EFFECTS OF ROTATION ON CONVECTION IN A POROUS LAYER DURING ALLOY SOLIDIFICATION

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ABSTRACT

The topic of this chapter is that of effects of an external constraint of rotation on convection, driven mainly by compositional buoyancy, in a porous layer adjacent to the solid-liquid interface during directional solidification of a binary alloy. In solidification literature such a porous layer is referred to as a mushy layer. The analyses carried out by several studies in the past on the subject of this chapter and the subsequent results are reviewed first, the latest results unpublished elsewhere are presented briefly, and then in the second-half of the chapter, investigation of effects of rotation on convection in a horizontal porous layer of melt and dendrite solids is carried out subject to a simple model. The results based on this simple model indicate that Coriolis force can have stabilizing effects on both stationary and oscillatory mode of convection, while the oscillatory mode of convection experiences an additional destabilization due to the Coriolis force effect.

KEYWORDS

Convection, rotating convection, porous medium, alloy solidification, Coriolis effects, natural convection, porous layer, rotating fluid

INTRODUCTION

Convection effects during alloy solidification are known to be important (Davis 1990). The convective flow affects the solid-liquid content within a porous layer, which exists adjacent to the solid-liquid interface, and influences the flow pattern and the critical conditions for the generation of flow instabilities in the solidification system. It is important to reduce the undesirable effects of convection as much as possible for the solidified system and also find ways to prevent formation of localized chimneys within the porous layer for such systems, since it is known that chimney convection can lead to defects and imperfections in the final produced crystals.

The rotational effects on the convective flow instabilities during alloy solidification have been of interest to the crystal growers for a number of years. In industrial crystal growth processes it has been desirable to impose certain external constraint, such as rotation, in an optimized manner, upon the solidified system in order to reduce the effects of flow instabilities or oscillations which can lead to microdefect density in the crystal and thus reduce the quality of the produced solidified material. Theoretical results on the effects of rotation about a vertical axis on the flow of melt during alloy solidification and in the normal gravity environment (Riahi 1993, 1994), indicated conditions under which rotation may stabilize the convective flow. Computational studies about the effects of rotation about a vertical axis of a horizontal layer of flow of melt during alloy solidification (Neilson and Incropera 1993) indicated stabilization of vertical plumes and their lack of meandering due to such rotational constraint. Sample and Hellawell (1982, 1984) did Nh_4cl alloy experiment in a cylindrical mold with a chilled bottom surface where solidification was induced. They applied a rotation and tilting technique to change the orientation of the force of gravity relative to the bottom surface of the cylinder. They observed that for slow and steady rotation of the mold about the vertical axis, which coincided with the axis of the cylinder, the chimney formation and development was about the same as in the case without rotation. However, for slow and steady rotation of a tilted mold about a vertical axis, the number of chimneys was reduced substantially and, under some conditions, completely eliminated. Fast rotation case is not considered beneficial to the crystal growers, as, for example, experimental results due to Kou et al. (1978) indicated that if the rate of rotation became too large, then segregates were formed along a ring between the axis and the outer edge of the ingot in their experimental apparatus.

The experimental results referred to in the previous paragraph and the indication for the possible usefulness of inclined rotational constraint applied on the solidified system, where the axis of rotation is inclined with respect to the direction of the gravity force, led to further research studies. As described in some details in the next section, some recent studies at the onset of convection in a layer of melt rotating about an axis, which was inclined with respect to the effective gravity vector (Sayre and Riahi 1996, 1997; Okhuysen and Riahi 2000), aimed at understanding centrifugal or Coriolis force effects and investigated, in particular, flow instabilities, due to either stationary or oscillatory mode of disturbances of the porous layer adjacent to the solidification front during alloy solidification. Results of numerical computations of these studies indicated preference of particular modes of convection, and rotational constraint was found to be effective only if rotation axis was inclined with respect to the effective gravity vector. Here by the effective gravity vector it is meant the resultant normal gravity vector and an average component vector of the centrifugal force in the direction perpendicular to the rotation axis. Another interesting result of these studies was that the spatial locations in the porous layer, which had tendency to form chimneys, changes as the rate of rotation was changed. This result suggested a possible industrial operational procedure for elimination of chimney's formation tendency to be application of an external constraint of rotation with a variable rate of rotation acting on the solidified system.

A different type of studies, based on scaling analyses and asymptotic methods in the limit of sufficiently large solutal Rayleigh number, for the natural convection in the porous layer adjacent to the liquid-solid interface, were found to be an appropriate procedure in a high gravity environment (Regel and Wilcox 1997) where porous-Rayleigh number can become sufficiently large for sufficiently high rotation rate of a centrifuge system which can carry the solidified layer. Such studies, which are described with some details in a later section in this chapter, were carried out by Riahi (1997, 1998, 1999) and provided qualitative results about the effects of centrifugal and Coriolis forces on the compositional convection in the porous layer during alloy solidification. A particularl and notable result of these studies, which indicated that Coriolis' force effect can have different types of influence on the flow stability depending on the rotation sense of the centrifuge, were found to agree with some recent experimental and computation studies on the subject (Ma et al. 1994, Tao et al. 1994).

DOUBLE-LAYER MODEL

The investigation carried out by Sayre and Riahi (1996, 1997) and the corresponding results are described briefly in this section followed by some ongoing research investigation and preliminary results due to Okhuysen and Riahi (2000). A layer of a binary alloy melt of some composition C_0 and temperature $T\infty$ is considered which is solidified at a rate V_0, with the eutectic temperature T_e at the position z=0 held fixed in a frame moving with the solidification speed in the z-direction, where the z-axis is assumed to be anti-parallel with the high gravity vector (Figure 1).

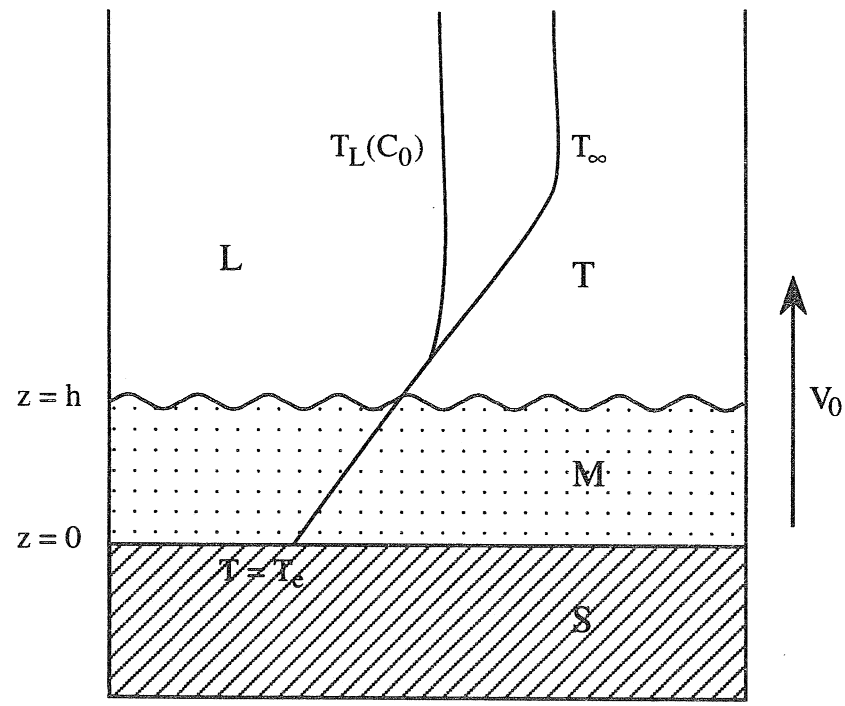

Figure 1. A diagram representing the directional solidification of an alloy at speed V_0. A porous layer between a solid region, where $T < T_e$ and a liquid region. The profiles for dimensional temperature and the local liquidus temperature T_L are also shown. L, M and S denote, respectively, liquid, mush (porous) and solid regions.

The investigated double-layer model, which consists of a non-porous layer, referred to as the liquid layer, and a porous layer, referred to as the mushy layer, at normal gravity condition is based on the assumptions of the type considered by Worster (1992), and the extension under a high gravity condition in based on the assumptions of the type considered by Arnolds et al. (1992) for solidification system in a centrifuge. The porous layer adjacent to the solidifying surface is of thickness h(x,y,t) where t is the time variable and the x and y axes are in a plane (z=0) perpendicular to the z-axis. The solidifying system is placed in a centrifuge basket rotating at some angular velocity Ω about the centrifuge axis which makes an angle γ with respect to the z-axis. The centrifuge axis is anti-parallel to the earth gravity vector (Figure 2).

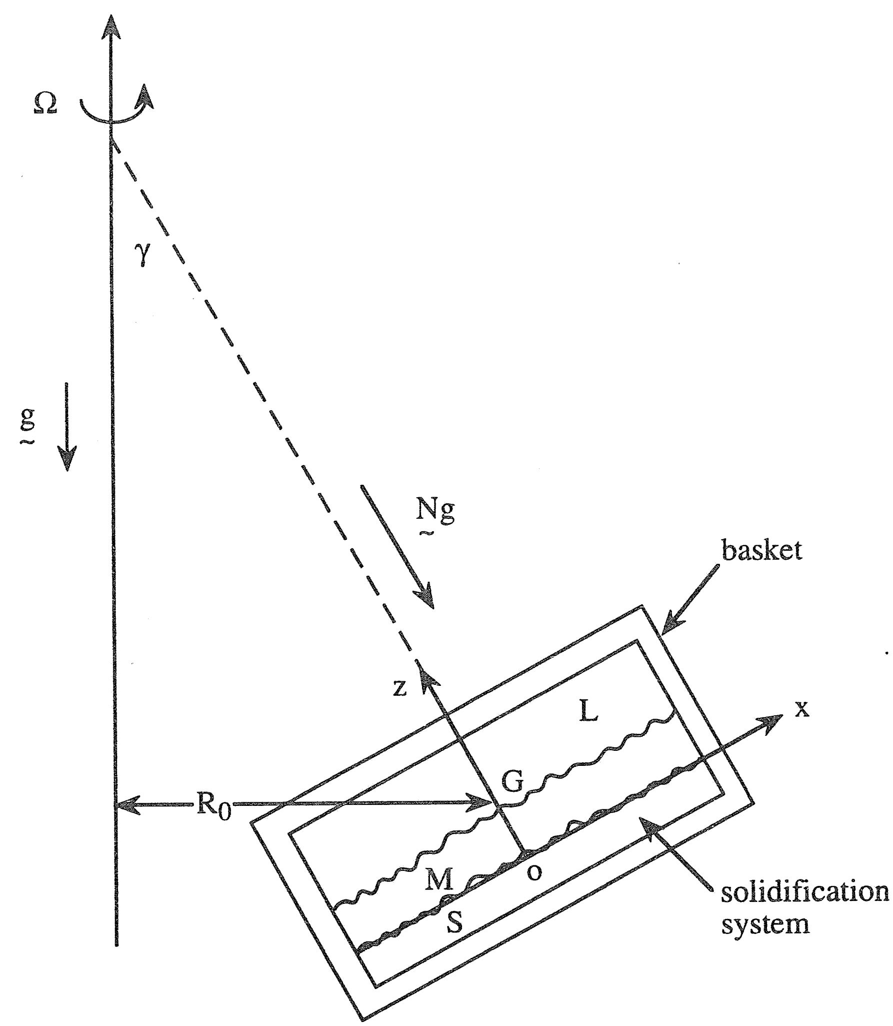

Figure 2. Solidification system in a centrifuge. G denotes the center of gravity of the centrifuge basket.

Next, the equations for momentum, continuity, heat and solute are considered for both non-porous layer ($z > h$) and porous layer ($0 < z < h$) in the moving frame oxyz whose origin o is centered on the solid-mush interface (z=0). The governing system of these equations for the solidifying system rotating with the centrifuge basket (Arnold et al. 1992) and translating with the solidification front at speed V_0 is non-dimensionalized by using $V_0, K/V_0, K/V_0^2, \beta\Delta C\rho_0 gK/V_0, \Delta C$ and ΔT as scales for velocity, length, time, pressure, solute and temperature, respectively. Here K is the thermal diffusivity, ρ_0 is a reference density, $\beta = \beta^* - \Gamma\alpha^*$, where α^* and β^* are the expansion coefficients for heat and solute respectively and Γ is the slope of the liquidus curve, $\Delta C = C_0 - C_e, C_e$ is the eutectic concentration of the alloy, $\Delta T = T_L(C_0) - T_e$ and T_L is the local liquidus temperature. Due to the variations of density with respect to both solute concentration and temperature, the centrifugal acceleration terms in the momentum equation cannot be converted into passive gradient terms and become important at significant rotation rate. The centrifugal acceleration term in the momentum equation is split into an average term, which is superimposed on the normal gravity term and a so-called gradient acceleration term (Arnold et al. 1992). For the porous layer Darcy's law is adopted in the governing equations.

The non-dimensional form of the governing equations are subjected to the appropriate boundary conditions of the type given in Worster (1992). Simplifying assumptions of negligible temperature contribution in the buoyancy force, $\beta \approx \beta^*$ and $K >> D$ are then considered, where D is the solute diffusivity. The non-dimensional system contains a number of variables and parameters which are defined as follows. The vector $\underset{\sim}{u}$ is the velocity vector, P is the pressure, C is the solute concentration, θ is the temperature, $P_r = \nu/K$ is the Prandtl number, ν is the kinematic viscosity, $R = \beta\Delta C N_g K^2/(V_0^3\nu H)$ is the solutal Rayleigh number, $N_g = (g^2 + \Omega^4 R_0^2)^{1/2}$ is the acceleration due to high gravity, $N_g = g$ corresponds to normal gravity case while $N_g > g$ indicates level of high gravity, g is the acceleration due to normal gravity, R_0 is the perpendicular distance from the center of gravity G of the centrifugre basket to the rotation axis, R_0 is a function of γ and $R_0 = 0$ where $\gamma = 0$ degree, $H = K^2/(V_0^2\Pi_0)$ is a non-dimensional parameter representing ration of R in the liquid zone inside the chimneys or above the porous layer to that in the porous layer outside chimneys, Π_0 is a constant reference value of the permeability $\Pi(\phi)$ of the porous medium, ϕ is the solid fraction of the porous zone, $T = 2\Omega K^2/(V_0^2\nu H)$ is the Coriolis parameter, which is square root of a Taylor number, $A = \beta\Delta C\Omega^2 K^3/(V_0^4\nu H)$ is the gradient acceleration parameter due to the centrifugal force, $\theta_\infty = T_\infty/\Delta T, E = D/K$ is the inverse of Lewis number, $S_t = L/(\tilde{C}\Delta T)$ is the Stefan number, $\tilde{C}$ is the specific heat per unit volume, L is the latent heat of solidification per unit volume, $C_r = (C_s - C_0)/\Delta C$ is a concentration ratio, and C_s is the composition of the solid phase forming the dendrites in the porous layer. Due to the liquidus relationship, which holds to a good approximation in the porous layer, $\theta = C$ in the porous zone outside the chimneys.

The flow solution examined by Sayre and Riahi (1996, 1997), was in the limit of small rotation rate and for zero Coriolis effect. The solution as sum of a base flow solution, which was at most a function of z (Worster 1992), and a normal mode type solution for disturbances whose dependence in the plane perpendicular to z-axis was of the form exp (B), where $B = i(wt + \alpha_1 x + \alpha_2 y)$. Here i is the pure imaginary number ($\sqrt{-1}$), ω is the frequency of the disturbances and $\underset{\sim}{\alpha} = (\alpha_1, \alpha_2)$ is the wave number vector of the disturbances. The linear system for disturbances is considered in the limit of sufficiently small amplitude of disturbances. Each equation and boundary condition is multiplied by exp(-B) and then average of the resulting system is obtained, with respect to the x and y variables in both layers. The solution of the esulting system were then determined numerically for both stationary (Sayre and Riahi 1996) and oscillatory (Sayre and Riahi 1997) disturbances.

Due to the complexity of the disturbance system, a numerical code, of the type applied by Worster (1992), was developed by Sayre and Riahi (1996), to solve the eigenvalue problem and determine the eigenfunctions and eigenvalues of the linear system in the neutral stability limit. Here the numerical approach is described briefly. A new independent variable $\tau = \theta_\infty - \theta_0 (0 \leq \tau \leq \tau_e)$ is defined, where θ_0 is the base flow solution for $\theta, \tau_e \equiv 1 + \theta_\infty$ and $\tau = 0$ and τ_e correspond, respectively, to $z = \infty$ and 0. Using this new variable, the governing system becomes a system of ordinary differential equations for the disturbance variables as functions of the independent variable τ. The new form of the stability system is conveniently over a finite domain in τ, but it has a regular singular point at $\tau = 0$. To take into account this feature of the system, any disturbance variable is assumed to be product of τ^m and a function of τ in the liquid region, $0 < \tau < \tau_i$, where $\tau_i \equiv \theta_\infty/(1-E)$ corresponds to value of $z = h_0$ and m is a root of the indicial equation. Here h_0 is the constant value of h in the absence of disturbances. Using this numerical procedure in the liquid region, four linearly independent solutions for the dependent variables were found which satisfy the boundary conditions at $\tau = 0$ and with the corresponding values $m_i (i = 1, 2, 3, 4)$ of m. When $m = m_i$, the corresponding boundary values of the scaled dependent variables at $\tau = 0$ are fouond from the governing system for these variables. In addition, for each value of $m = m_i$, a Taylor-series expansion of the governing equations for these variables about $\tau = 0$ was applied to determine the first three derivatives of these variables at $\tau = 0$. These results allowed numerical evaluation of the governing equations for the scaled variables in the liquid region to be started from the asymptotic expressions for the scaled dependent variables near $\tau = 0$. Sayre and Riahi (1996, 1997), applied an efficient fourth-order Runge-Kutta scheme for this purpose. For each value of m_i, the governing equations were integrated from $\tau = 0$ to $\tau = \tau_i$. Next, the interface boundary conditions between the liquid and mush regions were used to relate the dependent variables in the porous layer at $\tau = \tau_i$ to the dependent variables in the non-porous layer. These values were used to start the numerical integration of the equations for the dependent variables in the porous layer at $\tau = \tau_i$, which continued until $\tau = \tau_e$. However, it was found that the resulting solution does not, in general, satisfy all the boundary conditions. Thus the remaining boundary conditions were used to compute the so-called residuals $r_{ij} (i, j = 1, 2, 3, 4)$ corresponding to the index m_i (Worster 1992). The determinant det of the matrix $[r_{ij}]$ is then computed, and R is varied until det=0. The corresponding solutions are eigenfunctions of the stability system which represent the marginally stable states of the system.

To determine the results for the stationary disturbances, Sayre and Riahi (1996) set $\gamma = 30^o, S_t = C_r = \theta_\infty = 1, P_r = 10$ unless otherwise stated and $\Pi(\phi) = 1$. The eigenvalue relation Det=0 then provided a marginal stability curve $R(\alpha), \alpha \equiv (\alpha_1^2 + \alpha_2^2)^{1/2}$ for each choice of the parameters E, H and A. The parameter E is the inverse of the Lewis number and is typically very small. The parameter H is a representative of the square of the ratio of the thermal length scale, on which the depth h of the porous layer depends, to the average spacing between dendrites within the porous layer (Worster 1992). This parameter is typically very large. The gradient acceleration parameter A was assigned the values 0.0, 0.1, 0.3 and 0.6. The values chosen for $S_t, C_r, \theta_\infty, P_r$, E and H are similar to those values chosen by Worster (1992). The results for the neutral stability curve, R versus α, were found as function of A and for values of E=0.025 and $H = 10^5$. The system is unstable in the region above the neutral curve and stable below the curve. Just as in the case of zero rotation (Worster 1992), the marginal curve for each value of A has two minima, corresponding to two distinct modes of convection. The first mode, corresponding to the smaller α value, was called long wavelength mode since its wavelength was comparable to h and causes flow throughout the porous layer, while the second mode, corresponding to the larger α value, is called short wavelength mode since its wavelength is comparable to the depth of the compositional boundary layer ahead of the mush-liquid interface and leaves the fluid within the interstics of the porous medium essentially stagnant (Worster 1992). These properties were confirmed by the observation of the streamlines to be discussed below. It was found that R increases with A for a given α and the wave numbers of the two modes increase with A. Hence, rotation has the familiar effects of increasing the critical values of R and α (Chandrasekhar 1961). Streamlines for the two convection modes corresponding to the local minimima of the neutral stability curves were determined as functions of A. It was found from these results that, for A=0.0 case, fluid flows deeply in the two layers for the long wavelength mode, while flow is restricted to a thin region about the interface between the porous and non-porous zones for the short wavelength mode. For $A \neq 0.0$ cases, fluid flows in both layers and is stronger in the porous layer for the long wavelength mode. As rotation increases, flow is stabilized more strongly in the non-porous layer. For the short wavelength mode, the flow is restricted to the porous layer only, but such flow is stronger close to the interface between the porous and non-porous zones for larger rotation rate. Vertical and horizontal velocity data were also determined in both layers for both of the convection modes. It was found that the flow speed in the non-porous layer is generally larger than the corresponding one in the porous layer, for zero-rotation case, while the opposite is generally true for the non-zero rotation cases. Also for each case the flow speed for the long wavelength mode is generally larger than the corresponding one for the short wavelength mode. Density data of the solid fraction in the porous layer for both convective modes and for different values of A were also determined. It was found that the short wavelength mode causes less perturbation to the solid fraction than the long wavelength mode. The long wavelength mode was, thus, mostly associated with the solid fraction perturbations. For this mode, there was a substantial decrease in the solid fraction in the interior of the porous layer in regions of up flow which indicates a tendency to form chimneys. The information provided by these data as function of A indicated that the spatial locations in the porous regions which have a tendency to form chimneys, that is regions corresponding to negative perturbation to the solid fraction that represent local melting of the dendrites, changed as rotation rate changed. This interesting result suggested an important operational procedure for possible elimination of chimneys formation tendency to be a variable rotational rate constraint applied on the solidifying system. The relative stability of the two convection modes were found to vary considerably with the values of H and E. A particular interpretation of H is as a measure of the relative mobility of fluid in the melt region to that in the porous layer. Thus, increasing H causes the melt region to be more unstable relative to the porous layer. The results from the marginal stability curves for various values of H with E=0.025 held fixed confirmed such interpretation of H. It appeared that the long wavelength mode was the most critical (unstable) one for sufficiently strong rotation or for sufficiently small H, while the short wavelength mode was the most unstable one for weak rotation provided H was sufficiently large. In regard to variation of E, it should be stated that the main effect of such variation is to change the thickness of the compositional boundary layer ahead of the mush-liquid interface relative to the depth of the porous layer, and the thickness of the compositional boundary layer decreased with decreasing E. The results for the marginal stability curves for various values of E and A with $H = 10^5$ indicated that the convection modes become more stabilized as E decreased. The wavelength of the short wavelength mode decreased with decreasing E. Rotational effects were seen to be minimized for some intermediate values of E. The results described so far were for the cases of aqueous solutions where $P_r = 10$ is representative. However, metallic alloys, which are of commercial interest, have a representative value of 0.02 for P_r. Marginal stability curves for two cases of $P_r = 10.0$ and $P_r = 0.02$ indicated that the system is more stable at lower P_r and that the wavelength of the long wavelength mode for the lower P_r is smaller than the corresponding one for the higher P_r. Since the inverse Prandtl number $(1/P_r)$ measures the strength of the advection or diffusion of vorticity generated by buoyancy, due to the solidification growth rate velocity V_0 and thus, the smaller the value of P_r, the larger is the advection of such vorticity towards the solid boundary, which acts as a sink of vorticity due to the no-slip condition. Thus the system tends to be more stable for smaller P_r.

To determine the results for the oscillatory disturbances, Sayre and Riahi (1995, 1997) set $\gamma = 30^o, S_t = C_r = \theta_\infty = 1$ and $\Pi(\phi) \equiv 1$. The eigenvalue relation det=0 then provided a marginal stability curve $R(\alpha)$ or $R(\omega)$ for each choice of values of other parameters. The results for neutral stability curves, R versus α, in the absence of rotation indicated that the most critical modes are non-oscillatory for $P_r = 10$. However, for $P_r = 0.02$, the marginal stability curve was found to have a minimum which corresponded to an oscillatory mode of convection. This oscillatory mode was found to be preferred over the stationary modes and corresponded to $\omega = 1.6$ or $\omega = -1.6$ since the stability system was found numerically to be insensitive with respect to sign of ω as long as rotational effects were absent. The system for $P_r = 0.02$ was found to be more unstable than the one for $P_r = 10.0$ in a certain range of α of the convection modes. For non-zero rotation rate it was found that there are two preferred oscillatory modes of convection with distinct different wavelengths and periods. It was found that rotation was destabilizing for the oscillatory modes and that R values for the marginal curve were no longer symmetric with respect to $\omega = 0$ axis. The preferred oscillatory mode had a small wavelength and α increases with A. The case with $P_r = 0.02$ corresponded to smaller R without rotation, while the case with $P_r = 10.0$ corresponded to smaller R with rotation. The rotational effect made the convection cells slightly inclined with respect to z-axis. The preferred oscillatory modes tended to be more concentrated close to the solid-mush interface. The fluid was found to flow in both layers for the preferred oscillatory mode with longer wavelength, while flow was restricted to the porous zone for the preferred oscillatory mode with shorter wavelength. The flow speed was found to be more significant in the porous zone, unless the wavelength of the preferred oscillatory mode was sufficiently long. The rotational constraint, low P_r value, and short wavelength modes all led to some decrease in the amount of negative perturbations to the solid fraction, which meant less tendency for chimney formation in the porous zone. The spatial location in the porous layer, which tended to form chimneys, changed as the rotation rate changed. Rotational constraint reduced the destabilizing effect of H, and the preferred oscillatory modes of convection became more stabilized as E decreased.

Okhuysen and Riahi (2000) developed a more sophisticated numerical code to determine the eigenvalue and eigenfunction of the neutral stability system with higher accuracy. They included the effects of the Coriolis force. Their preliminary investigation for the stationary disturbances considered the case $A = 0, \gamma = 30^o, P_r = 10.0, E = 0.025, H = 10^5, S_t = C_r = \theta_\infty = 1$ and $\Pi(\phi) \equiv 1$. They found, in particular, that rotation resulted in higher critical value R_c, local increase in solid fraction near the liquid-mush interface was reduced from the normal gravity case, and regions of reduced solid fraction that were surrounded by regions of increased solid fraction in the porous zone were 'opened up' under rotation.

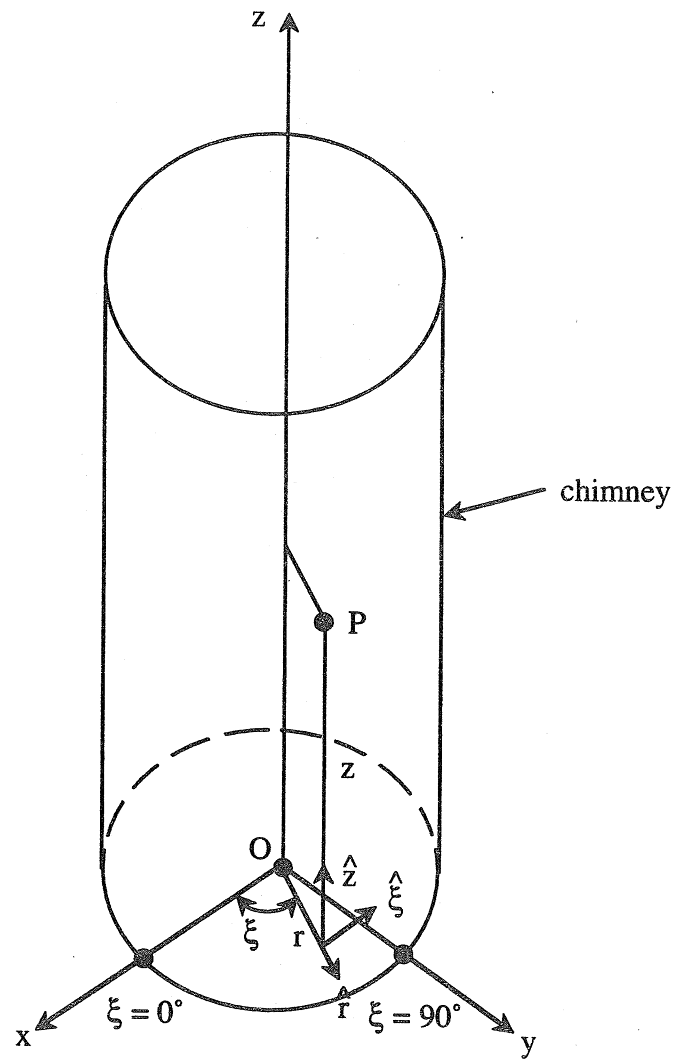

Figure 3. This figure provides sketch of a cylindrical chimney in the porous layer, coordinate system, whose z-axis coincides with the axis of the chimney, cylindrical coordinate (r, ξ, z) of a point P of the chimney, unit vectors $(\hat{r}, \hat{\xi}, \hat{z})$ and two locations of the chimney where the azimuthal angle ξ equals 0 and 90 degrees.

CHIMNEY MODEL

The investigation carried out by Riahi (1997, 1998, 1999, 2001a) and the corresponding results are described briefly in this section. The same type of double-layer model described in the previous section is considered here. But here we are concerned mainly with the convection in any cylindrical chimney, whose axis is assumed to be parallel to the z-axis, within the porous layer. The governing equations are, thus, considered in a cylindrical coordinate whose axial direction is along the z-axis. Riahi (1997) considered weakly nonaxisymmetric convection in a cylindrical chimney, whose axis coincided with the z-axis (Figure 3), and in the porous layer in the asymptotic limit of the strong compositional buoyancy force, negligible thermal buoyancy and sufficiently large P_r and E^{-1}. Asymptotic and scaling analyses were applied to chimney convection to determine the results qualitatively about the effects of both Coriolis and centrifugal forces on the convective motion within chimneys in the porous layer. It was found that, for some moderate values of the rotation rate and $\gamma \neq 0$, axial convection in the chimneys decrease rapidly with increasing A above some azimuthally dependent axial level. The axial convection was also found to decrease with increasing T in certain range which depended on γ, T and R. The Coriolis force effect was destabilizing for T outside this range of values. The work carried out in Riahi (1997) was based on the assumption that the flow of melt was under certain derived parameter regimes and the radius a of any chimney was an independent parameter with prescribed value. Riahi (1999) investigated theoretically, using asymptotic and scaling analyses, the effects of centrifugal and Coriolis forces, due to arbitrary inclined rotational constraint, on a nonaxisymmetric chimney convection at finite values of P_r and presented the qualitative results for the case where the derived parameter regime implied that the chimney's radius a was a function of the other parameters. Riahi (1999) found that there were P_r and rotation-dependent ranges where the wall of chimneys may or may not be vertical. There were also $\gamma-$, P_r- and rotation-dependent ranges, where convection in the chimneys increased or decreased with increasing rotation rate. An interesting aspect of the results presented in Riahi (1999) was the way the inclined angle γ affected the chimney convection. Within certain range controlled by the axial component of the Coriolis force in the (P_r, T, A)-parameter space for small P_r melt convection in the chimneys decreased with increasing T if $\gamma < 180^o$, while chimney convection increased with T if $\gamma > 180^o$ keeping all the other parameter values the same as in the $\gamma < 180^o$ case. These results agreed with some earlier experimental studies for small P_r (Ma et al. 1994) under centrifugation, where Coriolis effect was found to have such different types of influence on the flow stability depending on the rotation sense of centrifuge. These results were also in agreement with some computational studies for $P_r = 0.02$ (Tao et al. 1994) under centrifugation, where enhancement of convection was found if centrifuge rotated counter-clockwise $(sin\gamma < 0)$, while convection was reduced if the centrifuge rotated clockwise $(sin\gamma > 0)$.

Riahi (2001a) extended his earlier asymptotic and scaling analyses to the case of weakly unsteady mode of nonaxisymmetric chimney convection in a high gravity environment and determined new analytical results for the leading order magnitudes of the azimuthal averages for the radial velocity, axial velocity and the volume flux of the vertical flow in the chimney as functions of R, P_r, T, A and γ, and for given small values of a.

SINGLE-LAYER MODEL

The oscillatory instability, detected by Anderson and Worster (1996) in a porous layer during alloy solidification and in the absence of any external constraint, was based on a simple one-layer model developed earlier by Amberg and Homsy (1993) in which the dynamics of the porous layer were decoupled from the dynamics of the overlying liquid layer. Here, we extend the model treated by Anderson and Worster (1996) by imposing an external constraint of rotation on the solidification system and study the effects of the Coriolis force on the linear convective instability which is present once R exceeds its critical value R_c.

It should be noted that the single-layer model of this section, which takes into account the rotational effects through the presence of the Coriolis force only, is relevant both in the geophysical applications where the centrifugal force is insignificant and in the engineering applications where understanding the Coriolis effects alone can be of primary interest before the combined Coriolis and centrifugal force effects can be understood.

We consider a binary alloy melt that is cooled from below and is solidified at a constant speed V_0. The solidifying system is assumed to be rotating at a constant speed Ω in the vertical direction anti-parallel to the normal gravity vector. Following Amberg and Homsy (1993) and Anderson and Worster (1996), we consider porous layer of thickness h adjacent and above the solidification front to be physically isolated from the overlying liquid and underlying solid zones. Thus, it is assumed that the horizontal porous layer is bounded from above and below by rigid and isothermal boundaries. We consider governing system in a moving frame translating at the speed V_0 with the solidification front and rotating with the speed Ω along the vertical axis.

The non-dimensional form of the governing system contains the non-dimensional parameters R, T, S_t and C_r and, in addition, the present model contains the non-dimensional thickness δ of the porous layer as a parameter which is assumed to be small (Amberg and Homsy 1993).

Following Anderson and Worster (1996) in reducing the model asymptotically, we follow their formulation, rescale R, the dependent and independent variables, based on $\delta(\delta << 1)$, and assume that $\delta C_r = C^*$ and $\delta S_t = S$ are order one quantities as $\delta \to 0$. As discussed in Anderson and Worster (1996), the assumption of thin porous layer ($\delta << 1$) is associated with large non-dimensional far-field temperature $\theta_\infty >> 1$, which can occur when the initial concentration is close to C_e. The assumption of order one quantity C^* corresponds to the near-eutectic approximation (Fowler 1985), which allows to describe the porous layer of constant permeability to the leading order. The assumption of order one quantity S allowed Anderson and Worster (1996) to detect oscillatory instability from their analytical porous-layer model.

The above rescalings were used in the governing system. This system admits a motionaless steady basic state of the form given in Anderson and Worster (1996). The basic state solutions contains the parameters δ, C^*, R and $\lambda \equiv (S/C)+1$. Since the basic state solid fraction is found to be small, an expansion for $\Pi(0)/\Pi(\phi)$ in powers of ϕ is assumed (Amberg and Homsy 1993), where ϕ is the solid fraction of the infinitesimal disturbances superimposed on the motionless basic state. The system for such disturbances admits normal mode type solution whose non-vertical dependence is of the form exp (B). Applying such normal mode form in the disturbance system yields a system of ordinary differential equations in variables z for the z-dependence coefficients of the disturbance's dependent variables. This system contains parameters R, T, $\delta, \omega, \alpha, \lambda$ and C^*. Presence of small parameter δ in the system suggests expansions of the dependent variables, ω and R in powers of δ. The systems up to order δ are then solved to determine the results for the neutral stability system and its critical conditions.

The results for the single-layer model are given as follows. The solution $\omega = 0$ always satisfies the eigenvalue relation, so that stationary (non-oscillatory) disturbances can always be admitted by the single-layer model. The neutral stability system implies that the case $\omega_r \neq 0$ can always be possible for particular non-zero values of T regardless of the value that a parameter combination $\lambda_0 \equiv (\lambda - 1)/(C^* \lambda^2)$ may take. Here ω_r is the real frequency and it turns out that λ_0 enters the eigenvalue relation as a parameter which combines both C^* and λ. Hence, in this sense presence of rotational constraint enhances the existence of oscillatory mode since Anderson and Worster (1996) showed that $\omega_r \neq 0$ cannot exist for $\lambda_0 \leq 0.4$ in their non-rotating system. The critical value R_c of R and value $\alpha = \alpha_c$, at which $R = R_c$, are found to increase with T for both stationary and oscillatory modes. The eigenvalue relation also indicates that both ω_r and $-\omega_r$ can be the solutions, and, thus, oscillatory mode for particular values of T can be in the form of a left travelling wave, a right travelling wave, a combination of left and right travelling waves, which remains a travelling wave, or a standing wave. Presently a nonlinear stability analysis is carried out by the author (Riahi 2001b) to determine which of these and what type of the horizontal flow pattern are selected by the flow system due to the single-layer model.

CONCLUDING REMARKS

In this chapter effects of rotation on convection in a porous layer during alloy solidification was considered. The work that has been done on this subject area in the last several years was reviewed and new results of very recent investigation on this subject were reported. Different models and methods of approach have been used to determine the effects of the centrifugal force alone, Coriolis force alone, or combination of these two forces on the convective flow within the porous type layer adjacent to the solidification front. It appears that both centrifugal and Coriolis force have non-trivial stabilizing and destabilizing effects on the fluid motion within the porous layer.

From a practical point of view, chimneys convection is undersirable since it produces freckles in the final form of the solidified material. Freckles are imperfections that reduce the quality of the solidified materials. The results based on the studies of the types described in this chapter indicate that suppression or enhancement of chimneys convection can be accomplished by either the centrifugal force or the Coriolis force in particular ranges in the parameters space which are all non-trivial and can be explored by investigations of the type described in this chapter.

As experimental evidence presented in this chapter indicate, the double-layer and chimney models under high gravity environment are useful models to determine ways to control the convective flow in the porous layer during alloy solidification. It should also be noted that such inclined porous layer under high gravity condition is equivalent to the case of a horizontal porous layer subjected to an inclined rotational constraint. It is known (Chandrasekhar 1961) that for such a system two-dimensional rolls parallel to the horizontal component of the rotation vector are preferred flow pattern for an extensive range of R above R_c. Hence, in the case of double-layer model described in this chapter, it can be expected that two-dimensional rolls, parallel to the component of the rotation vector in the plane perpendicular to the z-axis, can be the preferred flow parameter over an extensive range of values for $R > R_c$.

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