



COMPLETELY BRITTLE FAILURE AND THE TECHNICAL COHESIVE STRENGTH

BY

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LIST OF EQUATIONS

<u>Equation Number</u>	<u>Page</u>	<u>Equation Number</u>	<u>Page</u>
1	18	12	34
4	26	12a	64
5	26	13	12
5.1	27	14	65
5.2	27	15	65
5.21	27	16	65
5.22	27	17	65
5.3	48	18	65
5.4	48	19	66
5.5	49	20	66
5.6	49	21	66
5.7	49, 52	22	66
5.71	49	23	63
5.72	52	24	63
5.8	49	26	63
5.9	55	27	64
6	26	28	52
7	28	28a	9
8	29	29	9, 52
9	29	30	57
9.1	40	30.1	9
9.2	40	30.2	9
9.25	41	32	74
9.3	42	1 HZ	45
9.4	42	2 HZ	45
9.5	43	3 HZ	46
9.6	44	4 HZ	46
9.7	44	5 HZ	46
9.8	44	6 HZ	47
10	29	7 HZ	47
11	33, 51	8 HZ	47
11a	51	9 HZ	47
11b	9, 51	10 HZ	47
11c	10		

TABLE OF CONTENTS

	Page
I. Introduction	
1. Definition of the Term "Technical Cohesive Strength."	1
2. General Discussion of the Problem	1
3. Purpose, Organization, and Scope of the Thesis	3
4. Acknowledgments.	4
II. Proposed Methods for the Determination of the Technical Cohesive Strength	
1. Justification for the Methods	5
2. Direct Method for Materials Whose Critical Temperature is Above the Room Temperature, i.e. For Materials Com- pletely Brittle at Room Temperature	5
3. Direct Method For Materials Ductile at Room Temperature but Fracturing in a Completely Brittle Manner Even in a Statical Tension Test of Unnotched Specimens at a Temperature Equal to or Higher Than the Temperature of Liquid Air	6
4. Direct Method For Materials which are not Yet Completely Brittle in a Statical Tension Test of Unnotched Specimens at the Temperature of Liquid Air	6
5. Criterion for "Complete Brittleness"	6
6. Suggestion of One Possible Method for Determining the Break- ing Stress in a Tension Impact test at Liquid Air Temperature .	7
7. First Indirect Method for Estimation of the Technical Co- hesive Strength of a Ductile Material	8
8. Summary of the Preceding Method	9
9. Second Indirect Method for Estimation of the Technical Cohesive Strength of Ductile Materials	10

TABLE OF CONTENTS (continued)

Page

10. Third Indirect Method for Estimation of the Technical Cohesive Strength of Ductile Materials	11
III. General Considerations for Explanation of the Proposed Methods	
1. Approaches to Technical Cohesive Strength	13
2. Approach to Technical Cohesive Strength Through Brittle Strength	13
3. The Technical Cohesive Strength is Identical with the Ultimate Strength Overcome at a Completely Brittle Fracture, No Matter How the Complete Brittleness is Caused. . . .	14
4. Definitions of the Terms "Critical Temperature of Brittleness" and "Brittle Strength"	14
5. The Technical Cohesive Strength of Materials Having a Critical Temperature of Brittleness Higher than the Temperature of Liquid Air	15
6. Test Results Showing that Brittle Strength Does not Vary with Test Temperature	15
7. Conclusions Drawn from the Tests	16
IV. Approaches to the Technical Cohesive Strength of Materials With Critical Temperatures Lower Than Those Commonly Obtainable in the Laboratory.	
1. Means of Increasing the Degree of Brittleness of Fracture . .	18
2. Increasing the Tri-Axiality of Stress State	18
3. Use of Notched Specimens to Increase the Tri-Axiality of Stress-State.	19
4. Effect of Plastic Flow on Yield Strength and Technical Cohesive Strength	20

TABLE OF CONTENTS (continued)

Page

5. Definition of "Current Cohesive Strength"	21
6. Two Possible Manners of Failure of a Notched Specimen of Ductile Metal at Room Temperature	21
7. Progressive Crack Under Single Load Application	22
V. High Rates of Straining (e.g. Impact)	
1. Increasing the Degree of Brittleness of Fracture by High Rates of Straining.	24
2. Some Experimental Data	24
3. Comparison of Embrittlement by Lowering the Test Temperature and by Increasing the Rate of Straining	27
4. Method for the Determination of the Critical Temperature of Brittleness for any Rate of Straining	28
VI. Low Test Temperatures	
1. Influence of Low Test Temperature on Yield Strength	29
2. Some Experimental Results Showing Physical Significance of Such Concepts as "Critical Temperature" and "Brittle Strength"	29
3. Comparison of "Strength vs. Temperature" Curve with "Energy Absorbed vs. Temperature" Curve	30
4. Comparison of "Energy Absorbed vs. Temperature" Curves for Different Rates of Straining.	31
5. Some Theoretical Conclusions from the Experimental Results Described Earlier	32
6. Further Experimental Results Showing the Variation of Yield Strength with Temperature	34
7. Some Experimental Results Indicating that the Technical Cohesive Strength Does not Vary with Temperature	36

TABLE OF CONTENTS (continued)

Page

8. A Proposed Test of Ioffe's Scheme	37
9. Some Experimental Results Showing That the Ultimate Tensile Strength Increases with Increasing Rate of Straining	37
VII. The Effect (upon Certain Measures of Strength) of an Increase in Straining Rate is Equivalent to the Effect of a Decrease in Temperature.	
1. Theoretical Basis for an Expression for the Rate of Plastic Deformation	40
2. Proposition of Hollomon and Zener	45
3. Analytical Justification of the First Indirect Method for Estimation of the Technical Cohesive Strength of Ductile Materials	50
4. Determination of Constant b_1 in Eqs. (11a) and (11b)	51
5. Determination of Equivalent Rates and of Q	54
6. Direct Investigations of the Influence of the Speed of Straining on the Yield Point in Mild Steels	62
7. Analytical Justification of the Third Proposed Method for Determination of the Technical Cohesive Strength of Ductile Materials Without Direct Measurement of Brittle Strength. . .	64
VIII. Experimental Test of the "Notch and Extrapolation" Method for Determination of the Technical Cohesive Strength (e.g. Kuntze's Method).	
1. Determination of the Technical Cohesive Strength of a Metal With a Critical Temperature Higher Than the Critical Temper- ature of Liquid Air	67

TABLE OF CONTENTS (continued)

Page

2. Actual Comparison of the Values of the Technical Cohesive Strength Obtained by Two Different Methods	67
3. Possible Sources of Error	68
4. Conclusion About the Validity of Kuntze's Method	69
IX. Another Experimental Test, Less Direct, of Kuntze's Method	
1. Difference in Kuntze's Values of Tech. Coh. Strength for Metals Cold Worked in Tension and in Compression	70
2. An Attempt to Check Kuntze's Results	70
3. Results of Additional Tests on Unnotched Specimens and Ex- planation of the Difference Found by Kuntze	70
X. Physical Meaning of Technical Cohesive Strength	
1. Conclusions from the Experimental Material Mentioned Earlier	76
XI. Vita	78
Bibliography	80

1. Introduction

1. Definition of the Term "Technical Cohesive Strength."

By technical cohesive strength (s_0) is understood that value of the normal unit stress (as obtained by the commonly used formulae) under the action of which a material breaks by a brittle separation fracture (i.e. all plastic deformation is absent).

2. General Discussion of the Problem.

Elastic behavior of materials has been investigated, experimentally and analytically, to a great detail. Not so much work has been done in the field of plastic behavior of materials, possibly because of the increased complexity of the process. The complexity of the phenomena of final fracture of materials, involving both elastic and plastic action (and something more in addition), is probably the reason why comparatively so little is known about this subject. Knowledge of the conditions under which materials fail by separation, and of the resistance (measured by the technical cohesive strength) which they are able to offer under various conditions, is certainly of paramount importance in engineering and it seems to be superfluous to stress it here any further.

A few more recently realized problems, in which the questions of the nature and magnitude of the technical cohesive strength are dominant, will be mentioned.

It is generally known that even materials classed usually as ductile fail, under certain conditions, in a brittle manner. Some of the conditions which have this "embrittling" influence are low temperature, impact, tension stress in three mutually perpendicular directions, and repetition of stress (fatigue). The tendency of a member (made from a material of a given chemical composition) to

fail in brittle manner can be controlled in several ways. Some of the ways are the thermal and mechanical processing of the material as a whole and of the surface of the finished member, the geometrical shape given to the member, manner of the cutting or shaping the member, polish and surface coatings applied to it, etc. To be able to devise and to use intelligently processes which extend the limits within which the member can resist fracture, a yardstick for measuring this ability to resist brittle fracture is obviously needed. This measure of the resistance to brittle fracture is the technical cohesive strength. For the same purpose, an insight into the physical mechanism of separation fracture is needed.

The yield and ultimate strength values and the measures of ductility, etc., of materials obtained by the usual means, do not furnish by themselves sufficient information about the resistance to fracture under the various sets of conditions at which the material can be used. The resistance to brittle fracture of the material is under certain conditions higher, under other conditions considerably lower than the ultimate strength. Ultimate strength can be sometimes regarded as the technical cohesive strength which the material possesses under a definite set of conditions which do not necessarily correspond to those under which the material will be used.

It is clear that all the above mentioned extremely important problems cannot be solved or even discussed in one paper or by one person. The following are necessarily very limited remarks having only a bearing on the problems mentioned.

3. Purpose, Organization and Scope of the Thesis.

The principal purpose of this thesis is to state a number of original methods, proposed by the author, for the determination of the technical cohesive strength of materials of engineering use. The proposed methods are briefly summarized in the next chapter. The experimental and analytical justifications for the methods are given in the remainder of the thesis. Included also are some discussions of other methods for the determination of the technical cohesive strength, and some conclusions about the physical nature of strength of materials, drawn from the experimental data reviewed herein.

It should be stressed that the methods for the determination of technical cohesive strength proposed by the author, are of tentative nature.

For several years, the author has been following closely the work done by a number of investigators in this field. Other work has kept the author from running a rather extensive series of tests to check the conclusions obtained by him when interpreting the experimental work of others in a different way.

The author has been encouraged by others to feel that the (merely) theoretical conclusions (and the description of the experimental evidence for them, derived from the work of a number of investigators) are of sufficient importance to warrant their publication as a thesis. It has been pointed out that the work of practically all of the men in this field (e.g. Kuntze, MoAdam, and others) started by publication of a paper (or several papers) reviewing and reinterpreting in a new way the experimental work done by others. The author begs for forgiveness for lack of modesty

consisting in that he believes (upon encouragement of others) that there are a few original contributions in this thesis.

4. Acknowledgments.

This thesis was prepared under the supervision of Professor H. F. Moore to whom the author is deeply indebted for encouragement, inspiration and guidance, as well as introduction to this general field of endeavor.

Deep appreciation is expressed to Professor F. B. Seely, whose quest for clarification of this complex problem of Mechanics of Materials made this thesis possible, and who extended to the author help, advice and favors too numerous to list here.

In the body of the thesis proper (which follows the brief statement of the steps involved in following the proposed methods for determination of the technical cohesive strength) the acknowledgments to sources are indicated by numbers in parentheses. These refer to the references in the Bibliography at the end of the thesis.

For a reader, who is not interested in following the derivations and reasoning leading to the proposed methods, but who merely wishes to see what the proposed methods amount to, it is suggested that he read Chapter II.

To someone, who wishes to follow the reasoning and derivations, it is suggested that he omit Chapter II for the time being and start with Chapter III.

For easier finding of pages with the equations referred to in the text, the List of Equations is inserted in front of the thesis. The author begs forgiveness for the clumsy system of numbering equations.

II. Proposed Methods for the Determination of the Technical Cohesive Strength.

1. Justification for the Methods.

The proposed methods are stated in this Chapter in a summary manner, arranged in numbered sequence of steps that are suggested to be carried out in the determination of the technical cohesive strength. The body of the thesis following this chapter gives the description of experimental data and analytical reasoning on which the proposed methods are based. It was attempted to include enough material to make clear the reasons and justifications for each proposed method or step.

2. Direct Method* For Materials Whose Critical Temperature is Above the Room Temperature, i.e. For Materials Completely Brittle at Room Temperature.

Determine the value of the ultimate strength S_u of a well centered unnotched tensile specimen in statical test. This value is also the value of the technical cohesive strength, because no plastic flow takes place (the fracture is completely brittle). This method is further discussed in (III, 2.)**

Example: Some varieties of gray cast iron, materials like "plaster of Paris".

* The author begs to be excused for including this very obvious and trivial procedure here. As it may serve as the first obvious stepping stone to lead possibly to other less obvious conclusions, and as its very simplicity might cause overlooking it, it is included here.

** This notation means: Chapter III, Art. 2.

3. Direct Method For Materials Ductile at Room Temperature but Fracturing in a Completely Brittle Manner Even in a Statical Tension Test of Unnotched Specimens at a Temperature Equal or Higher Than the Temperature of Liquid Air.

Determine the value of the ultimate strength S_u of a well centered unnotched tensile specimen in statical test performed at the temperature of liquid air. This value of S_u is also the value of the technical cohesive strength of the material.

Example: High phosphorus steel, 0.1 per cent C and 0.25 per cent P.

Further discussion: Chapter III, Art. 3.

4. Direct Method For Materials Which Are not Yet Completely Brittle in a Statical Tension Test of Unnotched Specimens at the Temperature of Liquid Air.

Determine the value of breaking stress S_b at the liquid air temperature during an unnotched impact tension test with sufficiently high rate of straining to cause completely brittle fracture. This value of S_b is identical with the value of the technical cohesive strength of the material.

Example: Mild Steel.

Further discussion: Chapter III, Art. 3 and following.

5. Criterion of "Complete Brittleness".

For fractures produced at rates of straining obtainable with screw power or hydraulic testing machines (statical rates of straining), the criterion of "complete brittleness" is "elastic behavior of the specimen up to break."

In the case of impact tests, the criterion of "complete brittleness" is the fact that the energy absorbed is zero when the break is completely brittle. As the ordinary impact tests (Charpy, Izod)

involve a considerable loss of energy, one (of several) possible ways out is to use a cable-suspended impact tester, e.g. like the one described by Southwell.⁽²⁰⁾

6. Suggestion of One Possible Method for Determining the Breaking Stress in a Tension Impact Test at Liquid Air Temperature.

Use unnotched tensile specimens with, e.g., carbon strip⁽²³⁾ or Baldwin Southwark SR-4, dynamic type, wire resistance strain gauges of shortest possible gauge length (or similar gauges made of materials suitable for low temperatures) cemented directly across the critical cross section. Record the readings of strain from a cathode ray oscilloscope or oscillograph⁽²⁴⁾ on a film moving with high speed. Use assumption that the modulus of elasticity is unchanged at high rates of straining⁽²¹⁾ and at low temperatures.

To obtain breaking stress, multiply the breaking unit strain by the modulus of elasticity E . This method eliminates the measurement of the force, which is the stumbling block of the high-speed tests. The short gauge length is suggested to eliminate the influence of any variation of strain (and stress) along the specimen from the stationary end to the end which is forced to move.

De Forest⁽³⁰⁾ states that the electric strain-sensitive strips used by him appear to give reliable measurements up to the rate of transmission of strain waves in solid metal.

No appreciable temperature rise during high speed tests⁽²⁵⁾ should take place if the fracture is completely brittle, since then no plastic flow takes place and therefore no energy is dissipated in heat. The heat dissipated in damping is small.

7. First Indirect Method For Estimation of the Technical Cohesive Strength of a Ductile Material.*

- (1) Use an ordinary tensile testing machine, preferably with a recording device for load-deformation.
- (2) Obtain (see Fig. 0.1) $s_{yv'T_0}$, $s_{yv'T_\ell}$, $s_{yv''T_0}$, $s_{yv''T_\ell}$, viz. the lower yield points or yield strengths (offset) for each two head speeds (lowest rate of straining v' , highest easily attainable rate v'') at each of two temperatures, room temperature T_0 deg. absolute and some low temperature T_ℓ (as low as possible).
- (3) Plot the logarithms of the yield strengths $s_{yv'T_0}$, $s_{yv''T_0}$ at room temperature (see Fig. 0.2) as OA and OB against the logarithms of the rates of straining v' , v'' , as OC, OD. Through the points E and F thus located, draw a straight line. Mark off on the straight line the points G and H, whose ordinates equal the logarithms of $s_{yv'T_\ell}$ and $s_{yv''T_\ell}$ (OJ and OK). Read off on the axis of abscissas the magnitudes OL, OM of those two velocities which (according to Chapter VII, Article 3) in combination with the room temperature T_0 are equivalent to the combination of v' at temperature T_ℓ (abscissa reading $\log_{10} v_{T_0}$ equiv. to v' at T_ℓ) and to the combination of v'' at temperature T_ℓ (abscissa reading $\log_{10} v_{T_0}$ equiv. to v'' at T_ℓ).

Find Q from the equation (30)**

* Based on discussion in Chapter VII, especially in Article 3.

** For pages, on which the equations referred to in text are located, please refer to the List of Equations in front of the thesis.

$$Q = 4.515 \frac{\log_{10} v(T_0 \text{ equiv. to } v' \text{ at } T_\ell) - \log_{10} v'}{1/T_\ell - 1/T_0} \quad (30.1)$$

Check by using the other set of values, viz:

$$Q = 4.515 \frac{\log_{10} v(T_0 \text{ equiv. to } v'' \text{ at } T_\ell) - \log_{10} v''}{1/T_\ell - 1/T_0} \quad (30.2)$$

- (4) Find the constant b_1 from equation (29)

$$b_1 = \frac{\log_{10}(s_{yv, T_\ell}) - \log_{10}(s_{yv, T_0})}{1/T_\ell - 1/T_0}$$

- (5) Use a number of unnotched tensile impact specimens. Use the lowest temperature obtainable in the laboratory (e.g. temperature of liquid air). Designate this temperature as T_0''' . Gradually increase the impact straining rate (without necessarily measuring the rate yet) until the breaks become completely brittle. Then repeat once with this (so far unknown as to its magnitude) critical impact straining rate and measure this rate v''' (e.g. by an electrical resistance strain gage and cathode ray oscillograph).
- (6) Solve for $1/T_0'$, the reciprocal of the critical temperature of brittleness at the low head speed strain rate v' , from equation (28), viz.

$$1/T_0' = 1/T_0''' - \frac{4.515 \log_{10} \frac{v'}{v'''}}{Q} \quad (28a)$$

- (7) Obtain the technical cohesive strength $s_{o, p=0}$ (for explanation of terms see Chap. IV, Art. 4) from eq. (11b),

$$\log_{10} s_{o, p=0} = \log_{10} s_{yv, T_0} + b_1 (1/T_0' - 1/T_0) \quad (11b)$$

8. Summary of the Preceding Method.

The previously mentioned equations (11b), (29), (5.72) and 10.
(30) boil down to one equation, namely the following Eq. (11c).

For the estimation of the technical cohesive strength when using Eq. (11c) alone, the following steps are necessary:

- (1f) Values of yield points $s_{yv'T_0}$, $s_{yv'T_\ell}$, $s_{yv'T_0'}$, $s_{yv'T_\ell'}$ must be determined by testing at head speeds v' and v'' , at each of the temperatures T_0 (room temperature and T_ℓ in degrees absolute (same as steps (1) and (2))).
- (2f) The value v_{T_0} equiv. to v' at T_ℓ is to be found from a graph similar to Fig. C.2 (which is equivalent to Fig. 15), by the procedure given in Step (3) above and justified in Article 3, Chapter VII.
- (3f) The value of T_0'' is to be chosen as some easily maintained low temperature and the corresponding critical rate of straining v''' determined experimentally as explained in Step (5) above.
- (4f) Substitute the above determined values in the equation (11c) as follows:

$$\log_{10} s_{e,p=0} = \log_{10} s_{yv'T_0} + \log_{10} \frac{s_{yv'T_\ell}}{s_{yv'T_0}} \frac{T_\ell(T_0 - T_0''')}{T_0'''(T_0 - T_\ell)} - \frac{\log_{10} \frac{v'}{v'''}}{\log_{10} \frac{v_{T_0} \text{ equiv. to } v' \text{ at } T_\ell}{v'}} \quad (11c)$$

9. Second Indirect Method for Estimation of the Technical Cohesive Strength of Ductile Materials.*

- (1s) Determine by test $s_{yv'T_0}$, the statical yield strength of the

* Based upon the discussion of Chap. VII, Art. 6.

material in tension at room temperature and at some low rate of straining v' .

- (2s) Using some high (impact) rate of straining v'' , find T_0'' , the critical temperature of complete brittleness of unnotched tensile specimens of the material considered. This is the temperature when the energy absorbed during the impact test becomes practically equal to zero.
- (3s) Determine T_0''' , as in (2s), but with a different rate of straining v''' .
- (4s) Plot the result of (2s) and (3s), using T_0'' and T_0''' in degrees absolute, on a graph similar to Fig. 3. Extrapolate and determine $\log_{10} v_{T_0}$.
- (5s) Make a tensile test, with unnotched specimen of the material considered, at as high a rate of loading v'''' as possible, and determine $s_{yv''''T_0}$.
- (6s) Plot the pairs of values $(v', s_{yv'T_0})$ and $(v''''', s_{yv''''T_0})$ on a graph similar to Fig. 14 and draw a straight line. On this straight line find a point corresponding to the ordinate $\log_{10} v_{T_0}$. The abscissa of this point is the reciprocal of the technical cohesive strength of the material, $s_{c,p=0}$.

10. Third Indirect Method For Estimation of the Technical Cohesive Strength of Ductile Materials.*

- (1t) Determine by test $s_{yv'T_0}$, the statical yield strength of the material in tension at room temperature and at some low rate of straining v' .
- (2t) Determine, also by test, $s_{yv'T_0}$, the statical yield point of the

* Based on the discussion contained in Chap. VII, Art. 7.

material in tension at some temperature T_ℓ , as low as possible.

- (3t) Plot the results of (1t) and (2t) on a graph similar to that shown on Figure 5, using T_0 and T_ℓ in degrees of absolute temperature scale. Determine the slope b_1 of the straight line connecting the two points.
- (4t) Using some high impact rate of straining v'' , find T_0'' , the critical temperature of complete brittleness of unnotched tensile specimens of the material considered. This is the temperature when the energy absorbed during the impact becomes practically equal to zero.
- (5t) Determine T_0''' , as in (4t), but with a different rate of straining v''' .
- (6t) With the results of (4t) and (5t), using T_0'' and T_0''' plot a graph similar to Fig. 3. Determine the slope B_1 of the straight line connecting the two points. Extrapolate and determine $\log_{10} v_{T_0}$.
- (7t) Find the technical cohesive strength $S_{o,p=0}$ from the equation

$$\log_{10} (s_{o,p=0}) = \log_{10} (s_{yv,T_0}) + \left(\frac{b_1}{B_1}\right) (\log_{10} v_{T_0} - \log_{10} v') \quad (13)$$

III. General Theory for Explanation of the Proposed Methods.

1. Approaches to Technical Cohesive Strength.

Kuntze^{(1)*} was the first to work out a method aiming at the determination of the technical cohesive strength by attempting to prevent plastic deformation in tension specimens by deep and sharp notches. Whether he (and other investigators who rely upon notches alone for prevention of plastic flow) succeeded in doing so and found the technical cohesive strength, is open to question.

There is also a somewhat different approach to the question of determination of the technical cohesive strength s_c , namely through "brittle strength".

This approach is treated in the next article.

2. Approach to the Technical Cohesive Strength through Brittle Strength.

From the generally accepted definition of the term "technical cohesive strength" (see I, 1.), as the stress at which the material breaks without plastic deformation, it follows that if a tension specimen of a material (like plaster of Paris) fails in a completely brittle manner at ordinary temperatures, its conventional ultimate strength s_u is the technical cohesive strength s_c of the material at that temperature.

The above reasoning from the definition may seem to be trivial, but its very simplicity may be the reason why even men like Kuntze, to whom the above definition is due, have not apparently drawn such an extremely obvious conclusion.

Most workers in the field of technical cohesive strength either use notched specimen method for estimation of the technical cohesive strength or test Kuntze's method and possibly criticize it. This indicates that they

* Numbers in parentheses refer to the bibliography at the end of the paper.

understand under the term of technical cohesive strength something similar to Kuntze's concept. The above seems to apply e.g. to McAdam (who defines technical cohesive strength in a rather general manner, as the "technically estimated resistance to fracture"). It applies to Davidenkov and other Russian investigators, to Gensamer, Sachs, and others. If someone starts by checking Kuntze's tests and uses notched specimens (clearly for the purpose of reducing plastic deformation) they are implicitly, if not explicitly, operating on the basis of the above definition (I., 1.).

The above simple conclusion is the basis for the very trivial recommendation of (II, 2.).

3. The Technical Cohesive Strength is Identical with the Ultimate Strength Overcome at a Completely Brittle Fracture, no Matter How the Complete Brittleness is Caused.

If a material, ductile in statical tension at ordinary temperatures, fails in a completely brittle manner in statical tension due to a sufficiently low temperature, again by the same definition of tech. coh. strength (I, 1), it follows that the normal stress at which the material fails is the technical cohesive strength of that material at that low temperature. This statement and the discussion below is a "justification" (if any such high sounding word can be applied to such a trivial statement) for the procedure of Chap. II, Arts. 3, 4, and 6.

4. Definitions of "Critical Temperature of Brittleness" and of "Brittle Strength".

Let the highest temperature at which completely brittle fractures occur (in the case of a specimen of a certain shape, made out of a certain material) be called the "critical temperature of statical brittleness in tension ($= t_{est}$)" for that material and specimen shape. Also let the normal stress in

an unnotched tensile specimen failing in a completely brittle manner be called "brittle strength (s_b)" of the material at the temperature of the test.

5. The Technical Cohesive Strength of Materials Having a Critical Temperature of Brittleness Higher Than the Temperature of Liquid Air.

Using the above term it can be said that if an unnotched tension specimen of any material fails in a completely brittle manner at any temperature, the brittle strength s_b of that material equals the technical cohesive strength s_c of that material at that temperature.

If the brittle strength s_b of a material (which can be determined only for temperatures below t_{ost}) could be proved to be independent of temperature, the determination of the technical cohesive strength s_c for all temperatures below t_{ost} would be a relatively simple matter in the case of all materials whose t_{ost} is attainable in the laboratory. The test temperature would have to be equal or lower than the critical temperature of statical brittleness t_{ost} , i.e. lowered sufficiently until a completely brittle fracture was obtained. The ultimate strength s_u obtained in the case of the completely brittle fracture (or the brittle strength s_b) would then be the technical cohesive strength s_c of the material for all temperatures below t_{ost} .

6. Test Results Showing That Brittle Strength Does Not Vary with Test Temperature.

There are some experimental results that seem to indicate that the brittle strength s_b , at least for some materials, is independent of temperature. From Ioffe's⁽²⁾ results it appears that the brittle strength s_b of rock-salt is constant at least from -315 deg. F to +383 deg. F (-193 deg. cent. to +195 deg. cent.) The maximum deviation in the results of Ioffe was 5 per cent. The results of Davidenkov and Shevandin⁽³⁾ appear to indicate little, if any, change in brittle strengths of dry rock-salt between the temperatures of

-148 deg. F and 68 deg. F (-100 deg. cent. and +20 deg. cent.). The same investigators⁽⁴⁾ found that the brittle strength of tungsten is constant over a range of temperatures from -328 deg. F to +302 deg. F (-200 deg. Cent. to +150 deg. Cent.). Tungsten however fails in the brittle manner by an intercrystalline fracture. The above result therefore indicates that the brittle strength of the grain-boundary material is constant, but does not give any information about the constancy of the brittle strength of the material of the grains proper.

Sauerwald and his collaborators⁽⁵⁾ found that the brittle strength of iron monocrystals is constant over a range of temperatures from -301 deg. F to -144 deg. F (-185 deg. cent. to -98 deg. cent.). There is a need of further tests to establish definitely that the brittle strength of all materials is independent of temperature.

7. Conclusions Drawn From the Tests.

The technical cohesive strength s_0 is the same as the brittle strength s_b for all temperatures below the critical temperature at which completely brittle fractures occur. Should it be proved that the brittle strength s_b is constant for all the temperatures for which it can be determined (below $t_{critical}$) there would be no apparent reason why there should be a change at the critical temperature of brittleness, i.e. why the technical cohesive strength should not remain independent of temperature also above the critical temperature of brittleness.

If however further experimental work should show that brittle strength s_b is a certain function of temperature, it may still be possible to estimate the technical cohesive strength at any temperature, once the brittle strength is known for one low temperature. For the time being, though, brittle strength s_b and technical cohesive strength s_0 can be considered as identical and independent of temperature. Further evidence of this will be given below,

in connection with Fig. 9 and 10.

The technical cohesive strength s_c of many materials can be consequently obtained by merely lowering the testing temperature below the critical temperature and determining the ultimate strength, in statical tension, of an unnotched specimen.

IV. Approaches to the Technical Cohesive Strength of Materials with Critical Temperatures Lower Than Those Commonly Obtainable in Laboratory.

1. Means of Increasing the Degree of Brittleness of Fracture.

The lowest temperature that can commonly be reached in laboratories is that of liquid air, about -315 deg. F (-193 deg. Cent.). Unnotched tensile specimens of many materials do not break in a completely brittle manner in statical test at the temperature of liquid air. In those cases some other means of causing a completely brittle fracture (without changing the original technical cohesive strength of the material) must be used. The following two may be investigated: (A) Lowering of the ratio of the maximum shear stress to maximum normal stress $\frac{s_{s \text{ max.}}}{s_{n \text{ max.}}}$ developed in the specimen tested; (B) increasing the rate of straining $\frac{d\varepsilon}{dt} = v$.

2. Increasing the Tri-Axiality of Stress State.

The lowering of the ratio $\frac{s_{s \text{ max.}}}{s_{n \text{ max.}}}$ reduces the tendency of the specimen to flow plastically due to the shear stress and increases its tendency to fail by separation, due to the normal stress. The maximum shear stress in any elastic stress state is known to be equal to one-half of the difference of the maximum and minimum principal stresses.*

$$s_{s \text{ max}} = \frac{s_{\text{normal max.}} - s_{\text{normal min.}}}{2} \quad . \quad . \quad . \quad (1)$$

In an unnotched tensile specimen, the maximum principal stress is the uniform axial stress and the other two principal stresses are everywhere equal to zero. The max. shear stress is therefore $s_{s \text{ max}} = \frac{s_{\text{normal max.}}}{2}$ and the ratio of maximum shear stress to the maximum normal stress is $\frac{s_{s \text{ max}}}{s_{n \text{ max}}} = \frac{1}{2}$.

This ratio can be reduced, (and thus the tendency to plastic flow decreased) if the minimum normal stress in equation (1) can be increased. If

* See e.g. Seely, Advanced Mechanics of Materials, John Wiley and Sons, 1932, p. 24.

one could make a specimen with a state of equal tension in three perpendicular directions, all the principal normal stresses would be equal and the shear stress would be reduced to zero as seen from eq. (1). Then all plastic flow (which is caused by shearing stresses) would be prevented, and a completely brittle failure would take place. Unfortunately, there are technical difficulties which so far have prevented the construction of a test specimen with three equal tensions in three perpendicular directions.

3. Use of Notched Specimens to Increase the Tri-Axiality of Stress State.

Kuntze⁽¹⁾ showed (for material in elastic condition) that deep and sharp notches cause some degree of three-axial tension. Brittle condition is elastic condition. However, the complicating circumstance is the fact that, in the outermost zone (around the circumference) of the reduced section, the minimum principal stress (namely radial stress) is zero.

Fig. 1a, the stress distributions of which were taken from (7), shows distributions of axial stress (s_{ax}), tangential stress (s_t), radial stress (s_r) and max. shear stress (s_{smax}) at any point across the root section of a tensile specimen with a deep notch. The ordinates of the distribution curve of s_{smax} at any point are drawn downward from the section through the notch merely to eliminate confusion with s_t and s_r . The maximum principal stress at the bottom of the notch is the axial stress s_{ax} of the magnitude $k \cdot s_{ax \text{ ave}}$ (where "k" is the stress concentration factor due to the notch). Therefore at the bottom of the notch the maximum shearing stress s_{smax} has, according to Eq. (1), the magnitude $s_{s \text{ max}} = \frac{k s_{ax \text{ ave}}}{2}$. The max. shearing stress $s_{s \text{ max}}$ in the regions located inward from the circumference is reduced, because of the presence of a radial tensile stress, s_r different from zero. In these regions the difference of the principal stresses of eq. (1) is reduced. However the max. shearing stress $s_{s \text{ max}}$ at the surface (bottom of the notch) is not reduced and

plastic flow can take place in these outermost fibers as soon as the shear stress $s_{s \max}$ at the bottom of the notch reaches the shearing yield point (or, according to the more common way of saying the same, as soon as the maximum normal stress $s_{ax \max}$ reaches the yield point in tension s_y)*.

4. Effect of Plastic Flow on Yield Strength and Technical Cohesive Strength.

It is well established that the stress at which further yielding of plastically prestrained steel takes place, increases with the amount of plastic prestrain (measured as unit permanent set p , inches per inch). Let the stress at which further yielding will take place in a tension specimen prestrained to a unit set p be called the "current yield strength s_{yp} ". This current yield strength s_{yp} is a function of p and its dependence on p is represented by the curve of effective actual axial unit stress vs. the corresponding unit strain for a tension specimen (accurate only up to the beginning of necking down since thereafter a complex state of stress exists in the reduced portion). The original yield point of the material is designated $s_{y,p=0}$.

The consequences of plastic flow at the bottom of a notch are as follows: (a) The load carried by the outer zone is redistributed and the stress distribution shown in Fig. 1a is changed to that shown schematically in Fig. 1 (b), (curves labeled s_{ax} and s_{yp} for the normal stress, s_{smax} for the shear stress), for the case of a material whose yielding is accompanied by strain-hardening. (b) Cohesive strength s_c of the material in the regions which have undergone plastic flow is raised. From the values of s_c obtained by Kuntze's⁽¹⁾ method it appeared that the cohesive strength of a virgin material is raised by

* According to some authorities,⁽⁸⁾ basing their views mainly on tests by Eiselin⁽⁹⁾ and Bierett⁽¹⁰⁾, plastic flow is not supposed to take place, in regions with stress-gradient, until a considerable portion of the critical cross-section can flow. However, Sidebottom and the author found⁽¹¹⁾ that yielding took place in steel specimens with a stress concentration up to $k=2.5$ as soon as the peak stress reached the yield point of the material.

increasing the degree of plastic deformation p (up to a certain limit, after which additional plastic strain was supposed to cause a gradual reduction in s_c , possibly even to a value lower than $s_{c,p=0}$.) More recently, Sakharov,⁽¹²⁾ found by means of static tensile tests on unnotched specimens of coarse-grained 0.2 per cent C steel in liquid air that the brittle strength is raised by plastic straining in tension which preceded the determination of brittle strength (see Fig. 2).

5. Definition of "Current Cohesive Strength."

Suppose the original cohesive strength of the virgin metal has been raised by a cold prestraining, accompanied by a unit plastic strain of magnitude p , to some higher cohesive strength. Let the value of this larger cohesive strength, reached after a pre-strain p , be called "the current cohesive strength $s_{c,p}$ ", and let the original cohesive strength before any plastic flow takes place, i.e. in the virgin metal, be designated $s_{c,p=0}$.

The prestraining was not carried by Sakharov⁽¹²⁾ to such high values of p as to observe any fall in brittle strength, as postulated by Kuntze.⁽¹⁾ The rise in the cohesive strength in the regions of a notched specimen that has undergone local yielding is schematically represented in Fig. 1b by the lines marked $s_{c,p}$. In the region marked R_0 , where no plastic flow can take place (due to absence of any shear stress), the cohesive strength will remain at its initial value $s_{c,p=0}$.

6. Two Possible Manners of Failure of a Notched Specimen of Ductile Metal at Room Temperature.

As the tensile test of a notched steel specimen progresses beyond the stage shown in Fig. 1b, the following two possibilities or alternatives exist;

- (a) The rising current yield strength s_{yp} in the regions near the

bottom of the notch (where plastic flow is taking place) reaches the current cohesive strength s_{op} before s_{ax} in the region R_0 (where no flow is taking place) reaches the unchanged original cohesive strength $s_{c,p=0}$ of the virgin metal.

(b) The axial stress s_{ax} in the middle regions of the reduced cross section where the $s_{\theta \max}$ is zero (marked R_0 on Fig. 1b), reaches the value of the unchanged (original) cohesive strength $s_{c,p=0}$ before the rising current yield strength s_{yp} reaches the current cohesive strength s_{op} in the regions near the bottom of the notch where flow is taking place.

Whether the phenomenon (a) or (b) takes place depends upon the particular material as well as upon the depth $\frac{D-d}{2}$, the radius of curvature of the notch bottom ρ , and the flank angle of the notch w .

If the alternative (b) is present, there is a separation break starting in the middle regions of the specimen. If the other alternative (a) occurs, the failure starts in the outer zone, at or near the surface. In this outer zone the failure may follow the 45° planes of max. shear. In region R_0 the failure is of the separation type.

7. Progressive Crack Under Single Load Application.

In the case (a), no approximation of the technical cohesive strength of the virgin material (unchanged by yielding) is possible. The sharp crack progressing in this case from the outside and inward, has an unknown high stress concentration factor, and the strength of the material is overcome progressively, grain by grain. No approximation of the strength can be obtained from the known load at fracture P_f , due to the fact that the strength of each "fiber" is overcome separately. Not even an approximation of the raised cohesive strength s_{op} of the material in the plastically deformed outer zone can be obtained.

In the case (b), an approximation of the technical cohesive strength of the material would be possible, if the stress distribution across the critical section at failure were known. This stress distribution at failure is entirely different from that present at that section under elastic conditions.

V. High Rates of Straining (e.g. Impact)

1. Increasing the Degree of Brittleness of Fracture by High Rates of Straining.

As mentioned above, there is another means of causing completely brittle fractures without changing the original technical cohesive strength $s_{0,p=0}$ of the material, viz. the case (B), increasing the rate of straining $\frac{d\varepsilon}{dt} = v$.

From a considerable number of investigations of the influence of the speed of straining v upon the critical temperature of brittleness t_c and the yield point $s_{y,p=0}$ (for shortness designated also by s_y) only a few investigations, which allow selection of an empirical equation to fit the results, will be mentioned.

2. Some Experimental Data.

Witman and Stepanov⁽¹⁴⁾ used unnotched cylindrical specimens of 0.2 per cent carbon steel annealed for 2 hours at 1832 deg. F (1000 deg. centigrade), with an average grain diameter of 53 microns. This steel exhibited brittle breaks at moderate rates of straining at the temperature of liquid air. The specimens were tested in bending at various temperatures and at various speeds of head (ranging from 0.024 inches per second to 3270 inches/sec., i.e. 83.3 meters per second). Table 1 shows a portion of their results.

TABLE 1

Variation of the Upper* Critical Temperature of Brittleness
of Coarse Grained 0.2 per cent C Steel with the Rate of
Straining (after Hitman and Stepanov) (14)

Speed of head, inches/sec.	.024	.165	197	327	3270	
Rate of strain v in the most stressed region., per cent per second.	0.95	6.43	7,650	12,750	127,500	
Rate of strain v in the most stressed region, in. per in./sec.	.0095	.0643	76.5	127	1275	
Upper* critical temperature of brittleness, bending of unnotched specimens (upper limit of the critical interval of brittleness)	t deg. Cent.	-165	-155	-110	-105	-75
	T_k deg. Kelvin.	108	118	163	168	198
	t_{ou} deg. F	-265	-247	-166	-157	-103
	T_R deg. Rankine	195	213	294	303	357
$\log_{10}(\frac{\text{equivalent strain rate at room temp.}}{\text{actual strain rate at low temp.}})$	7	6.2	3.1	2.9	1.9	

* For definition of "upper" critical temperature see Chapter VI, Art. 4.

Fig. 3 is the graph of these results using coordinates $\log_{10} v$ and $\frac{1}{T_c}$.* The axis of ordinates is drawn at a value of abscissa equal to $\frac{1}{T_0}$, where T_0 is room temperature (20 deg. Cent.) expressed in deg. absolute. The meaning of the graph can be expressed as follows. Suppose the critical temperature of brittleness (of the steel considered) is required, i.e. the temperature at which completely brittle break will occur at a given rate of straining. Locate on the Y axis the given rate of straining, draw a horizontal line with that constant ordinate until it intersects the curve of results. The abscissa of that point of intersection is the reciprocal of the critical temperature which, in combination with the given rate of straining, will produce brittle fracture. The experimental data are fitted by the equation

$$\log_{10} v_{T_c} = \log_{10} v_{T_0} - B_1 \left(\frac{1}{T_c} - \frac{1}{T_0} \right) \quad . \quad . \quad . \quad . \quad (4)$$

where $\log_{10} v_{T_0}$ and $\frac{1}{T_0}$ are the coordinates of a point on the straight line of brittle failures.

In this expression, the values $(\log_{10} v_{T_0})$ and B_1 are constants characteristic of the material. Equation (4) can be rewritten in the following form:

$$v_{T_c} = v_{T_0} \cdot e^{-B \left(\frac{1}{T_c} - \frac{1}{T_0} \right)} \quad . \quad . \quad . \quad . \quad (5)$$

The constant B in equation (5) has the following relation to constant B_1 in equation (4):

$$B_1 = B \log_{10} e \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

* The symbol T_0 stands for "critical temperature of brittleness at a given rate of straining v , expressed in absolute units." The absolute units used in Fig. 3 are deg. Rankine, in Fig. 11 these units are degrees Kelvin. Therefore T_0 is designated in Fig. 3 by the symbol T_R , in Fig. 11 by T_K . In the equations like (4) or (5), either T_R or T_K can be used for T_0 , if the proper value of B_1 is used.

The constant v_{T_0} in equation (5) now acquires a meaning: it is the minimum rate of strain at which the material breaks in a brittle manner even at room temperature T_0 .

Equation (5) can be rewritten as

$$v_{T_0} = (v_{T_0} e^{\frac{B}{T_0} - \frac{B}{T}}) e^{\frac{B}{T}} \quad (5.1)$$

or

$$v_{T_0} = A e^{-\frac{B}{T}} \quad (5.2)$$

Thus the constant A has the value $v_{T_0} e^{\frac{B}{T_0}}$.

Equation (5.2) can also be written* as

$$\ln v_{T_0} = \ln A - B \frac{1}{T} \quad (5.21)$$

or

$$\log_{10} v_{T_0} = \log_{10} A - B_1 \frac{1}{T} \quad (5.22)$$

This form of equation is the equation of the data in Fig. 3 when the upper scale of abscissas ($\frac{1}{T}$) is used.

3. Comparison of Embrittlement by Lowering Test Temperature and by Increasing the Rate of Straining.

It now becomes possible to compare quantitatively the effectiveness, upon embrittlement, of lowering the temperature as against increasing the rate of strain. The mild steel tested⁽¹⁴⁾ can be caused to break in a completely brittle manner at v of about 0.01 in/in per sec. at the temperature of -265° F (-165 degrees centigrade). In order to cause it to break in a completely brittle manner at the temperature of 55° F ($+13$ degrees centigrade, room

* "ln" designates a logarithm to the base e.

temperature) the rate of straining would have to be about 10^5 in./in. per second. That means that to produce the same embrittlement, which is produced by lowering the temperature by 320° F (178 degrees centigrade), an increase of rate of straining 10^7 times would be necessary. This shows that temperature is a far more dangerous embrittling agent than the rate of straining.

4. Method for the determination of the Critical Temperature of Brittleness For Any Rate of Straining.

Witzan and Stepanov⁽¹⁴⁾ conclude that, if it is proved that relation between the critical temperature of brittleness and the rate of straining is of the above exponential form for all materials, it will only be necessary to obtain the values of the critical temperature of brittleness for two rates of deformation easily reached and then draw a straight line similar to that shown in Fig. 3. From this the constants $\log_{10} v_{T_0}$ and B_1 can be read and also the rate of straining necessary to cause a completely brittle fracture at any desired temperature. Or, if the critical temperature of brittleness T_0' for some low rate of straining v' is known, the critical temperature of brittleness T_0'' for any given rate of straining v'' can be found from the following expression, derived from equation (4) or (5.22):

$$T_0'' = \frac{T_0'}{1 - \frac{B_1}{\log_{10} \frac{v''}{v'}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

VI. Low Test Temperatures.

1. Influence of Low Test Temperature on Yield Strength.

Witman and Stepanov⁽¹⁴⁾ also determined the variation of the statical yield strength s_y (in bending) with the varying temperature t , for the same steel. The results are shown in Fig. 4. If the data is replotted to coordinates $\log_{10} s_y$ and $\frac{1}{T}$ (where T is absolute temp.), a straight line relation is obtained. This is shown in Fig. 5. If the values along the axis of abscissae in Fig. 5 are expressed in the form $(\frac{1}{T} - \frac{1}{T_0})$, where T_0 is the room temperature (55°F) expressed in absolute degrees, the straight line can be expressed by the relation

$$\log_{10} s_{yT} = \log_{10} s_{yT_0} + b_1 \left(\frac{1}{T} - \frac{1}{T_0} \right) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\text{or} \quad s_{yT} = s_{yT_0} e^{b \left(\frac{1}{T} - \frac{1}{T_0} \right)} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The relation between the empirical constants b_1 and b is

$$b_1 = b \log_{10} e \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The constant s_{yT_0} is the value of the statical yield strength in bending at room temperature.

The above exponential form (Eq. 9) of the relation between test temperature and yield strength also fits the data obtained by Goerens and Mailander⁽¹⁵⁾ for high carbon steels, Gruschka⁽¹⁶⁾ for Armco iron, Shevandin and Kisin⁽¹⁷⁾ for high phosphorus iron. Some of the results of the latter two investigators appear in Fig. 6.

2. Some Experimental Results Showing Physical Significance of such Concepts as "Critical Temperature" and "Brittle Strength."

Shevandin and Kisin⁽¹⁷⁾ tested statically (in bending) cylindrical

unnotched specimens of a high-phosphorus iron. The average grain size was 0.1 mm. The specimens were tested in a 2 ton Amsler testing machine at various low temperatures. Load-deflection diagrams were obtained by autographic attachment. Stress corresponding to a break in the load-deflection curve was taken for yield strength s_y of the specimen* and that corresponding to the maximum load was taken for the ultimate strength s_u . Fig. 6 shows the variation of the yield strength s_y with temperature, for the specimens with ductile behavior, and the variation of the ultimate strength s_u , with temperature, for the completely brittle specimens (empty circles).

The results shown in Fig. 6 illustrate well two of the phenomena, mentioned previously, viz.:

- (1) The statical yield strength s_y increases with the decreasing temperature.
- (2) When the temperature is low enough and completely brittle failure occurs, the ultimate strength seems to remain fairly constant for all temperatures lower than this "critical temperature of brittleness."

3. Comparison of "Strength vs. Temperature" Curve with "Energy Absorbed vs. Temperature" Curve.

It is interesting to compare Fig. 7 with Fig. 6. In Fig. 6, yield stress or ultimate strength is plotted against the temperature as abscissa. In Fig. 7 energy absorbed by the same specimens is plotted against the temperature as abscissa. The scales on the axes of abscissae are the same. The comparison shows that the failure is not completely brittle until the energy absorbed becomes zero.

* This arbitrary assumption introduces an error in all the ordinates of the s_y curve I on Fig. 6. However this error is of constant magnitude, so that the shape of the curve is unchanged.

4. Comparison of "Energy Absorbed vs. Temperature" Curves for Different Rates of Straining.

It is interesting to go one step further and compare Fig. 8 with Fig. 7. In Fig. 8, against the same scale of temperatures, is plotted the energy absorbed during an impact test (speed of head 197 inches/sec. or 5 meters per sec.). It is seen that the temperature, at which completely brittle fractures take place under the influence of impact (Fig. 8), is raised with respect to that at which completely brittle fractures take place under static conditions (Fig. 7). Let the temperature at which completely brittle fractures occur under impact conditions (Fig. 8) be designated by the term "critical temperature of impact brittleness t_{ci} ". The temperature at which completely brittle fractures occur during the ordinary static tension tests (Fig. 6 and 7) has already been designated by the term "critical temperature of static brittleness t_{ost} ". The superposition or comparison of Figs. 7 and 8 illustrates the well known phenomenon, viz. that an increase in the rate of straining shifts the critical temperature of brittleness upward, closer to room temperature (see e.g. Table 1).

From Figs. 7 and 8 it is clear that the upper limits of the shaded scatter regions (intervals of transition from ductile to mixed and finally completely brittle failures) are easier to determine than the temperatures at which the energy absorbed is zero (in an impact test, extraneous energy losses occur easily). As it is more important to know the temperature at which the first brittle or semi-brittle breaks occur, this upper limit of the transition region may be considered the really "critical" temperature of brittleness in which practicing engineers are interested. However, in this thesis, the temperature at which only completely brittle failures can occur is of more interest, because at that temperature and below that temperature the brittle strength s_b can be obtained. Therefore the former temperature (at which the first brittle and

semi-brittle breaks occur) will be called in this thesis the "upper critical temperature of brittleness t_{cu} ", whereas the term "critical temperature of brittleness t_0 " is retained for the latter temperature, i.e. maximum temperature at which only completely brittle breaks occur.

It is probable that, if the change from statical to impact test shifts the upper critical temperature t_{cu} by a certain number of degrees, the "lower" critical temperature t_0 is shifted by approximately the same number of degrees. It should be noted that the values shown in Table 1 and Fig. 3 are the upper critical temperatures in bending of unnotched specimens. It was observed above that the ordinates of curve I are too large due to arbitrary determination of the yield strength. The two errors tend to cancel each other.

5. Some Theoretical Conclusions from the Experimental Results Described Earlier.

A number of investigators have attempted to obtain stress readings during impact tests (e.g. Mann⁽²⁹⁾). However, up to now, there are no reliable values of the breaking stress at completely brittle failures of materials during impact tests. As was mentioned above, there are tests which indicate that the brittle strength of a material does not change with changing temperature below the critical temperature of brittleness t_0 ^{(2), (3), (4), (5)}. This is also apparent from Fig. 6 for high-phosphorus iron⁽¹⁷⁾, although only in a narrow interval of temperatures. If the brittle strength s_b is constant up to the t_0 and is in this region equal to the technical cohesive strength $s_{c,p=0}$ and if t_0 can be shifted upward by increasing the rate of straining v , there seems to be no reason why the brittle strength s_b (and the technical cohesive strength $s_{c,p=0}$ which is equal to s_b as long as brittle failures with no plastic flow take place) should not follow the dotted horizontal line in Fig. 6 above the critical temperature. This is better illustrated by the schematical diagram in

Fig. 9.

Curve 1 represents the variation of the yield strength s_y with varying temperature T (in degrees absolute), for a given rate of straining v' . The mean curve 1 of the data of Shevandin and Kisin⁽¹⁷⁾ shown in Fig. 6, was used for drawing curve 1 in Fig. 9. These data happen to be obtained from statical tests. The curve fits equation (9) which can be therefore rewritten in the form

$$s_{yv'T} = s_{yv'T_0} \cdot b\left(\frac{1}{T} - \frac{1}{T_0}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where T_0 is room temperature in deg. absolute and $s_{yv'T_0}$ is the yield point at room temperature when the rate of straining is v' (in the present case statical).

When the temperature falls below the critical temperature of brittleness T'_0 , for that rate of straining v' , the failures are completely brittle (Fig. 6) and the energy absorbed during the test is zero (Fig. 7). The brittle strength s_b , which in this case is the same as the technical cohesive strength with no plastic prestraining $s_{0,p=0}$, then follows the horizontal line AB.

When the tests on the same specimens are performed at a higher rate of straining v'' (e.g. impact with speed of head 197 inches/sec.), no absolutely reliable values of yield strength are available. It is known, however, that the resistance to flow is raised by increasing v . The critical temperature of brittleness at v'' can be obtained either roughly from Fig. 8 (when values of energy absorbed during impact test become zero) or more exactly from equation (4), (5), or (7) or a graph similar to that shown in Fig. 3. Such a graph can be obtained for the material under consideration by finding the upper critical temperatures for two rates of straining, e.g. statical and impact. In Fig. 9 this critical temperature at rate of straining v'' is T'_0 . In the above

case of rate of straining v' (statical), it is clear that the critical temperature T'_0 is the temperature at which the curve of yield strengths $s_{yv'}$ (curve I for rate of straining v'), intersects the straight horizontal line BA of the brittle strengths s_b . Although there are no unimpeachable quantitative values for the ordinate of the point D, let, for the time being, the reasoning proceed by analogy. Let us say that the critical temperature of brittleness T'_0 for the higher rate of straining v'' is the temperature at which the curve II, the locus of yield strengths $s_{yv''}$, at rate of straining v'' , intersects the continuation of the horizontal line of brittle strength BA.

It remains to construct the curve II. It is known that the yield strength $s_{yv''T_0}$ at room temperature T_0 at the rate of straining v'' is higher than the statical yield strength at room temperature $s_{yv'T_0}$. It is further known that the relation between the yield strength and temperature is expressed by Equation (9) for various materials and at various speeds of testing (15), (16), (17), (1). The constant b in Eq. (9) seems to be a property of the given material. (This point requires experimental checking.) If this is so, the curve II in Fig. 9 must be of the same general shape as Curve I, only with larger ordinates, so that it agrees with equation (9), viz.

$$s_{yv''T} = s_{yv''T_0} \cdot b \left(\frac{1}{T} - \frac{1}{T_0} \right) \quad (12)$$

The only real justification for drawing the line AD (and even further DG) as a continuation of the straight line BA of the brittle strength, can be found in the work of Ioffe⁽²⁾ and collaborators on rock-salt (discussed in the following article).

6. Further Experimental Results Showing the Variation of Yield Strength with Temperature.

In Fig. 10 are shown some of Ioffe's results. Rock-salt is brittle

at temperatures below $+392^{\circ}\text{F}$ ($+200$ degrees Centigrade). The horizontal line BA is the locus of brittle strength of the rock salt specimens. The curve AM is the locus of the elastic limits of the specimens at different temperatures. The elastic limits are the stresses calculated from loads read at the instant the X-ray interference spots (obtained by Laue's method), of the uniformly strained specimen, changed (i.e. started elongating into streaks). The accuracy was about 1 per cent. From Fig. 10 it can be seen that the elastic limit becomes equal to zero at the point M, about $+1500^{\circ}\text{F}$. This is the melting point of rock salt. Accordingly, Ioffe concludes that it is possible that the melting point can be defined as that temperature at which the elastic limit or yield point of the material decreases to zero, and that melting can be explained as slip in crystals.

From Fig. 10, it can be observed that, during the loading of the specimen, the brittle strength is reached without any plastic deformation, if the temperature of test is below the critical temperature of brittleness. Above the critical temperature, the specimen flows plastically before it tears at a higher stress. In this second case there appears an additional phenomenon, viz. the work-hardening (work-strengthening, cold-working, or prestraining "p") of the material, which raises the technical cohesive strength.

Above the critical temperature Ioffe observed plastic flow and necking down of the specimens. When finally the necked down specimens broke, the breaking stress calculated on the basis of the reduced cross-section exceeded considerably the normal brittle strength of 640 psi.

As the amount of plastic flow, which preceded the break, increased (at successively higher temperatures of test), the breaking stress reached higher values (approached values as high as 7,100 psi). The higher strength is not the consequence of higher temperature, but of the amount of plastic flow.

This was shown by deforming the specimens at high temperature plastically, then cooling and breaking. The same high strength was obtained from tests of specimens treated in this manner as was obtained in the case of specimens broken at the high temperature. This increase of the technical cohesive strength due to preceding plastic flow is the same phenomenon as that indicated above in connection with Fig. 2.

7. Some Experimental Results Indicating that the Technical Cohesive Strength does not Vary with Temperature.

Now we finally come to the only two experimentally obtained results, available to author, which justify the assumption that the technical cohesive strength $\sigma_{o,p=0}$ (if no plastic flow p precedes the separation break) is constant even at temperatures above the critical temperature of brittleness. By increasing the rate of loading (i.e. also rate of strain v), Ioffe succeeded in raising the elastic limit of rock-salt at 752° F ($400^{\circ}\text{ centigrade}$) sufficiently so that it broke in a brittle manner even at this temperature. This temperature of 752° F ($400^{\circ}\text{ Centigrade}$) is higher than the critical temperature of static brittleness. However, it is equal to or lower than the critical temperature of brittleness at the higher rate of straining which Ioffe used to fracture that particular specimen of rock-salt.

Referring to the schematic diagram of Fig. 9, Ioffe's curve AM (Fig. 10) corresponds to the Curve I (for static, low rate of straining v') of Fig. 9. Ioffe's critical temperature of static brittleness ($200\text{ deg. centigrade} = 392^{\circ}\text{ F}$ on Fig. 10) corresponds to T'_0 of Fig. 9. Ioffe's higher rate of straining, which he used at 752° F ($400\text{ deg. centigrade}$) corresponds to v'' of Fig. 9. This technical cohesive strength $\sigma_{o,p=0}$ of the specimen at $400\text{ deg. centigrade}$ failing without any plastic flow, corresponds to a point on the line ADG of Fig. 9, e.g. point D.

Ioffe tested another specimen at 650 deg. centigrade (1200° F) at such a high rate of loading that it was sufficient to shift the critical temperature of brittleness (corresponding to that particular rate of loading) upward, viz. to or above 1200° F (650° Centigrade). He obtained a brittle break and the breaking strength very nearly equal to that observed below the temperature of statical brittleness. (With no plastic flow, breaking strength is identical with the technical cohesive strength.) The rates of strain used were not published and there were only two specimens tested in such a manner. The evidence is not conclusive, but on its strength the horizontal line of constant technical cohesive strength of material not plastically prestrained (BG on Fig. 9) is continued tentatively above the critical temperature of statical brittleness (to the right of point A on Fig. 9). The scheme illustrated on Fig. 9 is justly called Ioffe's scheme.

8. A Proposed Test of Ioffe's Scheme.

Ioffe's scheme will be automatically checked directly as soon as values of cohesive strength are obtained by direct readings of brittle strength at high rates of straining. It might be advisable at first to use materials for which a moderate increase in the rate of straining shifts the critical temperature into the range obtainable in the laboratory. At such moderate rates of strain it might be possible to measure loads by ordinary means. Materials that might satisfy the above requirement might be some plastics or high phosphorus steel.

9. Some Experimental Results Showing That the Ultimate Tensile Strength Increases with Increasing Rate of Strain.

Nadai and Manjone⁽²⁵⁾ tested copper, aluminum, mild steel, pure iron and stainless steel in tension at rates of strain varying from 10^{-4} per cent per second to 10^5 per cent per second.

The results of the tests show that the ultimate strengths (referred to original area) increase slowly with increasing rate of strain, both for room temperature and for temperatures higher than room. In the case of copper, a somewhat similar increase with the rate of strain is shown also for the true stress at ultimate.

It should be emphasized that in none of these tests the fracture was brittle. There was plastic flow of varying extent at the critical section at different rates of strain at the same temperature. Varying extent of plastic flow causes varying change in the technical cohesive strength. Even authorities on cohesive strength who otherwise differ in their views, as MoAdam⁽¹³⁾, Kuntze⁽¹⁾, and Sakharov⁽¹²⁾, agree on this point, viz. that plastic deformation changes the technical cohesive strength.

Therefore at the testing conditions used by Nadai and Manjoine⁽²⁵⁾, because the amount of plastic flow "p" also was probably changing with speed, one should not be surprised even if it should appear that the values of the technical cohesive strength varied, (as they are values of $s_{0,p}$ for varying p). However the data shown are ultimate strengths after plastic flow and not technical cohesive strengths.

Therefore, the published data of Nadai and Manjoine⁽²⁵⁾ cannot be used as evidence either for or against the proposition that the virgin technical cohesive strength is independent of rate of strain during a test.

In connection with their admirable experimental setup there arises a question. They measured the load at high speeds by the elastic extension of a force-measuring bar located in series with the specimen. The force-measuring bar was supported by the fixed head at one end and coupled to the specimen at the other. The question is whether the force, measured near the fixed end and on one side of a mechanical coupling, may not be considerably different, near

the upper limit of the speeds used, from the force at the cross section of the specimen where the flow took place (on the other side of the coupling).

VII. The Effect (upon Certain Measures of Strength) of An Increase in Straining Rate is Equivalent to the Effect of A Decrease in Temperature.

1. Theoretical Basis for An Expression for the Rate of Plastic Deformation.

Figure 13(a) shows a simplified schematic picture of one plane of atoms in an undisturbed hypothetical crystal. If shear were applied to this crystal and if the crystal distorted in a homogenous manner, the situation shown in Fig. 13(b) would exist just before any plastic action would take place. By Hooke's Law, under elastic conditions the shearing unit stress s_s is proportional to the shearing unit strain γ ($s_s = E_s \gamma$, where E_s = shear modulus of elasticity). The energy w absorbed by a unit volume of the material strained elastically by an amount γ , is

$$w = \frac{1}{2} s_s \gamma = \frac{1}{2} \frac{s_s^2}{E_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.1)$$

This equation can be also written as

$$w = \frac{1}{2} E_s \gamma^2 \quad \text{or approximately } w = \frac{E_s e^2}{2l^2} \quad . \quad . \quad . \quad (9.2)$$

This shows that the elastic energy which has to be absorbed by a unit volume of the material in order to shift one row (or better one plane) of atoms homogeneously with respect to another, by a displacement e , is roughly proportional to the square of that displacement e . However, as soon as e is larger than $\frac{l}{2}$, the distance between atoms A1 and B2 (Fig. 13b) becomes smaller than that between A1 and B1. Consequently the attraction between A1 and B2 is larger than that between A1 and B1 and atom A1 jumps to column B, bending to make the distances between A1 and B2 minimum. Similarly B1 jumps to column C, etc. If the load is released after the jump, atom A1 does not return to Column A, the material does not return to its original state. The jump therefore constitutes plastic action. When one atomic row is displaced (with respect to the neighboring one) by half

the dependency of the velocity of plastic deformation on temperature, as given by Equation (9.4). However this equation does not account for the very low value of actual shearing yield strength compared with that expected for the simple model of the ideal crystal. Smekal⁽³⁴⁾ suggested that stress concentrations around internal flaws in a crystal start plastic flow at a stress lower than $s_{\text{sy}t}$, e.g. at $\frac{s_{\text{sy}t}}{k}$, where k is the stress concentration factor. Crowan⁽³⁵⁾ substituted in Becker's formula the value $(k s_{\text{sy}appl} T)$ for $(s_{\text{sy} appl} T)$, thus getting for Eq. (9.4) the relation

$$v = c \cdot \frac{V(s_{\text{sy}t} - k s_{\text{sy}appl} T)^2}{2E_s RT} \quad (9.5)$$

This relation is an expression for the velocity of plastic deformation.

Crowan also arrived at a relation between the temperature T and the magnitude of the externally applied shear stress $s_{\text{sy}appl} T$ which will cause yielding at the temperature T . He pointed out that this critical magnitude of shear stress, which will cause yielding when applied, is that magnitude of shear stress at which the velocity of deformation v suddenly and rapidly increases from a negligible value to some definite and noticeable value. If the stress is little lower than $s_{\text{sy}appl} T$, the velocity v is negligibly small. If the stress is only a few per cent higher, the velocity v is already so large that sometimes the ordinary testing machines cannot keep up with it. In this range of velocities, the stress varies only a little. One could possibly think of Crowan's condition better in this way: Yield stress is that limiting magnitude of stress at which, if an infinitesimally small external disturbance creates an additional impulse, the plastic deformation thus started will continue at a constant rate, i.e. $v = \text{constant}$. If the expression in Eq. (9.5) is to be constant, the exponent must be constant, and

$$\frac{(s_{\text{sy}} - k s_{\text{sy appl}} T)^2}{T} = \text{Constant} \quad (9.6)$$

because all the other quantities are constant. If the constant is designated by say C^2 , the dependence of shearing yield strength on temperature is expressed by the relation

$$s_{\text{(sy appl at T)}} = s_{\text{(sy theor at T = 0)}} - C T^{\frac{3}{2}} \quad (9.7)$$

$$\text{or } s_{\text{(sy appl at T)}} = A - C T^{\frac{1}{2}} \quad (9.8)$$

The author found that Eq. (9.8) fitted experimental results for the critical shear stress on zinc monocrystals in the range of temperatures from 30 to 530 deg. Kelvin, which were reported by Schmid and Boas⁽⁶⁾. The value of the constant C was found to be 4.75 and that of A was 157 (using s_{sy} in grams per mm²). The same source⁽⁶⁾ gives for the shear modulus of elasticity of zinc single crystals the values of $E_s \text{ max} = 4,970,000$ and $E_s \text{ min} = 2,780,000$ grams per mm² (depending on direction of slip). Because in the simplified estimation of theoretical strength of crystals (see discussion connected with Fig. 13 above) only the order of magnitude was sought, let the value of E_s for rough check be taken as 4,000,000 grams per mm.² The expected theoretical s_{yt} should be therefore by Eq. (9.25) of the order of 2,000,000 grams per mm.² From Eq. (9.7) the value of k , the internal stress concentration factor postulated by Snokai,⁽³⁴⁾ would have to be (on the basis of above figures) of the order of magnitude of 10,000.

It should be mentioned that the above formula Eq. (9.8) was tried by the author on the yield points of a polycrystalline iron, which were obtained at different temperatures, by Witman and Stepanov⁽¹⁴⁾. It was found, as was to be expected, that it did not fit well, as it is designed for monocrystals. It is easily seen that the grain boundaries may add additional complications to the

above simplified picture.

The agreement of equations (9.4), (9.5) and (9.8) with experimental results on monocrystals may be taken as an indication that the underlying concept of rate of plastic flow of metals as having to do with what amounts to a sort of diffusion in these solids (supported by thermal motion) has some justification. This furnishes to the author a basis for understanding why Hollomon and Zener⁽²¹⁾ use, apparently with success, an equation amounting practically to the same as Eq. (9.5) above.

2. Proposition of Hollomon and Zener.

Zener and Hollomon⁽²¹⁾ postulated, on the basis of theoretical considerations, a quantitative relationship between the effect (upon certain properties of a material connected with its strength) of an increase in rate of straining and the effect of lowering the temperature. As Hollomon and Zener's presentation of the theory underlying the proposed relation of equivalence is rather abbreviated, the author begs to be excused if he starts by a recapitulation of the reasoning they might have used.

Zener and Hollomon⁽²¹⁾ related the unit stress s at any stage of a tensile test of a material (i.e. some measure of its strength) to the corresponding unit strain ϵ by the formal expression

$$s = s_p(\epsilon) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1) \text{ HZ}$$

in which the parameter p is defined by:

$$p = \frac{\epsilon \cdot \frac{Q}{RT}}{f_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2) \text{ HZ}$$

where ϵ is the strain rate, Q the heat of activation of the material which has apparently to be supplied to cause plastic flow, expressed in calories per gram mol, R the molal gas constant in deg./cal; and T the absolute temperature in

degrees Kelvin.

The reason for defining p in this manner is as follows. Since the parameter p must be dimensionless function $p(\dot{\epsilon}, T)$, it must be expressible as a ratio of the rate of deformation and of some other time rate connected with the material. Nearly all physico-chemical time rates associated with materials are governed by heats of activation and their dependence on temperature is given by an equation of the form

$$\text{time rate} = f_0 e^{-\frac{Q}{RT}} \quad (3) \text{ HZ}$$

where the constant f_0 has the dimension of a reciprocal of time (t^{-1}).

The fundamental similarity of this equation with Eq. (9.5) should be noted. Equation (3) HZ thus seems to be based also on Eq. (9.3). From diffusion problems, Hollomon and Zener⁽²¹⁾ took the value usually assigned to a constant of the type of f_0 , namely

$$f_0 = \frac{JQ}{Nh} \quad (4) \text{ HZ}$$

where J is the mechanical equivalent of heat, N is Avogadro's number and h is Planck's constant. When f_0 is so interpreted, the time rate of equation (3) HZ is essentially the number of times per second any given atom acquires the heat of activation Q for diffusion (change of place). That explains why the parameter p is to be expected to be of the form shown in Eq. (2) HZ. Summarizing,

$$p = \frac{\text{rate of deformation}}{\text{time rate connected with diffusion}} = \frac{\dot{\epsilon}}{f_0 e^{-\frac{Q}{RT}}} = \frac{\dot{\epsilon} e^{\frac{Q}{RT}}}{f_0} \quad (5) \text{ HZ}$$

To show that a certain change in the rate of straining $\dot{\epsilon}$ is, in its effects upon some mechanical properties of materials, equivalent to a certain

Ad Eq. (4) is the equation of the straight line on Fig. 3 or Fig. 11, it appears that the curves in Fig. 12 would be straight lines when plotted with similar coordinates as on Fig. 11 or Fig. 3.

The value of slope B_1 on Fig. 11 is 1.2×10^3 . Therefore, from Eq. (5.8) the value of Q can be calculated as $Q = 5,400$ cal./gm mol.

From Fig. 11 can be seen that the equivalent strain rate v_{T_0} is practically 10^5 inches per inch per second for the steel shown. Dividing this equivalent strain rate v_{T_0} by the actual strain rates v_{T_0} of Table 1 and taking the logarithms of these ratios, the values in the last line of Table 1 were obtained by the author. These values when plotted in Fig. 12 appear as the full circles. They are seen to fall near the curve for $Q = 5,000$ cal./gm mol postulated theoretically by Zener and Hollomon.⁽²⁾

Thus, the relation proposed by them⁽²¹⁾ is seen to agree with the experimental results of Witman and Stepanov⁽¹⁴⁾ shown in Fig. 11. The significant thing is that these tests are of entirely different nature than those performed by Zener and Hollomon (for the latter see Fig. 15). The values of the heats of activation which determine the coefficient of diffusion in solids are usually considerably higher than the value of Q found here. However, it is to be expected that the potential barrier determining the speed of the process of relaxation present in yielding phenomena should be lower than the barrier in diffusion. In diffusion, atom goes from one knot of the lattice to the next, (or from one column to the neighboring one, as seen in Fig. 13), whereas in relaxation it just goes from between the knots to the nearest knot (or from the midposition between the columns A and B in Fig. 13 to the nearest column).

3. Analytical Justification of the First Indirect Method for the Estimation of the Technical Cohesive Strength of Ductile Materials.

Schematic curve 1 in Fig. 9 represents the variation of the static yield point in tension with change of temperature. It is expressed by the

$$b_1 = \frac{\log_{10} s_{yV,T_l} - \log_{10} s_{yV,T_0}}{\frac{1}{T_l} - \frac{1}{T_0}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

The remaining unknown necessary for solution of Eq. (11b) is T'_0 . This critical temperature of brittleness at a low rate of straining is difficult, if not impossible, to obtain by testing, as the temperature is probably lower than can be realized easily in the laboratory. However T'_0 can be obtained in the following way, if the Zener and Hollomon's⁽²¹⁾ proposition is utilized.

If a sufficiently high (impact) rate of straining $\dot{\epsilon}'''$ is chosen, the corresponding critical temperature of brittleness T_0''' can be shifted upwards so as to be realizable with ordinary laboratory facilities.

The effect of increasing the rate of straining to some high rate v''' and simultaneously decreasing the temperature to T_0''' is equivalent to the effect of the simultaneous action of only a moderate rate of straining v' (statical) coupled with some (very low) temperature T_0' , and this equivalence is based on equation (5.7) of Art. 2 of this chapter. Substituting in Eq. (5.7) the terms v''' , T_0''' and v' , which are experimentally obtainable, one can determine the unknown quantity T_0' as follows:

$$\log_{10} \frac{v'}{v'''} = \frac{Q}{R} \left(\frac{1}{T'''} - \frac{1}{T'} \right) \log_{10} e \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (28) \text{ or } (5.7)$$

and

$$\frac{1}{T_1} = \frac{1}{T_2} - \frac{R \log_{10} \frac{v_1}{v_2}}{Q \log_{10} e} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (5.72)$$

In this equation the quantity Q appears, its determination will be explained presently. However, first will be discussed one way to determine T_0''' (reverse process). Using the lowest temperature obtainable in the laboratory (e.g. the temperature of liquid air), choose that particular temperature as T_0''' and

increase gradually the impact straining rate (without necessarily measuring the straining rate as yet), until the fractures become completely brittle. When that happens, the straining rate is the sought rate v''' , which, in conjunction with the known temperature T_c''' , causes complete brittleness. It remains then to measure this straining rate. That can be done by repeating the impact test once more with this strain rate and measuring it by some means (e.g. using electrical resistance strain gages and a cathode ray oscillograph). One way to do this may be to attach the resistance strain gages directly to the critical region of the specimen and obtain a timed oscillogram of the impact test. The record up to the fracture of the specimen (as well as of the gages attached to it) would not only give the rate of strain, but possibly even a readable value of the maximum strain at the instant of the break ($=\epsilon_f$). If so, because the fracture is brittle (no heating due to yielding -- isothermal) and the material is elastic up to the break, a product of the breaking strain ϵ_f and the modulus of elasticity E can be used as a direct measure of the cohesive strength

$$s_{c,p=0} = \epsilon_f \times E,$$

to be checked with the analytical result from Eq. (11b) of Art. 3 of this Chapter, or Eq. (11c).

For criterion of "complete brittleness" in impact tests may be taken the condition when the energy absorbed becomes zero. As the ordinary impact testing machines (Charpy, Isod) involve a considerable loss of energy, one way out would be to use a cable-suspended impact tester, e.g. similar to that described by Southwell⁽²⁰⁾.

In equation (5.72) the constant Q is so far unknown. The meaning of the constant Q was discussed above in connection with equation (2) HZ to (4) HZ. As for its physical meaning, Q can be considered as the heat of activation

that must be added before an atom in the strained metal can change place. It is a quantity of the type of the energy W discussed in connection with Eq. (9.3). Nevertheless the value of the constant Q for a given material can be obtained indirectly without consideration of the process of relaxation or diffusion in solids.

The theoretical basis for determination of Q will be first given and then a simplified procedure described.

One way of determining the value of Q is from a series of tests of the type performed by Witman and Stepanov⁽¹⁴⁾ (Chap. V, Art. 2). The results of Witman and Stepanov's tests are shown in Fig. 3 and 11. The value of Q can be calculated from the slope (e.g. B_1 on Fig. 11), as was shown in connection with Eq. (5.8) of Art. 2 of this Chapter. It is not necessary to run a whole series of such tests, because a straight line is uniquely specified by two points (except that the slope is subjected to error if the coordinates of one or both of the points contain experimental error). However, such tests are not simple and therefore an easier method is desirable.

5. Determination of Equivalent Rates and of Q .

Fig. 3 and Equation (4) of Art. 2, Chap. V show that the logarithm of the rate of straining is a linear function of the reciprocal of temperature. Fig. 5 and Eq. (8) of Chap. VI, Art. 1, on the other hand, show that the logarithm of the yield strength is also a linear function of the reciprocal of temperature. Therefore, the logarithm of the yield strength should be expected to vary linearly with the rate of straining plotted on logarithmic scale. This is shown schematically in Fig. 0.2. The expression for the straight line of Fig. 0.2 is obtained by simultaneously solving Eqs. (4) and (8). The resulting equation is

$$\log_{10} s_{yT} - \log_{10} s_{yT_0} = \frac{b_1}{B_1} (\log_{10} v_{T_0} - \log_{10} v_{T_0}) \quad . \quad . \quad . \quad (5.9)$$

Fig. 0.2, in addition, might throw some light upon the physical significance of the term "equivalent rate of straining." One can describe the terms also thus: simultaneous and combined action of a rate of straining v_2 and temperature T_2 is equivalent to the combined action of another rate of straining v' and temperature T_0 , if either pair has the same effect upon the material, (e.g. either pair causes beginning of yielding at the same stress, or either pair causes completely brittle fracture at identical values of stress, etc.).

The pair of values v' and T_0 is a pair of specific values of the variables v and T , which cause, when acting together, beginning of yielding at a stress of magnitude s_{yv, T_0} . The pair of values of v'' and T_0 is another pair of magnitudes of v and T , such as to cause the beginning of yielding at a stress of the magnitude s_{yv'', T_0} . These two pairs of values are not equivalent, because the stresses at which they cause yielding are not identical. However they give two points on Fig. 0.2, through which must pass the straight line locus of all yield points (when plotted in the manner as in Fig. 0.2).

Suppose the temperature is decreased to T_2 , a somewhat lower value than T_0 , and suppose a series of tension tests is performed, each with a different, successively lower rate of straining v . Suppose the rate of straining is changed by small steps. The tests then give a series of values of yield points. At some one of the rates of straining, say v_2 , the yield point is bound to be found numerically equal to s_{yv'', T_0} . Because this specific value of the yield point is obtained at a temperature of T_2 and rate of straining v_2 , and because this value of yield point is of the same magnitude as that obtained at the temperature of T_0 and rate of straining v'' , the two pairs of values, namely

(v_2, T_2) and (v'', T_0) , are equivalent to each other as far as their influence on the yield point of the material (and supposedly also on other properties, e.g. brittleness, technical cohesive strength) is concerned. It can be therefore said that the rate v_2 at temperature T_2 is equivalent to the rate v'' at room temperature T_0 . Thus the length OD on the axis of abscissas of Fig. 0.2 can be either called $\log_{10} v''$ or $\log_{10} v''_{(T_0 \text{ equiv. to } v_2 \text{ at } T_2)}$. The expression $v''_{(T_0 \text{ equiv. to } v_2 \text{ at } T_2)}$ is supposed to mean in "human" language the following: that rate of strain v'' , which, when acting at a room temperature T_0 , is equivalent to the rate of strain v_2 acting at the temperature T_2 . Hollomon and Zener (see the text following Equation 5.7 of Art. 2 of this Chapter) chose to call such a rate as the v'' discussed here by the name "equivalent rate."

Conversely, suppose a value of yield stress of magnitude $s_{yv'T_\ell}$ is obtained at the rate of straining v' and temperature T_ℓ (as was suggested in steps (1) and (2) of Chapter II, Article 7). This value, say OJ in Fig. 0.2, must be the ordinate of a point on the straight line of yield points of Fig. 0.2, because it was proved above in connection with Eq. (5.9) that, when the yield points are plotted in the manner of Fig. 0.2, they fall on a straight line. The point G on the straight line of Fig. 0.2 corresponds to the ordinate OJ and the corresponding abscissa OL is the logarithm of the strain rate, which when applied to a tension specimen at room temperature T_0 , will cause beginning of yielding at the same stress $s_{yv'T_\ell}$ as the strain rate v' at the temperature T_ℓ . Such a strain rate, read off the axis of abscissas ^{as} OL, is the strain rate at room temperature, which is equivalent to the rate v' at the low temperature T_ℓ .

It is hoped that the preceding discussion explained the procedure suggested in Chapter II, Article 7, Step (3) for finding to what rate at room temperature the rates v' and v'' at temperature T_ℓ are equivalent. Such an explanation will be attempted once more in a somewhat different manner in the

following.

The actual and equivalent strain rates are identical at room temperature, by definition of the equivalent strain rate, given in connection with Eq. (5.7) in this Chapter, Art. 2. The change in strain rate which is equivalent to the lowering of the temperature from T_0 to T_ℓ can be obtained by a reversed process. The ordinate of the point which should represent the yield strength (of the material under consideration) at the low temperature of T_ℓ and actual strain rate v' , is obtained from a test. On a graph of Fig. (0.2) this point is made to lie on the straight line EH, the locus of the yield strengths. This point G is marked on the straight line EH at the (experimentally obtained) value of ordinate OJ representing s_{yv, T_ℓ} to a logarithmic scale. The abscissa OL corresponding to this point G is then $\log_{10} v(T_0 \text{ equiv. to } v' \text{ at } T_\ell)$ (the equivalent strain rate necessary to produce the same mechanical effect on the material at room temperature as the actual strain rate v' produces at the low temperature T_ℓ). Having found this, the value of Q can be found from Eq. (5.71) modified in the following form:

$$Q = \frac{R}{\log_{10} e} \times \frac{\log_{10} v(T_0 \text{ equiv. to } v' \text{ at } T_\ell) - \log_{10} v'}{\frac{1}{T_\ell} - \frac{1}{T_0}} \quad . \quad . \quad (30)$$

The value of Q obtained in this manner should be checked by finding $v(T_0 \text{ equiv. to } v'' \text{ at } T_\ell)$ (the strain rate at room temperature, which is equivalent to the strain rate v'' at the low temperature T_ℓ). An average of the two values of Q, if different, should be more reliable than merely one value.

The substitution of Q, obtained in the above manner, in Eq. (28) gives $\frac{1}{T_0}$, as shown in step (6) of Article 7, Chapter II.

The substitution of the latter value ($\frac{1}{T_0}$) in Eq. (11b) gives the

value of the technical cohesive strength.

The above is the theory behind the seven steps suggested in Chapter II, Article 7 as a procedure for indirect estimation of the technical cohesive strength.

At this stage, it might be profitable to examine some experimental data concerning the preceding theory.

Hollomon and Zener⁽³⁷⁾ measured tensile strengths and yield strengths of several materials for several strain rates (for which the deformation was essentially isothermal) at various temperatures. The strain rates were approximated as the head speeds of the testing machine divided by the uniform gage length of the specimen (1.4"). Values obtained in this manner of course show only the relative (comparative) magnitudes of the strain rates, not the actual magnitudes.

The values of Q for each material were determined by trial and error. The criterion for the correctness of the chosen Q was the fact that all the values of the yield strength fell on a straight line (see e.g. Fig. 15).

The data for a cast steel of Fig. 15 are given in the table below marked $Q = 12,000$ cal./gm. mol.

Q = 12,000

Heat Treatment: Normal 2200°F.
Anneal 1600°F; Rough machined.
Quenched H₂O 1650°F.
Drawn 1100°F 7 hours.
Furnace Cooled. All in
10 in. rounds, 2 in. hole.

Steel Making
Electric Ind. Furnace
Forming:
Centrifugal Casting.

Composition:
C .24 Cr 1.05
P .009 Ni --
S .019 Mo .57
Mn .275 V .075
Al --

Temp. °C	Strain Rate x10 ⁴	Type of S - S Curve	Yield Strength		Tensile Strength		Fracture		Strain		Ep # Lb/in	R.A. %	Type of Fracture
			.01* or LP	.1 or LP	Orig. Area	Load Act.	Load Act.	Stress Lb/in ²	T. S. F. S.	to F. S.			
20	2.2	R**	136.0	144.0	153.4	169.5	169.5	186.0	.10	.288	110.	25.2	Trans. Cavities
20	43	R	140.0	148.0	159.8	175.9	175.9	184.0	.09	.170	110.	15.9	"
20	330	R	144.0	151.0	162.0	178.0	178.0	220.0	.08	.410	110.	35.5	C.P. cavities
-10	2.2	R	140.5	147.0	161.0	172.0	172.0	185.0	.095	.216	110.	19.8	Trans. Cavities
-12	43	R	147.5	156.0	164.0	173.0	173.0	196.0	.09	.240	110.	22.0	"
-12	330	R	151.0	158.5	167.8	183.0	183.0	190.0	.09	.154	--	14.2	"
-70	2.2	R	150.0	159.0	169.0	181.3	181.3	192.0	.08	.120	--	11.3	"
-70	43	R	156.5	165.0	173.0	187.0	187.0	198.0	.08	.184	--	17.0	"
-95	84	R	160.0	170.0	180.0	196.0	196.0	220.0	.08	.270	120.	22.9	"

*By the method employed - Difficult to determine accurately.

**R - rounded - no drop in Load at Yield.

#Ep is the slope of the strain-hardening curve.

All stresses - 1000 P.S.I.

The present author considers the line of yield strengths of Fig. 15 more significant than the line of the tensile strengths. Hollomon and Zener's Equation (1) HZ (Art. 2 of this Chapter) expresses the stress s as a function of the corresponding strain ϵ . For all the yield strengths, the value of ϵ was constant, or nearly so (offset 0.01 in./in. + elastic strain). With ϵ constant, the value of s in Eq. (1) HZ is therefore influenced only by the value of the parameter p , and the fact that the results lie on a straight line is evidence for the correctness of the proposition.

Expressed in different words, eq. (1) HZ can be said to postulate that the vertical height position of the stress-strain curve (i.e. its elevation, or shifting up or down) as a whole is completely specified or influenced by the single parameter p , but the stress s corresponding to the different points along the stress strain curve varies as a function of strain ϵ .

The effective stresses at rupture were reached after different amounts of plastic strains ϵ . Therefore they should be, according to equation (1) HZ, influenced both by the parameter p and by the amount of plastic deformation preceding the fracture. That explains why the effective rupture stresses are not specified by the parameter p alone. This can be seen from Fig. 2 obtained by Sakharov⁽¹²⁾. It shows that the rupture stress, at constant temperature and strain rate, increases with the amount of preceding plastic deformation.

Why the tensile strengths also fall upon a straight line in Fig. 15 would be clear if it should prove to be true that the maximum loads at different temperatures and strain rates correspond to approximately the same strains.

The conventional tensile strength, being a useful value in practice, has not much physical meaning. For practical approximation of tensile strengths at high rates of straining, the fact that the tensile strengths vary

linearly with $\log_{10} p$ as seen in Fig. (5) would be gratifying, if this phenomenon proved to be of general occurrence.

Additional experimental results, due to Hollomon and Zener⁽³⁷⁾, having a bearing upon the material discussed in this chapter are shown in Fig. 16. In it a hint of the physical meaning of brittle fracture, similar to that postulated in connection with Figs. 7, 8, 9, and 10 in Chapter VI, can be recognized. The yield point of the tempered pearlite is seen to rise with the increase in the equivalent strain rate until it reaches the fracture strength, when brittle fracture results. That the fracture at this point is brittle can be seen from the top part of Fig. 16, where the plastic deformation is seen to have dropped to zero at the rate of straining corresponding to the point of intersection of the yield strength and fracture strength curves.

The other steel, tempered martensite, does not break in a brittle manner at the rates of strain shown. In fact, cursory inspection shows that the curves of yield strength and fracture strength do not show much of a tendency to approach and intersect. Two things can be pointed out, however: (1) The yield strength and the fracture strength curves of the tempered pearlite are also practically parallel at the lower rates of strain. Similar behavior of the curves of the martensite does not preclude their approach and intersection at rates of straining higher than those shown in Fig. 16. (2) The top part of the Figure 16 (even at the rates of straining reached by Hollomon and Zener) already shows a drop in plastic deformation of the martensite, hinting at an approach of brittleness.

It should be pointed out that the curve of the fracture strength should not be expected to follow a horizontal course similar to the curve of the brittle strength of Figs. 9, 10, and 0.1. Except at the rate of 10^{17} inches per inch sec⁻¹, plastic deformation changed the initial tech. cohesive strength, in

the manner indicated in Fig. 1 or 2.

If in the top part of Fig. 16 the per cent elongation was plotted instead of per cent reduction of area, one might see an initial rise in ductility values obtained with increase of the rate of straining.

However, at higher rates, per cent elongation must drop and finally reach zero, as indicated by the drop to zero in the value of per cent reduction in the top part of Fig. 16. Thus the observations of some investigators⁽²⁹⁾, namely that the amount of plastic deformation might increase with increase in the rate of straining, can be understood. If still higher equivalent rates of straining were used, a region would be reached where the amount of plastic deformation drops and tends to become zero (brittle fracture).

6. Direct Investigations of the Influence of the Speed of Straining on the Yield Point in Mild Steels.

Winlock and Leiter⁽²⁷⁾ ran a set of tests which can be utilized to estimate directly the influence of rate of straining upon the yield point. (Earlier, in Art. 5 of this chapter, a relation intended to do likewise was shown as Equation (5.9). That equation was derived in a roundabout manner.)

Kanter, in his discussion of Winlock and Leiter's test results⁽²⁶⁾, plots them on a diagram, the abscissa of which is (according to the terminology of present paper) $\frac{1}{s_{yvT_0}}$ and the ordinate $\log_e v$. The abscissa is the reciprocal of the yield point s_{yvT_0} at the variable rate of straining v and room temperature T_0 . The ordinate is the variable rate of straining v on logarithmic scale. The five sets of data of Winlock and Leiter⁽²⁷⁾, when plotted to such a scale, fall each on a reasonably straight line. Kanter uses the results of Mann's high velocity impact tests⁽²⁹⁾ for his further reasoning. However it may be noted that e.g. Zener and Hollomon⁽²¹⁾ believe that these earlier observations of Mann are uncertain because of inherent difficulties in the apparatus.

Therefore from this point on a different procedure from Kanter's⁽²⁶⁾ is used. Fig. 14 shows Winlock and Leiter's results⁽²⁷⁾ replotted with $\log_{10} v$ along the axis of ordinates and $\frac{1}{s_y}$ along the axis of abscissas. The slope "m" of the straight line diagrams like that in Fig. 14, for a given material, may be obtained by making a few tests at different rates of straining. The slope of the straight line may then be written as

$$m = - \frac{\log_{10} v'' - \log_{10} v'}{\frac{1}{s_{yv''T_0}} - \frac{1}{(s_{yv'T_0})}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where v'' and v' are two different rates of straining (v'' larger than v'). Consequently, the yield point at any rate of straining v'' and room temperature T_0 can be expressed as

$$s_{yv''T_0} = \frac{1}{\frac{1}{s_{yv'T_0}} + \frac{1}{m} (\log_{10} v'' - \log_{10} v')} \quad . \quad . \quad . \quad . \quad (24)$$

Therefore the yield point at the high rate of straining v'' , for which the critical temperature is T_0'' , (see Fig. 9) is obtained by Eq. (24). From Equation (12) of Art. 5 of Chapter VI and from Equation (24) above, the technical cohesive strength is then

$$s_{c,p^0} = \frac{1}{\frac{1}{s_{yv'T_0}} + \frac{1}{m} (\log_{10} v'' - \log_{10} v')} \cdot b\left(\frac{1}{T_0''} - \frac{1}{T_0}\right) \quad . \quad . \quad . \quad (26)$$

where $s_{yv'T_0}$ is the statical yield point at rate of straining v' and room temperature T_0 , m is an experimentally determined slope on a diagram similar to Fig. 14, v'' is a sufficiently high impact rate of straining at which completely brittle fractures are obtained below and up to a critical temperature of T_0'' .

However, as outlined in connection with equation (22) below (Article 7 of this chapter), instead of v'' in Eq. (26) the rate of straining v_{T_0}

can be used. Then T_0'' becomes T_0 and the exponent of e becomes zero. Then equation (26) simplifies to

$$s_{c,p=0} = \frac{1}{\frac{1}{s_{yv''T_0}} + \frac{1}{m} (\log_{10} v_{T_0} - \log_{10} v')} \quad . \quad . \quad . \quad . \quad . \quad (27)$$

To estimate the technical cohesive strength on the basis of this equation, actual substitution in the equation is not necessary. The work can be done graphically as described under the heading "Second Indirect Method for Estimation of the Technical Cohesive Strength of Ductile Materials" (II., 9).

7. Analytical Justification of the Third Proposed Method for Determination of the Technical Cohesive Strength of Ductile Materials Without Direct Measurement of Brittle Strength.

This article serves as a basis for the method outlined in Chapter II, Article 10.

The derivation of the equation (13) used in Chapter II, Article 10, is based on the work of Witman and Stepanov⁽¹⁴⁾ and Fig. 9. From this Fig. 9, it can be seen that the technical cohesive strength $s_{c,p=0}$ is equal e.g. to the ordinate of point D. But the ordinate of point D, which lies on curve II can be obtained from the equation of that curve, viz. eq. (12) of Chapter V, Art. 6. Therefore

$$s_{c,p=0} = (s_{yv''T_0}) e^{b(\frac{1}{T_0''} - \frac{1}{T_0})} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12a)$$

The difficulty now is the determination of the yield strength $s_{yv''T_0}$ at room temperature but with some high rate of straining v'' . It is hard to measure reliably the breaking stress under impact with completely brittle fracture directly, it is almost equally difficult to measure reliably the yield strength during impact.

However, an expression can be derived which does not require any

Therefore, from (17), (18) and (10) of Chapter VI, Art. 1,

$$\log_{10} X = \frac{b_1}{B_1} \log_{10} \frac{v''}{v'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and

$$X = \frac{s_{yv''T}}{s_{yv'T}} = \left(\frac{v''}{v'}\right)^{\left(\frac{b_1}{B_1}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Using equations (12a), (14), (10) and (20), the technical cohesive strength is

$$s_{o,p=0} = s_{yv'T_0} \left(\frac{v''}{v'}\right)^{\left(\frac{b_1}{B_1}\right)} e^{2.3b_1\left(\frac{1}{T_0''} - \frac{1}{T_0}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Equation (21) can be used directly instead of the simpler equation (13) of Chapter II, Article 10, for the determination of $s_{o,p=0}$. The same steps (1t) to (6t) as given in Article 10 of Chapter II would have to be followed to get the necessary data for substitution in Equation (21).

However, if the rate of straining v_{T_0} is used for v'' in equation (21), T_0'' becomes T_0 , the exponent of e becomes zero and Equation (21) becomes

$$s_{(o,p=0)} = s_{(yv'T_0)} \cdot \left(\frac{v_{T_0}}{v'}\right)^{\left(\frac{b_1}{B_1}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Equation (13) of Chapter II, Art. 10 is another form of equation (22).

The order of magnitude of the errors that might be connected with such a determination, using extrapolation through two points only, may be considerable. The relation has not been practically tested as yet, as mentioned before. The above is intended to be the justification for the 6 steps of Chapter II, Article 10.

VIII. Experimental Test of the "Notch and Extrapolation" Method for Determination of the Technical Cohesive Strength (e.g. Kuntze's Method).

1. Determination of the Technical Cohesive Strength of a Metal With a Critical Temperature Higher Than the Temperature of Liquid Air.

It was endeavored to show earlier that if a material has a fairly high temperature of statical brittleness, high enough to be attainable in a laboratory, a foolproof method for determining the technical cohesive strength $s_{(c,p=0)}$ of that material is this: Find the value of brittle strength s_b of unnotched tensile specimens tested at or below the critical temperature. The value of $s_{(c,p=0)}$ is equal to that obtained for s_b .

The $s_{(c,p=0)}$ obtained by this method can then be used to check the values obtained by any other method.

2. Actual Comparison of the Values of the Technical Cohesive Strength Obtained by Two Different Methods.

Such a test was made by Sakharov⁽¹²⁾. He compared the results of Kuntze's method for determination of technical cohesive strength s_T against the value of brittle strength s_b of unnotched specimens tested statically at the temp. of liquid air. The steel used had the following chemical composition: 0.2 per cent C; 0.45 per cent Mn; 0.027 per cent P; 0.026 per cent S. The average diameter of the grains was about 0.003 inches (0.08 mm). This coarse grain was obtained purposely by long annealing, in order to raise the critical temperature. In addition, the steel used for specimens was pre-strained in tension, before the specimens were machined. Complete brittleness of the specimens was checked by automatically drawn load-deformation curves. Only the results of those tests were used, for which the load-deformation curves consisted of straight lines up to fracture. Table 2 shows the results. Symbol s_T designates the value obtained for the technical cohesive strength by Kuntze's method,

whereas s_b denotes the average breaking stress of unnotched specimens failing in a brittle manner at the temperature of liquid air.

Table 2 (After Sakharov⁽¹²⁾)

Comparison of Values of Tech. Cohes. Strength Obtained by Kuntze's Method and the Brittle Strength Method.

Degree of Prestraining in tension per cent	Ultimate Strength room temp., psi.		s_T Technical Cohesive Strength, psi. by Kuntze's Method	s_b Brittle Strength of Unnotched Specimens at the Temp. of Liquid Air, psi.
	Unnotched Specimens	Notched Specimens		
		90° V $k = \frac{d^2}{D^2} = 0.64$		
2	51,700	74,700	176,000	107,000
5	55,000	78,000	185,000	116,600
10	58,300	83,000	195,000	129,300
20	66,700	91,700	206,000	141,000

3. Possible Sources of Error.

The recommendations of Kuntze⁽¹⁾ (regarding the notch, the machining, and the centering of the specimen during test) were not literally followed by Sakharov. However, with the exception of the machining, the differences of Sakharov's procedure from the recommendations of Kuntze should make the s_T values of the technical cohesive strength obtained by Kuntze's method (fourth column of Table 2) too low, if anything. The notch used deviates from that recommended by Kuntze⁽¹⁾ in the direction of increasing danger of "premature fracture", which would make the values s_T shown in the fourth column, at worst, lower than the values of s_T which Kuntze would get. Lack of proper centering of the specimens during the test would have a larger influence on the notched specimens than on the unnotched ones, which would make the values of s_T shown in

the fourth column, at worst, lower than the values of s_T Kuntze would obtain. Suppose that the machining procedure used by Sakharov did cold-work the notch bottom more than that recommended by Kuntze would. From Table 2 it can be seen that the average (and almost constant) difference between the values of s_T obtained by Kuntze's method and the values of s_b obtained by the brittle strength method is about 67,000 psi. From Fig. 2 it can be estimated by extrapolation that, to cause such an increase of the technical cohesive strength, the cold working at the bottom of the notch would have to be of the order of magnitude of an additional 30 per cent permanent set. Such a degree of cold-working is hardly to be expected from improper machining alone. In fact, if Kuntze's own qualitative curve of variation of s_T with cold-work is used, the conclusion would have to be that no degree of additional cold work could increase by 65,000 psi. the tech. coh. strength of the metal prestrained already previously to a permanent set of 20 per cent (last line in Table 2). However this is the amount by which the value s_T of tech. coh. strength obtained by Kuntze's method exceeds $s_b = s_{c,p=0}$ obtained by the brittle strength method.

4. Conclusion about the Validity of Kuntze's Method.

It must therefore be concluded that the values s_T of the technical cohesive strength obtained by Kuntze's method in this case are excessively high, or that Kuntze's method, possibly due to phenomena discussed earlier in connection with Fig. 1, does not give the true value of the technical cohesive strength.

IX. Another Experimental Test, Less Direct, of Kuntze's Method.

1. Difference in Kuntze's Values of Tech. Coh. Strength for Metals Cold-Worked in Tension and in Compression.

From one of Kuntze's papers⁽¹⁸⁾, it appears that, if a virgin metal (e.g. 0.7 per cent C steel) is cold worked by extension, the value of its cohesive strength rises, but if the same virgin metal is cold worked in compression, its cohesive strength, as indicated by s_T , obtained by Kuntze's method, falls by about 30 per cent.

2. An Attempt to Check Kuntze's Results.

Davidenkov and Shevandin⁽¹⁷⁾ tested in tension notched specimens (120° V notch) of two metals. The machining and notching of the specimens was done after subjecting them to severe cold-work. The results of the tests are shown in Table 3. These results seem to indicate that the average notched strength (s_N = maximum load divided by root area), at room temperature, of those specimens (from both metals) which were cold worked in compression before machining ("pre-compressed" specimens for shortness) was lower than s_N , at room temperature, of the specimens (of the same metals) which were cold worked in tension previous to notching ("pre-extended" specimens). The cold working in tension was carried to a degree equivalent to the degree to which cold working in compression was carried. However, at the temperature of liquid air, similar notched specimens gave the reversed results: The average notched strengths s_N of the pre-compressed metals were about 20 per cent higher than the s_N of the pre-extended metals.

3. Results of Additional Tests on Unnotched Specimens and Explanation of the Difference Found by Kuntze.

Supplementary tests by Davidenkov and Shevandin on unnotched

Table 3

Difference in Notched Strength s_N of Pre-Extended and Pre-Compressed Steels at Room Temperature and at the Temperature of Liquid Air.

Davidenkov and Shevandin⁽³⁶⁾
(17)

Steel	s_N , Notched Strength, psi.*	
	Room Temperature	(120° V notch) at Liquid Air Temp.
0.2 per cent C		
Pre-extended	133,300	91,500
Pre-compressed	132,000	108,700
Per cent difference	+1 per cent	-19 per cent
0.8 per cent C		
Pre-extended	186,500	129,000
Pre-compressed	170,000	156,000
Per cent difference	+9.5 per cent	-21 per cent
* Average of several determinations		

specimens lead to several interesting results and conclusions:

(a) The unnotched specimens machined from metal previously strained in tension did not show any uniform axial elongation and uniform lateral contraction during the tensile test. They started necking down.

(b) The unnotched specimens machined from the metal previously strained in compression had considerable uniform elongation and lateral contraction during the subsequent tension test. The local elongation and lateral contraction during the necking down portion of the tension test was about the same for both kinds of pre-strained metals. The following Table 4 shows the per cent total reduction in area at break at room temperature:

Table 4.

		Steel	
		0.2 per cent C	0.8 per cent C
Per cent Reduction of Area at Break	Pre-extended	62.1 per cent	31.6 per cent
	Pre-compressed	67.9 per cent	45.9 per cent

(c) The yield stress of the unnotched specimens machined from the pre-extended 0.2 per cent C steel was 45 per cent higher than the yield stress of the precompressed metal (similar difference of 65 per cent was observed for the 0.8 per cent C steel).

(d) Due to (a) and (b), the actual cross-sectional area at failure of the pre-compressed specimens was smaller than that of the pre-extended specimens. Also the actual cross-sectional area of the pre-compressed specimens at maximum load was smaller than that of the pre-extended ones.

(e) As one of the values utilized in the extrapolation to s_T , Kuntze uses the conventional ultimate strength s_u based on the maximum load and the original cross-sectional area. As a consequence of (d), the values used by Kuntze for the ultimate strength s_u of the unnotched specimens machined from the precompressed metal were lower than those of the specimens machined from the pre-extended metal. The s_u of the pre-extended 0.2 per cent C steel exceeded that of the same pre-compressed material by 3 per cent. In the case of the 0.8 per cent C steel, the excess of s_u of the pre-extended metal over the s_u of the pre-compressed metal was 17 per cent. The effective stresses at maximum loads, however, were practically identical for the pre-extended and the pre-compressed varieties of either steel.

(f) The effective breaking stress s_f of the pre-compressed 0.2 per cent C steel is 6 per cent larger than s_f for the pre-extended variety of the same metal. The s_f of the pre-compressed 0.8 per cent C steel is 10 per cent larger than the s_f for the pre-extended variety of the same steel. This result is, as far as sign is concerned, in agreement with the s_H at liquid air temperature. This result is, however, diametrically opposite (in sign) to that obtained by Kuntze's method.

(g) A phenomenon similar to that described under the item (e) above probably took place during the tension test of notched specimens at room temperature. The cross-sectional area of the notched specimens machined from the pre-compressed metal, at the instant of maximum load, was smaller than that of the specimens machined from the pre-extended metal.

(h) As a consequence of (g), the values of notched strength s_N , calculated on the basis of the maximum load and the original cross-sectional area, (which were used by Kuntze together with the s_H for extrapolation to s_T) were lower for the case of the notched specimens machined from the precompressed metal.

(j) As both the values of s_H and s_N used by Kuntze for extrapolation to s_T were too low for the pre-compressed metal, the value of s_T extrapolated from them was also too low. The lower value of s_T for pre-compressed metal, found by Kuntze⁽¹⁶⁾, is therefore not due to any actual decrease of technical cohesive strength due to pre-compressing the metal, but rather to the shortcomings in Kuntze's method for determining the tech. coh. strength. Davidenkov and Shevandin⁽¹⁷⁾ state that, when the actual areas of cross-section at the maximum load are used, the values of s_T are practically identical (within 2 per cent) for the pre-extended and the pre-compressed metals.

(k) The following question then comes up: Why could one not use the effective stresses s_{true} based on the maximum load and on the actual area at

eye. The only condition under which the failure can be completely brittle is this: initial yield points must be higher than the tech. cohesive strength $s_{c,p=0}$ of the virgin metal. This is possible only below the critical temperature t_0 of brittleness of unnotched specimens.

It was mentioned in (c) above that s_y of the pre-extended 0.2 per cent C steel is 45 per cent higher than that of the pre-compressed metal of the same composition. Therefore, as can be seen from Fig. 9, the critical temperature of the pre-extended variety of the metals is higher, and the amount of plastic flow as well as the raising of the current technical cohesive strength by this plastic flow (see Fig. 2) is thus smaller. This may at least partly account for the difference between the notched strengths s_y at the temperature of liquid air.

X. Physical Meaning of Technical Strength.

1. Conclusions from the Experimental Material Mentioned Earlier.

Ioffe⁽²⁾ tested (in tension) rock-salt crystals (Fig. 10) at temperatures higher than critical. In those cases he observed plastic flow and necking down of the specimens. The plastic flow was the more pronounced the higher was the temperature. When he calculated the breaking stress on the basis of the cross-sectional area at fracture, he obtained stresses increasing with the amount of preceding plastic flow and approaching 7,100 psi. This value exceeds considerably the value of the brittle strength (640 psi.). When the rate of straining was sufficiently high to raise the yield point to the level of the $s_{o,p=0}$, no plastic flow could take place before $s_{o,p=0}$ was reached. The absence of plastic flow meant no raising of the technical coh. strength to continuously higher values (designated as the current cohesive strength s_{ep}). The material therefore broke as soon as $s_{o,p=0}$ (of about 640 psi.) was reached.

After considering a number of proposed explanations of this behavior, Ioffe comes to the following conclusion: It is possible that the value of the physical cohesive force theoretically derived from the well-proven electrical theory of crystal structure (amounting in this case to about 280,000 psi.) is the actual strength of the rock-salt, rather than the experimentally attained values. If this idea is correct, there must be another, incidental cause of the fracture (apparent low strength).

The phenomenon of increase of strength of a material due to cold-work (strain-hardening) can be therefore ascribed to some partial removal of the incidental cause of fracture.

Following the ideas of Griffith⁽³¹⁾ and Smekal⁽³⁴⁾, such incidental cause of premature tearing of Ioffe's single crystal specimens can be found in stress concentration which can be either (a) present at the surface of the

crystal before the test (in the form of defects), or (b) emerging on the surface as traces of slip planes connected with plastic flow during the test. It is possible also that both (a) and (b) can be present simultaneously.

The technical cohesive strength of a single crystal can be thought of, accordingly, as that average stress which, when multiplied by the stress concentration factor of the surface stress raisers (either present originally or appearing during the plastic flow), reaches the physical cohesive strength of the crystal and causes a progressive growth of cracks. Thus the physical cohesion forces are apparently not overcome simultaneously along a whole crystallographic plane, but consecutively, one by one.

Such an explanation was tested by Ioffe. The salt crystals were found to be brittle in hot oil or even in boiling saturated salt solution, as long as their surface could not be dissolved away. But in hot unsaturated water, or even in a colder water not saturated with salt, where the stress-concentrations present or appearing on the surface could be swiftly rounded off or removed by dissolution, the same crystals were plastic and flexible.

Salt crystals were tested in tension in hot water far below 200 deg. centigrade (392°F). The latter temperature is the critical temperature of brittleness for rock-salt. Still the crystals did not break at the stress of 640 psi., but exceeded safely the yield point and flowed plastically, with simultaneous dissolution. At the instant of the fracture, the water was automatically spilled out, so that the cross-sectional area was not appreciably changed after break. The quotients of the breaking load divided by the final cross-section area gave values as high as 43,000 and even as high as 230,000 psi. These values approach in order of magnitude the theoretical cohesion strength of about 280,000 psi. Such tests allowed Ioffe to continue the curve I (Fig. 10) even above the brittle strength line BD. The continuation (without the actual Ioffe's values) is shown on Fig. 10 by the dot-dash curve.

XI. Vita.

The author was born on May 11, 1911, in Prague, Czechoslovakia. After graduating in 1930 from 1st Czech State Real Gymnasium, Prague, he took a year's course (1930-31) in the Academy of Commerce, Prague, and attended the University of Prague in 1932-33. In the years 1933 to 1937, he attended the University of Southern California in Los Angeles, California. The year 1937-38, he spent on a fellowship in Mechanical Engineering at Rensselaer Polytechnic Institute, Troy, New York, and in 1938-39 he held the Sibley Fellowship in Mechanical Engineering at Cornell University, Ithaca, New York. During the first semester of 1939-40 he held a graduate School Fellowship at the University of Illinois. From February 1940, to September 1941, he was Teaching Assistant in Mechanical Engineering at the Illinois Institute of Technology in Chicago, Illinois. From September 1941 to February 1942 he was a graduate research Assistant, from February 1942 to September 1943 an Instructor, and since September 1943 an Associate in Theoretical and Applied Mechanics at the University of Illinois, Urbana, Illinois. His professional experience, besides the research and teaching mentioned above, was three years' work in costs, systems and auditing with H. D. Campbell, C. P. A., Los Angeles, and consulting work in testing, design and stress analysis for several steel and machinery manufacturing companies.

He received B.S. (B.A.) degree in 1934 at the University of Southern California, M.B.A. in 1935, B.S. (M.E.) in 1937. He is member of the following honorary societies: Beta Gamma Sigma (Commerce), Phi Kappa Phi, and Sigma Xi.

The titles of two publications of the author in collaboration with Professor H. F. Moore as the senior author are:

"Progress Report on the Effect of Size of Specimen on Fatigue Strength of Three Types of Steel," A.S.T.M. Proceedings, Vol. 42, 1942, pp. 145 and foll.

"Second Progress Report on the Effect of Size of Specimen on Fatigue Strength of Three Types of Steel," A.S.T.M. Proceedings, Vol. 43, 1943.

The following are the published discussions of the author:

"A Discussion of Plastic Flow as an Unstable Process" (discussion of paper by L. H. Donnell), Journal of Applied Mechanics, Transactions A.S.M.E., Vol. 10, No. 1, March 1943, pp. A-49 and foll.

"A Discussion of Plastic Flow and Rupture of Metals" (discussion connected with a paper by C. Zener and J. H. Hollomon) Transactions of the American Society for Metals, Vol. 33, 1944, pp. 221 and foll.

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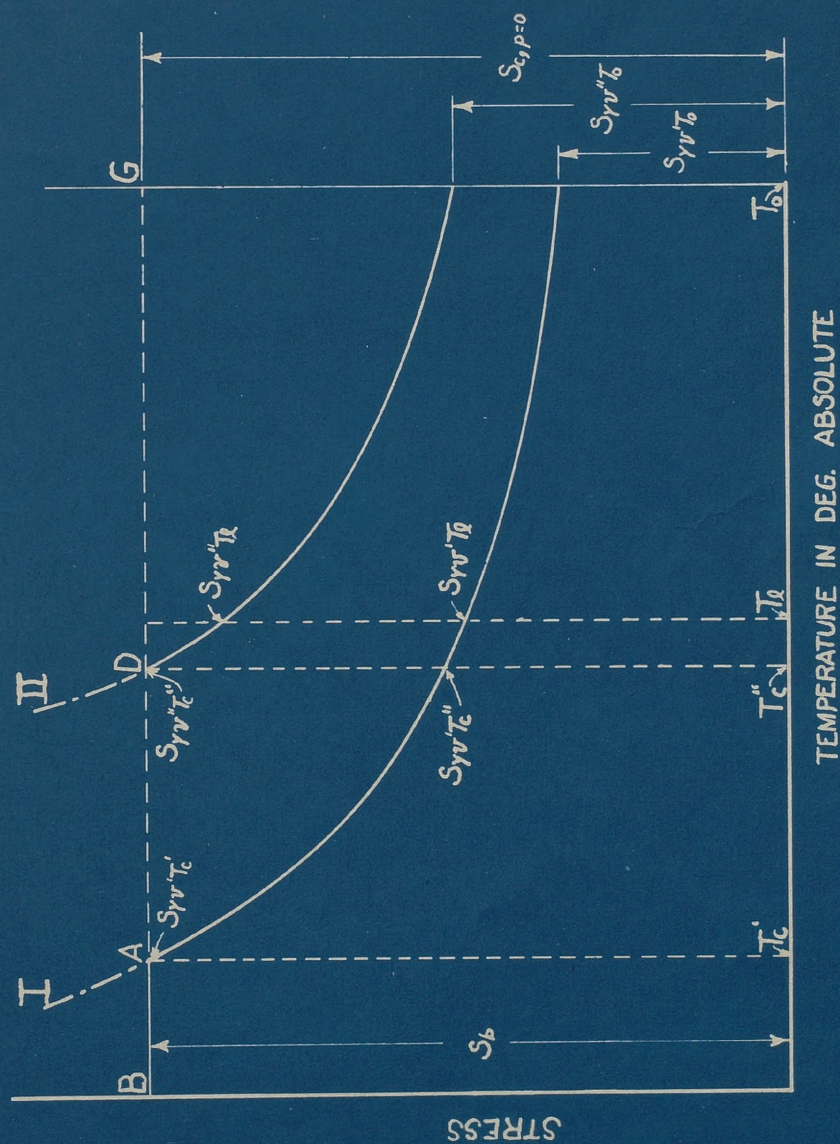


FIG. 0.1 SCHEMATIC DIAGRAM OF VARIATION OF YIELD STRENGTH S_y AND BRITTLE STRENGTH S_b WITH TEMPERATURE T AND RATE OF LOADING v .

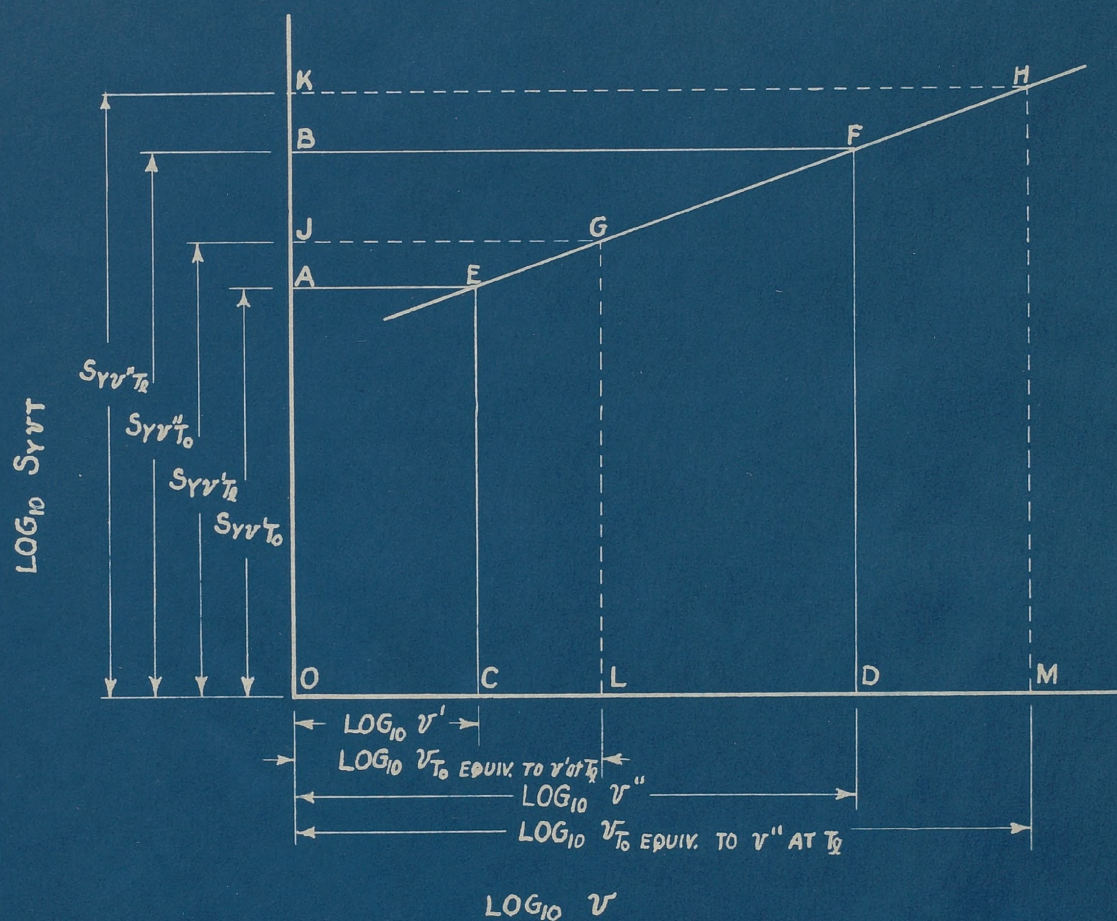
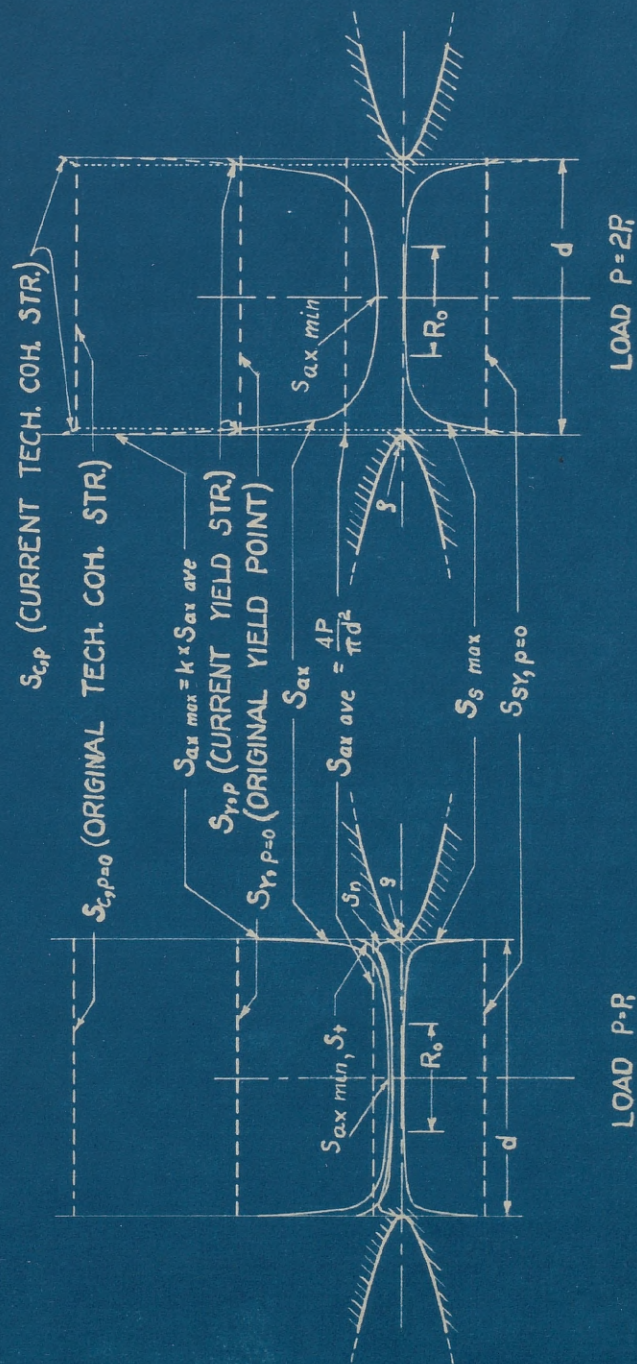


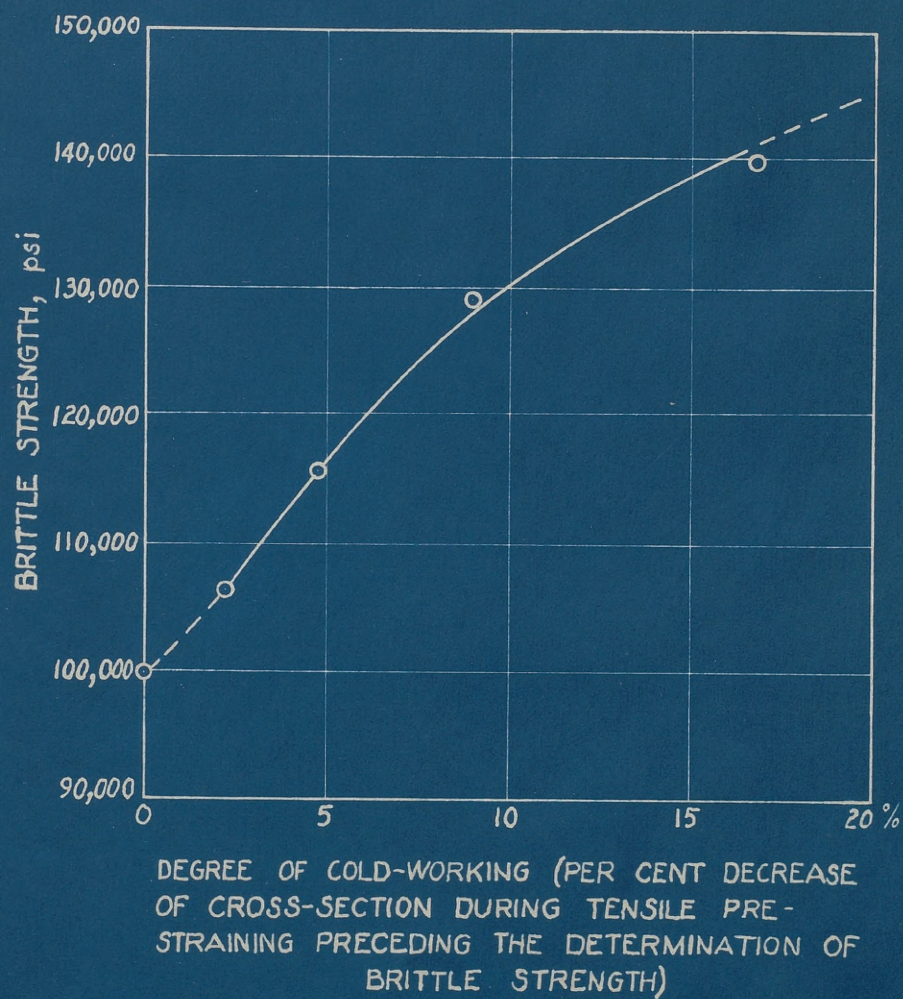
FIG. 0.2 SCHEMATIC DIAGRAM SHOWING THE DETERMINATION OF THE RATE OF STRAINING (V_{T_0} EQUIV. TO V' AT T_0) WHICH, IF USED AT ROOM TEMPERATURE T_0 , IS EQUIVALENT (IN ITS EFFECT UPON YIELD STRENGTH S_Y) TO A STRAIN RATE V' AT TEMPERATURE T_0 . BASED UPON THE EXPERIMENTAL RESULTS OF ZENER AND HOLLomon (SEE E.G. FIG. 15), WHICH SHOW THAT THE CURVE OF YIELD STRENGTHS VS. THE RATES OF STRAINING IS A STRAIGHT LINE WHEN PLOTTED TO LOG SCALE.



DISTRIBUTION OF STRESS ACROSS THE
MINIMUM CROSS-SECTION OF A NOTCHED
TENSILE SPECIMEN ($\frac{d}{2g} = 24$)

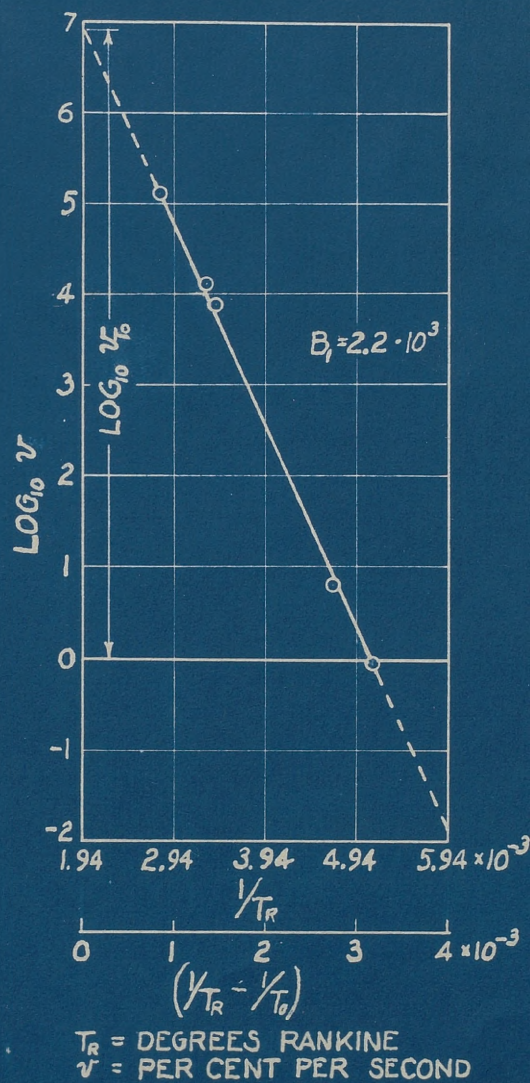
FIG. 1a.

FIG. 1b.



DEPENDENCE OF BRITTLE STRENGTH ON THE
AMOUNT OF PRECEDING PLASTIC DEFORMATION
IN TENSION (SAKHAROV)

FIG. 2.



VARIATION OF THE CRITICAL TEMPERATURE OF
 BRITTLENESS T_R WITH THE RATE OF STRAINING ν
 (VITMAN AND STEPANOV)

FIG. 3.

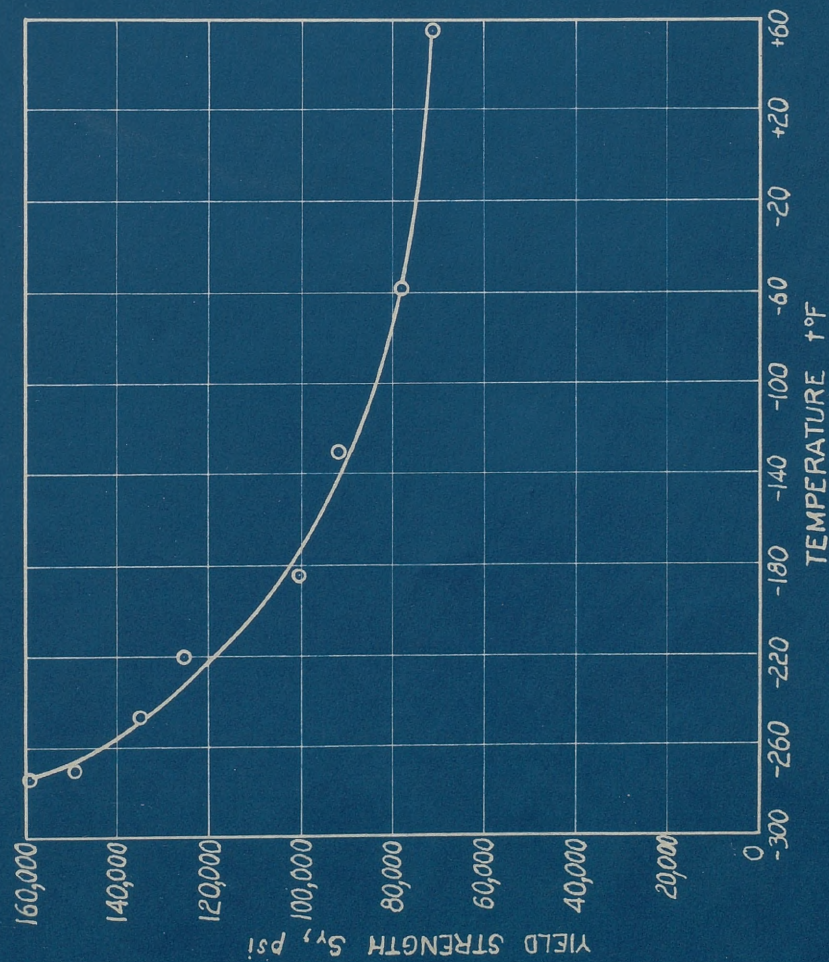


FIG. 4.

VARIATION OF YIELD STRENGTH WITH TEMPERATURE
(VITMAN AND STEPANOV)

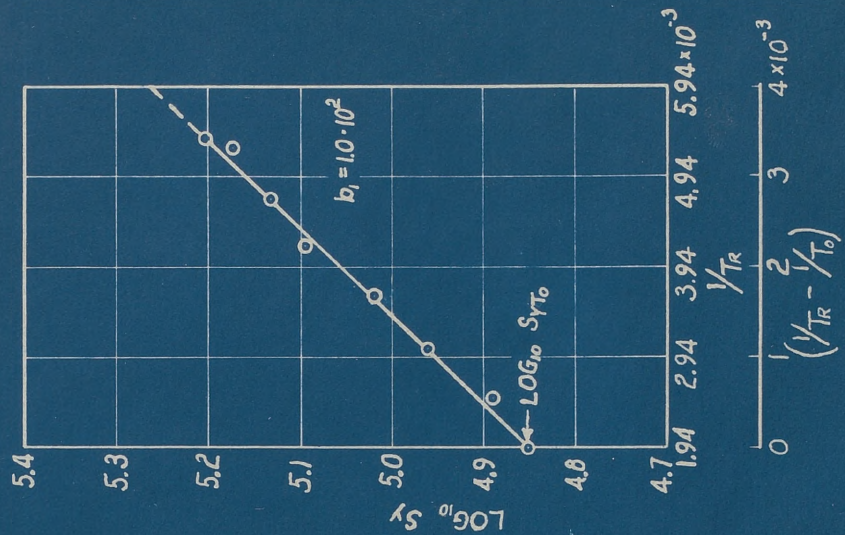


FIG. 5.

STRESS, psi

TEMPERATURE $^{\circ}\text{F}$

200,000

160,000

120,000

80,000

40,000

0

-320 -240 -160 -80 0 80

A

B

I

$t'_c = \text{CRITICAL TEMP. AT } \nu'$

ABSORBED, FT.-LB.

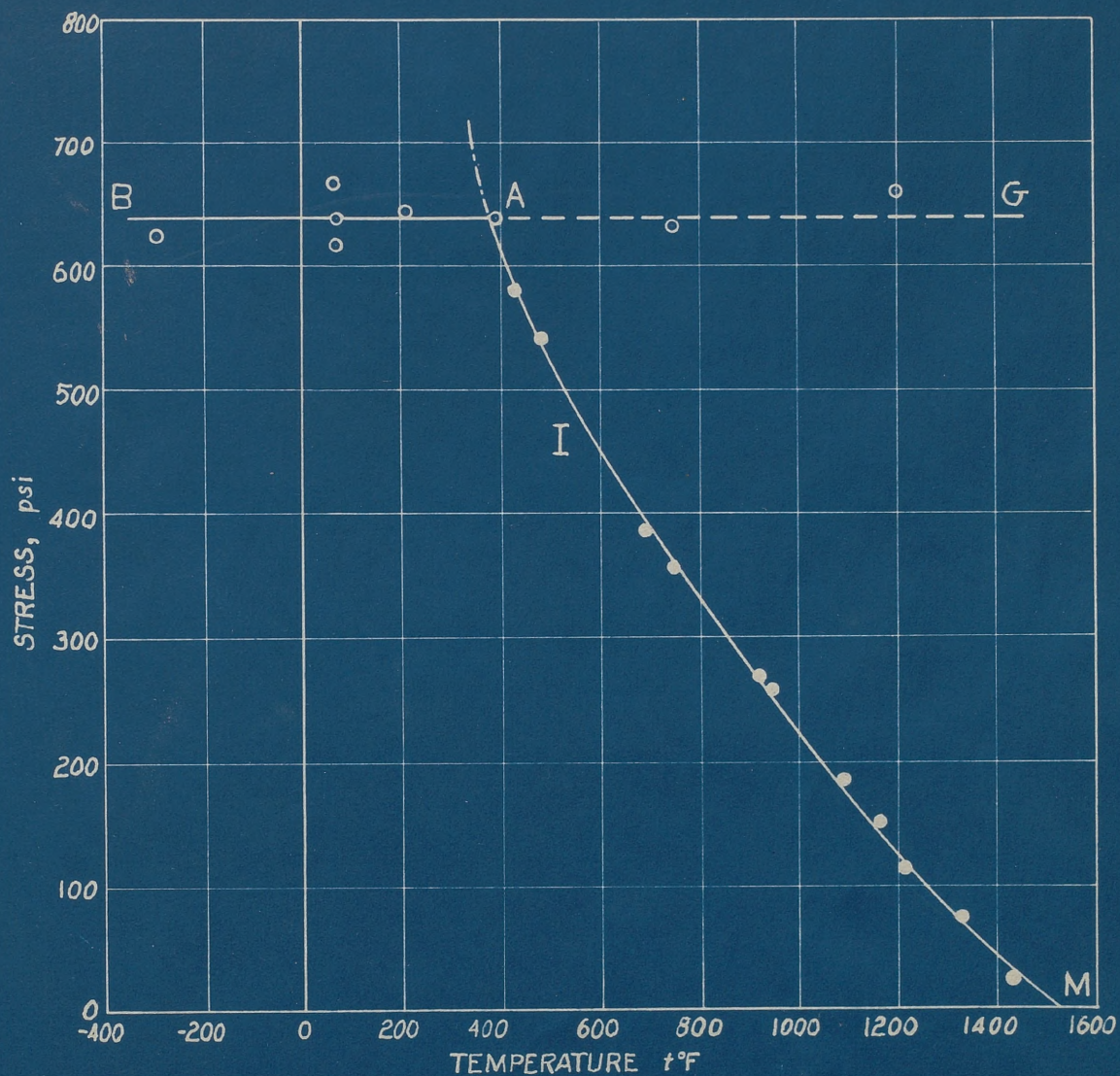
$t'_c = \text{CRITICAL TEMP. AT } \nu'$

t'_{cu}

ENERGY

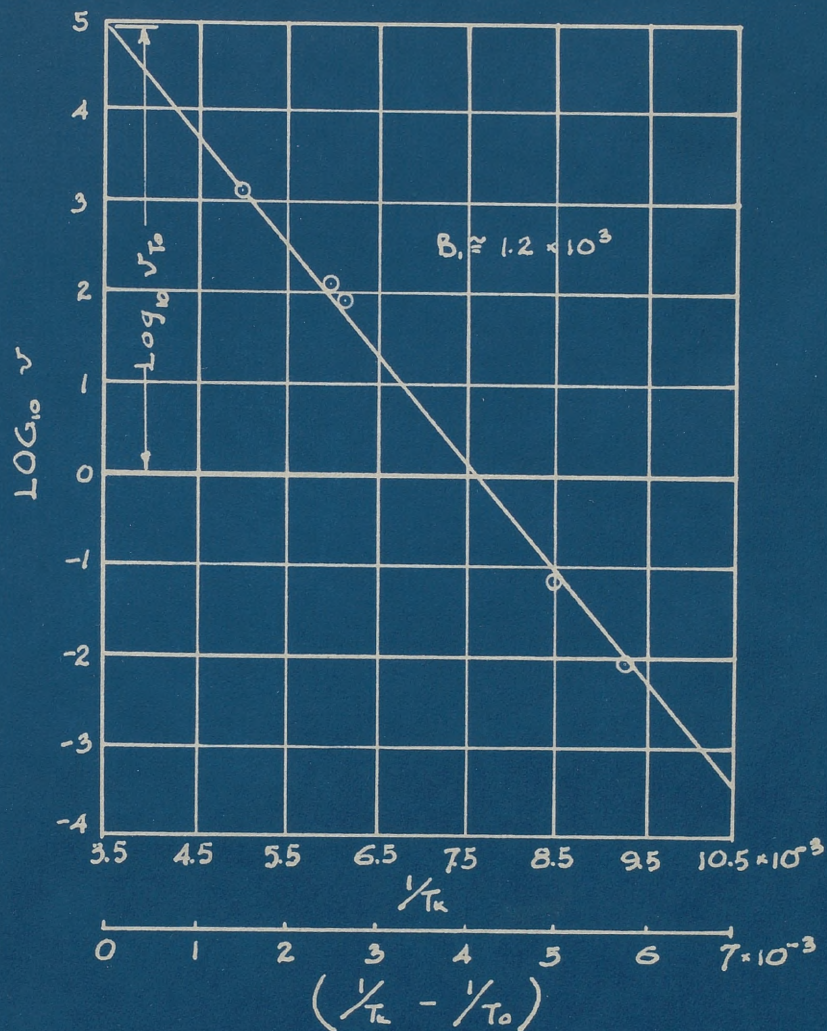
$t_c'' = \text{CRIT. TEMP AT } v''$

HIGH PHOSPHORUS STEEL (SHEVANDIN AND KISIN)



VARIATION OF YIELD STRENGTH • AND BRITTEE
STRENGTH ° OF ROCK SALT WITH TEMPERATURE
(IOFFE')

FIG. 10.



T_k = DEGREES KELVIN.

v = INCHES PER INCH PER SEC.

FIG. II VARIATION OF THE UPPER CRITICAL TEMPERATURE OF BRITTLENESS WITH THE RATE OF STRAINING.

(WITMAN AND STEPANOV)

C. ZENER AND J. H. HOLLLOMON

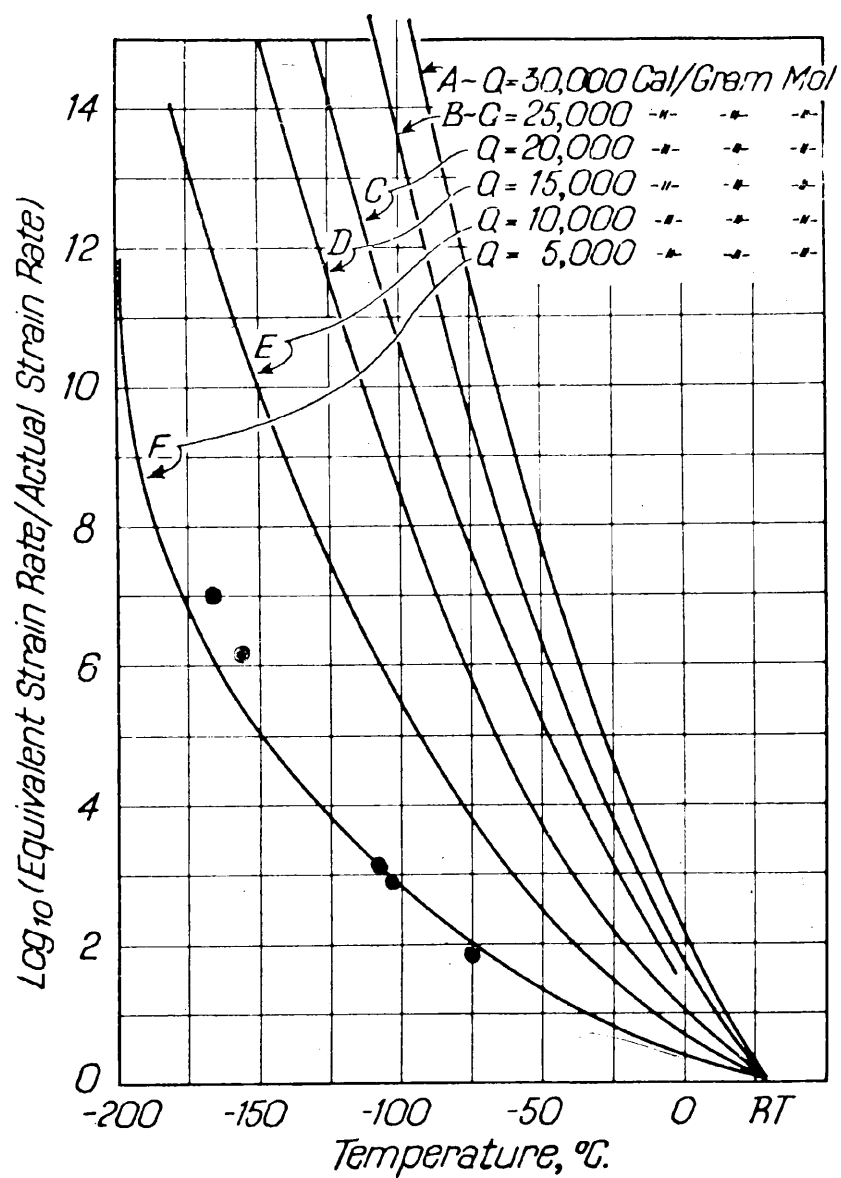
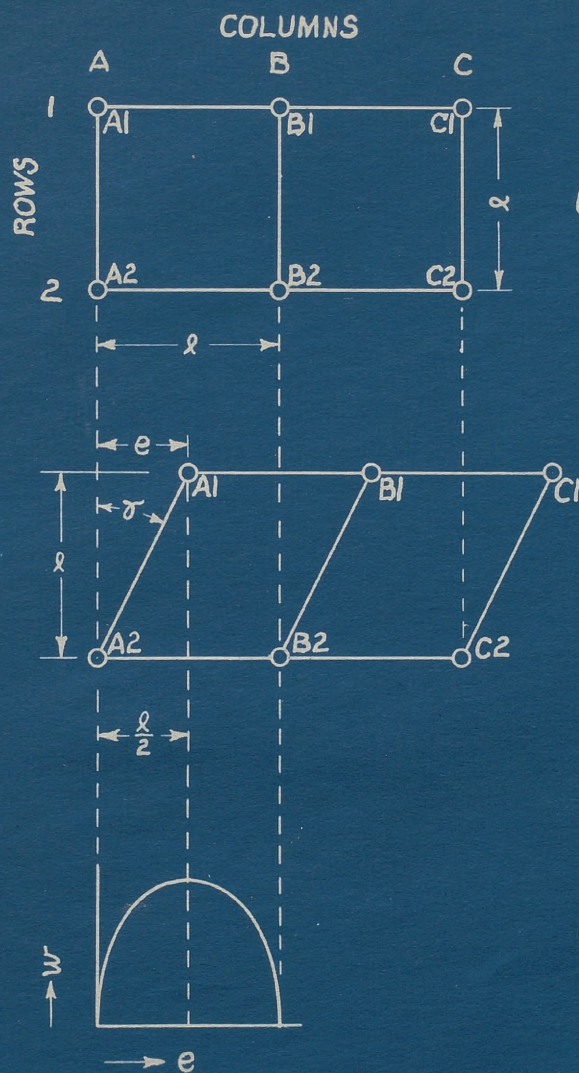


Fig. 12.—Equivalence of a Decrease of Temperature to a Rise in Rate of Strain.



(a) SIMPLIFIED SCHEMATIC PICTURE OF ONE PLANE IN AN UNDISTURBED CRYSTAL LATTICE. (l =LATTICE CONST.)

(b) ONE PLANE OF CRYSTAL LATTICE DISTORTED HOMOGENOUSLY SO THAT A1 IS READY TO JUMP TO COLUMN B, B1 TO COLUMN C, ETC.

(c) ELASTIC STRAIN ENERGY w ABSORBED BY UNIT VOLUME WHEN ROW (1) IS DISPLACED HOMOGENOUSLY WITH RESPECT TO ROW (2) BY AN AMOUNT e = LINEAR STRAIN (OR DISTORTS BY AN ANGLE γ = SHEAR STRAIN)

FIG. 13

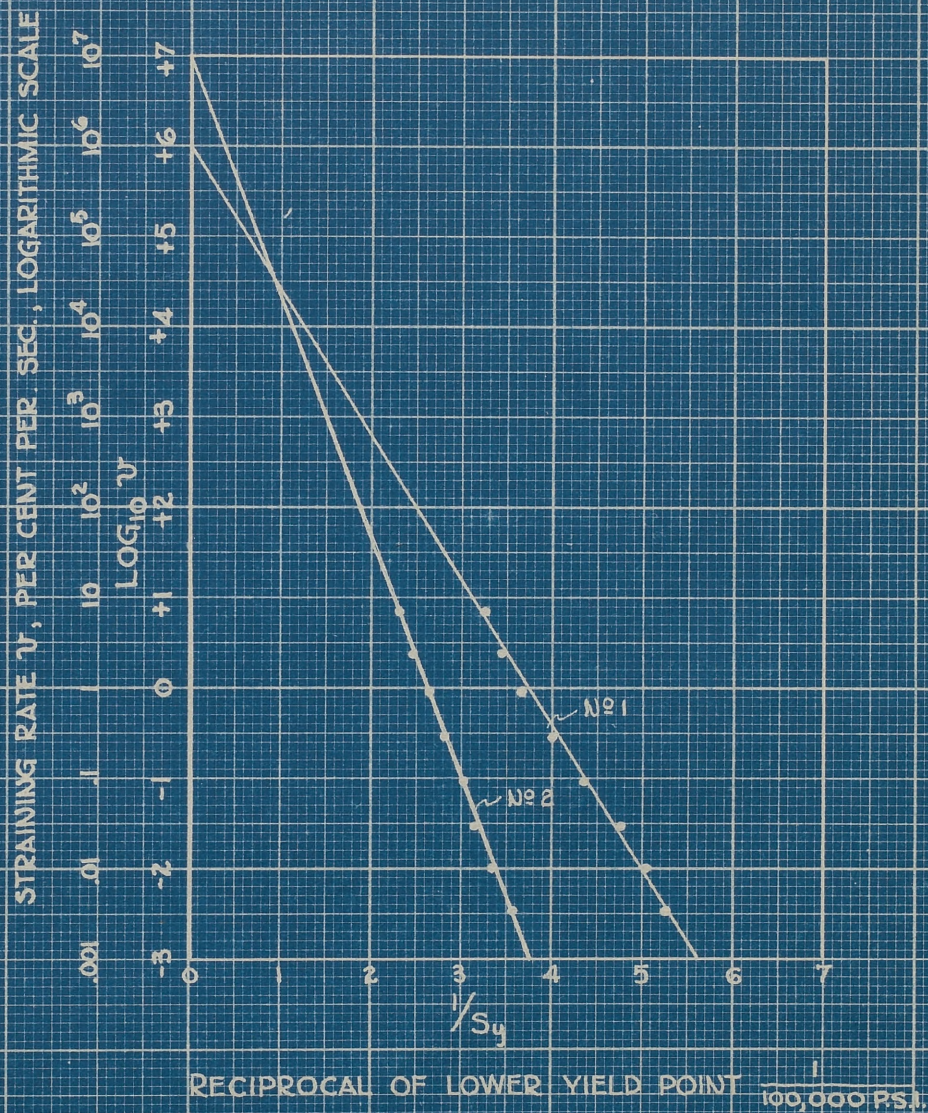
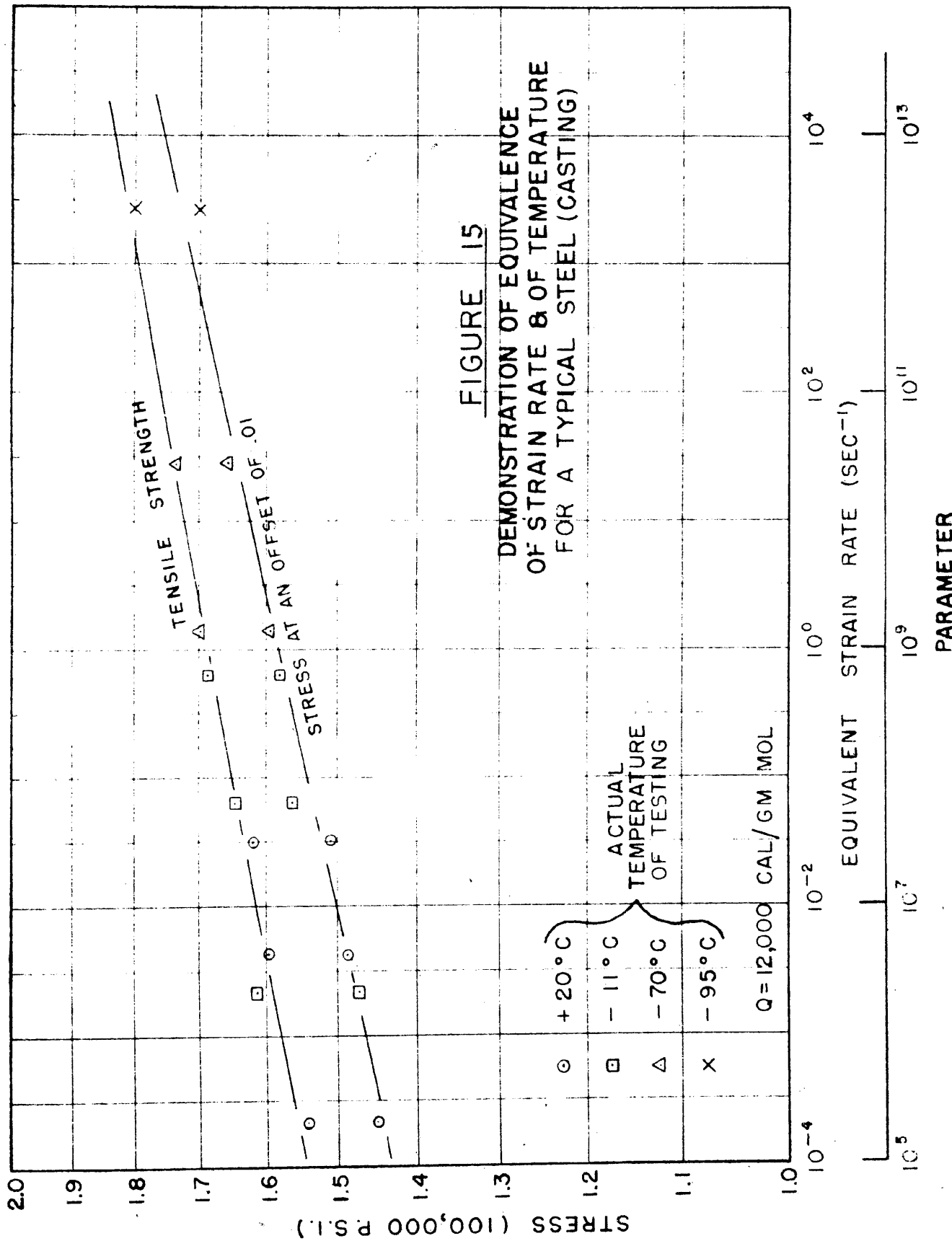
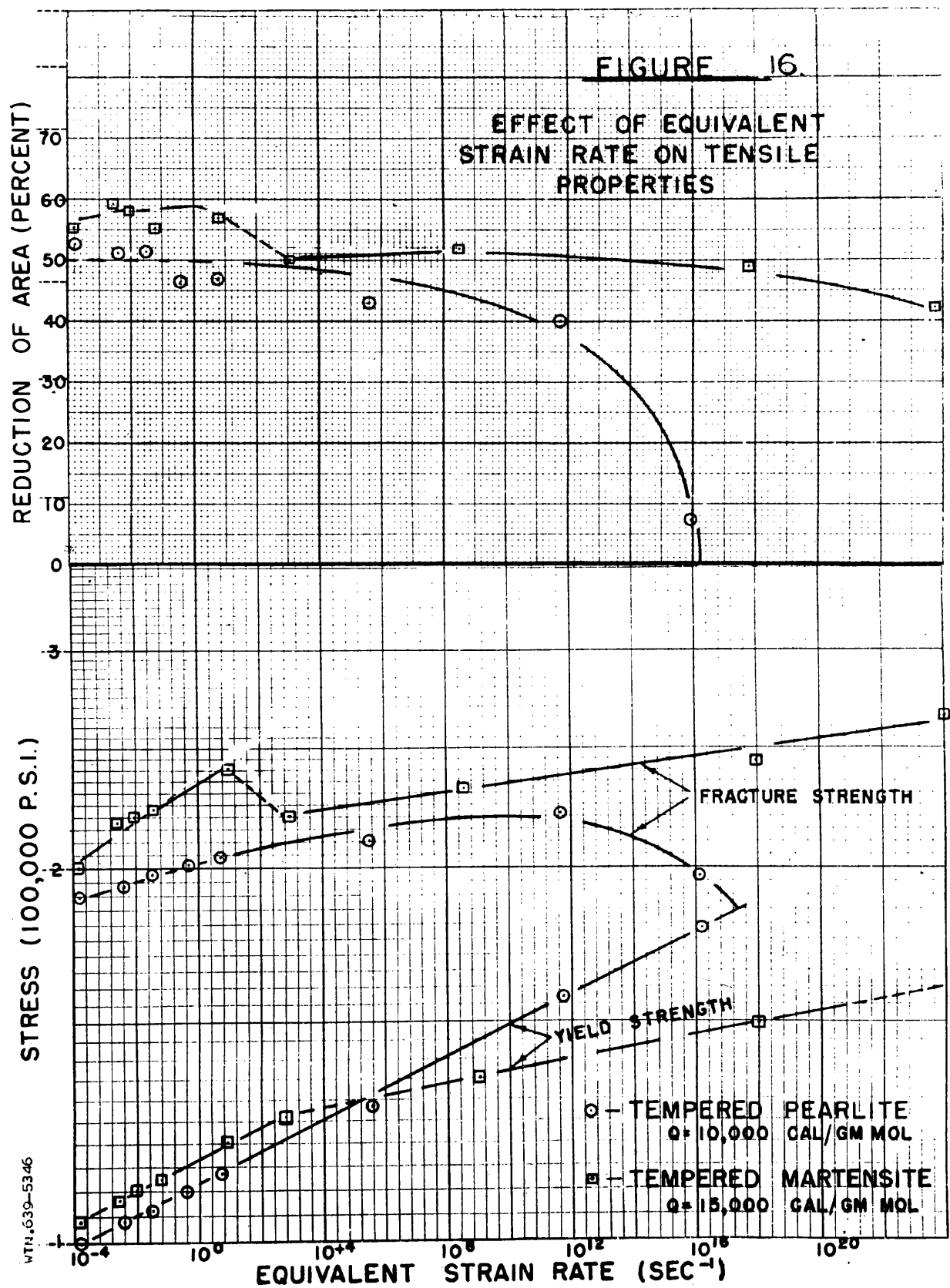


FIG. 14.

VARIATION OF YIELD POINT S_y OF MILD STEEL
WITH THE RATE OF STRAINING v (WINLOCK & LEITER)

No. 1 - 20 GRAINS PER IN.² AT 100X
No. 2 - 200 GRAINS PER IN.² AT 100X





C. ZENER AND J. H. HOLLOMON

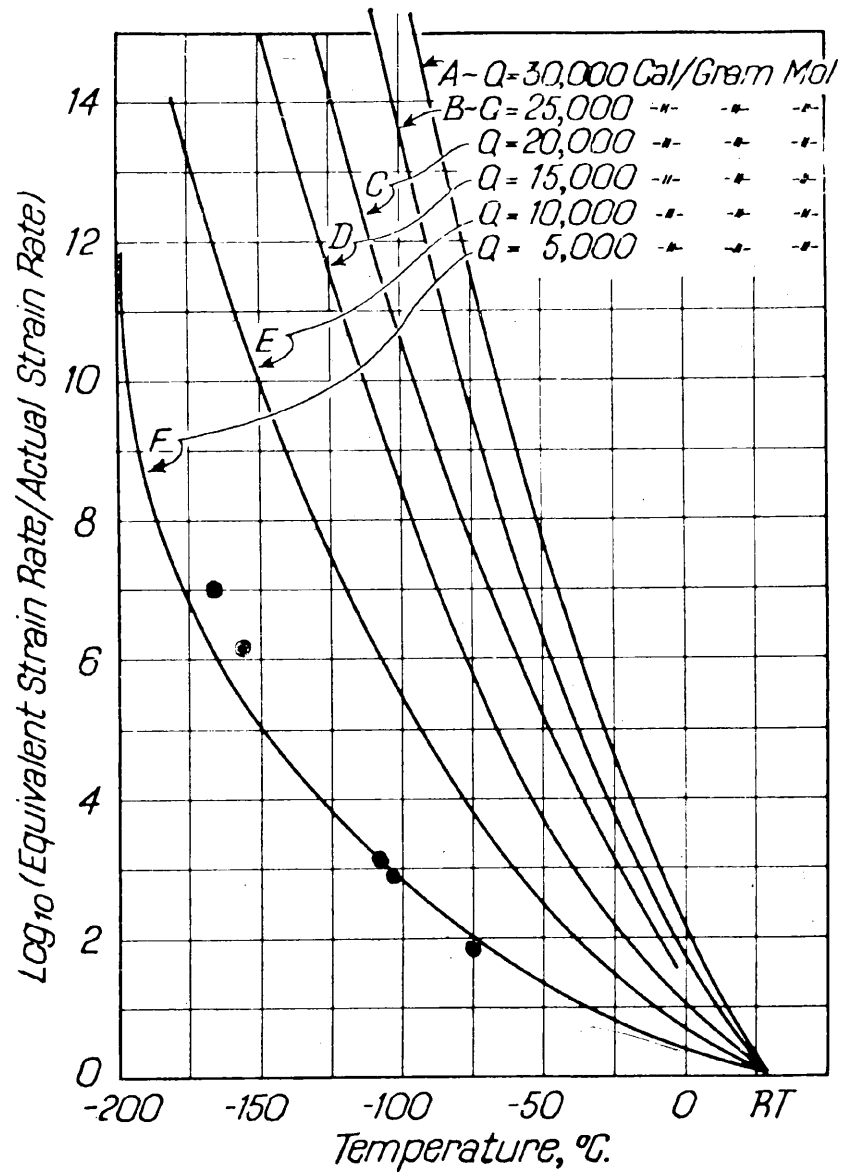
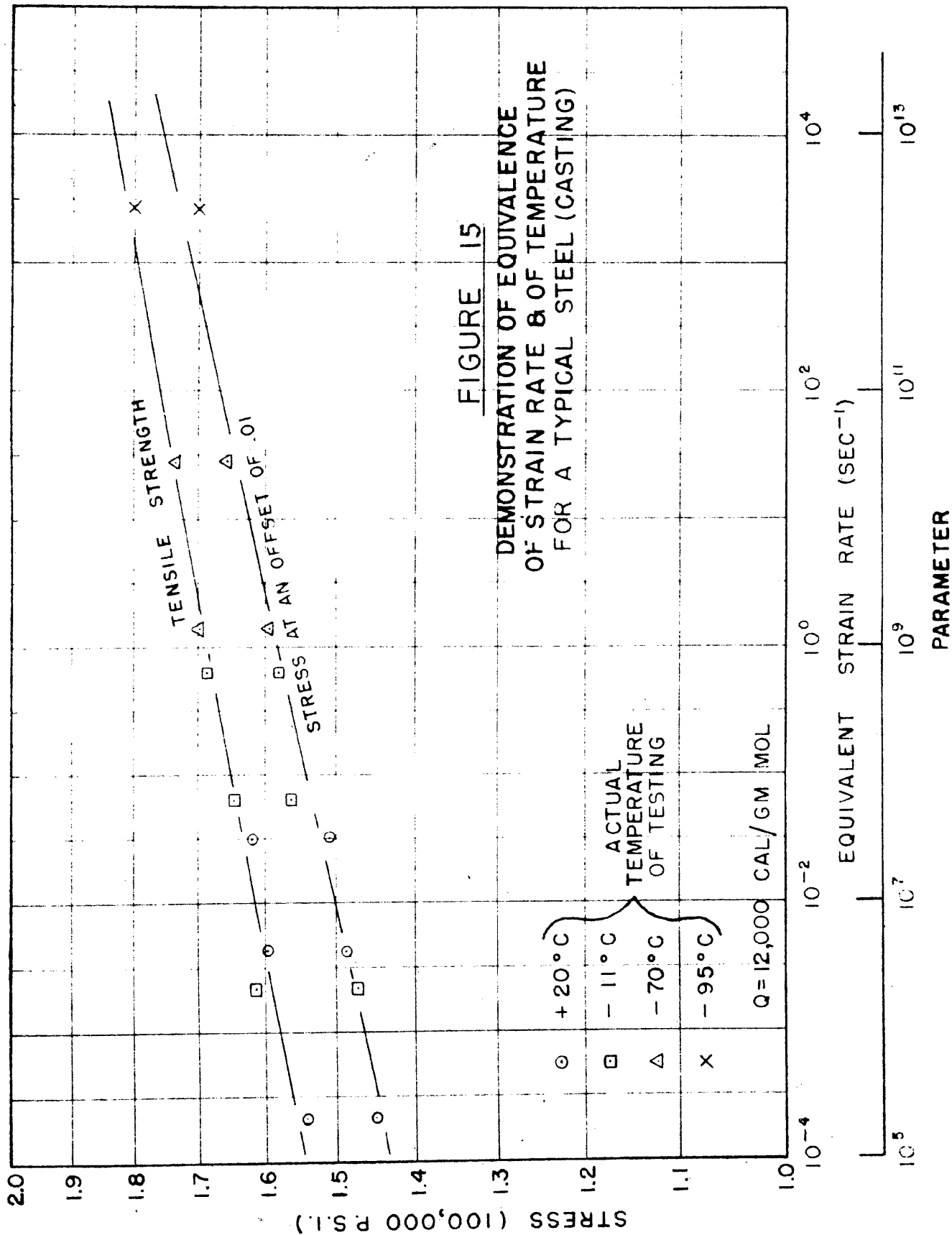
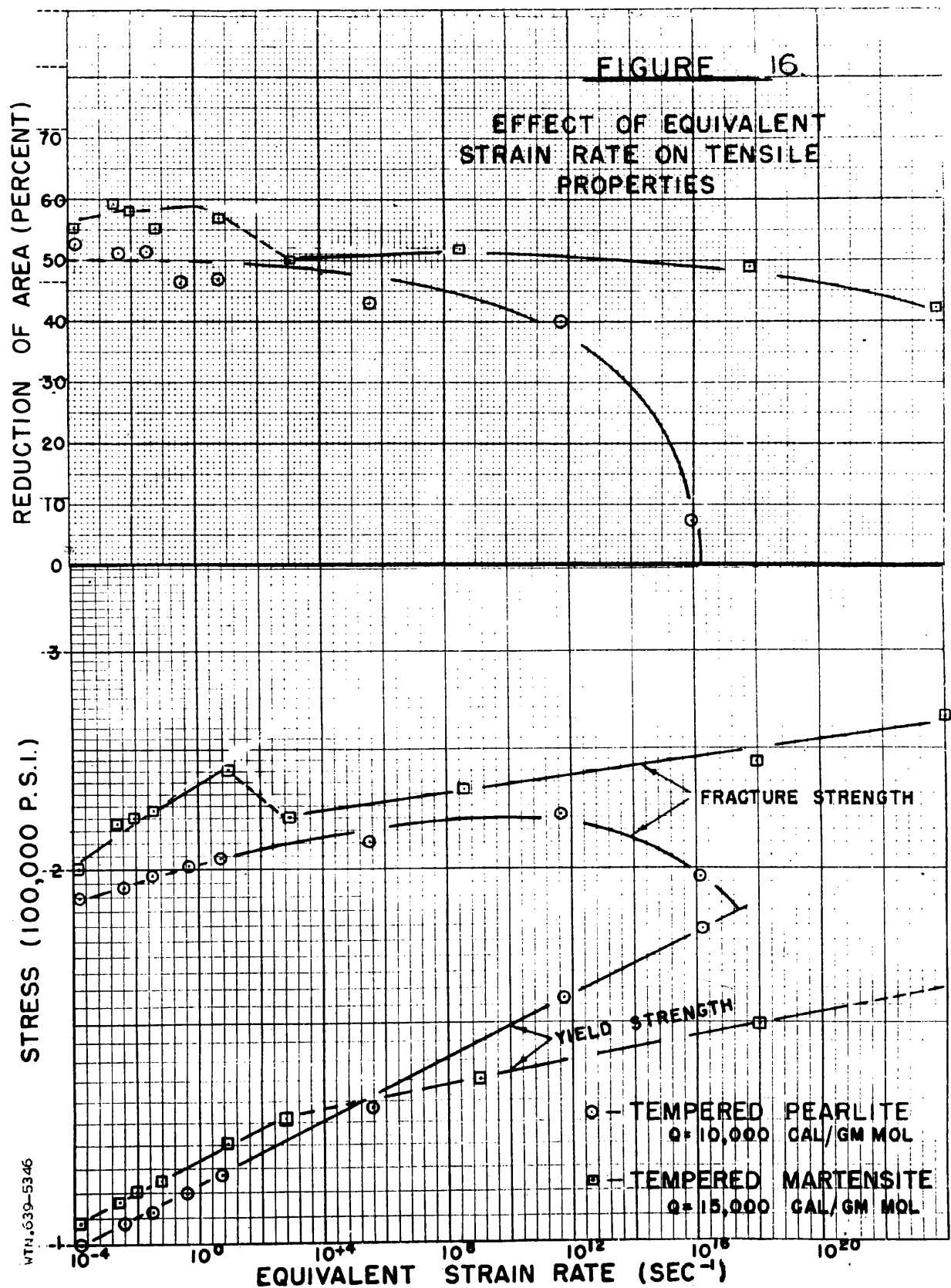


Fig. 12.—Equivalence of a Decrease of Temperature to a Rise in Rate of Strain.





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