit form of the functional dependence of Π on ϕ is not needed in our present analysis. In any case, Π decreases with increasing ϕ.

We consider a thin mushy layer adjacent to solidifying surface of a binary alloy melt and of thickness $\tilde{h}$. The binary alloy melt of constant composition c_0 and constant temperature T_∞ is solidified at a constant rate V_0, with the eutectic temperature T_e at the position $z = 0$ held fixed in a frame moving with the solidification velocity in the vertical z-direction which is anti-parallel to the direction of the gravity force. A representative figure for the solidification system is the figure 1 given in [3] and will not be repeated here.

The magnetohydrodynamic aspect of the physical model is based on the Maxwell equations combined with the governing equations for the convective flow that are given in Chandrasekhar [6]. The governing system of equations for the solidifying system, subjected to an external magnetic field $B\hat{z}$ in the vertical direction of uniform strength B, is non-dimensionalized using $V_0, K/V_0, K/V_0^2, \beta\Delta c\rho_0 gK/V_0$, Δc, ΔT and B as scales for velocity, length, time, pressure, solute, temperature and magnetic field, respectively. Here K is the thermal diffusivity, $\Delta c = c_0 - c_e, \Delta T = T_L - T_e, c_e$ is the eutectic concentration, T_L is the local liquidus temperature at $c = c_0$, $\hat{z}$ is a unit vector in the positive z-direction and β is the expansion coefficient for solute.

Since we will be concerned mainly with convection in the melt and in any cylindrical chimney, whose axis is assumed to be parallel to the z-axis, we consider the governing equations in a cylindrical coordinate whose axial direction is along the z-axis. We also consider weakly non-axisymmetric flow of the type developed by Riahi [3]. For the analysis of convection in a cylindrical chimney, we shall assume that the chimney's axis coincides with the z-axis. The investigation will be based on asymptotic and scaling analyses in the limit of strong compositional buoyancy, negligible thermal buoyancy and large Lewis number K/D, where D is the solute diffusivity.

The non-dimensional form of the basic equations for the hydromagnetic convective flow of the melt in the liquid zone above the mushy zone as well as inside the chimneys are given below in their steady state form which will be analysed in this paper

$$\frac{1}{P_r}\left(-\frac{\partial}{\partial z} + \boldsymbol{u}\cdot\nabla\right)\boldsymbol{u} = -HR(\nabla p + s\hat{z}) + \nabla^2\boldsymbol{u} + \frac{HQ}{\tau}\left(\frac{\partial}{\partial z} + \boldsymbol{b}\cdot\nabla\right)\boldsymbol{b}, \tag{1a}$$

$$\left(-\frac{\partial}{\partial z}+\boldsymbol{u}\cdot\nabla\right)\boldsymbol{b}=\left(\frac{\partial}{\partial z}+\boldsymbol{b}\cdot\nabla\right)\boldsymbol{u}+\frac{1}{\tau}\nabla^2\boldsymbol{b}, \tag{1b}$$

$$\nabla\cdot\boldsymbol{u}=0, \tag{1c}$$

$$\nabla\cdot\boldsymbol{b}=0, \tag{1d}$$

$$\left(-\frac{\partial}{\partial z}+\boldsymbol{u}\cdot\nabla\right)\theta=\nabla^2\theta, \tag{1e}$$

$$\left(-\frac{\partial}{\partial z}+\boldsymbol{u}\cdot\nabla\right)s=0, \tag{1f}$$

where $\boldsymbol{u}=u\hat{r}+v\hat{\xi}+w\hat{z}$ is the velocity vector, u is the radial component of $\boldsymbol{u}$, $\hat{r}$ is a unit vector in the radial r-direction, v is the azimuthal component of $\boldsymbol{u}$, $\hat{\xi}$ is a unit vector in the azimuthal ξ-direction, w is the axial component of $\boldsymbol{u}$, $\boldsymbol{b}$ is the induced magnetic field, $\boldsymbol{b}=(b_r,b_\xi,b_z)$, $p=\tilde{p}/\rho_0+|(B\hat{z}+\boldsymbol{b})|^2\mu/(8\pi\rho_0)$ is the modified pressure, ρ_0 is the reference density (a constant), μ is the magnetic permeability, $\tilde{p}$ is the pressure, $s=c/\Delta c$, c is dimensional solute concentration, θ is the temperature, $P_r=\nu/K$ is the Prandtl number, ν is the kinematic viscosity, $R=\beta\Delta cgK^2/(V_0^3\nu H)$ is the solutal Rayleigh number, g is the acceleration due to gravity, $H=K^2/(V_0^2\Pi_0)$ is a non-dimensional parameter representing ratio of a liquid type Rayleigh number to that of a mush type Rayleigh number, Π_0 is a reference value of the permeability Π of the porous medium, $Q=\mu B^2K^2/(4\pi\rho_0\nu\eta V_0^2H)$ is the Chandrasekhar number, $\tau=K/\eta$ is the Roberts number and η is the magnetic diffusivity.

The non-dimensional steady state form of the basic equations for the mushy zone outside the chimneys are

$$\boldsymbol{u}/\Pi=-R(\nabla p+s\hat{z})+\frac{Q}{\tau}\left(\frac{\partial}{\partial z}+\boldsymbol{b}\cdot\nabla\right)\boldsymbol{b}, \tag{2a}$$

$$\left(-\frac{\partial}{\partial z}+\boldsymbol{u}\cdot\nabla\right)\boldsymbol{b}=\left(\frac{\partial}{\partial z}+\boldsymbol{b}\cdot\nabla\right)\boldsymbol{u}+\frac{1}{\tau}\nabla^2\boldsymbol{b}, \tag{2b}$$

$$\nabla\cdot\boldsymbol{u}=0, \tag{2c}$$

$$\nabla \cdot \boldsymbol{b} = 0, \tag{2d}$$

$$\left(-\frac{\partial}{\partial z} + \boldsymbol{u} \cdot \nabla\right) \theta = \nabla^2 \theta - S_t \frac{\partial \phi}{\partial z}, \tag{2e}$$

$$\frac{\partial}{\partial z}\left[(1-\phi)(c_r - s)\right] + \boldsymbol{u} \cdot \nabla s = 0, \tag{2f}$$

where $S_t = L/(\tilde{c}\Delta T)$ is the Stefan number, $\tilde{c}$ is the specific heat per unit volume, L is the latent heat of solidification per unit volume, $c_r = (c_s - c_0)/\Delta c$ is a concentration ratio, and c_s is the composition of the solid phase forming the dendrites. The boundary conditions for the solidification system are given in Worster [2] and for the induced magnetic field can be of the form given in Riahi [7]. These boundary conditions are not repeated here since their explicit use are not needed in the present analyses though the resulting leading order solutions are consistent with such boundary conditions. Analysis in [2] indicates that $\theta = s$ in the mushy zone outside the chimneys which is also valid here.

In the next section we shall proceed with asymptotic and scaling analyses for the equations given in this section in the asymptotic limit of sufficiently large R, to determine the strongly nonlinear steady state for weakly non-axisymmetric flows in the mushy zone and mainly in the chimneys in three ranges for high (or moderate) P_r values. It will be assumed mostly that the Roberts number τ is small as is the case in the laboratory experiments, though the order one value of τ will not be discarded. The value of the Chandrasekhar number Q will be assumed arbitrary and, as will be seen in the next section, certain parameter regimes for Q are predicted under which chimney convection behaves quite differently. The analyses presented in the next section are based on the assumptions of the type already given in [3, 5], and a number of results remain the same as those predicted in [3, 5]. Consequently, we shall refer to [3, 5] whenever there is no need to provide those already given in [3, 5] and, instead, we shall present explicitly those new results, which are mostly due to the presence of the magnetic field.

3 Analysis and results

Consider a chimney, whose axis coincides with the vertical z-axis, and its radius $a(\xi, z)$ is small ($a << 1$). The maximum orders of magnitude of r, ξ and z are considered to be a, 1 and 1 (or less than one), respectively. Assuming the magnitude of the flow velocity to be of order one in the mushy layer, then (2a) implies that to the leading term pressure field in the mushy layer is unaffected by the flow velocity and $\theta = s$ is independent of r and ξ. The equations (2b) and (2e)-(2f) for the assumed r and ξ independent leading order variables $\theta_0(z), \phi_0(z), w_0(z)$ and $b_{z0}(z)$ then imply equation (4a,b) given in [5] plus the following equations

$$-w_0' + (w_0 - 1)b_{z0}' = b_{z0} \cdot w_0' + \frac{1}{\tau} b_{z0}'', \tag{3a}$$

$$(1 - w_0)b_{z0}'' + w_0''(1 + b_{z0}) + b_{zo}'''/\tau = 0, \tag{3b}$$

where a prime denotes differentiation with respect to z. As in the earlier work in [2], $c_r >> \theta$ is assumed in the mushy zone.

A weakly non-axisymmetric flow assumes [3] that the azimuthal velocity does not exceed the order of magnitude of the radial velocity and the azimuthal derivative of any dependent variables is much smaller than the dependent variable itself. Under such assumption, (1c)-(1d) imply that, to the leading terms, stream functions $\psi(r, \xi, z)$ and $h(r, \xi, z)$ for the flow velocity and the induced field, respectively, can be introduced, so that

$$(u, w) = \left(-\frac{1}{r}\frac{\partial \psi}{\partial z}, \frac{1}{r}\frac{\partial \psi}{\partial r} \right), \tag{4a}$$

$$(b_r, b_z) = \left(-\frac{1}{r}\frac{\partial h}{\partial z}, \frac{1}{r}\frac{\partial h}{\partial r} \right). \tag{4b}$$

For flow in the chimney, it is assumed that $s \sim 1$ and $w >> 1$. Then scalings of the form (6) in [3] follow (1a). The assumption that the inertia terms in (1a) can be, at most, as large as the viscous terms in (1a) together with the above scaling in [3] imply that

$$P_r \geq 0(HRa^4). \tag{5}$$

This is the range under which the analysis of the present study is valid and is classified as the high (or moderate) P_r range [5].

3.1 Case of strong field

Due to a comparison of the magnetic field and buoyancy terms in (1a), under the assumptions that the nonlinear magnetic field term $\boldsymbol{b} \cdot \nabla \boldsymbol{b}$ is, at most, as large as the $\frac{\partial b}{\partial z}$ and $b_z \leq 0(1)$, we find

$$Q \geq 0(R\tau), \tag{6}$$

$$b_z \sim R\tau/Q, \quad b_r \sim aR\tau/Q, \quad h \sim a^2 R\tau/Q. \tag{7}$$

The range (6) is classified here as strong magnetic field range where the strength B of the external field is assumed to be sufficiently large that (6) is satisfied.

Designating $\theta_1(r, \xi, z)$ to be the deviation of θ from θ_0, using (1e) or (2e) and the condition on HR given by (7) in [3], we find $\theta_1 << 1$. Using these results, (1a), (1c), (1e) and (1f), we simplify (1e), integrate it in r from $r = 0$ to $r = a$ and follow ref. [2]. We then find the result (9) in [3].

Assuming

$$\tau \leq 0(1) \tag{8}$$

and designating $b_{z1}(r, \xi, z)$ to be the deviation of b_z from b_{z0}, then the simplified form of (1b) implies $b_{z1} << 1$. Integrating the simplified form of (1b) in r from $r = 0$ to $r = a$, we find

$$b_{z1} \sim -\tau\psi_a' 1_n r, \tag{9a}$$

where

$$2\pi\psi_1 = \int_0^a 2\pi r w d_r \tag{9b}$$

is the vertical volume flux in the chimney.

Using (4a), we find (10) in [3], while (4b) implies

$$b_r \sim -h_a'/r \quad \text{at} \quad r \to a, \tag{10a}$$

where

$$2\pi h_a = \int_0^a 2\pi r b_z d_r \tag{10b}$$

is the vertical magnetic flux in the chimney. Using (2a) and (10), we find

$$\Delta_r p \sim (\psi_a'/R)1_n a + [Q/(\tau R)][(h_a')^2/(2a^2) - (1_n a)(1 + b_{z0})h_a''], \tag{11}$$

where $\Delta_r p$ represents the radial pressure difference near the wall of the chimney. Using (9) and the result for θ_1 given in [3], we find that (2a) yield an expression for w which can be simplified to the form

$$w \sim -R(\psi_a + \psi_a'' Q/R) 1_n a. \tag{12}$$

This result holds near the wall of the chimney, and it implies the following condition in an average sense

$$Q \leq 0[\min(R, R/\lambda_1)], \quad \lambda_1 \equiv |\langle \psi_a'' \rangle / \langle \psi_a \rangle|, \tag{13}$$

which is also a range of the validity of the present analysis. Here an angular bracket denotes a vertical average from $z = 0$ to the top of the chimney at $z = z_0$.

Using (2f) and the condition $c_r >> 1$ [3], (2f) is simplified and (14)-(17) in [3] follow under further restrictions that $u << 1$ and $\phi c_r \sim 1$ [3] near the wall of the chimney and in the mushy zone. In particular

$$w \sim 1. \tag{14}$$

The following results in this paragraph hold near the wall of the chimney. Using (14), the scaling relations (6) given in [3], the result (9) in [3] and making use of (9) and (12), we find

$$a \sim \{[RH(R + \lambda_1 Q)] \, |1_n [RH(R + \lambda_1 Q)]|\}^{-1/4}, \tag{15}$$

$$\theta - \theta_0 \sim -1/(R + \lambda_1 Q), \tag{16}$$

$$b_z - b_{z0} \sim \frac{-\tau Q (\psi_a''/\psi_a)'}{(R + \lambda_1 Q)^2}. \tag{17}$$

$$u \frac{\partial \theta_1}{\partial r} \sim \left[\frac{HRa^2}{(R + \lambda_1 Q) 1_n a} \right]. \tag{18}$$

Thus, $u\frac{\partial \theta_1}{\partial r}$ term in the simplified form of (2f) is negligible if the right-hand-side in (18) is small. For

$$HRa^2 \geq 0[(R + \lambda_1 Q)|1_n a|], \tag{19}$$

all the main three terms in the simplified form of (2f) [3] must balance and, thus,

$$w \sim R^2 H^2 a^6. \tag{20}$$

The result in (20) is more restricted than the result in (14) since the former is under the condition of (19). Using (12), (20) and the scaling relations given by (6) in [3], we find

$$a \sim \left[\left(\frac{R+Q\lambda_1}{RH}\right)\left|1_n\left(\frac{R+Q\lambda_1}{RH}\right)\right|\right]^{1/2}. \tag{21}$$

Using (21) in (11) and taking into consideration the scalings (6) given in [3], we obtain

$$\Delta_r p \sim a^2\left[(1+\lambda_1 Q/R)\,(1_n a)^2\right] + \left(\frac{Q}{\tau R}\right)\left[(h_a')^2/(2a^2) - (1+b_{z0})h_a''(1_n a)\right]. \tag{22}$$

Vertical and horizontal advection of solute balance here in this regime where (19) holds. Using (2e), (20) and (21), we find

$$\frac{\partial \phi}{\partial z} \sim \frac{-(R+\lambda_1 Q)^3}{RH}\left[1_n\left(\frac{R+\lambda_1 Q}{RH}\right)\right]^3. \tag{23}$$

The right-hand-side in (23) is small if

$$\left[(R+Q\lambda_1)\,1_n\left(\frac{R+Q\lambda_1}{RH}\right)\right]^3 << RH. \tag{24}$$

Following [3], defining the wall of the chimney as zero values of $(1-\phi)$ and taking derivative with respect to z of ϕ, we find

$$a'/a \sim \frac{\partial \phi}{\partial z}. \tag{25}$$

Thus the wall of the chimney is in the axial direction to the leading order terms if (24) holds, while it can not be concluded such results from (23) and (25) if (24) does not hold.

Following [2, 4], we find that the total volume flux $2\pi\psi_a$ in the chimney, due to upward flow, is satisfied, to the leading terms, by the following equation:

$$\frac{\partial^2 \psi_a}{\partial z^2} + \tilde{s}^2 \psi_a = -16\gamma R(1+\theta_0)/(Q 1_n a), \tag{26a}$$

$$\tilde{s} = 4/[a^2(-HQ1_n a)^{1/2}], \tag{26b}$$

where γ is a positive constant of order one. The condition $\psi_a' \geq 0(\psi_a)$ then implies the following restriction on the parameter regime

$$a^4 HQ|1_n a| \leq 0(1). \tag{27}$$

The solution to (26a), subjected to the initial conditions

$$\psi_a = \frac{\partial \psi_a}{\partial z} + 16\gamma R B_0/(Q1_n a) = 0 \quad \text{at} \quad z = 0, \tag{28}$$

is found by standard methods [8] to be

$$\psi_a = \left(\frac{-16\gamma R}{Q1_n a}\right) \left\{ B_0 z + (1/\tilde{s}) \int_0^z \sin(\tilde{s}z - \tilde{s}\eta)[1 + \theta_0(\eta) - \tilde{s}^2 B_0 \eta] d\eta \right\}. \tag{29}$$

Here B_0 is a positive constant.

3.2 Case of moderate field

Due to a comparison of the magnetic field and buoyancy terms in (1a), under the assumptions that the nonlinear magnetic field term $\boldsymbol{b} \cdot \nabla \boldsymbol{b}$ dominates over $\frac{\partial b}{\partial x}$ term and $b_z >> 1$, we find

$$Q << R\tau, \tag{30}$$

$$b_z \sim (R\tau/Q)^{1/2}, \quad b_r \sim a(R\tau/Q)^{1/2}, \quad h \sim a^2(R\tau/Q)^{1/2} \tag{31}$$

The range (30) is classified here as moderate magnetic field range where the strength B of the external field is assumed to be moderately small that (30)-(31) are satisfied.

A number of results presented in the previous subsection are also valid here, and thus, we present here only those results which are new and different from those given in the subsection 3.1. But one result, which is essentially valid for both cases of moderate and strong field and was not presented in the previous subsection, is based on the induction equation (1b) for the flow in the chimney. Assuming $b_{z0}(z)$ to be the leading order variable for b_z then (1d) implies

$$b_{r0} = -(r/2)b'_{z0}, \tag{32}$$

where $b_{r0}(r, z)$ is the leading order variable for b_r. Using (30) in (1b), simplify, multiply by r and integrate in r from $r = 0$ to $r = a$ in the chimney. We then find

$$[a^2/(2\tau)]b''_{z0} + (a^2/2)b'_{z0} + \psi'_a(b_{z0} + 1) = 0. \tag{33}$$

The results (32)-(33) are valid for both moderate and strong field regimes.

Using the result for θ_1, referred to in the subsection 3.1, in (2a) and simplify, we find

$$w \sim -R\theta'_0\psi_a 1_n a + [Q/(2\tau)](b^2_{z0})'. \tag{34}$$

Following the analysis given in the subsection 3.1, the results (10)-(11) and (14) follow here as well.

The following results in this paragraph hold near the wall of the chimney. Using (14), the scaling relations (6) given in [3], the result (9) in [3] and making use of (34), we find

$$a \sim \left[\frac{1-\lambda_2 Q/(2\tau)}{R^2 H}\right]^{1/4}\left\{1_n\left[\frac{1-\lambda_2 Q/(2\tau)}{R^2 H}\right]\right\}^{1/2}, \lambda_2 \equiv |\langle(b^2_{z0})'\rangle|, \tag{35}$$

$$\theta - \theta_0 \sim [-1 + (b^2_{z0})'Q/(2\tau)]/R, \tag{36}$$

$$u\frac{\partial\theta_1}{\partial r} \sim (RHa^3)^2. \tag{37}$$

It is seen from (2f) and (37) that $u\frac{\partial\theta_1}{\partial r}$ term in the simplified form of (2f) is negligible if the right-hand-side in (37) is small. For

$$(RHa^3) \geq 0(1), \tag{38}$$

all the main three terms in the simplified form of (2f) [3] must balance and, thus (20) follows. The result (20) for the present moderate field case is more restricted than (14) since the former is under the condition of (38). Using (20), (34) and the scaling relations given by (6) in [3], we find

$$a \sim \begin{cases} [Q\lambda_2/(2\tau H^2 R^2)]^{1/6} & \text{for} \quad Ha^2 >> |1_n a| \\ \left[\left(\frac{Q\lambda_2}{2HR^2\tau}\right)/1_n\left(\frac{Q\lambda_2}{2HR^2\tau}\right)\right]^{1/4} & \text{for} \quad Ha^2 \leq 0(|1_n a|). \end{cases} \tag{39}$$

Using (39) in (9), (11), the expression for θ_1 given in [3] and taking into consideration the scalings (6) given in [3], we can obtain the result for $\Delta_r p, \theta_1$ and b_{z1} for condition under which (37) holds in the present case.

Vertical and horizontal advection of solute balance here in this regime where (38) holds. Using (2e), (20) and (39), we find

$$\frac{\partial \phi}{\partial z} \sim \begin{cases} \left(\frac{Q\lambda_2}{2\tau}\right) & \text{for} \quad Ha^2 >> |1_n a| \\ \frac{\sqrt{H}}{R}\left[\left(\frac{Q\lambda_2}{2\tau}\right)/1_n\left(\frac{Q\lambda_2}{2\tau HR^2}\right)\right]^{3/2} & \text{for} \quad Ha^2 \leq 0(|1_n a|). \end{cases} \tag{40}$$

Following the analysis of the previous subsection, it follows that the wall of the chimney is in the axial direction to the leading order terms if the right-hand-side in (40) is small in respective range of values for Ha^2, while it can not be concluded such result if the right-hand-side in (40) is not small.

Following [2, 4], we find that the total volume flux in the chimney, due to upward flow, is given by

$$2\pi\psi_a = 2\pi Ha^4[\gamma R(1+\theta_0) - Qb_{z0}b'_{z0}/(16\tau), \tag{41}$$

where γ is a positive constant of order one. It is seen that (41) is coupled with (33). A solution to these equations was attempted subjected to the appropriate boundary conditions for b_{z0} treating ψ_a a given function of z to the leading term. This solution is lengthy and will not be given here, but it indicates that ψ_a is independent of Q to the leading term after substituting back the solution for b_{z0} in (41).

3.3 Case of weak field

Under the assumption that the Lorentz force in (1a) is small in comparison with the buoyancy term, we find that (30) still holds, but (31) is replaced by

$$b_z << (R\tau/Q)^{1/2}, \quad b_r \sim ab_z, \quad h \sim a^2 b_z. \tag{42}$$

Following the analysis presented in the previous subsection, we find that various results are simplified under the conditions (42) and, in particular, (34) is replaced by

$$w \sim -R\theta_0'\psi_a 1_n a, \tag{43}$$

(35)-(36) are replaced by

$$a \sim [R^2 H 1_n (R^2 H)]^{-1/4}, \tag{44}$$

$$\theta - \theta_0 \sim -1/R, \tag{45}$$

(37)-(38) remain the same, (39)-(40) are replaced by

$$a \sim (H^{-1} 1_n H)^{1/2}, \tag{46}$$

$$\frac{\partial \phi}{\partial z} \sim (RHa^3)^2, \tag{47}$$

and, finally, (41) is reduced to

$$2\pi\psi_a = 2\pi\gamma a^4 RH(1+\theta_0). \tag{48}$$

The results (43)-(48) all agree with those in the absence of a magnetic field obtained by Worster [2].

3.4 Discussion of the results

The analyses, presented in the last three subsections, were based on the condition (5) for P_r which was classified earlier [5] as the high (or moderate) P_r regime. As can be seen from the presented analyses, all the results were found to be independent of P_r, to the leading terms, in this P_r regime. For the solidification problem under an external constraint of rotation [5], it was also found that the results were independent of P_r, to the leading terms, in the regime (5).

The equations (3a) and (3b), which are derived respectively from the axial and radial components of the induction equation (2b) for the flow in the mushy zone and outside the chimneys, represent the leading term for the axial component of the induced magnetic field for a given $w_0(z)$. The expression for $w_0(z)$ is given by [2, 4, 5]

$$w_0 = -2\pi\psi_a N,$$

where N is the number density of all the chimneys in the mushy zone. This result is based on the principle of mass conservation that the downward flow through the mushy zone and outside the chimneys must be equal to the total upflow through all the chimneys per unit horizontal area. It should also be noted that (3a) and (3b) are not independent and (3b) can be derived directly by taking derivative with respect to z of (3a).

For the strong magnetic field case, where the Lorentz force is mainly due to the externally imposed field and the range (6) holds, there are essentially two different regimes classified here as regime I, where mainly (14)-(15) hold, and regime II, where mainly (20)-(21) hold. The result (29) for the volume flux in the chimney hold for both of these regimes. It can be seen from (15),(21) and other results given for the strong field case that these results depend on λ_1 which is due to the result (29). Restricting the strong field limit to the case, under which only inequality sign in (27) holds, we determined the order of magnitude of different quantities, dependence on different parameters and dependence on the strength of the magnetic field of various results, by making use of approximated version of (29), and found, in particular, the following main results to the leading order terms. For the case where the regime I holds, $\lambda_1 Q$ is independent of Q, and the radius of the chimney is independent of Q but decreases with increasing either R or H. The non-azimuthal speed of the flow, defined by

$$V_n = \sqrt{u^2 + w^2},$$

decreases with increasing Q but increases with either R or H. The volume flux in the chimney decreases with increasing either Q or R (or H). For the case where the regime II holds, $\lambda_1 Q$ is again independent of Q, and the radius of the chimney is independent of Q and decreases with increasing H. However, a is independent of R for $\lambda_1 Q << R$, while it decreases with increasing R for $\lambda_1 Q \sim R$. Based on the results for λ_1 in the case of regime II, the condition (24) does not appear to be satisfied in the asymptotic range of sufficiently large R, and, thus, the radius of the chimney can depend on z. The non-azimuthal flow speed in the chimney decreases with increasing either H or Q and increases with R. The volume flux in the chimney decreases with increasing either Q or H and increases with R.

For the moderate magnetic field case, there are again two regimes I and II, where mainly (14) and (35) hold for the regime I and (20) and (39) hold for the regime II. The result (41) for the volume flux in the chimney hold for both of these two regimes. It can be seen from (35) and (39) that these results depend on the coefficient λ_2 which is due to the results (33) and (41). As we stated in the previous subsection, these later two equations for b_{z0} and ψ_a were solved subjected to the boundary conditions

$$b_{z0} = b'_{z0} = 0 \quad \text{at} \quad z = 0.$$

The results indicated that ψ_a is independent of Q to the leading order terms and $b_{z0} \sim [R\tau/Q]^{1/2}$ for both of regimes I and II. However, ψ_a decreases with either R or H in both of the regimes I and II, except for $Ha^2 >> |1_n a|$ in the regime II, where ψ_a actually increases with R. In both of the regimes I and II, the radius of the chimney decreases with increasing either R or H, but it is independent of Q. The quantity $\lambda_2 Q/(\tau R)$ is a constant of order one. The non-azimuthal flow speed in the chimney is independent of Q in both of the regimes I and II. However, it increases with either R or H in regime II, except that it is independent of H for $Ha^2 >> |1_n a|$ in the regime II. In the regime I, it decreases with increasing either R or H. It can be seen from (40) that the right-hand-side term for $Ha^2 >> |1_n a|$ is of order R and is definitely not small. Thus the wall of the chimney can not be in the axial direction in the range II for $Ha^2 >> |1_n a|$.

For the case of a weak magnetic field, $\lambda_2 < 0(\tau R/Q)$. The radius of the chimney decreases with increasing either R and H in the regime I, while it is independent of R and decreases with increasing H in the regime II. The volume flux in the chimney decreases with increasing either R or H in the regime I, while it increases with R and decreases with increasing H in the regime II. The non-azimuthal flow speed in the chimney increases with H and decreases with increasing R in the regime I, while it increases with R and decreases with increasing H in the regime II. All the main results to the leading order terms are independent of Q and τ. These results all agree with those in the absence of a magnetic field due to Worster [2].

4 Some conclusions

(i). Three cases of strong, moderate and weak magnetic field effects were detected and analysed in this paper which effectively correspond, respectively, to the conditions

$$Qb_z \sim R\tau(Q \geq 0(R\tau)),$$

$$Qb_z^2 \sim R\tau(Q << R\tau)$$

and

$$Qb_z^2 << R\tau(Q << R\tau).$$

The effect of the imposed magnetic field becomes significant only in the case where the field is strong.

(ii). The presence of the externally imposed magnetic field in the vertical direction and with a uniform strength is either stabilizing, in the case of the strong field, or is ineffective, in the case of the moderate (or weak) field as far as the leading order effects in the asymptotic limit of sufficiently large R are concerned.

(iii). The results of the present study, which are based on the moderate or small values of the Roberts number τ, indicated that the main results, such as those for the radius of the chimney, the flow speed in the chimney and the volume flux in the chimney, are mostly insensitive with respect to τ to the leading order terms, while results about the induced field and the pressure difference across the chimney depend significantly on τ.

(iv). The results of the present study, which are based on the moderate or large values of the Prandtl number P_r, indicated that all the main results are independent of P_r to the leading order effects. This result agrees with those in the absence of a field and in the absence [2, 4] or presence of an externally imposed rotation [3, 5] that P_r effect is insignificant so long as the condition (5) is satisfied.

(v). The results of the stabilizing effects of the strong magnetic field reported in this paper agree with the results of related magneto convection studies [7, 9, 10, 11, 12] that presence of externally imposed vertical magnetic field, with sufficiently strong strength, in buoyancy driven flows can lead to significant weakening of the convection. However, the extent that such stabilizing effects of the magnetic field can be practical or be desired in an actual crystal growth application will require quantitative studies, and optimized flow controlling procedure may also suggest applications of combined externally imposed magnetic field and rotation upon the solidification system.

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