ENGINEERING STUDIES
STRUCTURAL RESEARCH SERIES NO. 62

THE SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS

By
A. LAUPA

Approved
by
C. P. SIESS and N. M. NEWMARK

Technical Report
to
THE OHIO RIVER DIVISION LABORATORIES
CORPS OF ENGINEERS, U. S. ARMY
Contract DA-33-017-eng-222
Stage 1

September, 1953

UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

THE SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS

A Thesis by

A. Laupa

Approved by

C. P. Siess and N. M. Newmark

A Technical Report of a Research Project

Sponsored by

THE OHIO RIVER DIVISION LABORATORIES
CORPS OF ENGINEERS, U. S. ARMY

In Cooperation With

THE DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ILLINOIS

Contract DA-33-017-eng-222
Stage 1

Urbana, Illinois
August 1953

TABLE OF CONTENTS

TABLE OF CONTENTS (CONT'D)

TABLES

TABLES (CONT'D)

TABLES (CONT'D)

TABLES (CONT'D)

FIGURES

FIGURES (CONT'D)

THE SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS

I. INTRODUCTION

1. Introduction

Reinforced concrete, like other structural materials, has been the subject of extensive experimental and analytical research. The past sixty years have witnessed a steady advance in our knowledge of the behavior of reinforced concrete members under static loads. With the aid of numerous tests, a rather complete understanding has been obtained about the ultimate strength of such members in pure flexure and under pure axial compression. In addition, there have been developed theories for members subjected to combined flexure and axial compression. However, no such extensive information is available about members subjected to combinations of flexure and shear, or flexure, compression and shear.

In previous research, major emphasis has been placed on the evaluation of the contribution of web reinforcement and the shear strength of a reinforced concrete member has been interpreted in the light of a truss analogy. Experimental evidence, however, has forced certain modifications on the original truss analogy equation. The contribution of the beam itself, without the benefit of web reinforcement, has been taken into consideration. Furthermore, it has been found that the moment-shear ratio affects the ultimate strength in shear. These modifications, suggested by different authors, have retained essentially the truss analogy relation but have added new terms to account for effects other than that of web reinforcement. All the new equations, however, have been derived experimentally for each given series of test specimens and have failed to give good correlation with other test data, outside the range of test variables for which the equations were derived.

The current design specifications have been based apparently on certain minimum values obtained from tests. While these specifications yield satisfactory or even conservative values in most practical cases, there have been reported test specimens which failed in shear at a lower load than that given by the "safe working stresses". This indicates a definite need for a better understanding of the phenomenon of shear failures and for a more reliable set of design rules.

2. Object and Scope of Investigation

The object of this investigation was to review and correlate the results of previous research in the field of shear and diagonal tension, to determine the modes and characteristics of shear failure of reinforced concrete beams, and to establish a general expression for the shear strength of reinforced concrete beams under different loading conditions. The investigation was limited to members subjected to combinations of shear and flexure only.

More than one thousand tests of beams with a wide range of physical properties and under different types of loading were studied. A basically new empirical equation was derived for the shear strength of simple-span rectangular beams without web reinforcement and under one or two symmetrical concentrated loads. It is shown herein that the basic equation can be interpreted with the aid of the conventional theory of compression failures of reinforced concrete beams. This equation was first presented in a previous technical report (1)*.

The basic empirical equation was extended to include beams with web reinforcement and the amount of web reinforcement required to prevent shear failures was determined. Furthermore, the same equation was modified to apply to simple-span T-beams and restrained beams under symmetrical concentrated loads. It was found also that the basic equation could be used to determine the shear strength of a reinforced concrete beam under uniform load and, possibly, under any type of distributed loading.

3. Acknowledgment

The studies reported herein were made as a part of a research program to establish by analysis and by studies of the available test data criteria for the structural design of reinforced concrete box culverts. This research program is conducted by the Structural Research Laboratory in the Engineering Experiment Station of the University of Illinois in cooperation with Ohio River Division Laboratories, Corps of Engineers, U. S. Army, under Contract DA-33-017-eng-222. This investigation was initiated in May 1953.

The program of the investigation is guided by Dr. N. M. Newmark, Research Professor of Structural Engineering. The immediate supervision of the program is provided by Dr. C. P. Siess, Research Associate Professor of Civil Engineering, whose critical study of the manuscript of this report and helpful comments are gratefully appreciated.

---

* Numbers in parentheses refer to corresponding entries in Bibliography.

This report was written as a thesis under the direction of Dr. Newmark and his assistance is gratefully acknowledged.

4. Notation

The following notation is used in this report:

a = distance from end support to concentrated load in simple-span beams

α = angle of inclination of web reinforcement with respect to axis of beam

A = given by Eq. (44)

A_c = compressive area of concrete as determined by "straight line" theory

A_w = area of web reinforcement

b = width of rectangular beam or width of flange of T-beam

b' = width of web of T-beam

C = internal compressive force in concrete; (also various numerical coefficients as defined in text)

d = distance from centroid of tension reinforcement to compression face of beam

D = total depth of beam

e = thickness of flange of T-beam

ϵ_u = ultimate compressive strain in concrete, taken as 0.004

ϵ_y = strain in steel at yield point

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of reinforcing steel

f_c = compressive stress in extreme fiber of concrete, given by straight line theory

f'_c = compressive strength of 6 by 12-in. concrete cylinders

f'_{cu} = compressive strength of concrete cubes

f_r = modulus of rupture

f_s = stress in tension reinforcement

f'_s = stress in compression reinforcement

f_y = yield stress of tension reinforcement

f'_y = yield stress of compression reinforcement

f_w = stress in web reinforcement

f_{yw} = yield stress in web reinforcement

F = total force in web reinforcement, see Fig. 3

F_t = shape factor of T-beams, given by Eq. (34)

g = see Fig. 12

I_{cr} = moment of inertia of "straight line" cracked transformed section, either rectangular or T-section

I_R = moment of inertia of uncracked rectangular section

I_T = moment of inertia of uncracked T-section

jd = internal moment arm

kd = depth of compression zone of concrete as determined by "straight line" theory

$k_s d$ = depth of compression zone of concrete at shear failure

k_1 = $\frac{C}{k_3 f'_c k_s bd}$, a parameter which determines the magnitude of the compressive force C. It is the ratio of the average compressive stress to the maximum compressive stress in concrete

k_2 = fraction of the depth of compression zone which determines the position of the compressive force C in concrete

k_3 = ratio of maximum compressive strength of concrete in beam to compressive strength of standard test cylinders

K = $(\sin\alpha + \cos\alpha)\sin\alpha$

L = span length of test beam

L' = total length of test beam

M = bending moment

M_s = shear-compression moment of beam without web reinforcement, given by Eqs. (18), (35), (43)

M_{sw} = shear-compression moment of beam with web reinforcement, given by Eq. (28)

n = $\frac{E_s}{E_c}$ = elastic modular ratio, taken as $5 + \frac{10,000}{f'_c}$

n' = $\frac{f_y}{f'_c}$ = plastic modular ratio

p = $\frac{A_s}{bd}$, where A_s = area of tension reinforcement

p' = $\frac{A'_s}{bd}$, where A'_s = area of compression reinforcement

p_o = given by Eq. (42)

p_t = given by Eq. (46)

P = total load on beam

P_s = load which corresponds to M_s

P_{sw} = load which corresponds to M_{sw}

q = $\frac{pf_y}{f'_c}$ = reinforcing index

q_{cr} = value of q which determines the boundary between initial flexural failure by crushing of concrete and by yielding of reinforcement, given by Eq. (32)

$r \quad = \dfrac{A_w}{b\ s\ \sin\alpha}$ for rectangular beams

$\quad\;\; = \dfrac{A_w}{b'\ s\ \sin\alpha}$ for T-beams

s = spacing of web reinforcement along axis of beam

td = distance between centroids of tension and compression reinforcement

v = nominal shearing stress in concrete, $\dfrac{V}{bjd}$, $\dfrac{V}{bk_s d}$, or $\dfrac{V}{bD}$ as defined in text

v_u = nominal shearing stress at ultimate load

v_c = nominal shearing stress at ultimate load for shear-proper, given by Eq. (47)

V = shearing force

x = clear distance between two load blocks

II. SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT AND UNDER ONE OR TWO SYMMETRICAL CONCENTRATED LOADS

5. Review of Earlier Research

Shear failures have been treated conventionally as failures in diagonal tension. Since the real value of diagonal tension stress was generally difficult to determine, the unit shearing stress

$$v = \frac{V}{bjd} \tag{1}$$

was considered as a measure of diagonal tension. The effect of web reinforcement was taken into account by considering a beam acting as a truss, in which the top chord was formed by the compression zone of the concrete, the bottom chord by the longitudinal reinforcement, the tension web members by the web reinforcement, and the compression web members by the concrete in the web of the beam. From these assumptions the following equation was derived to represent stress in the web reinforcement:

$$f_w = \frac{v}{rK} \tag{2}$$

where the value of K depended on the angle of inclination of web reinforcement.

It was realized that the above equations were approximate in nature and empirical data were used to correlate the real behavior of test beams with the above theoretical considerations. It was observed that the measured stresses in the web reinforcement were, in general, considerably less than predicted by Eq. (2); this discrepancy was attributed to the fact that a portion of the total shear was carried by the concrete. In 1927 Richart (2) modified the truss analogy equation (2) in the following form:

$$v = C + Krf_w \tag{3}$$

where the constant C was found to vary from 90 to 200 psi and it was stated that C probably depended "upon the percentage of web reinforcement used and also on the quality of the concrete".

More complete conclusions about the contribution of the concrete to resist shear had been reached by Talbot some twenty years earlier. In 1909 Talbot (3) reported that for beams without web reinforcement the ultimate nominal shearing stress v increases as the quality of concrete increases, as the amount of longitudinal reinforcement increases, and as the span length L decreases. These conclusions, however, were apparently disregarded in favor of the unmodified concept of truss analogy by most later investigators. Only in relatively recent years new attempts were made to evaluate in quantitative terms the contribution of the various elements of a beam to its strength in shear. In 1945 Moretto (4) presented the following equation for the shearing strength of a simply supported beam:

$$v = Krf_{yw} + 0.10\ f'_c + 5000\ p \tag{4}$$

which was essentially an improvement of Eq. (3) suggested by Richart. In 1951 Clark (5) reported the following formula:

$$v = 2500\ \sqrt{r} + 0.12\ f'_c\ (d/a) + 7000\ p \tag{5}$$

which was the first to account quantitatively for all of the variables listed by Talbot in 1909 as influencing the shearing strength of reinforced concrete beams.

In a previous report (1) attempts were made to correlate the results of previous research and to investigate the validity of the above and some additional empirical equations in the following form:

$$v = Krf_{yw} + C_1 f'_c + C_2 p \ (d/a) \tag{6}$$

$$v = Krf_{yw} + \left[C_1 f'_c + C_2 p \right] \ (d/a) \tag{7}$$

All these attempts to relate the nominal shearing strength of simple-span reinforced concrete beams to a function consisting of the truss analogy term Krf_{yw} and linear terms of f'_c and p failed to give good correlation with test results. Thus, all empirical equations which were derived for a certain range of test variables were not applicable outside that particular range.

Since the introduction of the concept of truss analogy some 50 years ago, major emphasis has been placed, in general, on the evaluation of the contribution of web reinforcement to shear strength. The contribution of the beam itself, without the benefit of any web reinforcement, has remained a relatively unknown quantity. Furthermore, any uncertainties with regard to the contribution of web reinforcement have reflected directly on the contribution of the beam itself, thus rendering both questionable.

Our first problem, therefore, should be the evaluation of shear strength of a beam without web reinforcement. In the following section a general expression for the shearing strength of such beams is derived.

6. Derivation of Basic Empirical Equation

After the formation of a diagonal tension crack, a reinforced concrete beam which does not fail in tension, will fail either in the compression zone of the concrete or in bond. Although the cause of these two types of failure is different, their appearance is often very nearly the same. When a beam fails by the destruction of the compression zone, the shear force which was previously carried by the concrete is transferred to the level of the longitudinal reinforcement. This leads to splitting of the concrete along the reinforcing bars. When a beam fails in bond, however, slipping of the longitudinal reinforcement produces cracking of the concrete along the bars and reduces effectively the bending resistance of the section. This causes a concentrated angle change at the end of the diagonal crack and leads to a premature destruction of the compression zone of the concrete. Since the above phenomena take place simultaneously, it has often been difficult to determine the real cause of failure. Especially in the early tests when plain bars without any end anchorage were used as tension reinforcement, bond failures were frequently considered as diagonal tension failures. In many of the recent experimental investigations, however, the possibility of bond failure has been eliminated by the use of some special type of end anchorage in addition to deformed bars of good bond characteristics. Splitting along the longitudinal reinforcement has still been observed and sometimes even considered as a primary mode of failure. This phenomenon, however, is secondary to the failure of the beam by destruction of the compression zone.

Failure by destruction of the compression zone takes place under a concentrated load, at the section of maximum moment and maximum shear. The real cause of failure has not been generally understood. It has been suggested that this failure is the result of the principal stresses, compressive or tensile, or of the maximum shearing stress. As was seen before, the conventional theory, treating shear failures as diagonal tension failures, considered the nominal shearing stress $\underline{v}$ as a measure of diagonal tension. Previous research has indicated that $\underline{v}$ is a function of the following variables:

$$v = \frac{V}{bjd} = F(p, f_c^*, \frac{d}{a}, Krf_y) \tag{8}$$

All empirical equations suggested by different investigators, however, have failed to give good correlation with all of the available test results. Furthermore, the conventional theory pictures the nominal shearing stress $\underline{v}$ distributed over the entire cross-section of a beam, uniform from the level of tension reinforcement to the neutral axis. The formation of a diagonal crack, however, radically changes the state of stress in a reinforced concrete beam. There cannot be any transfer of stress across a crack. Thus, the nominal shearing stress cannot possibly be the criterion of shear failure and the state of stress in the uncracked compression zone should be investigated in order to determine the real cause of failure.

A basic equation for the shear strength of a simple-span rectangular beam without any form of web reinforcement and under one or two symmetrical concentrated loads was derived by considering the state of stress in the compression zone of the concrete. It was first assumed that the total shearing force C is resisted solely by the compression area of the concrete. For beams without compressive reinforcement the area of the compression zone is given by $k_s db$, where the quantity $k_s d$ refers to the depth of the compression zone at shear failure. Thus the average shearing stress is given by $v = \frac{V}{k_s db}$. It was further assumed that the ultimate shearing unit stress, v_u, was a function of f'_c. Test results have shown that the shear capacity of the compression zone decreases as the moment-shear ratio, M/V, increases. This effect has usually been taken into consideration by the $\frac{d}{a}$-ratio, and there seems to be a linear relationship between this ratio and the shear capacity of the beam. Since both the horizontal compressive stresses and the vertical shearing stresses are assumed to be resisted by the same compressive area, it seems more reasonable to consider the shear-compressive force ratio, V/C, rather than the M/V-ratio as influencing the ultimate load in shear. For the type of beam under consideration it can be written that V/C = jd/a. Thus the ultimate shearing stress v_u can be expressed as follows:

$$v_u = \frac{V}{k_s db} = \frac{jd}{a} F_1 (f'_c) \tag{9}$$

It is noticed that this expression can be rewritten in a different way:

$$\frac{Va}{bd^2f'_c} = k_s jF(f'_c) \quad \text{or} \quad \frac{M}{bd^2f'_c} = k_s jF(f'_c) \qquad (10)$$

These equations are in a form which suggest that the criterion for shear failure is a limiting moment rather than an ultimate shearing stress. There is some supporting evidence for this observation in previous test results. Beams with no web reinforcement tested by Clark (5) had the d/a-ratio as the only variable; all these beams failed at a nearly constant moment, although the total shear force at failure depended upon the location of the loads on the beams. Turneaure and Maurer (6) reported a series of tests on small mortar beams with the d/a-ratio as the only variable, and their results again show that the ultimate moment was nearly the same for all positions of loads. Thus the so-called shear failures seem to be failures in compression; the criterion of failure being a limiting average compressive stress or a limiting total compressive force in the compression zone of the concrete. This type of failure differs from flexural compression failures only because the compressive area is reduced because of diagonal tension cracking.

In Eq. (10) there are two main unknowns: the depth of the compression zone, $k_s d$, and the limiting average compressive stress, related to $F(f'_c)$. The quantity j can be considered as a constant since it does not vary over a great range.

The depth of the compression zone can be determined accurately for flexural failures, both in tension and in compression, by considering statical equilibrium and the strain relations involved. For shear failures, however, no theoretical relationship relating the extent of diagonal tension cracking and the physical properties of the beam has been found. Consequently, the depth of the compression zone must be determined empirically. From previous investigations it can be shown qualitatively that k_s is a function of f'_c and p. Furthermore, this function must be a complex one since different empirical equations considering v as a linear function of f'_c and p have failed to agree with test results. In order to facilitate the empirical evaluation of k_s, it was deemed advantageous to consider the ratio k_s/k rather than k_s alone. The value of k as determined by the straight line theory is also a function of f'_c and p. It was felt that there might be some similarity between the functions representing k_s and k, so that the ratio k_s/k might be easier to evaluate than k_s alone. It was considered that if the ratio k_s/k is either a constant or a function of f'_c, Eq. (10) can be written as

$$\frac{M}{bd^2 f'_c} = k\,F(f'_c) \qquad (11)$$

and the unknown function $F(f'_c)$ can be evaluated directly from available test data. If this cannot be done, the ratio k_s/k must also depend on p and Eq. (11) must be rewritten as

$$\frac{M}{bd^2 f'_c} = k\,F(f'_c, p) \qquad (11a)$$

Equation (11) was derived for beams without compression reinforcement. For beams with both tension and compression reinforcement, Eq. (11) must be modified to take into account the added effect of the compression reinforcement. If it is assumed that a beam fails before the compression reinforcement yields, an expression for the limiting moment of shear failure can be derived by considering that the presence of compression reinforcement increases the compression area of the transformed section by an amount of $np'bd$, the steel area transformed to concrete:

$$A_c = bkd + np'bd = bd\,(k + np') \tag{12}$$

This modified compression area leads to the following equation which corresponds to Eq. (11) for beams without compression reinforcement:

$$\frac{M}{bd^2 f'_c} = (k + np')\,F(f'_c) \tag{13}$$

The quantity <u>k</u> refers to the actual depth of the compression zone, as determined from an equivalent section transformed to concrete. For beams with only tension reinforcement the numerical value of <u>k</u> is obtained from the well-known equation

$$k = \sqrt{(pn)^2 + 2pn} - pn \tag{14}$$

For beams with both tension and compression reinforcement the following equation can be derived to determine <u>k</u>:

$$k = \sqrt{\left[n\,(p + p')\right]^2 + 2n\,(p + p' - p't)} - n\,(p + p') \tag{15}$$

where td is the distance between the centers of the tension and compression reinforcement.

In all subsequent calculations the value of the modular ratio n used in the above equations was determined by Jensen's formula (7)

$$n = 5 + \frac{10,000}{f'_c} \tag{16}$$

which has been found to give reliable results.

7. Test Data

In order to determine the unknown function $F(f'_c)$ in Eqs. (11) and (13), experimental results of previous research were analyzed. Attention was directed first only to simple-span rectangular beams without web reinforcement and subjected to one or two symmetrical concentrated loads. All known tests of such beams were included in the analysis except some of the very early beams for which there was some doubt about the compressive strength of concrete used.

A total of 125 beams from 15 different investigations were considered in the analysis. These beams were tested over a period of 43 years and had a wide variation in their physical properties. Table 1 names the different investigations and gives their entry in the Bibliography and the number of the table in which they are analyzed. This table summarizes also the range of test variables for the different groups of beams.

One hundred and eleven of these beams failed in shear, seven of them, however, yielding before failure. The remaining fourteen beams failed in bond, although their mode of failure was reported as diagonal tension. These beams are discussed later in this section. Thirty beams were provided with both tension and compression reinforcement; all other beams were reinforced in tension only.

The test results for the different groups of beams are analyzed in Tables 2 through 15. Both physical properties as reported by the investigators and calculated quantities are given for each individual beam. All dimensions are given in inches and the compressive strength of concrete in pounds per square inch. In most cases, concrete strength was determined from tests on 6 by 12-in. standard cylinders. In a few cases, tests either on cubes or on modulus of rupture beams were employed; these cases are marked in the tables and the concrete strength is reduced to that of a standard cylinder by the formulas

$$f'_c = 0.75\ f'_{cu} \qquad \text{for cubes}$$

and

$$f'_c = 6.7\ f_r \qquad \text{for modulus of rupture beams.}$$

In order to evaluate the function $F(f'_c)$, the quantity $M/bd^2f'_c\ (k + np')$ was calculated for each beam. For beams without compression reinforcement the term $(k + np')$ reduces to $\underline{k}$. In Fig. 1a the above quantity is plotted against f'_c. It is seen that the concrete strength varies from about 1000 to 6000 psi. Within these limits the function $F(f'_c)$ can be approximated by a linear equation:

$$F(f'_c) = 0.57 - \frac{4.5\ f'_c}{10^5} \qquad (17)$$

where f'_c is the compressive strength of a standard cylinder in pounds per square inch. Substitution of Eq. (17) into Eq. (13) yields an equation for moment, subsequently called the shear-moment, at which a simple-span reinforced concrete beam without web reinforcement and under one or two symmetrical concentrated loads fails in shear:

$$\frac{M_s}{bd^2f'_c} = (k + np')\left(0.57 - \frac{4.5\ f'_c}{10^5}\right) \qquad (18)$$

The agreement between test results and Eq. (18) is satisfactory. The average ratio of M/M_s for the 111 beams which failed in shear is 0.993; the standard deviation 0.120. The group of beams which were loaded through a column stub at midspan (1) failed at a somewhat higher load than that given by Eq. (18). It is possible that the column stub had a strengthening effect against shear failure. Six beams from four different investigations failed at a considerably lower load than predicted by Eq. (18). However, all these beams had companion specimens which failed in close agreement with the predicted values.

In this connection, it must be pointed out that all compression failures are sensitive to the compressive strength of the concrete at the section of failure. The compressive strength reported for a test beam is the average strength obtained from control cylinders. Since even control cylinders can have wide differences in their strength, it is not expected that a test specimen is of uniform strength. If the concrete strength at the section of failure happens to be different from the average strength of the control cylinders, the test beam may fail at a load different from the predicted load. It is believed that most of the scatter in test results can be attributed to the variation of the concrete strength from the average value. This is especially so since in some cases only the average concrete strength was reported for the whole test series or for a group of companion specimens. Furthermore, it is recalled that the beams were tested over a period of almost a half of a century, and that the beams were both made and cured under greatly different conditions.

No systematic difference can be detected between beams reinforced in tension only and beams reinforced both in tension and compression. If the last group of beams is considered separately, the average ratio of M/M_s for thirty such beams is 0.940 and the standard deviation 0.14. It is interesting to note, however, that five of the six beams which fell considerably lower than the predicted values were provided with compression reinforcement. This explains also why the average ratio for this type of beams is somewhat lower than that for all beams combined; if these beams are excluded, the average ratio is 0.986 and the standard deviation 0.084.

Equation (18) was based on assumptions made in deriving Eq. (11); that is, the ratio k_s/k is a function of f'_c alone and does not depend on $\underline{p}$. In order to check this assumption and to investigate whether Eq. (11a) might not represent better the moment at shear failure, the ratios M/M_s are plotted against $\underline{p}$ in Fig. 1b. Although the steel percentages used in the test beams vary over a large range of values, no consistent relationship can be detected for the ratio M/M_s and $\underline{p}$. Consequently, the ratio k_s/k does not seem to be influenced by $\underline{p}$ and Eq. (18) is thereby applicable for beams with any amount of longitudinal reinforcement.

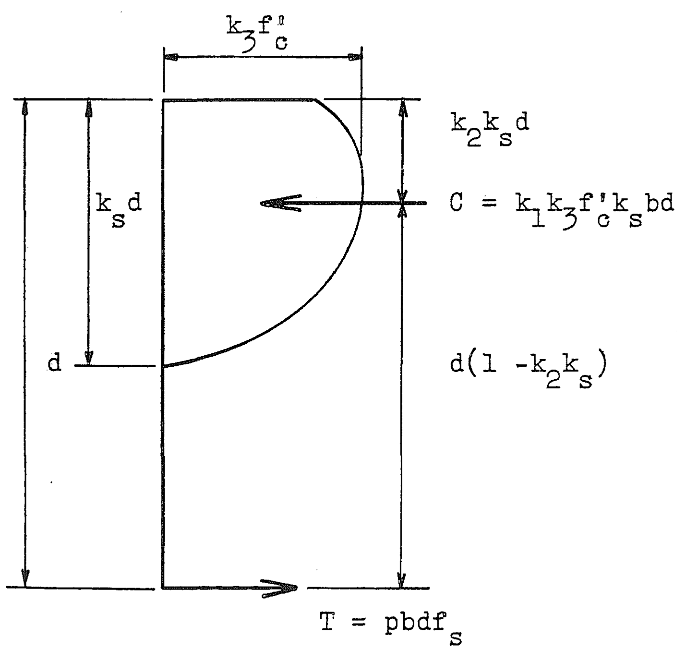
Series 1917 of the beams tested by Richart (Table 5) is very illuminating for a study of the mechanism of shear failure. These beams were provided with a 4-in. thick layer of high-strength concrete at the top of each beam "as a precaution against premature failure of the beam by crushing of the concrete". Table 5 gives an analysis of these beams using both the actual concrete strength in the compression zone and the concrete strength used in the lower portions of the beams. It is seen that the use of the actual concrete strength gives very good agreement with Eq. (18) whereas the use of the concrete strength not in the compression zone gives up to 59 percent differences from the predicted values. Thus is it clearly evident that the load at failure is controlled solely by the strength of the compression zone of the concrete. The remaining part of the concrete section does not greatly influence the shear strength of a beam.

Fourteen beams, although reported as diagonal tension failures, failed in bond. Those were beams tested by Richart; three from Series 1910 (Table 2), ten from Series 1911 (Table 3), and one from Series 1913 (Table 4). A total of 18 beams of Series 1911 were without web reinforcement. These beams were very nearly the same in every respect except for the end anchorage of the tension reinforcement. All beams with well-anchored longitudinal steel, either by conventional hooks, by overhang, or by an end plate tightened against the end of the beams, failed in shear at a load in good agreement with Eq. (18). All other beams, however, either with unanchored straight bars or with end plates not tightened, failed at a much lower load, suggesting bond failures. Some typical beams of this group were checked as to their bond strength by a procedure suggested by Mylrea (13). Mylrea gives an empirical relationship between the length of embedment of a plain round bar in a simple-span beam and the cumulative bond stress the bar can develop before bond failure. By using as the length of embedment the distance from the end of the bar to the 45-degree diagonal crack, the these beams using both the actual concrete strength in the compression zone and the concrete strength used in the lower portions of the beams. It is seen that the use of the actual concrete strength gives very good agreement with Eq. (18) whereas the use of the concrete strength not in the compression zone gives up to 59 percent differences from the predicted values. Thus is it clearly evident that the load at failure is controlled solely by the strength of the compression zone of the concrete. The remaining part of the concrete section does not greatly influence the shear strength of a beam.

Fourteen beams, although reported as diagonal tension failures, failed in bond. Those were beams tested by Richart; three from Series 1910 (Table 2), ten from Series 1911 (Table 3), and one from Series 1913 (Table 4). A total of 18 beams of Series 1911 were without web reinforcement. These beams were very nearly the same in every respect except for the end anchorage of the tension reinforcement. All beams with well-anchored longitudinal steel, either by conventional hooks, by overhang, or by an end plate tightened against the end of the beams, failed in shear at a load in good agreement with Eq. (18). All other beams, however, either with unanchored straight bars or with end plates not tightened, failed at a much lower load, suggesting bond failures. Some typical beams of this group were checked as to their bond strength by a procedure suggested by Mylrea (13). Mylrea gives an empirical relationship between the length of embedment of a plain round bar in a simple-span beam and the cumulative bond stress the bar can develop before bond failure. By using as the length of embedment the distance from the end of the bar to the 45-degree diagonal crack, the cumulative bond stress as given by Mylrea agreed closely with the steel stress calculated from the load at failure. This indicates that the ultimate bond resistance was reached and that the beams failed in bond before developing their ultimate shear capacity. The three beams of Series 1910 with unanchored straight bars failed in bond also. The only beam of Series 1913 for which concrete strength was reported was reinforced with hooked plain bars. However, it failed at a low load and a photograph at failure indicated a possible bond failure.

8. Theoretical Interpretation of Basic Empirical Equation

(a) Beams Reinforced in Tension Only. Equation (18) where the quantity np' reduces to zero for beams without compression reinforcement, can be interpreted in the light of the conventional theory of compression failures of reinforced concrete beams. The only modification is in the depth of the compression zone. The following stress block is assumed:



$$C = T$$

$$M = Cd(1 - k_2 k_s)$$

$$M = k_1 k_3 f'_c k_s bd^2 (1 - k_2 k_s)$$

$$\frac{M}{bd^2 f'_c} = k_1 k_3 k_s (1 - k_2 k_s) \quad (19)$$

The parameters k_1k_3 and k_2 at flexural failure conditions have been determined experimentally by previous investigators. In Fig. 2 the values of k_1k_3 as obtained by Gaston (11) and Billet (14) have been plotted against f'_c. There is considerable scatter in the measured values as would be expected in an investigation of this kind. A reasonable approximation, however, can be obtained by a linear relationship between k_1k_3 and f'_c. When f'_c is within the limits of 1000 and 6000 psi, k_1k_3 can be approximated as follows:

$$k_1k_3 = 1.37 - \frac{10.8\ f'_c}{10^5} = 2.4\ (0.57 - \frac{4.5\ f'_c}{10^5}) \qquad (20)$$

Substitution of this function into Eq. (19) gives:

$$\frac{M}{bd^2f'_c} = 2.4\ (0.57 - \frac{4.5\ f'_c}{10^5})\ k_s\ (1 - k_2k_s) \qquad (21)$$

It is noticed that this equation is in the same form as Eq. (18) previously derived for the shear-moment. Equating the two yields a relationship between k_s and $\underline{k}$:

$$k_s = \frac{k}{2.4(1 - k_2k_s)} \qquad (22)$$

from which, using k_2 as 0.45:

$$k_s = 1.11 - \sqrt{1.23 - 0.926\ k} \qquad (23)$$

Since $\underline{k}$ remains usually within 0.2 and 0.5, Eqs. (22) and (23) show that k_s is practically a constant fraction of $\underline{k}$, the depth of the compression zone computed by the "straight line" theory. This finding is based on the assumption that the value of k_1k_3 is the same for both flexural and shear failures. If there should be any difference, it is still likely that k_s remains practically proportional to $\underline{k}$, although it might be a smaller fraction of $\underline{k}$ than Eq. (22) indicates.

Test observations have shown that a beam fails in flexure when the concrete crushes at a limiting strain of about 0.004 and that the strain distribution over the depth of the beam remains practically linear up to the final failure. With the aid of this information, the parameter k_1k_3 which is the ratio between the average strength of the concrete in flexure and the cylinder strength has been evaluated from test results. This quantity was approximated by Eq. (20) for the purposes of this investigation.

The use of Eq. (20) yielded an empirical expression for the depth of the compression zone at shear failure, k_sd. Equation (22) shows that the neutral axis is considerably higher for shear failures than for flexural failures. This fact has also been observed in tests. Since the load for a shear failure is smaller than the flexural capacity of the beam, the steel stress must be smaller at shear failures than that at flexural failures. Assuming a linear strain distribution, the magnitude of the steel strain and the location of the neutral axis specify a very small concrete strain at shear failures, much smaller than the ultimate strain at flexural failures. However, this is incompatible with the assumed value of the parameter k_1k_3. Test observations have shown that for small values of strain there is a linear stress-strain relationship in concrete. This gives a triangular stress block and the parameter k_1k_3 equal to 0.5 for all values of the concrete strength. Since Eq. (20) gives values of k_1k_3 in the neighborhood of 1.0, the new value of k_1k_3 would approximately double the depth of the compression zone as given by Eq. (22). Furthermore, this depth would be equal to or greater than that at flexural failures.

Since k_s must be considerably smaller at shear failures than at flexural failures, the above assumption of linear strain distribution cannot be applicable in this case. This agrees with the observed behavior of beams failing in shear. The formation of a diagonal crack disrupts the normal distribution of the steel strain along the tension reinforcement. Since there can be no transfer of stress across the diagonal crack, consideration of moment shows that the steel stress must be the same both at a vertical section through the upper end of the crack and at the intersection of the reinforcing bars and the crack. Thus, the steel strain must be practically uniform over this distance. Furthermore, in order to preserve the continuity of the beam, the total elongation of steel between these two sections must have a corresponding total shortening of the top concrete fiber at the location of the diagonal crack. This requires a concentration of the concrete strain in that region. Consequently, the strain distribution over the depth of the beam cannot be linear at the section of failure. This hypothesis is compatible with the previous findings. Because of a nonlinear strain distribution, the parameter k_1k_3 can remain essentially the same both for shear and flexural failures, although the magnitude of steel stress and the location of the neutral axis are different. Since the value of k_1k_3 at flexural failures was determined from the condition of a limiting concrete strain, the use of the same value for shear failures implies that the criterion for this type of failure is also a limiting strain in the concrete. Zwoyer has verified this observation for prestressed concrete beams by actual tests (29). He observed high concentration of the concrete strain in the region of the diagonal crack and an average ultimate strain of 0.00385. This is in good agreement with the previously found values for flexural failures. Likewise, the observed location of the neutral axis was higher for shear failures than that for flexural failures.

However, since the actual distribution of strain cannot be determined, no theoretical relationship can be written for the depth of the compression zone and the ultimate concrete strain at shear failures. In order to interpret the test results and to determine a general expression for shear-compression failures, either the value of k_s, or the magnitude of steel stress, or some relationship between the average strains in the reinforcement and in the concrete must be determined empirically. In this investigation it was chosen to evaluate k_s empirically. Using the parameter k_1k_3 as determined from flexural failures, it was shown that there existed a simple relation between k_s and $\underline{k}$. This shows why the previous attempt to use $\underline{k}$ as a measure of k_s gave satisfactory results. However, since the relationship between the two was determined empirically, it can be only speculated why these two quantities are related. Zwoyer used in his investigation (29) an empirical relationship between the average values of the concrete strain on the top surface of the beam and at the level of the reinforcing steel. In addition, the parameter k_1k_3 was determined from the measured values for prestressed concrete beams and the same value of k_1k_3 was used later for ordinary reinforced concrete beams. Moody used the parameter k_1k_3 as obtained from flexural failures and evaluated the magnitude of steel stress from test results (12). Two different expressions were obtained for the steel stress; one for simple-span and another for restrained rectangular reinforced concrete beams under symmetrical concentrated loads.

(b) Beams Reinforced Both in Tension and Compression. Equation (19) was derived for beams without compressive reinforcement. For beams reinforced both in tension and in compression it can be modified as follows:

$$M = k_1k_3\ f_c^*k_sbd^2\ (1 - k_2k_s) + f_s^*\ p^*bd^2t \qquad (24)$$

where td is the distance between the centers of the tension and compression reinforcements, f_s^* is the stress in the compression reinforcement, and p^* is the ratio of compression reinforcement.

Since the ultimate strain in the concrete is approximately 0.0040 and the yield strain for reinforcing bars around 0.0017, yielding of the compression reinforcement precedes crushing of the concrete in most flexural compression failures. For shear compression failures, however, diagonal cracks extend higher than the vertical cracks caused by flexural tension. It is conceivable that a beam can fail in shear either before or after the compression reinforcement yields. Expressions for the ultimate shear moment for both of these two cases are derived in the following paragraphs and the validity of these equations are determined by the help of experimental data.

If it is first assumed that the compression reinforcement has reached its yield stress f'_y at shear failures and that k_s is still given by $k_s = \frac{k}{2.4(1 - k_2 k_s)}$, Eq. (24) for maximum shear moment can be written as:

$$\frac{M_s}{bd^2 f'_c} = k(0.57 - \frac{4.5\ f'_c}{10^5}) + n'p't \tag{25}$$

Since this equation assumes that compression reinforcement has yielded while the tension reinforcement is still elastic, the elastic modular ratio <u>n</u> is to be used for the tension reinforcement and the plastic modular ratio, $n' = \frac{f'_y}{f'_c}$, for the compression reinforcement while computing the quantity <u>k</u>.

An expression for the maximum shear moment for the second case, a beam failing in shear before the compression reinforcement yields, was derived previously:

$$\frac{M_s}{bd^2 f'_c} = (k + np')\ (0.57 - \frac{4.5\ f'_c}{10^5}) \tag{18}$$

In this expression the elastic modular ratio n is used for both tension and compression reinforcement in computing the quantity k.

Equations (25) and (18), based on different assumptions, are greatly different. Equation (25) gives a much higher ultimate moment than Eq. (18). In the analysis of previous test data, thirty of the 111 beams under consideration were provided with compression reinforcement. If these beams are considered as having failed after their compression reinforcement had yielded, the internal resisting moment given by the compression reinforcement acting at its yield stress is almost as large and in several cases even larger than the total external moment. Thus it must be concluded that these beams failed in shear before their compression reinforcement yielded. Furthermore, since the thirty beams with compression reinforcement gave good agreement with Eq. (18), this equation can be used to take the effect of compression reinforcement into consideration. According to Eq. (18) the shear strength of a beam with compression reinforcement is but little greater than that of the same beam without; p' decreases the value of k while adding the term np', so that the quantity $(k + np')$ is but little greater than the value of k for a beam without compression reinforcement.

9. Properties and Limitations of Basic Empirical Equation.

The basic empirical equation was derived for simple-span rectangular beams without web reinforcement and under one or two symmetrical concentrated loads. Different variables have the following effect on Eq. (18):

(a) Ratio of a/d. Equation (18) considers shear failures as compression failures. The load at failure is determined by a limiting shear-moment. In that sense, the ratio a/d loses its usual meaning; that is, as affecting the shearing strength of a beam. The quantity a relates the magnitude of the applied load to the moment at failure, $M = Va$, and the effective depth d affects both the lever arm of the internal moment and the area of the compression zone. For the beams analyzed, the ratio a/d varied from 1.17 to 4.80. This variation did not seem to have any effect on the agreement between the test results and the predicted values. It is conceivable, however, that as the ratio a/d increases and the relative magnitude of the shearing stresses decreases, a beam will fail either in shear at a higher load than given by Eq. (18) or, for still higher values of a/d, in flexure. This phenomenon is discussed further in Section 18. Conversely, as the ratio a/d decreases to a very small value, it is expected that the mode of failure changes from that in shear-compression to shear-proper. This question is discussed in more detail in Section 19.

(b) Tensile Reinforcement. The amount of tensile reinforcement affects the size of the compression area of the concrete. It was found empirically that the moment at failure could be related to k and that the actual depth of the compression area was practically a constant proportion of k, or $k_s = \frac{k}{2.4\,(1 - k_2 k_s)}$. This procedure implies that the parameter $k_1 k_3$ which is a measure of the total compressive force in concrete remains essentially the same both for flexural and shear compression failures and that the failure criterion is still a limiting compressive strain in the concrete.

(c) Concrete Strength. The shear strength of a beam is directly proportional to the following function of f'_c: $f'_c(0.57 - 4.5\, f'_c/10^5)k$. It is seen that as f'_c increases, both the quantity $(0.57 - 4.5\, f'_c/10^5)$ which represents the effect of $k_1 k_3$, and the value of $\underline{k}$ decrease. Thus the shear strength is not a linear function of f'_c. As an example, for a beam with one percent tension reinforcement, an increase of f'_c from 2500 to 5000 psi increases the shear strength 36 percent.

(d) Compressive Reinforcement. The contribution of compression reinforcement to the shear strength is rather small and can be included in the analysis by considering $\underline{p}'$ in computing both the elastic $\underline{k}$ and the transformed concrete area. This procedure led to Eq. (18).

(e) Column Stub. Beams which had a column stub cast integrally with the beam at midspan failed consistently at a slightly higher load than beams without a column stub. This increase in strength was somewhat larger for lower values of concrete strength than for higher values of concrete strength.

III. SIMPLE-SPAN RECTANGULAR BEAMS WITH WEB REINFORCEMENT AND UNDER ONE OR TWO SYMMETRICAL CONCENTRATED LOADS

10. General Considerations

In the previous chapter a rather simple expression was derived for the shear strength of a simple-span beam without web reinforcement. Here an attempt is made to extend the above procedure to beams which are provided with web reinforcement.

The contribution of web reinforcement to the shear strength can be pictured in different ways. As was seen before, the conventional theory originally assumed that all shear was carried by the web reinforcement. Later modifications of the concept of truss analogy, prompted by experimental evidence, allocated a certain proportion of the shear to be resisted by the concrete. Essentially, even the modified expressions for the shear strength implied that the contribution of web reinforcement was determined by the properties of the web reinforcement alone, as expressed by the term Krf_y, and not influenced by the shear strength of the beam without web reinforcement.

Another approach to the effect of web reinforcement is to consider that its contribution is determined by both the properties of the web reinforcement and the shear strength of the beam itself. The two alternatives are examined in more detail in the following paragraphs.

Test observations show that, in general, web reinforcement which crosses the main diagonal crack yields before the beam fails in shear. Figure 3a shows a simple-span beam shortly before shear failure.

For convenience, only the main diagonal crack is shown while in reality numerous cracks appear as the beam is being loaded. Figure 3b shows the portion of the beam to the left of the crack as a free-body diagram and Fig. 3c shows the approximate locations of the internal forces at the assumed 45-degree diagonal crack. The force F is the resultant of all stirrup forces crossing the crack. It has been projected down to the level of the tension reinforcement and divided into horizontal and vertical components. The other symbols have their usual meanings.

If it is first assumed that the contribution of web reinforcement is independent of the shear strength of the same beam without web reinforcement, then it must be possible to determine the increase of the shear capacity of the beam solely from the amount and physical properties of the web reinforcement. This can be calculated as follows:

$(1/s)\ (\cot \alpha + 1)\ jd$ = number of stirrups crossed by crack

$(1/s)\ (\cot \alpha + 1)\ jd\ A_w f_{yw} = F$ = total tension force in stirrups

$(1/2)\ \cos 2\alpha\ (jd)^2\ brf_{yw}$ = moment given by stirrups acting at their yield stress, about point A

The moment due to all forces about point A is then:

$$Va = Cjd - (1/2)\ b(jd)^2 \cos 2\alpha\ rf_{yw}$$

It is seen from this equation that the total internal resisting moment is made up of two parts: the web reinforcement resists directly a part of the applied moment, the remainder being resisted by the compressive force C. The direct contribution of the web reinforcement is influenced by the angle of inclination of the stirrups. For vertical stirrups, $\cos 2\alpha = -1$, and the moment of the stirrup forces is added to Cjd. As the angle α decreases, the direct contribution of the web reinforcement decreases also. At $\alpha = 45$ degrees, this contribution is zero. For α less than 45 degrees cos 2α reverses its sign; this indicates that the direct contribution is detrimental to the shear strength of the beam since the moment of the stirrup forces is subtracted from Cjd. The remaining part of the internal resisting moment is given by Cjd. For vertical stirrups the horizontal component of the stirrup force F reduces to zero. Consequently, the term Cjd is equal to that of the beam without web reinforcement, given by Eq. (18). As the value of α decreases, the horizontal component of F increases and, consequently, the value of C increases. This increases the part of the internal resisting moment given by Cjd.

The above assumptions about the effect of web reinforcement can be easily checked for vertical stirrups where the magnitude of C is determined by Eq. (18). For this case the above equation can be rewritten as follows:

$$Va = (1/2)b(jd)^2\ rf_{yw} + bd^2 f'_c\ k\ (0.57 - 4.5\ f'_c/10^5)$$

or

$$M = M_s + (1/2)b(jd)^2 rf_{yw}$$

In this equation all quantities can be determined and the validity of the equation can be checked against test results. This was done for Clark's and Moretto's beams with vertical stirrups. It was found, however, that the increase of the shear capacity of the beams was much greater than the direct contribution of the web reinforcement as given by the above equation. Furthermore, the difference between the two was consistently larger than could be accounted for by inaccuracies in the assumed locations of the internal forces, e.g. as given by the 45-degree crack in Fig. 3c. Consequently, it was concluded that the above assumptions of the effect of web reinforcement were not valid.

The above reasoning had one useful purpose. It showed not only that the shear strength is affected by the internal forces in the stirrups but also that the presence of web reinforcement changes the location of the neutral axis. Web reinforcement hinders the development of diagonal cracks; thus a larger compression area is available to resist the compressive stresses in the concrete. The combined effects of web reinforcement on the shear-compression capacity are (1) to contribute directly a portion of the internal resisting moment which can be either beneficial or detrimental, depending on the angle of inclination of the web bars, (2) to provide a larger ultimate compression force through a larger compression area in the concrete, and (3) to decrease the moment arm of the larger compression force through lowering the neutral axis of the beam. It is also conceivable that the presence of web reinforcement restricts the concentration of the compressive concrete strain in the region of the main diagonal crack.

In estimating the total effect of web reinforcement, only the direct contribution of stirrup forces can be determined rationally. However, even this contribution depends on the assumed angle of inclination between the main diagonal crack and the axis of beam. As this angle of inclination decreases, both the total force in the stirrups and its moment arm increase. This casts a doubt on the reliability of assuming a definite value for the angle between the diagonal crack and the axis of beam. The other two effects of web reinforcement cannot be determined rationally. Moreover, there is no theoretical basis for estimating the effect of stirrups on restricting the concentration of concrete strain in the region of the diagonal crack. For these reasons it was deemed desirable to express the total effect of web reinforcement empirically rather than to attempt to separate the different effects. This is done in the following section by assuming that the shear strength of a beam with web reinforcement is greater than that of the same beam without web reinforcement by an amount that is a function of the strength of the unreinforced beam and the amount and yield strength of web reinforcement provided.

11. Stirrups as Web Reinforcement

The findings of the previous section suggest that the shear strength of a beam with a reinforced web is affected by both the amount and properties of web reinforcement and the shear strength of the beam itself. Since the most important function of web reinforcement appears to be its resistance to the extension and widening of diagonal cracks, it is logical to assume that a given amount of web reinforcement will increase the shear strength of a beam in proportion to that of the same beam without web reinforcement; furthermore, test results show that in most cases web reinforcement yields before the beam fails in shear which indicates that both the amount of web reinforcement and its yield strength influence the load at failure.

All available test data on simple-span beams with stirrups as web reinforcement were analyzed in the light of the above assumptions. A total of 179 beams from 11 different investigations were included, 87 of them failed actually in shear, 91 in flexure, and one additional beam failed because of insufficient anchorage of stirrups. Different groups of beams are analyzed in Tables 17 through 27; Table 16 summarizes the range of test variables. In addition to shear failures, it was found advantageous to consider beams which failed in flexure also.

Several empirical expressions for the shear strength of such beams were investigated. The most consistent results were obtained by plotting the ratio P/P_s, where P is the measured load and P_s the load corresponding to the shear capacity of the same beam without web reinforcement, Eq. (18), against the quantity rf_{yw}. Figure 4 shows such a plot for the 87 beams which failed in shear. Satisfactory agreement with test results was obtained with the following linear equation:

$$P_{sw}/P_s = 1 + \frac{2rf_{yw}}{10^3} \qquad (26)$$

where P_{sw} is the shear strength of a beam with stirrups, P_s, that of the same beam without web reinforcement, and f_{yw} is expressed in pounds per square inch.

It is seen that most beams fall within ±15 percent of the value predicted by Eq. (26). Only 7 beams failed at a considerably lower load. All these beams had a very small a/d-ratio, and for two of them, tested by Moody, it was reported that the stress in stirrups was but 83 and 67 percent of their yield strength. It is likely that these beams did not fail in shear-compression but in shear-proper. This mode of failure is discussed in more detail in Section 19.

The average ratio between the load at failure and that given by Eq. (26) is 1.012 for the 80 beams which failed in shear-compression; the standard deviation 0.085. This agreement is somewhat better than that obtained previously for beams without web reinforcement.

As a further check on Eq. (26), the ratio P/P_s is plotted against rf_{yw} in Fig. 5 for beams which failed in flexure, either in tension or in compression. It is well known that beams which have been tested to obtain information about their shear strength have frequently failed in tension. Some of these beams, however, were rather close to their shear capacity at failure, as was indicated by well-developed diagonal cracks. Furthermore, some beams tested to yield data on their flexural strength can also be utilized to obtain information about their shear capacity. Figure 5 is used with the following criterion in mind: a beam will fail either in flexure or in shear, whichever capacity is reached first. Beams which failed in shear were used to derive Eq. (26) for their shear strength. Beams which failed in flexure, however, must fall below the line representing their strength in shear in Fig. 5. If they fall above, Eq. (26) cannot be true; if it was true, the beams should have failed in shear rather than in flexure since their shear capacity was smaller than their flexural capacity.

Figure 5 proves the general validity of Eq. (26); all beams with a few exceptions fall below the line representing this equation. The flexural capacity of the beams was reached at different ratios of P/P_s, Eq. (26) being the limit. Only four of the 91 beams fall substantially above this limit. Two of these beams were tested by Johnston and Cox and had only the average concrete strength reported for 20 beams;

it is likely that the actual value of f'_c for the individual beams had increased P_s sufficiently to bring the ratio P/P_s in agreement with other test results. Two other beams in this category were tested by Slater and Lyse. One of the beams had one companion specimen which failed in shear and another which failed at a much lower load. Both companion specimens of the other beam failed at a considerably lower load.

Figures 4 and 5 can also be used to determine the relative effectiveness of different angles of inclination and the yield strength of web stirrups. Most of the beams considered in the analysis had vertical stirrups; there were, however, beams with stirrups inclined at 67.5, 45 and 20 degrees. The effect of different angles of inclination was taken into consideration in plotting Figs. 4 and 5 by computing the ratio of web reinforcement from the conventional expression:

$$r = \frac{A_w}{bs \sin\alpha} \tag{27}$$

The conventional theory considers that the parameter Krf_{yw} is the measure of shear strength. Since the concept of truss analogy is disregarded by the present analysis, there is no justification for employing the quantity $\underline{K}$. Furthermore, while the variation in $\underline{K}$ is rather small for $\underline{\alpha}$ between 45 and 90 degrees, for smaller values of $\underline{\alpha}$ the coefficient $\underline{K}$ decreases rapidly. For beams of Slater and Lyse which had stirrups inclined at 20 degrees, $\underline{K}$ is equal to 0.44. The use of this low value of $\underline{K}$ would shift these beams considerably to the left in Fig. 5. Consequently, the beams would lie above the shear strength line. Since the beams failed in flexure, the use of Krf_{yw} rather than rf_{yw} is not justified.

The yield strength of the web reinforcement varied from about 44,000 to 73,400 psi for beams which failed in shear and from about 40,000 to 93,300 psi for beams which failed in flexure. The majority of the beams, however, had their yield strength between 45,000 and 55,000 psi. This variation is perhaps not large enough to bring out the effect of yield strength. However, beams of Slater and Lyse were reinforced with stirrups of relatively high yield strength, $f_{yw} = 73{,}400$ psi. If the ratio P/P_s was plotted against $\underline{r}$ alone, these beams would again fall above the shear strength line determined from other test results. This shows that the quantity rf_{yw} is a more correct measure of shear strength than the ratio $\underline{r}$ alone. It seems reasonable to believe that stirrups with higher yield strength offer greater resistance to the extension and widening of the diagonal cracks than stirrups of low yield strength.

12. Bent-Up Bars as Web Reinforcement

Relatively few simple-span beams with bent-up bars as web reinforcement have been tested to determine their strength in shear. The only source of experimental data are beams tested by Richart (2); practically all these beams failed in tension, however.

Series 1917 included 32 beams with hooked bent-up bars. The variables were the amount, angle of inclination, and spacing of the web bars. The main body of the beams was made of concrete from 2450 to 3770 psi; at the top center of each beam, however, there was a 4-in. deep zone of higher strength concrete, $f'_c = 4770$ psi. The beams were tested twice: they were first loaded to yielding with loads placed at

48 in. from the end supports, and then they were retested with loads at 36 in. from the supports. All beams failed in tension.

In order to obtain some indirect information about the shear strength of these beams, some of the beams with the smallest ratio of web reinforcement are analyzed in Table 28. Their P/P_s-ratios are plotted against rf_{yw} in Fig. 6. This figure shows that four beams with rf_{yw} equal to 210 psi were very close to shear failures, provided that Eq. (26) holds true for beams reinforced with bent-up bars. Photographs taken of these beams after failure show well-developed diagonal cracks. In all probability the beams were very close to their shear capacity.

Two beams of Series 1922 were also provided with bent-up bars as web reinforcement. These beams are analyzed in Table 19 and shown in Fig. 6. Both beams failed in tension and, as seen in the figure, lie below their strength in shear as given by Eq. (26).

Three beams of Series 1911 had one longitudinal bar bent up at a rather small angle so as to reinforce the entire shear span, 24 in. long. These beams are reported to have failed in diagonal tension. Table 29 analyzes the beams by using $s = a$ in Eq. (27) in order to calculate their ratio of web reinforcement. Undoubtedly, this procedure is rather approximate and these beams fall somewhat low in Fig. 6. However, a sketch of one of the beams after failure shows extensive cracking at the end hooks of the reinforcement and indicates a possible failure in anchorage.

With the help of Fig. 6 and more numerous tests on T-beams which are analyzed later, it was concluded that the contribution of bent-up bars to the shear strength of a beam is the same as that of stirrups. Consequently, Eq. (26) can be used in both cases:

$$P_{sw}/P_s = 1 + \frac{2rf_{yw}}{10^3} \tag{26}$$

Some additional information about the effectiveness of bent-up bars is available from tests of Series 1917. One of the variables investigated was the distance from the load point to the first bent-up bar. This distance varied from 9.6 to 16.8 in., or up to 1.68 times the effective depth of the beams. The analysis of some of these beams was included in Table 28. It is seen that even these beams failed in tension, although a considerable part of the shear span just in the region of maximum moment was without any direct web reinforcement. The highest P/P_s-ratio at failure was 1.51. Consequently, well-anchored bent-up bars, although not covering the entire shear span, appear to be beneficial with respect to resisting the development of diagonal cracks. This phenomenon was also observed for beams of Series 1910 which had vertical and diagonal stirrups supplemented by some bent-up bars. It is seen in Table 17 that the addition of only one layer of bent-up bars, not covering the entire shear span, increased the shear strength of the beams sufficiently to permit a tension failure.

13. Maximum Useful Amount of Web Reinforcement

Excluding bond, a reinforced concrete beam can fail either in flexure or in shear. Flexural failures can be initiated either by yielding of tension reinforcement or by crushing of concrete on the top of the beam, depending on the physical properties of the beam. Since the flexural capacity of a beam can be determined accurately, the purpose of this analysis is to find the amount of web reinforcement necessary to force a beam to fail in flexure rather than in shear.

Expressions for the shear capacity of a simple-span rectangular beam under one or two symmetrical concentrated loads were derived previously. Equation (26) can be rewritten as:

$$\frac{M_{sw}}{M_s} = 1 + \frac{2rf_{yw}}{10^3} \tag{28}$$

where

$$M_s = bd^2 f'_c\, k(0.57 - \frac{4.5f'_c}{10^5}) \tag{18}$$

Expressions for the flexural capacity of a beam are taken from a previous technical report (11). The ultimate flexural moment is given as:

$$\frac{M_f}{bd^2 f'_c} = \frac{pf_s}{f'_c}\left(1 - \frac{k_2}{k_1 k_3}\frac{pf_s}{f'_c}\right) \tag{29}$$

When a beam fails in tension, the yield stress f_y is substituted for f_s in Eq. (29). For compression failures, the steel stress f_s is below its yield strength; it can be determined from the following equation:

$$f_s = \sqrt{\frac{E_s \epsilon_u k_1 k_3 f'_c}{p} + (\frac{1}{2}\epsilon_u E_s)^2} - \frac{1}{2}\epsilon_u E_s \tag{30}$$

Whether the stress in the tension reinforcement at failure is below or at its yield stress is determined by the following criterion. The reinforcing index q is defined as:

$$q = \frac{pf_y}{f'_c} \tag{31}$$

The critical value of q is given by

$$q_{cr} = \frac{k_1 k_3}{1 + \frac{\epsilon_y}{\epsilon_u}} \tag{32}$$

If $q > q_{cr}$, the steel stress at failure is below its yield stress and the beam fails in compression. If $q = q_{cr}$, the beam fails by crushing of concrete as soon as the tension reinforcement yields. If $q < q_{cr}$, the steel stress at failure is either at or above its yield stress and the beam fails initially by yielding of the tension reinforcement. Another critical value of q can be utilized to determine whether or not the steel stress reaches work hardening at failure; this, however, is an unnecessary refinement in the present analysis.

The following numerical values are used in the above equations:

$$k_2 = 0.45$$

$$k_1 k_3 = 2.4\left(0.57 - \frac{4.5\, f'_c}{10^5}\right) \tag{20}$$

$$\epsilon_u = 0.004$$

$$E_s = 30{,}000{,}000 \text{ psi}$$

The behavior of beams with different values of the reinforcing index is shown by Fig. 7. To facilitate the presentation of expressions for shear strength, the quantity $M/bd^2f'_c$ for the ultimate moment is plotted against the parameter p/f'_c rather than $\underline{q}$. The curves are drawn for $f_y = 45{,}000$ psi, $f'_c = 3000$ psi, and $f_{yw} = 45{,}000$ psi. If the beams have sufficient web reinforcement to fail in flexure, the ultimate moment is determined by Curves 1 and 2. For $p/f'_c < (p/f'_c)_{cr}$ the beams fail in tension according to Curve 1, obtained from Eq. (29) by substituting $f_s = f_y$. At $(p/f'_c)_{cr} = 1.69 \times 10^{-5}$ in^2/lb, computed from Eq. (32), the mode of failure changes from that in tension to compression. For $p/f'_c > (p/f'_c)_{cr}$, the ultimate moment is given by Curve 2, computed by Eq. (29) with steel stresses obtained from Eq. (30).

If, however, no web reinforcement is provided, the maximum load is governed by Curves 1 and 3. Curve 3 represents the shear strength of a beam without web reinforcement, given by Eq. (18). The intersection between these two curves determines the transition between tension and shear failures. When some web reinforcement is provided, the shear strength increases according to Eq. (28) and the transition between the two types of failures takes place at a larger value of p/f'_c. Curve 4 shows this for $r = 0.005$.

If it is desired that the beams fail in flexure for any value of p/f'_c, the shear strength must be larger than the flexural strength for the entire range of ultimate moment, Curve 1 for tension and Curve 2 for compression failures. It is seen that a shear strength Curve 5 passing through the intersection between Curves 1 and 2 satisfies this condition. Computations based on the value of p_{cr} obtained from Eq. (32)

show that the corresponding ratio of web reinforcement is 0.011 for the variables under consideration. Thus r = 0.011 corresponds to the maximum useful amount of web reinforcement for the values of f_c', f_y, and f_{yw} used in the above example. This limit was calculated for other combinations of f_c', f_y, and f_{yw} and is shown in Fig. 8 graphically.

The maximum useful amount of web reinforcement does not depend on the percentage of tension reinforcement. It forces a beam of any amount of tension reinforcement to fail in flexure, either in tension or in compression. However, for any value of p except that at the transition between tension and compression failures, this maximum useful amount is more than sufficient to insure flexural failures, see Curve 5 of Fig. 7. In practice, most beams are designed to fail in tension if loaded to destruction. These beams would fall considerably to the left of the transition point and, consequently, would require much less web reinforcement to prevent shear failures. Table 30 shows an analysis of beams designed according to the present ACI Code balanced design requirements. This analysis considers rectangular beams reinforced in tension only with the steel percentages taken for f_s = 20,000 psi and $f_c = 0.45\, f_c'$.

The amount of web reinforcement necessary to prevent shear failures is calculated with the aid of Eq. (28) for several values of f_y and f_{yw}. It is seen that as f_y increases and f_{yw} decreases, the amount of web reinforcement necessary to ensure flexural failures increases. For f_y equal to 50,000 psi and f_{yw} equal to 40,000 psi, about 0.35 percent web reinforcement is required while for both f_y and f_{yw} equal to 45,000 psi, about 0.20 percent will be sufficient.

IV. SIMPLE-SPAN T-BEAMS UNDER ONE OR TWO SYMMETRICAL CONCENTRATED LOADS

14. T-Beams Without Web Reinforcement

The basic empirical equation (18) was derived for simple-span rectangular beams. It was seen that this equation could be interpreted by means of the conventional theory of compression failures, as modified by diagonal tension cracking, and that the failure criterion was the ultimate compressive strain in the concrete.

The above concept of shear failures as shear-compression failures was extended to include T-beams. Since the moment-rotation relationship of a T-beam differs from that of a rectangular beam, a correction must be made to take into consideration the effect of the shape of the beam on the compressive strain in the concrete. But since the distribution of the concrete strain was not determined previously, the exact form of the shape factor cannot be established. If a linear strain distribution is assumed, strain in any fiber is given by

$$\epsilon = \frac{M}{EI} y$$

where y is the distance from the neutral axis to the fiber under consideration. Comparing a T-section with a rectangular section of the same width as the flange in the T-section, the following relationship can be written if the ultimate strain in the concrete is the same in both these two cases:

$$M_T = M_R \frac{I_{cT} y_R}{I_{cR} y_T} \tag{33}$$

where the transcripts R and T refer to rectangular and T-sections, respectively, I_c refers to the moment of inertia of a section transformed to concrete, and $\underline{y}$ refers to the distance from the neutral axis to the top fiber in the concrete, all quantities taken at the instant of failure. If the strain distribution were linear and all quantities could be determined, the above expression would give the relationship between shear moments of a T-section and a rectangular section of the same width. However, the formation of a diagonal crack produces a non-linear strain distribution. The stress in the tension reinforcement is approximately uniform from the lower end of the crack to a vertical section through the upper end of the crack. This affects also the distribution of concrete strain at the top of the beam, causing a certain concentration of strain at the end of the diagonal crack. Furthermore, since the section cracks progressively as load is applied, the exact values of $\underline{I}$ and $\underline{y}$ cannot be determined. Consequently, Eq. (33) cannot be applicable.

An approximate shape factor was derived by assuming that the effect of shape of a beam is determined primarily by its moment of inertia. In an uncracked state, the moment of inertia of a T-beam is considerably smaller than that of a similar rectangular beam. After extensive cracking, the value of $\underline{I}$ of a section transformed to concrete is very nearly the same in both cases. At the instant of failure, the relationship between the two is unknown; it was approximated by the ratio of the average values of $\underline{I}$ of the uncracked and the fully cracked state. Thus, the shape factor takes the following form:

$$F_t = \frac{I_T + I_{cr}}{I_R + I_{cr}} \qquad (34)$$

where I_R and I_T refer to the uncracked rectangular and T-sections, respectively, and I_{cr} refers to the "straight line" cracked transformed section of either a rectangular or a T-section since both have very nearly the same moment of inertia.

The above shape factor makes it possible to modify Eq. (18) for rectangular beams so that it applies to T-beams. The compressive area A_c of a T-section as determined by the conventional "straight line" theory is substituted for bkd and the equation is rewritten as follows:

$$\frac{M_s}{A_c d f'_c F_t} = 0.57 - \frac{4.5\ f'_c}{10^5} \tag{35}$$

The validity of Eqs. (34) and (35) must be determined with the help of test results. All available data on T-beams under one or two symmetrical concentrated loads was analyzed; the range of test variables is summarized in Table 31 and the physical properties and calculated quantities of individual beams are given in Tables 32 through 38. All units are given in inches and pounds. The width of the flange is marked by b, that of the web by b', and the thickness of the flange by e. Other symbols have their usual meaning. Some beams were reinforced with straight unanchored bars and failed in bond; these beams are not included in the analysis.

Beams without web reinforcement are considered first. Ferguson and Thompson (20 and 21) have reported tests on beams of a number of different shapes. Some of the beams were provided with shoulders; that is, the width of the upper part of the web was larger than that of the lower part. These beams are analyzed in Tables 35 and 36 and the quantity $M/A_c df_c' F_t$ is plotted against f_c' in Fig. 9. It is seen that in most cases Eq. (35) gives reasonable agreement with the test results. However, two series with the largest number of beams, Series A and B, indicate consistently lower shear strengths than those given by Eq. (35). This discrepancy could mean either that the shape factor given by Eq. (34) is fundamentally incorrect or that there are some other considerations besides the effect of the moment of inertia which determine the compressive strain in the concrete. It is noticed that beams reported by Ferguson and Thompson have, in general, very wide and thin flanges. It is known that in such beams parts of the flanges at some distance from the web do not resist their full share of the bending moment. This phenomenon is discussed in some detail for an elastic medium by Timoshenko (24). It can be seen in Fig. 9 and Tables 35 and 36 that beams which fall the greatest percentage below the predicted values have very large d/e and b/b'-ratios; that is, the depth of the beam is large relative to the thickness of the flange and the width of the flange is large relative to that of the web. The beams of Series A, B, C, and D had the same d/e-ratio, 5.5, while the b/b'-ratio was equal to 4.25 for Series A and 2.43 for Series D. The two beams of Series D failed at a load rather close to the predicted values, 85 and 93 percent, respectively, while the beams of Series A reached only about 70 percent of their predicted capacity at failure. The beams of Series B and C were provided with shoulders of the same width as the web width of the beams of Series D. The depth of the shoulder, e", was 4 in. for Series B and 7 in. for Series C. The addition of shoulders reduces the unsupported width of the flange; in that sense, it should have the same effect as a decrease in the b/b'-ratio. However, the beams of Series B failed but at a slightly higher load than those of Series A. The two beams of Series C were benefited more, they reached 75 and 84 percent, respectively, of their predicted strength at failure. These results show that for the same value of d/e, the agreement between the measured and calculated loads improves as the ratio b/b' decreases. The addition of shoulders has partially the same effect as that of decreasing the b/b'-ratio. Furthermore, deeper shoulders have a larger effect on the increase of the shear strength than shallower shoulders. This is apparently related to the formation and propagation of cracks in the tension zone of the concrete. However, since the shape factor of Eq. (34) was primarily intended for ordinary T-beams without shoulders, it is not expected it would apply equally well for more complex shapes of T-beams. The remaining beams had rather large b/b'-ratios, varying from 4.47 to 5.18, while the d/e-ratio varied from 2.11 to 4.67. All these beams except those of Series N failed at a load in good agreement with Eq. (35). The beams of Series N had the largest d/e-ratio, 4.67, and failed at a somewhat lower load than that predicted. In conclusion, the above findings suggest that the shape factor of Eq. (34) is applicable whenever beams with abnormally high d/e and b/b'-ratios are excluded.

In the above comparison, the beams of Series H-B and K-B were of composite tile-concrete construction. One 5/8-in. thickness of tile was included in the overall dimensions of the beams in calculating the shear strength of the beams. These beams were built with B-type tile which had but slightly higher compressive strength than that of the concrete used. It was seen that the beams failed at about the predicted load. Beams built with tiles of higher concrete strength are not included in the analyses. These beams failed at a somewhat higher load; the high strength tiles seemed to have acted as a form of web reinforcement in increasing the load at failure.

Figure 10 shows the above beams, except Series A and B, together with results from other investigations. The beams of Series A and B were excluded because of the simultaneous high ratios of d/e and b/b', and as was discussed before, the shoulders of the beams of Series B were not deep enough to increase their shear strength. It is seen that when beams with abnormally large d/e and b/b'-ratios are excluded, Eq. (35) gives satisfactory agreement with test results. In some beams of Bach and Graf, there is some doubt about the primary mode of failure, heavy cracking at the end hooks of the tension reinforcement indicated possible anchorage failure. This might explain why one of these beams is somewhat low. Beams of Richart and of Braune and Myers show good agreement with Eq. (35). Although the beams of Braune and Myers had a very high b/b'-ratio, 7.0, the d/e-ratio was rather small, 2.56, and the beams failed according to Eq. (35).

It is concluded that the shear strength of simple-span T-beams without web reinforcement as normally used in construction can be predicted by Eq. (35) where the shape factor is computed by Eq. (34). Beams with abnormally high d/e and b/b'-ratios are outside the scope of Eq. (35), their shear strength is lower because the effective width of such flanges is reduced. No attempt was made, however, to determine an expression for the effective flange width. Moreover, T-beams of such dimensions are not permitted by the present ACI Code requirements for isolated beams.* In the following section, it is shown that the use of transverse reinforcement in the flange effectively counteracts the reduction in the effective width of the flange and thereby increases the scope of Eq. (35). This phenomenon was also observed for beams of Braune and Myers in the present comparison.

15. T-Beams With Web Reinforcement

T-beams considered in this section are analyzed in Tables 32, 33, 34, 37 and 38. A summary of the test variables was included in Table 31. The ratio P/P_s, where P_s was obtained from Eq. (35), was calculated for each beam. This ratio is plotted against the parameter rf_{yw} in Fig. 11 for beams which failed in shear. The ratio of web reinforcement was computed with respect to the width of the web.

Heft 10 by Bach and Graf reports tests on 81 beams. The beams were tested in 28 groups, 25 groups of three and three groups of two companion specimens. All beams were reinforced with two tension bars. One group of three beams had 2.82 sq. in. of tension reinforcement and failed in tension. The remaining beams were provided with about 3.9 sq. in. of tension steel and failed either in shear or in bond. Beams with straight unhooked bars failed at a lower load than similar beams with hooked bars, apparently in bond. Beams with hooked bars failed mostly in shear, these beams are analyzed in Table 32. Figure 11 shows that most of the beams give good correlation with Eq. (26), originally derived for rectangular beams. Only two groups of beams failed at a somewhat lower load than that predicted. However, photographs of beams after

* ACI 318-51, Sec. 705-f.

failure show rather extensive cracking at the end hooks of the tension reinforcement. Beams with larger amount of web reinforcement resisted a higher load at failure and showed more marked cracking. It is possible that the two groups with the highest amount of web reinforcement failed in bond through excessive bending of the anchorage hooks.

Heft 67 by Graf reports tests on 8 T-beams under two symmetrical concentrated loads. These beams were provided with transverse reinforcement in the flanges and although the flanges were rather thin and wide, no reduction was noticed in the effective flange width. All beams were reinforced with the same amount of web reinforcement; the only variable was the arrangement of bent-up bars. Four different groups of two beams were investigated, the test results are given in Table 38. Beams of Group 6 were reinforced with regular bent-up bars, the horizontal part of the bends being carried over the transverse reinforcement in the flanges. This arrangement of web reinforcement was the most effective one; the beams failed in tension, and as seen in Fig. 11, the load at failure was about 20 percent higher than that predicted for shear. Beams of Group 8 were reinforced with "brought-back" bent-up bars: all longitudinal bars were first taken to the end of the beam, bent up there and then bent down at the desired spacing to serve as web reinforcement. The beams failed in shear at a load slightly higher than the predicted load; some crushing of concrete was observed at the end hooks of the "brought-back" bars. Beams of Group 9 were provided with conventional bent-up bars except that the bends had no horizontal extension at the top of the beam. This type of web reinforcement was about as effective as that of Group 8. Beams of Group 7 were reinforced with loose, "floating" type of inclined bars, hooked on both ends. These beams failed at a somewhat lower load than that predicted, indicating that this type of web reinforcement was not fully effective.

T-beams tested by Richart were provided with vertical stirrups. All these beams failed in tension. Beams tested by Braune and Myers had both vertical stirrups and bent-up bars as web reinforcement. These beams failed in tension also. However, the web reinforcement was sufficiently effective to permit high ratios of P/P_s at failure, the highest ratio being 3.19.

Beams reported by Bach and Graf in Heft 12 were reinforced with bent-up bars. A total of 87 beams were tested; the tension reinforcement consisted of from 4 to 7 bars of the same total area, and no transverse reinforcement was used in the flanges. All beams with unhooked longitudinal bars failed at a lower load than similar beams with hooked bars, evidently in bond. Beams with hooked longitudinal and bent-up bars are analyzed in Table 37. It is seen that despite the large ratios of web reinforcement only a few beams failed in tension. Some other beams might have had yielding of the lower layer of the tension reinforcement at failure. In most beams, failure was initiated by excessive cracking and crushing of the concrete at the hooks. The most effective arrangements of bent-up bars can be found from Table 37.

It was concluded that the shear strength of simple-span T-beams with web reinforcement can be determined by the same expression as that for rectangular beams:

$$P_{sw}/P_s = 1 + \frac{2rf_{yw}}{10^3} \qquad (26)$$

where P_s is determined from Eq. (35) and r from the following equation:

$$r = \frac{A_w}{b's \sin\alpha} \qquad (27a)$$

For bent-up bars, there is some danger of a premature failure because of cracking and crushing of concrete at the hooks. This can be protected against by using sufficiently large hooks and, especially, by using transverse reinforcement in the flanges of the beam.

V. RESTRAINED BEAMS UNDER SYMMETRICAL CONCENTRATED LOADS

16. Modes of Failure

Simple-span beams under concentrated loads fail at the location of an applied load, at the section of maximum shear and maximum moment. Shear stresses combined with flexural stresses are instrumental in producing a main diagonal crack; after this crack has formed, the beam fails in compression.

In restrained beams, shear and moment conditions are such as to permit, in general, the formation of three main diagonal cracks as shown in Fig. 12. The beam can fail at any of these three cracks, depending on the magnitude of shear and moment at the section under consideration and on the arrangement of both longitudinal and web reinforcement.

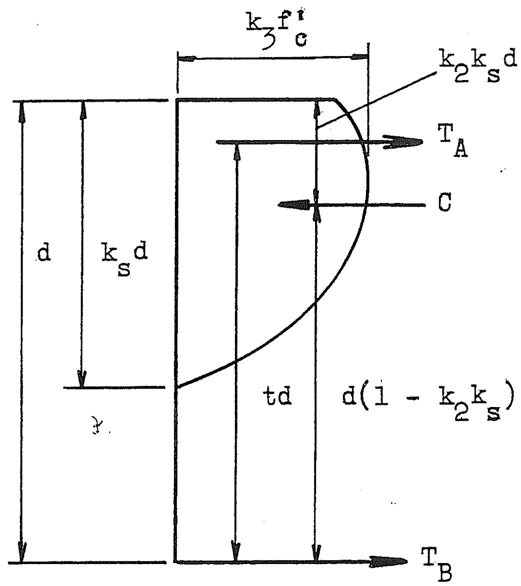
While the static moment is the same at both sides of section A, the magnitude of shear can be different in spans f and g. The crack at the section of greater shear forms first, for small shear ratios it is even conceivable that the beam fails at that section before the other crack has formed. Although span g has constant shear, the moments can be different at sections A and B. Depending mainly on the relative magnitudes of moment, either one or two cracks form. The crack at the larger moment develops first and the beam fails, in general, at that crack.

Various modes of failure with special emphasis on the arrangement of reinforcement are discussed below. It is assumed that span f has sufficient reinforcement so that the beam fails in span g.

(a) Continuous Top and Bottom Reinforcement. A free-body diagram for this arrangement of longitudinal reinforcement is shown in Fig. 13. It is assumed first that only one diagonal crack forms before failure. Figure 13 shows crack 2 and assumes that shear is resisted exclusively by the compression area of the concrete. The top longitudinal reinforcement is in tension at crack 2 and in compression at section B. If there is no possibility for bond failure between these two sections, e.g. if the span g is long relative to the effective depth of the beam and bars of good bond characteristics are used, a shear failure similar to that in simple-span beams is expected to take place. Thus, Eq. (18) can be employed directly to determine the shear strength of such beams without web reinforcement and Eqs. (18) and (28) that of such beams with web reinforcement. If two cracks are present, and bond failure does not occur between them, the mode of failure is unchanged and the shear capacity of the beam can be determined by the same equations at the section of maximum moment.

If, however, bond is destroyed between the reinforcing bars and the concrete in the middle portion of the span g, the above equations no longer represent the shear strength of the beam. Bond failures are likely to take place when the span g is relatively short. Then only a small distance separates the diagonal crack from either section A or B, and a change in stress from tension to compression in the reinforcement must take place over this length. If a bond failure results from the high bond stresses in this region, both the top and bottom reinforcing bars are in tension as shown in Fig. 14 for one crack and in Fig. 15 for two cracks at failure. For simplicity, it is assumed that the whole tensile force, T_A or T_B, is carried through the middle portion of the beam. This redistribution of internal forces is very unfavorable to the shear capacity, and the beam fails at a much lower load than it would if no "compressive" reinforcement was provided.

An approximate expression for the shear strength of a beam with both top and bottom reinforcement in tension can be derived as follows:



$$C = k_1 k_3 k_s d b f'_c$$

$$C = T_A + T_B \qquad (36)$$

$$M_s = Cd(1 - k_2 k_s) - T_A td \qquad (37)$$

Equation (36) determines the moment at shear failure. However, there are two unknowns, k_s and T_A, which must be evaluated before the shear moment, M_s, can be expressed quantitatively. If the tensile force T_A is determined by assuming that the moment arm is the same for both sections A and B, the following relationship can be written:

$$T_A/T_B = M_A/M_B \qquad (38)$$

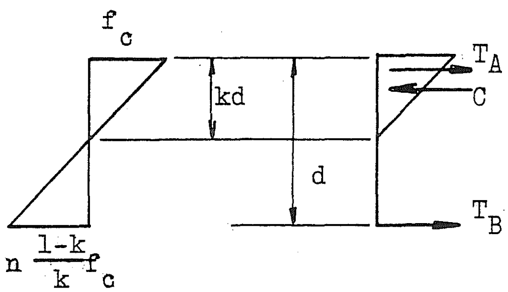
From Eqs. (36) and (38):

$$T_A = \frac{C}{\frac{M_B}{M_A} + 1} \qquad (39)$$

It is further assumed that the factor k_s as derived for simple-span beams remains valid for restrained beams:

$$k_s = \frac{k}{2.4(1 - k_2 k_s)} \qquad (22)$$

The quantity k is determined again by the "straight line" theory. This can be done as follows:



$$C = (1/2)\ bkd\ f_c \qquad (39)$$

$$T_B = pbdn \frac{1-k}{k} f_c \qquad (40)$$

From Eqs. (36), (38), (39), and (40):

$$k = (p_o n)^2 + 2p_o n - p_o n \qquad (41)$$

where

$$p_o = p\ (1 + \frac{M_A}{M_B}) \qquad (42)$$

The shear moment as given by Eq. (37) can now be rewritten as:

$$\frac{M_s}{bd^2 f'_c} = Ak(0.57 - \frac{4.5\ f'_c}{10^5}) - \qquad (43)$$

where

$$A = 1 - \frac{t}{\left(\frac{M_B}{M_A} + 1\right)\left(1 - k_2 k_s\right)} \quad (44)$$

k is given by Eq. (41)

k_2 is taken as 0.45

k_s is given by Eq. (23)

Equation (43) determines the shear strength of a restrained beam which fails at section B after bond has been destroyed from this section to crack 2 so that both the top and bottom reinforcing bars are in tension. This equation can be used for any section provided that the subscript B refers to the section under consideration and the subscript A to the adjacent section from which the tensile force T_A is carried through to the section B. It was derived by assuming that the longitudinal reinforcement was continuous throughout the entire length of the beam and that the whole tensile force at one section was carried through to the other section. This is a conservative estimate since it is likely that in some cases a part of the tensile force is resisted by frictional bond, although the reinforcing bar might be slipping in the entire region from section A to B. If the actual ratio between the top and bottom tensile forces can be determined for the section at failure, the actual ratio T_B/T_A should be substituted for M_B/M_A in Eqs. (43) and (44).

The first crack in span g will form at the section of maximum moment. If a beam fails in shear after only this crack has formed as shown in Fig. 14, redistribution of the internal forces has taken place at section B whereas the bottom longitudinal reinforcement is still in compression at section A. Although both the top and bottom reinforcing bars are in tension at section B, diagonal cracking has not reduced the compression area and the beam cannot fail in shear at that section. Consequently, section A is the critical section and the shear strength of the beam is determined by Eq. (18) at the section of maximum moment. If two cracks are present at failure and full redistribution of the internal forces has taken place as shown in Fig. 15, Eq. (43) is applicable at section A as well as section B. The shear strength of the beam is determined by Eq. (43) at the section of maximum moment. Under certain conditions it is conceivable that despite the formation of two cracks only partial redistribution of the internal forces has taken place. This may be the case if for example the moment at section A is much greater than the moment at section B. Then the bond stresses are much higher in the top reinforcement than those in the bottom reinforcement and local bond failure may take place only in the top longitudinal bars. The shear capacity of the beam is given by Eq. (43) at section B and by Eq. (18) at section A, the section of maximum moment. The beam fails at the section of the smallest shear strength. However, since the conditions for partial redistribution of the internal forces cannot be determined in advance, it is more conservative to assume full redistribution whenever two cracks are present at failure.

The validity of Eq. (43) is checked against test results in Section 17.

(b) Straight Bars Cut Off Beyond the Theoretical Point of Contraflexure. A diagram for this arrangement of longitudinal reinforcement is shown in Fig. 16a. When the length of embedment, both x and y, is sufficient to prevent a bond failure, it is expected that the shear strength of a restrained beam can be determined by Eqs. (18) and (28). However, when the length of anchorage is small or reinforcing bars of poor bond characteristics are used, the failure may be a sudden stripping out of the reinforcement and a complete destruction of the beam. Failures of this type have been reported by Richart and Larson (25) and by Moody (12). Figure 16b shows a sketch of a beam in this category after failure.

(c) Beams With All Bars Bent Up. Figure 17 shows this arrangement of longitudinal reinforcement. This arrangement appears to be an effective one; it prevents any possibility of bond failures and uses the bent-up bars as web reinforcement. When the bars are bent at some distance from the support, it seems advisable to use a few stirrups between the first bend and the load point. The shear strength of such beams is determined by Eqs. (18) and (28). While such an arrangement of reinforcement is very effective, care must be taken with the design and fabrication of bends. Richart and Larson (25) observed frequent crushing of the concrete at the bends after yielding of reinforcement.

(d) Beams With Both Bent-Up and Straight Longitudinal Bars. A diagram of such a beam is shown in Fig. 18. This type is similar to that discussed under (c). When bond failures are prevented, shear capacity is given by Eqs. (18) and (28). When, however, numerous bars are left straight, a premature bond failure similar to that discussed under (b) is possible.

17. Test Data on Restrained Beams

The only tests on restrained beams reported in the literature are those by Richart and Larson (25) and by Moody (12). These tests are analyzed and the validity of previously derived equations checked in the following paragraphs.

(a) Tests Reported by Richart and Larson. Richart and Larson reported tests on 59 beams, 17 in Series 1911 and 42 in Series 1917. Beams of Series 1911 failed either in tension or in bond and the concrete strength was not recorded for all beams. Thus, very little information is available about the shear strength of these beams and they are not included in the present analysis.

Beams of Series 1917 were designed to investigate the effect of various arrangements of bent-up bars in span g (Fig. 19). The type of beam is shown in Fig. 19, and Table 39 gives the arrangement of reinforcement for each individual beam. All beams had eight 5/8-in. round plain bars over the support A. The overhanging portions of the beam, span f, were heavily reinforced so as to produce failures in span g. Most beams of Series 1917 failed in tension. There are, however, a few beams which throw some light on the modes of shear failure as discussed under (b), (c), and (d) in the previous section.

Beams 380 represent beams with straight longitudinal bars cut off beyond the point of contraflexure. From the description and photographs of failure it appears that these beams failed in bond by stripping off the concrete above the bars at failure as shown in Fig. 16. This premature bond failure cannot be predicted by any of the shear strength equations of this report; it is a matter of bond characteristics of the reinforcing bars.

The rest of the beams were of the types discussed in sections 16(c) and (d), with some or all of the longitudinal bars bent down in span g. Beams which had four of the eight bars at the support A bent down in one layer, Beams 388, 389, and 400, appear to have failed in bond after yielding of the reinforcement. They correspond to a bond failure of the type discussed in section 16 (b). Beams which had four or more bars bent down in two or three layers failed in tension without any tendency for stripping of the concrete at the straight bars. However, crushing of the concrete inside the bends was frequently the cause of final failure. Furthermore, diagonal cracks were observed to intersect the reinforcement at the bends. These two phenomena were often responsible for a sudden shear-type final collapse of the beams. This occurred, however, well after the yielding of reinforcement. The ratios P/P_f in Table 39 were computed by Eq. (29), using $f_s = f_y$ and $k_2/k_1k_3 = 0.5$ in the calculation of the flexural capacity P_f. Since these ratios are greater than one, the reinforcement was stressed in the work-hardening region at final failure.

The main variables intended to be investigated were the angle of inclination and the number and spacing of bends. Even the largest spacing of bent-up bars gave a value of r which was sufficient to prevent shear failures. A few shear failures were obtained, however, when the first bar was located so far from support A that a diagonal crack could form without intersecting any inclined bars. Such failure was observed in Beam 381.2 where the first bend was removed 24 in. from the support.

As seen in Table 39, this beam failed before yielding and at a ratio P/P_s equal to 0.92. Thus, the load at failure was governed by Eq. (18). The companion specimen failed in tension at a higher load, however. Beams 398, 397, and 396 were similar to Beams 381 except that they were provided with vertical stirrups as additional web reinforcement. All of these beams failed in tension, although the final failure of Beam 398.1 was a sudden break, called diagonal tension by Richart and Larson.

In conclusion, it can be said that the behavior and strength of the restrained beams with bent-up bars in these tests is not inconsistent with the behavior of simple-span beams as predicted by Eqs. (18) and (28). Bond failures are outside the scope of these equations; beams must be designed so that the danger for the destruction of bond is eliminated. Care must be taken in the design of bends to avoid crushing of the concrete inside the bends in the reinforcing bars.

(b) Tests Reported by Moody. Moody reports tests on 96 restrained beams, tested in five series (12). The dimensions of the beams and the arrangement of reinforcement and loads are shown in Fig. 20. All beams were provided with equal amounts of top and bottom longitudinal reinforcement, four bars placed in two layers. In all but three beams the four top bars and the two lower bottom bars were continuous throughout the total length of the beam, the other two bottom bars were cut off 4 in. from the supports. In the remaining three beams the longitudinal reinforcement was cut off at the supports and the inner load points in accordance with the present ACI Code. The test variables included the percentage of longitudinal reinforcement, the

As seen in Table 39, this beam failed before yielding and at a ratio P/P_s equal to 0.92. Thus, the load at failure was governed by Eq. (18). The companion specimen failed in tension at a higher load, however. Beams 398, 397, and 396 were similar to Beams 381 except that they were provided with vertical stirrups as additional web reinforcement. All of these beams failed in tension, although the final failure of Beam 398.1 was a sudden break, called diagonal tension by Richart and Larson.

In conclusion, it can be said that the behavior and strength of the restrained beams with bent-up bars in these tests is not inconsistent with the behavior of simple-span beams as predicted by Eqs. (18) and (28). Bond failures are outside the scope of these equations; beams must be designed so that the danger for the destruction of bond is eliminated. Care must be taken in the design of bends to avoid crushing of the concrete inside the bends in the reinforcing bars.

(b) Tests Reported by Moody. Moody reports tests on 96 restrained beams, tested in five series (12). The dimensions of the beams and the arrangement of reinforcement and loads are shown in Fig. 20. All beams were provided with equal amounts of top and bottom longitudinal reinforcement, four bars placed in two layers. In all but three beams the four top bars and the two lower bottom bars were continuous throughout the total length of the beam, the other two bottom bars were cut off 4 in. from the supports. In the remaining three beams the longitudinal reinforcement was cut off at the supports and the inner load points in accordance with the present ACI Code. The test variables included the percentage of longitudinal reinforcement, the concrete strength, the dimensions of the beams, and the magnitude of moments and shear as reflected by different arrangements of loads. Sixty-one beams were tested without web reinforcement, 29 with vertical stirrups, and 6 with 45-degree stirrups.

Beams with no web reinforcement are analyzed in Tables 40a, 40b, and 40c. The beams of Series I, II, and IV had continuous longitudinal reinforcement and equal moments at sections A and B. From crack patterns and strain measurements recorded for Beam I-2c it was observed that bond was destroyed in span g so that both the top and bottom reinforcing bars were stressed in tension. The beams failed after developing, in general, two main diagonal cracks between sections A and B. Consequently, Eq. (43) should apply at both these sections. In order to apply Eq. (43), the weakest critical section must be determined first. Everything else remaining the same, the shear capacity of a section is determined by the square of its effective depth. This distance was always 0.25 in. larger at section B than that at section A, indicating that A was the critical section. However, it is recalled that there were some differences in the arrangement of the longitudinal reinforcement at these sections. This might have a larger effect on Eq. (43) than the small difference in the values of d. From the condition of equal moments and entirely continuous top reinforcement it can be concluded that $T_A = T_B$ at section B. This assumes that the total tensile force is carried through from section A to section B so that Eq. (43) can be used with $T_A/T_B = 1$ at section B. At section A, however, only half of the bottom reinforcement is continuous. After bond is destroyed, it is likely that the stress in the continuous bars is increased relative to its magnitude before bond failure. In order to transmit the total force T_A to section B, the cut-off bars must be completely inactive and the continuous bars must resist twice their former stress. However, stress measurements in Beam I-2c show that although the stress increased in the continuous bars, it never reached more than about 120 percent of its former value. This indicates, using the proper subscripts, that the ratio T_A/T_B is less than one at section A. The smaller is this ratio, the larger is the factor A in Eq. (43) and, consequently, the shear strength of the beam. Thus, section B must be considered as the critical section for Eq. (43), using $M_A/M_B = T_A/T_B = 1$.

Beams of Series I, II, and IV are analyzed in Tables 40a and 40b and the quantity $M/bd^2f'_ckA$ is plotted against f'_c in Fig. 21a. It is seen that, in general, test results give satisfactory agreement with Eq. (43). Thus it appears that the assumptions made in deriving this equation are essentially correct and that this equation can be used to determine the shear strength of restrained beams with continuous reinforcement whenever the shear failure takes place subsequent to destruction of bond. This type of failure is still a primary shear failure since the destruction of bond in the high bond stress region does not produce in itself a failure of the beam. It causes only a redistribution of the internal forces so that the new combination of the tensile forces at a certain section requires a larger compressive force than before. For Moody's beams the new compressive force is about two times larger than that before the destruction of bond. The greatly increased compressive force leads to a lower shear strength since the capacity of the compressive zone of the beam is but little greater than that for simple-span beams. Thus the factor A of Eq. (43) can be considered as a reduction factor for restrained beams which fail after local bond failure.

In deriving Eq. (43) it was assumed that the whole tensile force at one section is transmitted to the adjacent section. This assumption is, in general, a conservative estimate since some of the tensile force is resisted by frictional bond. Tables 40a and 40b show that the ratio M_{test}/M_s increases as the g/d-ratio increases or as smaller reinforcing bars are used as longitudinal reinforcement. In both cases the relative importance of partial bond is more pronounced and, consequently, not all of the tensile force is transmitted from one section to the adjacent section. This gives a smaller actual ratio T_A/T_B than that obtained from the bending moments and increases the shear strength of the beams. However, as seen in Tables 40a and 40b, the increase in the shear capacity was rather small even for the largest value of g/d and the smallest size of reinforcing bars used in the tests. Furthermore, beams of Series II which had the smallest value of g/d fall even somewhat low in Fig. 21a.

The limits of the applicability of Eq. (43) cannot be determined from Moody's tests. For beams shown in Fig. 21a the g/d-ratio varied from 1.52 to 4.0. All of these beams failed after redistribution of the internal forces. Thus, it appears that local bond failures are possible with g/d-ratios larger than four.

Beams of Series V are analyzed in Table 40c. These beams had their longitudinal reinforcement cut off at the supports and the inner load points. Failure took place by a sudden stripping out of the longitudinal reinforcement as discussed in Section 16(b). The beams were analyzed by Eq. (18) with the support as the critical section and the load at final bond failure was found to be about one-half the theoretical shear capacity.

Beams of Series VI had unequal bending moments, M_A being two times larger than M_B. The beams failed, in general, after developing only one main diagonal crack at section A. Thus it is likely that local bond failure had taken place only in the top reinforcement, so that both the top and bottom bars were in tension at section B whereas the bottom bars were still in compression at section A. This possibility was shown in Fig. 14. Consequently, the shear strength of these beams should be governed by Eq. (18) at section A and the beams are analyzed in Table 40c accordingly. The quantity $M/bd^2f'_c(k + np')$ is plotted against f'_c in Fig. 21b and it is seen that there is good agreement between the measured and calculated moments.

Beams with web reinforcement are analyzed in Tables 41a and 41b. For beams of Series I and IV the quantity M/M_s is plotted against rf_{yw} in Fig. 22. Both these test series had equal moments at sections A and B and the shear moment M_s was calculated by Eq. (43). Figure 22 shows that the beams of Series I which had 45-degree stirrups give very good agreement with Eq. (28), derived for simple-span beams. Beams with vertical stirrups were tested in two different groups, 6 beams in 1950 and 10 beams in 1952. Beams of 1950 had the closed ends of the stirrups placed toward the lower face of the beam whereas the beams of 1952 had the closed ends always in the compression zone. Beams of 1950 give good agreement with Eq. (28) except for the two beams with the largest amount of web reinforcement. These beams were observed to split longitudinally along the reinforcement and the stirrups did not reach yielding at failure. It is possible that longitudinal splitting destroyed the anchorage of stirrups so that they were unable to develop their full effectiveness. In the beams of 1952 longitudinal splitting took place in a more restricted scale, and then only in the region where the stirrups were open-ended. It appears that longitudinal splitting will not occur if the reinforcing bars are tied together in the transverse direction. Beams of 1952 give good agreement with Eq. (28) except for two beams with very high values of rf_{yw}. It is noticed, however, that at the value of rf_{yw} at which the beams of 1950 fell below the predicted ultimate moment, the beams of 1952 still agree with Eq. (28). Thus, the anchorage of stirrups was more effective in Series 1952 than in Series 1950; only with very high values of rf_{yw} did the vertical stirrups not develop their full strength at failure.

Among the beams of Series I two beams were provided with a 23 by 4-in. flange. Since the flange area increases the compression area of the concrete at section B, the beams are analyzed for section A as the critical section in Table 41a. In line with the previous discussion about the effect of cutting off one half of the bottom bars at the supports, the ratio T_A/T_B must be less than one at that section. The beams were analyzed with T_A/T_B equal to 0.5, or only one half of the tensile force at the inner load point transmitted to the section at the support. The use of this ratio gave satisfactory agreement with Eq. (28), the loads at failure being 7 and 18 percent more than the predicted loads. If the ratio T_A/T_B had been taken larger than 0.5, the calculated ultimate moment would have been still smaller. Three beams of Series IV were provided with vertical stirrups. These beams were included in Table 41b and Fig. 22. The test moments were found to be from 11 to 28 percent larger than the calculated moments. Beams of Series IV had the largest g/d-ratio used in these tests, g/d = 4. Since the beams without web reinforcement in this series had only slightly larger shear capacities than given by Eq. (43), the addition of web reinforcement appears to have restricted the development of diagonal cracks so that the relative importance of partial bond was increased. The shear capacity of the beams was thereby increased also.

Beams of Series II had the smallest g/d ratio of all beams, g/d = 1.52. It is seen in Table 41b that the beams failed at a considerably lower load than that given by Eq. (43). Since the beams failed at about 30 percent greater loads than similar beams without web reinforcement, an increase in the amount of web reinforcement apparently did not produce a corresponding increase in shear strength. These beams appear to have failed in shear-proper and they are analyzed accordingly in section 19.

VI. BEAMS UNDER OTHER TYPE OF LOADING

18. Limitations of Shear-Compression Failures

Equation (18), the basic equation of shear strength, was derived for simple-span rectangular beams without web reinforcement and under one or two symmetrical concentrated loads. This equation considers shear failures as compression failures. Shearing stresses together with flexural tension stresses are combined in the principal tension stresses and produce a diagonal crack which extends higher than the flexural tension cracks. After this crack has formed, compression failure takes place in the reduced concrete area of the concrete. From theoretical interpretation of the basic empirical equation, it was concluded that the criterion of failure was the ultimate compressive strain in the concrete.

In deriving Eq. (18), the unknown function $F(f'_c)$ was determined empirically. All available test data, a total of 111 beams were used in the analysis. The ratio a/d which corresponds to the compressive force-shear ratio in simple-span beams, $C/V = a/jd$, varied from 1.17 to 4.80 for the beams considered. This variation did not appear to have any effect on the agreement between test results and the values predicted by Eq. (18). Within these limits, consequently, the shear strength of a beam is determined entirely by the physical properties of the beam and is not a function of either the magnitude of shear or the moment-shear ratio at the section of failure.

There is practically no experimental data for beams with larger values of a/d. As the compressive force-shear ratio increases, however, the trajectories of the principal tension stresses become more and more horizontal in the region of maximum moment. They must still intersect the neutral axis at 45 degrees, but since the shear force is relatively small, the magnitude of the principal tension stresses at that location is relatively small also. It is conceivable that the cracks are but slightly inclined and that the beam fails in flexure rather than in shear-compression.

Figure 23 shows the magnitude of shear at failure as a function of the ratio a/d for a simple-span rectangular beam under one or two symmetrical concentrated loads. The magnitude of the total shear is obtained by Eq. (18) for shear-compression failures and by Eq. (29) for flexural failure. For $a < a/d < b$ it is assumed that the beam fails in shear-compression, for $a/d > c$ that the diagonal cracks have not sufficiently developed and that the beam fails in flexure. For $b < a/d < c$, however, there seems to be transition region between shear-compression and flexural failures. The behavior of a beam in the transition region can be pictured in several ways. It is possible that at the load given by Eq. (18) the shearing stresses are too small to produce sufficiently inclined tension cracks for shear-compression failure. As the load increases, the cracks both extend higher and become more inclined. Finally the beam fails in shear-compression at a higher load than that given by Eq. (18). If a beam is heavily reinforced in tension so that a flexural failure would take place in compression rather than in tension, it is conceivable that no cracks develop before a load higher than that given by Eq. (18). With increasing load it is possible that as soon as a diagonal crack forms, the beam fails suddenly in shear-compression.

One such failure was observed in tests of Slater and Lyse (16). Beam 8B failed in what was called diagonal tension without previous warning. The two companion specimens of this beam failed in compression "with no diagonal cracks or other indication of being near failure in diagonal tension". However, these beams were moist cured until testing and failed in compression, generally without any tension cracks before failure. Moist curing has been observed to increase the modulus of rupture of a beam.

In the absence of experimental data the true relationship between the shear capacity and the a/d-ratio of a beam cannot be determined for the transition region. Since Eq. (18) will always give the lower limit of the shear strength for that region, an exact relationship is perhaps not important for beams under concentrated loads. For beams under distributed loads, however, where the magnitude of both shear and moment change from section to section, knowledge of the relationship between the shear capacity and the compressive force-shear ratio is more important.

For very low values of a/d it is not expected that a beam fails through beam-action. The mode of failure seems to change from shear-compression to what can be called shear-proper; that is, actual shearing off of the concrete. This type of failure is discussed in the following section.

19. Shear-Proper

In the range of shear-compression failures a beam fails, after the formation of diagonal cracks, in compression. However, as the ratio a/d decreases, the mode of failure seems to change. With a concentrated load close to a support, the cracks open up near the load block in the tension zone of the concrete and progress towards the other load block in the compression zone. Since the load blocks are but a short distance apart, the cracks are almost vertical. The ultimate failure seems to take place by the actual shearing off of the remaining concrete in compression.

It is rather difficult to determine what is the true criterion of failure. Cracking of concrete is produced by the principal tension stresses. As load on a beam is increased, more cracks form and the existing cracks both widen and extend higher. Consequently, less and less concrete remains effective to resist the complicated state of stress. Since the shear span is short, the magnitude of the principal tension stresses is also affected by the presence of the compressive stresses in the vicinity of the end reaction and the concentrated load. These compressive stresses will reduce the magnitude of the principal tension stresses and will make them less inclined with the axis of the beam. The closer is a load to a support, the larger is the relative importance of the local compressive stresses. Consequently, the tensile stresses are smaller and it is expected that the cracks will form and the beam will fail at a higher load than it would if the load were farther from the support.

Some quantitative information on this type of shear failure can be obtained from tests reported by Graf in Heft 80 (26). A total of 26 beams were tested, 21 small rectangular beams with the outside dimensions and loading arrangement shown in Fig. 24a and 5 large T-beams as shown in Fig. 24b. The variables for the beams included the size of the bearing block for the concentrated load, the amount of longitudinal reinforcement, the amount and angle of inclination of bent-up bars, and to a minor extent the compressive strength of concrete. In all tests the distance x between the bearing blocks, Fig. 24a, was either zero or a very small fraction of the depth of the beams.

An analysis of the test results shows that everything else remaining equal, the size of the bearing block had no effect on the ultimate load. This is despite the fact than an increase in y produced a larger moment, the load at failure being the same. It was concluded, therefore, that the ultimate load depends on the magnitude of the shear force V and the clear shear span x rather than on the a/d-ratio. It also appears that the size of the bearing area was sufficiently large in all cases to produce shear-type failures; it is conceivable that local crushing of concrete can take place under the bearing block when the bearing area is too small.

Some of the small beams were without any reinforcement. The addition of longitudinal steel increased the ultimate load. Furthermore, it appears that the use of longitudinal steel was equally effective at any depth in the beam: in the bottom half, at mid-depth, or in the top half of the beam. The use of bent-up bars was more effective than the addition of longitudinal steel. The effectiveness increased as the angle of inclination increased. Judging from the load at failure, it seems that the effectiveness of the inclined reinforcement increases in proportion to the quantity $(1 + \sin\alpha)$, at least to the largest angle of inclination used in these tests, $\alpha = 62.7$ degrees. Since the cracks were almost vertical, the use of vertical stirrups, however, did not increase the ultimate load. Thus, there seems to be a maximum value of α which limits the usefulness of the bent-up reinforcement.

The ultimate load increased as the concrete strength increased. However, the range of f'_c varied generally only from 1500 to 2000 psi with but one beam of about 3000 psi concrete strength.

The above observations suggested that the ultimate load could be expressed in terms of a nominal shearing stress in the following form:

$$v = \frac{V}{bD} = C_1 + C_2 f'_c + C_3 p_t \qquad (45)$$

where

$$p_t = \frac{A_s(1 + \sin\alpha)}{bD} \qquad (46)$$

and the quantity $A_s(1 + \sin\alpha)$ refers to the total steel area crossing section A-A, Fig. 24. When both horizontal and inclined reinforcement is used, the reinforcement ratio p_t must be evaluated for each part separately and the total value used in the calculations.

This type of equation was checked against test results. Reasonable agreement was found with the following equation:

$$v_c = 200 + 0.188 f'_c + 21{,}300 p_t \qquad (47)$$

where both the value of v and f'_c is expressed in pounds per square inch. Since plain beams were included in the analysis, the nominal unit shearing stress was determined for the gross section of the beams. For the T-beams of Heft 80 the value of v was calculated by neglecting the flange area outside the web since the load was applied at a section in the end of the beam where the flange was being tapered off to the width of the web.

The physical properties of the beams and the ratios v/v_c are shown in Table 42 and in Fig. 25 the quantity v/v_c is plotted against x/D, the ratio of the clear distance between the load blocks to the total depth of the beams. It is seen that Eq. (47) gives satisfactory agreement with the test results; only two plain concrete beams with the largest bearing area fall more than 15 percent below the predicted load and three beams are slightly more than 15 percent above. The five large T-beams agree quite well with Eq. (47).

Among previously analyzed test data there were a few beams which failed at a lower load than predicted by the shear-compression equations. Those were the beams tested by Clark (5) which had the shortest shear-span and two simple-span beams and eight restrained beams of Series II by Moody (12); all these beams had a very small a/d-ratio, and were reinforced with vertical stirrups. The beams for which strain readings were reported, failed in general before yielding of the web reinforcement. These beams are reanalyzed in terms of shear-proper in Table 43.

The nominal shearing unit stress v_c as given by Eq. (47) was computed for each beam and the ratio v/v_c is plotted against the parameter x/D in Fig. 25. Some of Clark's beams failed in tension and are not included in this comparison. Figure 25 shows that the ratio v/v_c decreases as x/D increases. Because Eq. (47) was entirely empirical by nature and the number of tests is rather limited, e.g. there are no beams in the range of x/D from 0.1 to 0.8, no attempt was made to write an expression for the relationship between v/v_c and x/D. One possibility is shown by the dashed line in Fig. 25.

Beams of Heft 80 with the load very close to the supports showed no evidence that vertical stirrups increased their shear strength. This is understandable since the location of the load forced the formation of almost vertical cracks. However, as x/D increases in the region of shear-proper, cracks follow the edges of the bearing blocks and vertical stirrups crossing the cracks produce a slight increase in the ultimate load. This is seen in Table 43 where for any value of x/D the ratio v/v_c increases somewhat as the ratio of web reinforcement increases. The beams fail, however, before the vertical web reinforcement yields. When the load is removed sufficiently far from a support, a regular shear-compression failure takes place.

The transition between shear-compression and shear-proper, point $\underline{\alpha}$ in Fig. 23, seems to depend both on the ratio x/D and the amount of web reinforcement used. All beams of Table 43 had corresponding test specimens without web reinforcement and these beams failed in shear-compression in agreement with Eq. (18). Furthermore, Clark's beams with 24-in. shear span having x/D equal to 1.14 and reinforced with vertical stirrups failed in shear-compression. Thus the transition region between the two types of failures seems to lie approximately between x/D equal to 0.8 and 1.1, increasing as the amount of vertical web reinforcement increases. The use of inclined web reinforcement, however, increases the ultimate load in shear-proper according to Eq. (47). Consequently, whenever the clear shear span $\underline{x}$ approaches the total depth of the beam, inclined web reinforcement should be used instead of vertical stirrups.

For restrained beams the distance x was considered in the same way as for simple-span beams; the clear distance between two load blocks. For Series II of Moody's restrained beams this procedure gave good results. It is seen in Fig. 25 that both simple-span and restrained beams with the same x/D-ratio failed at about the same nominal shearing stress. However, if the ratio x/D is considered as a measure of principal tension stresses and the extent of cracking, the use of x as defined above is not strictly correct since the magnitude of flexural bending stresses for simple-span beams is generally different from that for restrained beams.

20. One Unsymmetrical Load or Several Concentrated Loads

Graf tested some simple-span T-beams under one unsymmetrically placed concentrated load. These beams are reported as Series II in Heft 67 (23). Four such beams were tested; the two beams of Group 1 were reinforced with bent-up bars along the entire length of the beams, the two beams of Group 2 had bent-up bars only in the short segment whereas the long segment was reinforced with a small amount of vertical stirrups. The beams are analyzed in Table 44.

The ratio a/d was 2.09 for the short segment and 8.18 for the long segment. The last value is much larger than the range of a/d for which Eqs. (18), (35), and (26) were derived. It is likely that this ratio corresponds either to the transition region between flexural and shear failures or to the region of flexural failures, Fig. 23. This observation is verified by the test results. The two beams of Group 1 failed in tension at a load 2.18 times larger than the shear strength of the long segment as given by Eq. (26). The two beams of Group 2 failed in shear and the load at failure was up to 2.44 times larger than that given by Eq. (26) for the long segment. It is interesting to note, however, that the beams did not fail under the concentrated load at the section of maximum moment but between the load point and the end reaction in the long segment. The final break took place about 68 in. from the support for Beam 1026 and about 116 in. for Beam 1024. The magnitude of the moment at the actual section of failure was 1.03 and 1.50 times, respectively, the shear-compression moment for the beams. In both cases, it was reported that the failure was sudden. Thus it appears that the ultimate load was governed primarily by shear. Because of the long shear span, the shearing stresses were relatively small at the load which corresponded to the shear-compression moment, M_{sw} from Eqs. (35) and (28), at the section of maximum moment in the more lightly reinforced long segment. This load was less than half the ultimate load. Photographs of the beams show that at that load all cracks were practically vertical. As the load increased, the magnitude of the shearing stresses increased also and the cracks started to incline. At a certain magnitude of shear force, cracks were sufficiently inclined to lead to a shear failure. Since at that load the moment was larger than the computed ultimate shear moment over most of the beam, any random occurrence of a diagonal crack could produce a shear failure. This might be the reason as to why the two beams failed at different sections.

Beams of Series I in Heft 67 were tested under three equal and symmetrical concentrated loads. Six T-beams were tested in three groups: beams of Group 1 had bent-up bars along the entire length of the beam, Groups 2 and 3 only between the end supports and the first load. All tension reinforcement was carried through the two middle segments of Group 2 whereas in Group 3 some of the bars were cut off beyond the moment requirement and hooked in the tension zone of the concrete. The beams are analyzed in Table 45.

The quantity a/d has been used as a convenient expression for the compressive force-shear ratio, C/V, of simple-span beams. An equivalent expression is given by a/d = M/Vd for other types of loading. This ratio is 8.52 for the beams of Series I, thus only a little greater than that for Series II. As a consequence, these beams failed in a manner similar to those of Series II. Beams of Group 1 failed in tension, beams of Group 2 in tension with a shear-type final collapse, and those of Group 3 in shear at the section of the center load before yielding of the tension reinforcement. The failure of the last group of beams appears to have been hastened by diagonal cracks which were initiated at the hooks on the cut-off tension bars. The ratios of the ultimate loads to the loads given by Eq. (26) are comparable to those of Series II since the M/Vd-ratios are nearly the same in both cases.

From the results of these tests it is evident that for high values of a/d = M/Vd a beam may fail either in shear at a greater load than that given by Eq. (28) or in flexure before developing any marked diagonal cracking. It appears that the strength and the behavior of a beam in this region of M/Vd is governed by both the actual value of M/Vd and the amount of web reinforcement provided. The parameter M/Vd expresses the ratio between the compressive force and the corresponding shear force at the section of maximum moment. It was seen before that in the shear-compression region of M/Vd, from a to b in Fig. 23, the strength of a beam in shear did not depend on either the ratio M/Vd or the magnitude of the shear force V at failure. The shear strength was determined by a limiting moment, M_s for beams without web reinforcement and M_{sw} for beams with web reinforcement. This limiting moment was reached by different combinations of V and a and the contribution of the shearing stresses was always large enough to permit sufficient diagonal cracking which was a prerequisite for this type of failure. However, as the ratio M/Vd increases beyond the limit of shear-compression, the shear stresses apparently become too small in proportion to the flexural stresses in the concrete to produce sufficient diagonal cracking. Consequently, the cracks are but little inclined at the load which corresponds to the shear moment M_{sw} at the section of maximum moment and the beam cannot fail in shear at that load. In order to produce a shear failure, the shearing stresses must be large enough to develop full diagonal cracking and the applied moment at the section of failure must be equal to or larger than the shear-compression moment M_{sw}. It is possible that the values of moment and shear satisfy both these conditions at some critical value of M/Vd, at a section other than that of the maximum moment. This implies that as soon as the shear-compression moment M_{sw} is reached at the section of the critical value of M/Vd, the contribution of the shearing stresses is sufficient for the development of full diagonal cracking. However, since the applied moment is larger than M_{sw} anywhere between the section of the critical M/Vd and the section of the maximum moment, the location of failure may be anywhere between these two sections, depending on the occurrence of the main diagonal crack.

Consequently, only the load at failure is controlled by the shear moment at the section of the critical M/Vd whereas the location of failure may be different in different beams.

If the above hypothesis about the behavior of beams in the region of high values of M/Vd is true, both the mode of failure and the ultimate load can be predicted in advance. This would involve only the calculation of both the flexural capacity, M_f from Eq. (29), and the shear-compression capacity, M_{sw} from Eq. (28). Both these quantities are determined solely by the physical properties of the beam. The applied moment at the section of maximum moment is compared with M_f and that at the section of the critical M/Vd with M_{sw}. The mode of failure and the ultimate load is determined by whichever theoretical moment capacity is reached first.

The results of the above tests give the critical value of M/Vd equal to 3.4-3.7. However, the tests are too limited both in number and in scope to check the validity of the above hypothesis. Furthermore, only T-beams made of rather low concrete strength, about 1000 psi, were tested. This combination leads to very high $P_f/P_s = M_f/M_s$ ratios, up to 2.79 as noticed in Tables 44 and 45. For rectangular beams without web reinforcement the flexural capacity rarely exceeds that in shear by more than 50-60 percent. This difference between the two types of beams could also influence the mode of failure which renders it impossible to draw any definite conclusions from these few test results.

21. Beams Under Uniform Load

It was seen previously that within certain limits of $a/d = M/Vd$ the shear strength of a beam under concentrated loads could be determined by Eqs. (18) and (28) for rectangular beams and by Eqs. (35) and (28) for T-beams. Under this type of loading, the beams tested failed at the section of maximum moment and maximum shear and the load at failure was determined by the magnitude of moment. As the value of M/Vd increased beyond these limits, however, the actual shear strength was found to be larger than that given by the above equations. Furthermore, the location of failure was not necessarily the section of maximum moment. The upper limit of M/Vd for the applicability of shear-compression equations and the shear strength of a beam in the transition region between shear and flexural failures could not be determined quantitatively because of insufficient experimental data for beams with high values of M/Vd.

For simple-span beams under uniform load the value of M/Vd ranges from zero at the section of no moment to infinity at the section of maximum moment. The beam cannot fail in shear at the section of maximum moment because there are no diagonal cracks at that section. Consequently, if a shear failure is to take place, it must occur at a section where the value of M/Vd is such as to permit diagonal cracking and the moment itself is sufficient to produce a shear-compression failure. In what follows, the available test data is analyzed in an attempt to find more quantitative information about the shear strength of beams under uniform loading.

No tests could be found of beams under actual uniform load. However, there are reports on tests where uniform loading was simulated by a large number of equal and equally spaced concentrated loads. These beams were tested by Bach and Graf in two series, one series under 16 equal loads as reported in Heft 48 (27), and the other series under 8 equal loads as reported in Heft 20 (28).

Beams of Heft 48 were five simple-span T-beams loaded with sixteen equal concentrated loads. The arrangement of loads and reinforcement is shown in Figs. 26 through 29, and Table 46 gives the physical properties and test results for these beams. Beam 1024 had no web reinforcement. It failed at a very low load, the maximum moment at midspan being only 68 percent of the shear-compression moment as given by Eq. (35). A diagonal crack formed at about the third-point of the span shortly before failure. Numerous longitudinal cracks ran from that crack towards the end support. It appears that this beam failed in bond. Beams 1026, 1025, and 1031 had almost identical arrangements of bent-up bars except that the size of the bars was different, the area varying as 1.00:0.53:0.36. Beams 1026 and 1025 failed in tension and Beam 1031 in shear. However, Beam 1025 was rather close to its shear strength at failure as indicated by marked diagonal cracking all along the beam. Beam 1032 had only two bent-up bars in the ends and failed in shear.

Figures 26 through 29 show the arrangement of loads and reinforcement and the main cracks at failure. Furthermore, the actual ratio M/M_s at failure, where M_s was computed by Eq. (35), and the corresponding predicted ratio, $1 + 2rf_{yw}/10^3$ from Eq. (28) are plotted along the beam for each individual beam. The shear-compression moment was calculated for the section at midspan, the reduction of the longitudinal steel area through bending up bars at other sections was not taken into consideration. The variation in $1 + 2rf_{yw}/10^3$ was calculated using values of r at mid-height of the beams. If the relationship between the actual and the predicted moment ratios is observed in these figures, it is seen that the beams failed in shear only when the ratio M/M_s approached the quantity $1 + 2rf_{yw}/10^3$ at about the fifth load point from the end of the beam. It is recalled that a beam under concentrated loads and in the shear-compression region of M/Vd would have failed in shear as soon as the value of M/M_s had exceeded that of $1 + 2rf_{yw}/10^3$ at the section of maximum moment, Eq. (28). This difference between the two types of beams suggests that it might be possible to determine empirically the value of M/Vd which limits the region of critical diagonal cracking capable of producing shear-compression failures.

Figure 30 shows the ratio between the actual moment at failure and the ultimate shear-compression moment of Eq. (28) plotted along the beams. The values of M/Vd at each side of the load points are also marked in the figure. This figure shows the effect of the M/Vd-ratio more clearly. Beam 1026 which failed in tension has the ratio M_{test}/M_{sw} less than one at the fifth load point. Beam 1025 which failed in tension while being very close to a shear failure, has the ratio just above one. Beam 1031 which failed in shear seems to have failed just as the ratio exceeded one. The ultimate flexural capacity of this beam is shown in the figure also. It is seen that this load, if reached, would have increased the ratio to considerably higher than one. Finally, Beam 1032 which failed in shear has the ratio somewhat more than one, 1.16. However, Fig. 29 shows that in the case of this beam there is some doubt in what to consider as the value of $1 + 2rf_{yw}/10^3$ at the fifth load. The bent-up bars do not cover that particular section; their presence in the vicinity undoubtedly offers some resistance to the formation of diagonal cracks. This, in a sense, would mean an increase in the value of rf_{yw} which would bring the ratio closer to one in Fig. 30.

Thus, it appears that the shear-compression equations are applicable for the beams under consideration. However, the section at which the shear moment is calculated is not at the maximum moment but at the value of M/Vd equal to about 4.5 which corresponds to the fifth load point of the beams of Heft 48.

Heft 20 reports tests on 51 simple-span T-beams, tested in groups of three companion specimens. Sixteen groups of beams were loaded with eight equal concentrated loads as shown in Figs. 31 and 32, one additional group had four loads omitted on one half of the span. The physical properties of the beams and the test results are given in Table 47.

The first four groups of beams were reinforced with two 1.57-in. plain round bars. The test variables included the effect of anchoring of the longitudinal bars, either straight or hooked, and the effect of web reinforcement which was vertical stirrups placed in accordance with the shear diagram along the entire length of beam. All these beams failed in bond as indicated by excessive end slip of the longitudinal bars which was measured in most beams. Bond failure led to longitudinal cracking along the reinforcing bars and to a final opening of a diagonal crack, generally between the first and the second load points.

Groups 55 and 56 were reinforced with four 1.10-in. plain round bars, two of which were bent up at 13 degrees. The ends of the bars were anchored with small 90-degree hooks. Beams of Group 55 had no additional web reinforcement and failed in bond, by excessive slipping of the bars. Beams of Group 56 had additional vertical stirrups placed according to the shear diagram and failed in tension.

The remaining beams were reinforced with 6 or 7 round bars of different sizes. Two bars were carried straight to the supports; the rest of the bars were bent up at different locations. The middle portion of the beams, not covered with bent-up bars, was reinforced with vertical stirrups. The beams were tested in companion groups; in one the two straight bars were left unhooked, in the other they were hooked. All bent-up bars were sufficiently hooked in all beams. All beams with the straight bars not hooked failed in bond by excessive end slip. This led to the opening of a diagonal crack at different locations in different beams. All beams with hooked straight bars failed in tension with a secondary crushing of the concrete at midspan.

Thus, no beams failed actually in shear. Some indication of the shear strength of the beams can be obtained, however, by analyzing the beams which had the smallest amount of bent-up bars. Figures 31 and 32 show beams of Groups 60 and 62 in this category. The arrangement of web reinforcement is shown together with the quantity $1 + 2rf_{yw}/10^3$ and the ratio M/M_s along the beams. The compressive force-shear conditions are represented by the ratio of M/Vd, given at both sides of each load point. It is seen that the M/M_s-curve intersects the web reinforcement curve near the third load point, at about M/Vd equal to 5. Since the beams failed in tension, the amount of web reinforcement was sufficient to prevent a failure in shear. Consequently, the critical value of M/Vd for shear failures must be less than 5, which agrees with the previous finding of about 4.5 for beams of Heft 48.

From the results of the above two series of tests it appears that the shear strength of beams under uniform load can be represented by the shear-compression equations (18) and (28) for rectangular beams and by Eqs. (35) and (28) for T-beams. Since there are no diagonal cracks in the region of maximum moment and the inclination of cracks is very small for high values of M/Vd, the beam cannot fail in shear unless the bending moment is higher than the shear strength as given by Eq. (28)

at a critical value of M/Vd. From the above results, the critical value of M/Vd is set tentatively at about 4.5. It is recalled, however, that the tests were far from being conclusive and that only simple-span T-beams were tested. The validity of the above concept of shear failures of beams under uniform load and a more reliable value of the critical M/Vd must be established by a more comprehensive test program.

It appears, however, that the conventional method of reinforcing simple-span beams under uniform load against shear-type failures is incorrect. Web reinforcement is placed to conform with the shear diagram. This means that the amount of web reinforcement at the critical region of M/Vd is smaller than that for lower values of M/Vd. The above findings suggest, however, that the web reinforcement should be placed at a uniform spacing between the end reaction and the critical region of M Vd, say 4.5. Only beyond that region it could be tapered off and reduced to zero at midspan. If it is desired to prevent shear failures altogether, the ultimate flexural and shear moments must be calculated from the properties of the beam by Eqs. (29) and (18) or (35). Then the ratio between the ultimate flexural moment at the section of the critical value of M/Vd and the shear moment of Eq. (18) or (35) must be substituted into Eq. (28) in order to find the necessary amount of web reinforcement which would force the beam to fail in tension at the section of the maximum moment rather than in shear at the section of the critical M/Vd.

VII. SUMMARY AND CONCLUSIONS

22. General Summary and Discussion

A general expression for the shear strength of reinforced concrete beams was derived by considering simple-span beams without web reinforcement. It was first assumed that the total shear force was resisted solely by the compression area of the concrete and that the criterion of failure was an ultimate shearing unit stress, related to the compressive strength of the concrete. These assumptions yielded an expression in a form which suggested that the real criterion for shear failures was a limiting moment rather than an ultimate shearing stress. This observation was supported by some test results reported in the literature (5,6). It was concluded that shear failures were actually a compression phenomenon. Shear-compression failures differ from flexural compression failures only because the compressive area of the concrete is reduced as the result of diagonal cracking. Diagonal cracks have been observed to extend higher than the flexural tension cracks at failure.

Treating shear failures as compression failures and assuming that the depth of the compression zone was related to k as determined by the elastic "straight line" theory, the following empirical equation was found to represent with good accuracy the shear strength of rectangular simple-span beams without web reinforcement and under one or two symmetrical concentrated loads:

$$\frac{M_s}{bd^2 f'_c} = (k + np') \left(0.57 - \frac{4.5 f'_c}{10^5}\right) \qquad (18)$$

where the value of k is determined for beams reinforced in tension only by

$$k = \sqrt{(pn)^2 + 2pn} - pn \tag{14}$$

and for beams reinforced both in tension and in compression by

$$k = \sqrt{\left[n(p + p')\right]^2 + 2n(p + p' - p't)} - n(p + p') \tag{15}$$

and where the modular ratio n is determined by

$$n = 5 + \frac{10,000}{f'_c} \tag{16}$$

Equation (18) was based on the test results from 15 different investigations involving 111 beams which failed in shear. These beams were tested over a period of 43 years and had a wide variation in their physical properties as summarized in Table 1. The average ratio of measured to computed moments was 0.993 and the standard deviation 0.120. The agreement between the measured and computed moments is also shown graphically in Fig. 1a.

Equation (18) was interpreted theoretically in the light of the conventional theory of compression failures of reinforced concrete beams. From previous test results at the University of Illinois (11,14) the value of k_1k_3 was approximated as follows:

$$k_1k_3 = 2.4(0.57 - \frac{4.5f'_c}{10^5}) \tag{20}$$

The use of this equation permitted the establishment of a relationship between k_s and k, where k_s refers to the depth of the compression zone at shear failures:

$$k_s = \frac{k}{2.4(1 - k_2k_s)} \tag{22}$$

Since k remains usually within the values of 0.2 and 0.5, Eq. (22) shows that k_s is practically a constant fraction of k. This finding explains why the previous attempt to use the value of k as a measure of k_s gave satisfactory agreement with test results and implies that the failure criterion is still a limiting compressive strain in the concrete. Equation (22) is based on the assumption that the value of $k_1 k_3$ is the same for both flexural and shear failures. If there should be any difference between the two, it is still likely that k_s remains practically proportional to k, although it might be a smaller fraction of k than Eq. (22) indicates.

The effect of web reinforcement was investigated next. It was found that the use of web reinforcement increased the shear strength of a beam more than would be accounted for by the internal forces in the stirrups. The total contribution of web reinforcement was determined empirically; the following equations were found to give good correlation with test results:

$$P_{sw}/P_s = 1 + \frac{2rf_{yw}}{10^3} \tag{26}$$

or

$$M_{sw}/M_s = 1 + \frac{2rf_{yw}}{10^3} \tag{28}$$

where M_{sw} is the shear-compression moment of a beam with web reinforcement, M_s that of the same beam without web reinforcement, Eq. (18), P_{sw} and P_s are the loads corresponding to M_{sw} and M_s, respectively, and r is given by

$$r = \frac{A_w}{bs \sin\alpha} \tag{27}$$

These equations were based on the test results for 80 beams. The average ratio between the measured and calculated moments was 1.012 and the standard deviation 0.085. The range of the physical properties of the beams is summarized in Table 16 and the ratios of P/P_s are shown graphically in Fig. 4. The equations were further checked by the help of beams which had failed in flexure. It is seen in Fig. 5 that although the flexural capacity of these beams was reached at different ratios of P/P_s, they always failed at a load lower than their strength in shear, given by Eq. (26).

Equations (26) and (28) were found to be applicable for all angles of inclination and for different values of yield strength of web reinforcement. It was also found that there was no noticeable difference between the effectiveness of bent-up bars and stirrups serving as web reinforcement.

Equations (26) and (28) show that a given amount of web reinforcement will increase the shear strength of a beam in proportion to its strength without web reinforcement rather than by an amount determined solely by the physical properties of the web reinforcement. It appears that by resisting the extension and widening of diagonal cracks, the presence of web reinforcement increases the compressive area of the concrete and, conceivably, restricts the concentration of the compressive strain of concrete in the region of the main diagonal crack.

The relationship between shear-compression and flexural failures was discussed in Section 13. It was found that the amount of web reinforcement necessary to prevent shear failures could be determined for any beam by Eqs. (29), (18), and (28). Rectangular beams reinforced in tension only and designed according to the present ACI Code balanced design requirements were found to require about 0.35 percent web reinforcement to ensure tension failures. This assumed that the yield strength of the tension reinforcement was 50,000 psi and that of web reinforcement 40,000 psi and that the beams were loaded under one or two symmetrical concentrated loads.

Since the moment-rotation relationship of a T-beam differs from that of a rectangular beam, Eq. (18) must be modified to apply for T-beams. This was done by the use of a semi-rational shape-factor in the following form:

$$F_t = \frac{I_T + I_{cr}}{I_R + I_{cr}} \tag{34}$$

Substituting the compressive area A_c of a T-section as determined by the "straight-line" theory for bkd and using the shape factor of Eq. (24), Eq. (18) was rewritten as:

$$\frac{M_s}{A_c d f'_c F_t} = 0.57 - \frac{4.5 f'_c}{10^5} \tag{35}$$

As seen in Fig. 10, Eq. (35) was found to give satisfactory agreement with test results when beams with abnormally large values of d/e and b/b' were excluded. These beams had a lower shear strength because the effective width of their flanges was reduced. However, no attempt was made to determine an expression for the effective flange width. Furthermore, it was found that the use of transverse reinforcement in the flange effectively counteracted the reduction in the effective width and thereby increased the scope of Eq. (35).

The shear strength of simple-span T-beams with web reinforcement could be determined by the same expression as that for rectangular beams:

$$M_{sw}/M_s = 1 + \frac{2rf_{yw}}{10^3} \tag{28}$$

where M_s is determined by Eq. (35) and r from the following equation:

$$r = \frac{A_w}{b's \sin\alpha} \tag{27a}$$

As seen in Fig. 11, the agreement between the measured and calculated quantities is satisfactory.

Simple-span beams under one or two symmetrical concentrated loads develop just one main diagonal crack under an applied load and fail at that section. In restrained beams, shear and moment conditions are such as to permit the formation of several diagonal cracks. The beam may fail at any of these cracks, depending on the magnitudes of moment and shear and the arrangement of both longitudinal and web reinforcement.

It was found that whenever the possibility of bond failures was precluded, the shear strength of a restrained beam was determined by the same equations as that of a simple-span beam, Eqs. (18) and (28). The critical section was the section of maximum moment.

When the longitudinal reinforcement was cut off at some section, a sudden and complete bond failure was possible by stripping out of the cut-off reinforcement. This type of failure was outside the scope of this investigation and was not examined in more detail. Evidently, this is a question of bond characteristics of the reinforcing bars and the length of embedment from a diagonal crack to the end of the bar.

Restrained beams with continuous top and bottom reinforcement may have another mode of failure. Under certain conditions, e.g., when the distance between a support and a load is short relative to the effective depth of the beam, a local bond failure may take place in the high bond-stress region between the sections of positive and negative moments. As a result of local destruction of bond, both the top and bottom longitudinal reinforcement is in tension at a certain section. This redistribution of the internal forces results in a reduced shear strength of the beam. Assuming that the whole tensile force was transmitted from one section to the adjacent section and that k_s was given by Eq. (22), the following equation was derived to represent the shear strength for this type of failure:

$$\frac{M_s}{bd^2 f_c' kA} = 0.57 - \frac{4.5\ f_c'}{10^5} \qquad (43)$$

where

$$A = 1 - \frac{t}{\left(\frac{M_B}{M_A} + 1\right)\left(1 - k_2 k_s\right)} \qquad (44)$$

$$k = \sqrt{(p_o n)^2 + 2p_o n} - p_o n \tag{41}$$

$$p_o = p(1 + \frac{M_A}{M_B}) \tag{42}$$

$$k_s = 1.11 - \sqrt{1.23 - 0.926k}, \text{ and} \tag{23}$$

$$k_2 = 0.45$$

In these equations the subscript B refers to the section under consideration and the subscript A to the adjacent section from which the tensile force T_A was transmitted to section B. The equations were derived for the assumption that the following relationship was valid:

$$T_A/T_B = M_A/M_B \tag{38}$$

If this expression is not correct, e.g., because of partial bond, and the actual ratio T_A/T_B can be determined, this actual ratio should be substituted for M_A/M_B in Eqs. (42) and (44).

The validity of Eq. (43) was checked against test results and satisfactory agreement was obtained. Figure 21a shows the measured and calculated moments graphically for all beams which failed after a local bond failure. Most of the test specimens show good agreement with Eq. (43); for some beams a small increase in the shear strength was noticed because of the effect of partial bond. This was discussed in more detail in section 17b.

All beams shown in Fig. 21a had equal positive and negative moments and developed, in general, two main diagonal cracks before failure. This resulted in a full redistribution of the internal forces and the shear strength of the beams was governed by Eq. (43). For unequal positive and negative moments, however, either one or two cracks may be present at failure. Two cracks will produce, in general, a full redistribution of the internal forces and the shear strength of a beam will be given by Eq. (43) at the section of maximum moment. One crack will lead to a partial redistribution of the internal forces so that the shear strength will be governed by Eq. (18) at the section of maximum moment. Beams of Series VI by Moody had unequal moments at sections A and B and failed at section A after developing only one crack in span g (Fig. 12). The beams were analyzed according to Eq. (18) at that section and Fig. 21b shows that good agreement was obtained between the measured and the calculated moments.

From the available test data, it was not possible to determine the limits of Eq. (43). The largest g/d ratio for which test results were available was 4.0. Since this ratio permitted a redistribution of the internal forces, the limiting g/d ratio must be larger than four. Furthermore, it is apparent that bond characteristics of the reinforcing bars have an effect on the limiting value of g/d. The above results were reported for beams reinforced with modern deformed bars, plain bars undoubtedly are more susceptible to local bond failures. Likewise, it was not possible to determine the conditions under which two cracks and, consequently, a full redistribution of the internal forces will occur for unequal positive and negative moments. Until such criteria can be established, the more conservative condition of full redistribution should be assumed in determining the shear strength of a restrained beam.

It was found that the contribution of web reinforcement could be determined in restrained beams, as in simple-span rectangular and T-beams, by Eq. (28). Beams reinforced with 45-degree stirrups gave very good agreement with Eq. (28); beams provided with vertical stirrups also agreed with this equation except for two beams with the largest values of rf_{yw}. It appears that in beams with relatively short shear span inclined stirrups are, in general, more reliable than vertical stirrups. It is conceivable that inclined stirrups have better anchorage conditions whenever diagonal cracks are forced to form in a restricted space and thereby can develop their full effectiveness. Conversely, the anchorage of vertical stirrups might be destroyed before their full effectiveness is reached.

All the above shear-compression equations were derived and checked for beams for which the a/d-ratio varied between 1.17 and 4.80. The a/d-ratio represents the compressive force-shear ratio for simple-span beams under one or two concentrated loads; for any other type of loading this ratio can be represented by the equivalent ratio M/Vd. Within these limits of M/Vd, the shear strength of a beam was found to be determined solely by the physical properties of the beam. It was not a function of either the magnitude of shear or the moment-shear ratio at failure.

However, as the M/Vd-ratio increases, the relative importance of shear in connection with the diagonal tension stresses decreases. Consequently, the extent of diagonal cracking is less pronounced and it was found that the shear strength of such beams was larger than that given by Eq. (28). It was also noticed that the location of shear failure was not necessarily the section of maximum moment. For sufficiently large values of M/Vd the beams failed in flexure rather than in shear. However, the upper limit of M/Vd for the applicability of shear-compression equations and the shear strength of a beam in the transition region between shear and flexural failures could not be determined quantitatively because of insufficient experimental data.

Conversely, for very small values of a/d the beams did not fail in shear-compression. The mode of failure appeared to be an actual shearing off of the compression zone of the concrete. This type of failure was tentatively called shear-proper. It was also found that the shear strength of such beams depended on the x/D-ratio rather than a/d-ratio, where x denotes the clear distance between the load-bearing blocks and D the total depth of beam. For x/D equal to zero, the shear strength of a beam could be related to a nominal shearing stress. The following entirely empirical equation was found to give satisfactory agreement with test results:

$$v_c = \frac{V}{bD} = 200 + 0.188\, f_c^{*} + 21{,}300\, p_t \qquad (47)$$

where

$$p_t = \frac{A_s(1 + \sin\alpha)}{bD} \qquad (46)$$

As the ratio x/D increased, the ratio between the test and calculated shearing stresses decreased as seen in Fig. 25. Since the number of tests was limited, no expression could be determined for the relationship between v/v_c and x/D.

For small values of x/D the location of the load-bearing blocks forced the formation of almost vertical cracks and, consequently, vertical stirrups were not found to contribute to the shear strength of the beam. Thus there is a limiting value of $\underline{\alpha}$ for which Eq. (46) is applicable. However, as x/D increased in the region of shear-proper, cracks followed the edges of the bearing blocks and vertical stirrups crossing the crack produced a slight increase in the shear strength. The transition region between shear-proper and shear-compression was estimated to lie approximately between x/D equal to 0.8 and 1.1, increasing as the amount of vertical stirrups increased. Since the contribution of vertical stirrups is very small, inclined stirrups should be used whenever the x/D ratio approaches unity.

For simple-span beams under uniform loading, the value of M/Vd ranges from zero at the section of no moment to infinity at the section of maximum moment. It is believed that with certain modifications the shear-compression equations (18), (35), and (28) could be used to determine the shear strength of such beams. Since there are no diagonal cracks in the region of maximum moment and the inclination of cracks is very small for high values of M/Vd, the beam cannot fail in shear unless the bending moment is higher than the shear strength given by Eq. (28) at a critical value of M/Vd. From test results studied, this critical value of M/Vd was set tentatively at about 4.5. However, since only a few T-beams have been tested under conditions which simulated uniform loading, the validity of the above concept of shear failures and a more reliable value of the critical M/Vd must be established by a more comprehensive test program.

It is conceivable that the same procedure can be used for any type of beam under either uniform or distributed loading to determine its strength in shear. It involves only the determination of critical sections for shear failures. Provided that the value of M/Vd is in the region of shear-compression, Eqs. (18) and (28) can be used directly at sections where maximum shear and maximum moment coincide. In regions of maximum moment and no shear the critical section at which the shear-compression equations should be used is given by the critical value of M/Vd.

Since shear-type failures result in a sudden and complete destruction of a structure, they should be avoided in actual construction. In order to determine the amount of web reinforcement necessary to ensure flexural failures, the flexural capacity of the beam should be determined first and the corresponding loading considered as applied loading. Then, both the applied moment and shear moment of Eq. (18) or (35) should be determined for critical sections of shear failure. The ratio between the two substituted for M_{sw}/M_s in Eq. (28) will determine the amount of web reinforcement required.

One additional problem is confronted in statically indeterminate structures whenever redistribution of moments near the ultimate load is considered. In order to utilize the full load-carrying capacity of the structure, its members must be so designed as to permit sufficient rotation at the plastic hinges. Consequently, not only primary but also secondary shear failures after yielding of the reinforcement must be prevented. This is a phase of the phenomenon of shear in reinforced concrete which has received very little attention in the past.

23. Conclusions

Shear-Proper. For $x/D = 0$ the shear strength of a beam is determined by the following expression:

$$v_c = \frac{V}{bD} = 200 + 0.188\, f'_c + 21{,}300\, p_t \qquad (47)$$

where

$$p_t = \frac{A_s(1 + \sin\alpha)}{bD} \qquad (46)$$

as x/D increases, the ratio v/v_c decreases. The relationship between x/D and v/v_c could not be determined although some information is available from Fig. 25. The transition region between shear-proper and shear-compression was estimated to lie between x/D equal to 0.8 and 1.1, depending on the amount of vertical stirrups. Otherwise, the effect of vertical web reinforcement is neglected in Eq. (46).

Shear-Compression. In the shear-compression range the shear strength of a beam without web reinforcement and under concentrated loads is given by the following equations for the maximum shear moment, M_s:

For rectangular beams:

$$\frac{M_s}{bd^2 f'_c\,(k + np')} = 0.57 - \frac{4.5 f'_c}{10^5} \qquad (18)$$

where $\underline{k}$ is given for beams reinforced in tension only by

$$k = \sqrt{(pn)^2 + 2pn} - pn \qquad (14)$$

and for beams reinforced in both tension and compression by

$$k = \sqrt{\left[n(p + p')\right]^2 + 2n(p + p' - p't)} - n(p + p') \qquad (15)$$

and where n is given by

$$n = 5 + \frac{10,000}{f'_c} \qquad (16)$$

For T-beams:

$$\frac{M_s}{A_c d f'_c F_t} = 0.57 - \frac{4.5\, f'_c}{10^5} \qquad (35)$$

where

$$F_t = \frac{I_T + I_{cr}}{I_R + I_{cr}} \qquad (34)$$

For restrained beams: the shear strength is given by Eq. (18), whenever bond failures are prevented, and by the following equation whenever redistribution of internal forces has taken place as a result of local bond failure in the high bond-stress region:

$$\frac{M_s}{bd^2 f'_c kA} = 0.57 - \frac{4.5\, f'_c}{10^5} \qquad (43)$$

where

$$A = 1 - \frac{t}{\left(\frac{M_B}{M_A} + 1\right)(1 - k_2 k_s)} \qquad (44)$$

$$k = \sqrt{(p_o n)^2 + 2p_o n} - p_o n \qquad (41)$$

$$p_o = p\left(1 + \frac{M_A}{M_B}\right) \tag{42}$$

$$k_s = 1.11 - \sqrt{1.23 - 0.926k} \tag{23}$$

$$k_2 = 0.45$$

The contribution of web reinforcement is determined in all cases by the following expression for the ratio of the maximum moment capacity M_{sw} of the beam with web reinforcement to the moment capacity M_s of the same beam without web reinforcement:

$$\frac{M_{sw}}{M_s} = 1 + \frac{2rf_{yw}}{10^3} \tag{28}$$

where

$$r = \frac{A_w}{bs\ \sin\alpha} \quad \text{for rectangular beams} \tag{27}$$

and

$$r = \frac{A_w}{b's\ \sin\alpha} \quad \text{for T-beams} \tag{27a}$$

The upper limit of $a/d = M/Vd$ for shear-compression failures could not be determined; the highest value used in tests was 4.8. For high values of M/Vd the shear strength is larger than that given by the above equations.

Distributed Loading. At a section where maximum moment and maximum shear coincide, the shear strength of a beam under distributed loading can be determined directly by the above shear-compression equations, provided that the value of M/Vd is in the range of applicability of these equations. However, in regions of maximum moment and no shear, the above equations should be used at a section given by M/Vd equal to about 4.5.

VIII. BIBLIOGRAPHY

1. Laupa, A., Siess, C. P., and Newmark, N. M., "The Shear Strength of Simple-Span Reinforced Concrete Beams Without Web Reinforcement", Civil Engineering Studies, Structural Research Series No. 52, University of Illinois, April 1953.

2. Richart, F. E., "An Investigation of Web Stresses in Reinforced Concrete Beams", Bul. No. 166, Eng. Exp. Station, University of Illinois, 1927.

3. Talbot, A. N., "Tests of Reinforced Concrete Beams: Resistance to Web Stresses", Bul. No. 29, Eng. Exp. Station, University of Illinois, 1909.

4. Moretto, O., "An Investigation of the Strength of Welded Stirrups in Reinforced Concrete Beams", M. S. Thesis, University of Illinois, 1944; also in ACI Journal, November 1945, Proc. Vol. 42, pp. 141-162.

5. Clark, A. P., "Diagonal Tension in Reinforced Concrete Beams", ACI Journal, October 1951, Proc. Vol. 48, pp. 145-156.

6. Turneaure, F. E. and Maurer, E. R., "Principles of Reinforced Concrete Construction", John Wiley and Sons, New York, 3rd Ed., 1919, p. 145.

7. Jensen, V. P., "Ultimate Strength of Reinforced Concrete Beams as Related to the Plasticity Ratio of Concrete", Bul. No. 345, Eng. Exp. Station, University of Illinois, 1943.

8. Richart, F. E. and Jensen, V. P., "Tests of Plain and Reinforced Concrete Made With Haydite Aggregates", Bul. No. 237, Eng. Exp. Station, University of Illinois, 1931.

9. Thompson, J. T., Hubbard, T. F., and Fehrer, J. N., "Concrete Beams With Sheet-Steel Web Plates", Civil Engineering, Vol. 8, No. 12, Dec. 1938, pp. 815-818.

10. Galletly, G. D., Hosking, N. G., and Ofjord, A., "Behavior of Structural Elements Under Impulsive Loads III", Department of Civil and Sanitary Engineering, Massachusetts Institute of Technology, July 1951.

11. Gaston, J. R., Siess, C. P., and Newmark, N. M., "An Investigation of the Load-Deformation Characteristics of Reinforced Concrete Beams Up to the Point of Failure", Civil Engineering Studies, Structural Research Series No. 40, University of Illinois, December 1952.

VIII. BIBLIOGRAPHY (CONT'D)

12. Moody, K. G., "An Investigation of Reinforced Concrete Beams Failing in Shear", Ph.D. Thesis, University of Illinois, 1953.

13. Mylrea, T. D., "Bond and Anchorage", ACI Journal, March 1948, Proc. Vol. 19, p. 521.

14. Billet, D. F., "Study of Prestressed Concrete Beams Failing in Flexure", Civil Engineering Studies, Structural Research Series No. 54, University of Illinois, June 1953.

15. Slater, W. A., Lord, A. R., and Zipprodt, R. R., "Shear Tests of Reinforced Concrete Beams", Technologic Papers of the Bureau of Standards, No. 314, Department of Commerce, 1926.

16. Slater, W. A. and Lyse, I., "Compressive Strength of Concrete in Flexure as Determined from Tests of Reinforced Beams", ACI Journal, June 1930, Proc. Vol. 26, pp. 831-74.

17. Johnston, B. and Cox, K. C., "High Yield-Point Steel as Tension Reinforcement in Beams", ACI Journal, Sept. 1939, Proc. Vol. 36, pp. 65-80.

18. Bach, C. and Graf. O., "Versuche mit Eisenbeton-Balken zur Ermittlung der Widerstandsfähigkeit verschiedener Bewehrung gegen Schubkräfte. Erster Teil", Deutscher Ausschuss für Eisenbeton, Heft 10, Berlin, 1911.

19. Braune, G. M. and Myers, C. C., "Tests on Ten Reinforced Concrete T-Beams", Concrete, April 1917, Vol. 10, No. 4, pp. 163-65.

20. Thompson, J. N. and Ferguson, P. M., "Shear Resistance of Tile-Concrete Floor Joists", ACI Journal, November 1950, Proc. Vol. 47, pp. 229-36.

21. Ferguson, P. M. and Thompson, J. N., "Diagonal Tension in T-Beams Without Stirrups", ACI Journal, March 1953, Proc. Vol. 49, pp. 665-76.

22. Bach, C. and Graf, O., "Versuche mit Eisenbeton-Balken zur Ermittlung der Widerstandsfähigkeit verschiedener Bewehrung gegen Schubkräfte. Zweiter Teil", Deutscher Ausschuss für Eisenbeton, Heft 12, Berlin, 1911.

23. Graf, O., "Versuche mit Eisenbeton balken zur Ermittlung der Widerstandsfähigkeit verschiedener Bewehrung gegen Schubkräfte. Sechster Teil," Deutsher Ausschuss für Eisenbeton, Heft 67, Berlin, 1931.

24. Timoshenko, S. and Goodier, J. N., "Theory of Elasticity", 2nd ed., 1951, McGraw-Hill Book Company, Inc., pp. 171-77.

VIII. BIBLIOGRAPHY (CONT'D)

25. Richart, F. E. and Larson, L. J., "An Investigation of Web Stresses in Reinforced Concrete Beams, Part II, Restrained Beams", Bul. No. 175, Eng. Exp. Station, University of Illinois. 1928.

26. Graf, O., "Versuche über die Widerstandsfähigkeit von Eisenbetonbalken gegen Abscheren", Deutscher Ausschuss für Eisenbeton, Heft 80, Berlin, 1935.

27. Bach, C. and Graf, O., "Versuche mit Eisenbetonbalken zur Ermittlung der Widerstandsfähigkeit verschiedener Bewehrung gegen Schubkräfte, Vierter Teil", Deutscher Ausschuss für Eisenbeton, Heft 48, Berlin, 1921.

28. Bach, C. and Graf, O., "Versuche mit Eisenbetonbalken zur Ermittlung der Widerstandsfähigkeit verschiedener Bewehrung gegen Schubkräfte, Dritter Teil", Deutscher Ausschuss für Eisenbeton, Heft 20, Berlin, 1912

29. Zwoyer, E. M., "Shear Strength of Simply-Supported Prestressed Concrete Beams", Ph.D. Thesis, University of Illinois, 1953.

NOTATIONS USED IN TABLES

Modes of Failure

B = bond

C = compression

Cr = crushing at hooks

DT = reported diagonal tension failures; most beams failed in shear, a few in bond as marked in the tables

S = shear

T = tension

T-S = tension with shear-type final collapse

Calculated Quantities

M_s = shear-compression moment of a beam without web reinforcement, given by Eq. (18) for rectangular beams, by Eq. (35) for T-beams, and by Eq. (43) for restrained beams with local bond failure in high bond stress region

M_{sw} = shear-compression moment of a beam with web reinforcement, given by Eq. (28)

M_f = ultimate flexural moment of a beam, given by Eq. (29)

P_s = load corresponding to M_s

P_{sw} = load corresponding to M_{sw}

P_f = load corresponding to M_f

v_c = ultimate nominal shearing stress for shear-proper type of failures, given by Eq. (47) for $x/D = 0$, where x is the clear distance between two load blocks and D the total depth of beam

RANGE OF TEST VARIABLES FOR SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT AND UNDER ONE OR TWO SYMMETRICAL CONCENTRATED LOADS

Test Series	Entry in Bibl.	Table No.	No. of Beams	No. of S,T-S Fail.	f'_c psi	p o/o	p' o/o	b in.	d in.	a in.	a/d	L in.	No. of Loads
Richart	(2)												
Series 1910		2	3	—	2030-2670	1.23-1.92	—	8	10	24	2.4	72	2
Series 1911		3	18	8	1490-2350	1.65-1.94	—	8	10	24	2.4	72	2
Series 1913		4	1	—	2180	1.47	—	8	15	40	2.67	120	2
Series 1917		5	6	6	4770	2.74-3.71	—	8.1	10	48	4.8	114	2
Series 1922		6	4	4	3696-4522	2.33	—	8	21	36	1.71	108	2
Richart and Jensen	(8)	7	6	6	2230-4760	2.80	—	8	21	32	1.52	96	2
Thompson, Hubbard, and Fehrer	(9)	8	5	5	2570	2.00	—	8	12	20	1.67	60	2
Moretto	(4)	9	4	4	3550-4640	3.98	0.50	5.5	18.25	32	1.75	96	2
Clark	(5)	10	12	12	3120-3800	0.98	—	8	15.37	18.36	1.17-2.34	72	2
M.I.T.	(10)	11	14	14	3130-4880	1.40-3.14	p'=p	4-6.25	7	30	4.28	60	1
Gaston	(11)	12	3	3	4020-4750	1.38-1.90	—	6	10.58	36	3.40	108	2
Laupa	(1)	13	9	9	2140-4690	0.93-4.11	—	6	10.5	48-51	4.48-4.79	108	1
Moody	(12)												
Series A		14a	12	12	880-4570	0.80-2.37	—	7	10.5	31.5	3	63	1
Series B		14b	16	16	1770-5970	1.90	—	6	10.56	36	3.41	108	2
Series III		15	12	12	2500-3620	2.72-4.25	p'=0.5p	7	21	32	1.52	96	2
Total			125	111	880-5970	0.80-4.11					1.17-4.8		

TABLE 2

TESTS BY RICHART, SERIES 1910 (2)*
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 8; d = 10; a = 24; a/d = 2.4; L = 72; L' = 78

Loading: 2 equal loads at 1/3-points

Reinforcement: Plain round bars; f_y = 38,500 psi for Beam 280.3; not given for others

Concrete Strength: Tests on 6-in. cubes; Reduced to cyl. strength by $f'_c = 0.75 f'_{cu}$

Age at Test: Around 60 days

Reported							Calculated			
Beam	f'_c psi	p o/o	Reinf. Bars No., Size	Anch.	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
280.1	2670	1.23	5-1/2	None	23.8	DT	0.380	0.352	0.78	B
280.2	2320	"	"	"	18.8	DT	0.378	0.322	0.69	B
280.3	2030	1.92	5-5/8	"	21.0	DT	0.456	0.340	0.71	B

* Numbers in parentheses refer to corresponding entries in Bibliography.

TESTS BY RICHART, SERIES 1911 (2)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 8; d = 10; D = 12; a = 24; L = 72; L' = 78

Loading: 2 equal loads at 1/3-points

Reinforcement: Plain round bars; f_y = 34,200 psi for Beam 293.3, not given for others

Concrete Strength: Tests on 6by8by40-in. control beams; Reduced to cyl. strength by $f'_c = 6.7\ f_r$

Age at Test: Around 60 days

	Reported						Calculated			
Beam	f'_c psi	p o/o	Reinf. Bars No., Size	Anch.	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
291.1	1690	1.65	3-3/4	Hooks	25.3	DT	0.446	0.503	1.02	S
291.2	"	"	"	"	22.5	DT	"	0.448	0.91	S
291.3	"	"	"	"	27.7	DT	"	0.551	1.12	S
294.1	1490	"	"	15-in.	25.0	DT	0.457	0.551	1.10	S
294.2	"	"	"	over-	20.2	DT	"	0.445	0.89	S
294.3	"	"	"	hang	24.7	DT	"	0.544	1.08	S
293.4	2350	"	"	Nuts*	27.4	DT	0.421	0.415	0.89	S
293.5	"	"	"	and	34.5	DT,T	"	0.523	1.13	S
293.6	"	"	"	Plates	19.3	DT	"	0.293	0.63	B
293.1	2040	"	"	Nuts**	20.0	DT	0.431	0.341	0.71	B
293.2	"	"	"	and	21.4	DT	"	0.365	0.76	B
293.3	"	"	"	Plates	24.8	DT	"	0.423	0.88	B
286.1	1660	"	"	None	18.0	DT	0.448	0.363	0.73	B
286.2	"	"	"	"	17.6	DT	"	0.355	0.72	B
286.3	"	"	"	"	22.5	DT	"	0.454	0.92	B
286.5	2160	1.94	5-5/8	"	17.4	DT	0.452	0.267	0.56	B
286.6	"	"	"	"	18.5	DT	"	0.284	0.60	B
286.7	"	"	"	"	22.1	DT	"	0.340	0.72	B

* = Nuts tightened

** = Nuts not tightened

TABLE 4

TESTS BY RICHART, SERIES 1913 (2)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 8; d = 15; D = 17; a = 40; a/d = 2.67; L = 120; L' = 126

Loading: 2 equal loads at 1/3-points

Reinforcement: 3/4-in. plain round bars, f_y = 36,300 psi

Concrete Strength: Tests on 6-in. cubes; Reduced to cyl. strength by $f'_c = 0.75 f'_{cu}$

Age at Test: 225 days

Reported							Calculated			
Beam	f'_c psi	p o/o	Reinf. Bars No., Size	Anch.	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
301.1	2180	1.47	4-3/4	Hooks	24.9	DT	0.409	0.311	0.66	B

TESTS BY RICHART, SERIES 1917 (2)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 8.1; d = 10; D = 12; a = 48; a/d = 4.8; L = 114; L' = 120

Loading: 2 equal loads

Reinforcement: Plain round bars; f_y = 45,700 psi for 7/8-in. bars, f_y = 40,600 psi for 3/4-in. bars

Age at Test: About 60 days

Reported							Calculated			
Beam	f'_c psi	p o/o	Reinf. Bars No., Size	Anch.	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
Analyzed With Actual Concrete Strength in Compression Zone										
16B20.1	4770	3.71	5-7/8	None	31.0	DT	0.523	0.368	1.04	S
16B20.2	"	"	"	"	29.6	DT	0.523	0.352	0.99	S
16B1.1	"	3.69	"	Hooks	32.0	DT	0.522	0.381	1.07	S
16B1.2	"	"	"	"	28.8	DT	0.522	0.343	0.97	S
16B2.1	"	2.74	5-3/4	"	26.6	DT	0.472	0.350	0.99	S
16B2.2	"	"	"	"	29.5	DT	0.472	0.388	1.09	S
Analyzed With Concrete Strength Used in Lower Portions of Beams										
16B20.1	3210						0.531	0.540	1.28	
16B20.2	3210						0.531	0.516	1.21	
16B1.1	2450						0.550	0.701	1.52	
16B1.2	2670						0.543	0.586	1.30	
16B2.1	2450						0.499	0.660	1.43	
16B2.2	2450						0.499	0.732	1.59	

TABLE 6

TESTS BY RICHART, SERIES 1922 (2)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 8; d = 21; D = 24; a = 36; a/d = 1.71; L = 108; L' = 120

Loading: 2 equal loads at 1/3 points

Reinforcement: 1 1/8 in. corrugated round bars; f_y = 52,400 psi

Age at Test: About 60 days

Reported							Calculated			
Beam	f'_c psi	p o/o	Reinf. Bars No., Size	Anch.	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
221.1	4076	2.33	4-1 1/8	None	149.4	B,DT	0.441	0.424	1.09	S
221.2	3696	"	"	"	148.0	B,DT	0.446	0.458	1.13	S
222.1	4522	"	"	Hooks	165.5	B,DT	0.435	0.429	1.17	S
222.2	4337	"	"	"	126.0	B,DT	0.437	0.339	0.91	S

TESTS BY RICHART AND JENSEN, 1931 (8)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Includes only those beams which were made of concrete with natural sand and gravel aggregates.

Dimensions: b = 8; d = 21; D = 24; a = 32; a/d = 1.52; L = 96; L' = 108

Loading: 2 equal loads at 1/3-points

Reinforcement: 1-in. plain round bars; f_y = 37,600 psi

Age at Test: 28 days (moist cured 28 days)

Reported							Calculated			
Beam	f'_c psi	p o/o	Reinf. Bars No., Size	Anch.	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
1	4760	2.8	6-1	Hooks	142.9	DT	0.463	0.294	0.83	S
2	4620	"	"	"	159.7	DT	0.463	0.339	0.94	S
3	4290	"	"	"	151.8	DT	0.467	0.344	0.91	S
4	3860	"	"	"	134.1	DT	0.473	0.333	0.84	S
5	2230	"	"	"	105.8	DT	0.510	0.422	0.90	S
6	2630	"	"	"	116.5	DT	0.498	0.403	0.89	S

TABLE 8

TESTS BY THOMPSON, HUBBARD, AND FEHRER, 1938 (9)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 8; d = 12; a = 20; a/d = 1.67; L = 60; L' = 74 for Series I, L' = 86 for Series II

Loading: 2 equal loads of 1/3-points

Reinforcement: 4-7/8 in. round bars; deformed (?); structural grade

Concrete Strength: The average value of f'_c reported for all beams

Age at Test: 28 days

Reported							Calculated			
Beam	f'_c psi	p o/o	Reinf. Bars No., Size	Anch.	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
I B-1	2570	2.0	4-7/8	Hooks	84.0	DT	0.445	0.510	1.12	S
I B-2	"	"	"	"	88.0	DT	"	0.534	1.18	S
I B-3	"	"	"	"	86.0	DT	"	0.522	1.15	S
II K-1	"	"	"	13-in.	88.0	DT	"	0.534	1.18	S
II K-2	"	"	"	over-hang	84.0	DT	"	0.510	1.12	S

TABLE 9

TESTS BY MORETTO, 1945 (4)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 5.5; d = 18.25; D = 21; a = 32; a/d = 1.75; L = 96; L' = 120

Loading: 2 equal loads of 1/3-points

Tension Reinforcement: Four 1-in. sq. deformed bars; f_y = 48,000 psi

Compression Reinforcement: Two 1/2-in. sq. deformed bars

End Anchorage: Hooks

Age at Test: 28 days

	Reported						Calculated				
Beam	f'_c psi	p o/o	p' o/o	t	P_{test} kips	Mode of Fail.	k	k+np'	$\frac{M_{test}}{bd^2 f'_c (k+np')}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
1N1	3550	3.98	0.50	0.932	70.0	DT	0.516	0.556	0.310	0.76	S
1N2	3620	"	"	"	88.0	DT	0.514	0.553	0.383	0.94	S
2N1	4340	"	"	"	78.5	DT	0.502	0.538	0.293	0.78	S
2N2	4640	"	"	"	90.5	DT	0.502	0.537	0.318	0.88	S

TESTS BY CLARK, 1951 (5)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 8; d = 15.37; D = 18; L = 72

Loading: 2 equal loads at various positions

Reinforcement: 2-No. 7 deformed bars; f_y = 53,710 psi

End anchorage: 1/2 by 8-in. steel plates 1/4 in. thick welded to the end of bars

Age at Test: 28 to 30 days; beams kept moist until the day prior to testing

Reported							Calculated			
Beam	f'_c psi	p o/o	a in.	a/d	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
AO-1	3120	0.98	36	2.34	40.0	DT	0.329	0.370	0.86	S
2	3770	"	"	"	48.5	DT	0.320	0.382	0.96	S
3	3435	"	"	"	53.5	T	0.324	0.457	1.10	T-S
BO-1	3420	0.98	30	1.95	54.4	DT	0.324	0.388	0.93	S
2	3468	"	"	"	42.4	DT	0.323	0.299	0.72	S
3	3410		"	"	57.6	DT	0.324	0.412	0.99	S
CO-1	3580	0.98	24	1.56	78.4	DT	0.322	0.430	1.05	T-S
2	3405	"	"	"	79.9	T	0.324	0.458	1.10	T-S
3	3420	"	"	"	75.1	DT	0.324	0.428	1.03	S
DO-1	3750	0.98	18	1.17	99.6	DT	0.320	0.394	0.98	S
2	3800	"	"	"	116.9	T	0.320	0.457	1.15	T-S
3	3765	"	"	"	100.4	DT	0.320	0.395	0.99	S

TESTS AT M. I. T., 1951 (10)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = variable; d = 7; D = 8; a = 30; a/d = 4.28; L = 60; L' = 65

Loading: One load at midspan

Reinforcement: Type of bars not given; f_y = 52,220 psi for 3/8-in., f_y = 48,370 psi for 1/2-in., f_y = 46,240 psi for 5/8-in. bars

End Anchorage: Not given

Age at Test: 8 days

	Reported								Calculated				
Beam	f'_c psi	p o/o	p' o/o	Reinf. Bars No., Size	t	b in.	P_{test} kips	Mode of Fail.	k	k+np'	$\frac{M_{test}}{bd^2 f'_c (k+np')}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
T-2b	3580	1.40	1.40	2-1/2	0.857	4	10.0	S	0.327	0.436	0.392	0.96	S
c	"	"	"	"	"	"	10.0	S	"	"	0.392	0.96	S
T-3a	3470	3.14	3.14	2-3/4	"	"	10.5	S	0.405	0.652	0.355	0.86	S
b	"	"	"	"	"	"	7.0	S	"	"	0.237	0.57	S
c	"	"	"	"	"	"	8.5	S	"	"	0.288	0.70	S
T-5a	3460	2.18	2.18	2-5/8	"	"	9.5	S	0.372	0.544	0.387	0.94	S
b	"	"	"	"	"	"	10.1	S	"	"	0.412	1.00	S
c	"	"	"	"	"	"	10.3	S	"	"	0.417	1.01	S
T-6b	3130	1.40	1.40	2-1/2	"	"	7.6	S	0.331	0.446	0.417	0.97	S
c	"	"	"	"	"	"	8.2	S	"	"	0.450	1.05	S
T-11b	4190	1.40	1.40	2-5/8	"	6.25	12.0	S	0.321	0.425	0.330	0.87	S
T-12a	4880	2.18	2.18	2-3/4	"	5.75	15.8	S	0.360	0.514	0.334	0.95	S
b	"	"	"	"	"	"	15.0	S	"	"	0.318	0.91	S
c	"	"	"	"	"	"	14.3	S	"	"	0.303	0.87	S

TABLE 12

TESTS BY GASTON, 1952 (11)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 6; d = 10.58; D = 12; a = 36; a/d = 3.40; L = 108; L' = 120

Loading: 2 equal loads at 1/3-points

Reinforcement: Deformed bars

Age at Test: Around 30 days

	Reported							Calculated			
Beam	f'_c psi	p o/o	Reinf. Bars No., Size	f_y ksi	Anch.	M_{test} kip-in.	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
T2Ma	4320	1.38	2-No.6	47.7	None	332.3	S	0.359	0.319	0.85	S
T2Mb	4020	"	"	48.3	Hooks	351.7	S	0.363	0.359	0.92	S
T2Mc	4470	1.90	2-No.7	46.8	None	450.2	S	0.405	0.377	1.02	S

TABLE 13

TESTS BY LAUPA, 1953 (1)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 6; D = 12; L = 108; L' = 120; distance a given from center of end support to edge of column stub

Loading: One load at center of 108-in. span, applied through 6 by 12-in. column stub, 6 in. high

Reinforcement: Deformed bars

End Anchorage: None, straight bars

Age at Test: Around 28 days

Reported										Calculated			
Beam	f'_c psi	p o/o	Reinf. Bars No., Size	f_y ksi	d in.	a in.	a/d	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
S-2	3900	2.08	3-No.6	41.2	10.58	48	4.54	19.1	S	0.415	0.421	1.07	S
S-3	4690	2.52	2-No.8	59.4	10.44	"	4.60	23.9	S	0.446	0.419	1.17	S
S-4	4470	3.21	2-No.9	44.8	10.37	"	4.63	25.0	S	0.478	0.435	1.18	S
S-5	4330	4.11	2-No.10	45.7	10.31	"	4.66	22.4	S	0.531	0.367	0.98	S
S-11	2140	1.90	2-No.7	47.5	10.51	"	4.57	15.2	S	0.450	0.571	1.20	S
S-13	3800	4.11	2-No.10	44.1	10.31	"	4.66	22.4	S	0.528	0.420	1.05	S
S-1	3940	1.46	3-No.5	44.6	10.65	51	4.79	16.8	T-S	0.361	0.443	1.13	T-S
S-9	2140	0.93	3-No.4	44.3	10.72	48	4.48	11.5	T-S	0.344	0.543	1.15	T-S
S-10	2280	1.39	2-No.6	41.8	10.58	"	4.54	15.4	T-S	0.396	0.608	1.30	T-S

TESTS BY MOODY, SERIES A, 1953 (12)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 7; d = 10.3-10.8; D = 12.0, a = 31.5; a/d = 2.92-3.06; L = 63; L* = 75

Loading: One load at midspan

Reinforcement: Intermediate grade deformed bars

End Anchorage: None, straight bars

Age at Test: About 28 days

	Reported						Calculated		
Beam	f'_c psi	d in.	p o/o	Reinf. Bars No., Size	P_{test} kips	Mode of Fail.	k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$
A1	4400	10.30	2.17	1-11	27.0	S	0.426	0.306	0.82
2	4500	10.50	2.15	2-8	30.0	"	0.423	0.322	0.88
3	4500	10.55	2.22	2-7;1-6	34.0	"	0.428	0.357	0.97
4	4570	10.63	2.37	4-6	32.0	"	0.437	0.319	0.88
B1	3065	10.50	1.62	1-8;2-4	25.3	"	0.401	0.420	0.97
2	3125	10.55	1.63	2-7	27.0	"	0.401	0.435	1.01
3	2785	10.63	1.60	2-6;1-5	25.0	"	0.404	0.442	0.99
4	2430	10.69	1.66	4-5	25.0	"	0.419	0.483	1.05
C1	920	10.55	0.81	1-7	9.0	"	0.395	0.501	0.95
2	880	10.70	0.83	2-5	11.0	"	0.403	0.608	1.14
3	1000	10.75	0.80	3-4	11.4	"	0.384	0.577	1.10
4	980	10.80	0.82	2-4;2-3	11.3	"	0.391	0.569	1.08

TABLE 14b

TESTS BY MOODY, SERIES B, 1953 (12)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 6; d = 10.56; D = 12; a = 36; a/d = 3.41; L = 108; L' = 120

Loading: 2 equal loads at 1/3-points

Reinforcement: 2-No. 7 intermediate grade deformed bars

End Anchorage: None, straight bars

Age at Test: About 28 days

Beam	Reported: f'_c psi	p o/o	P_{test} kips	Mode of Fail.	Calculated: k	$\frac{M_{test}}{bd^2 f'_c k}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
1	5320	1.9	26.0	S	0.397	0.334	1.01	S
2	2420	"	16.0	S	0.441	0.408	0.89	S
3	3735	"	23.5	S	0.414	0.413	1.03	S
4	2230	"	19.8	S	0.447	0.540	1.15	S
5	4450	"	23.4	S	0.395	0.362	0.98	S
6	2290	"	15.8	S	0.445	0.421	0.90	S
7	4480	"	23.0	S	0.395	0.354	0.96	S
8	1770	"	14.0	S	0.465	0.462	0.94	S
9	5970	"	24.0	S	0.393	0.278	0.92	S
10	3470	"	22.0	S	0.418	0.412	1.00	S
11	5530	"	27.0	S	0.395	0.336	1.05	S
12	2925	"	21.2	S	0.424	0.464	1.06	S
13	5480	"	25.0	S	0.396	0.313	0.97	S
14	3265	"	19.4	S	0.421	0.382	0.90	S
15	5420	"	23.0	S	0.397	0.291	0.89	S
16	2370	"	17.0	S	0.446	0.437	0.94	S

TESTS BY MOODY, SERIES III, 1953 (12)
SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

Dimensions: b = 7; d = 21; D = 24; a = 32; a/d = 1.52; L = 96; L' = 120

Loading: 2 equal loads at 1/3-points

Tension Reinforcement: Four deformed bars

Compression Reinforcement: Two deformed bars; t = 0.91

Age at Test: 28 days

	Reported							Calculated				
Beam	f'_c psi	p o/o	p' o/o	f_y ksi	Anch.	P_{test} kips	Mode of Fail.	k	k+np'	$\frac{M_{test}}{bd^2 f'_c (k+np')}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
24a	2580	2.72	1.36	45.7	Hooks	133	S	0.432	0.552	0.484	1.07	S
b	2990	"	"	"	"	136	S	0.424	0.538	0.439	1.01	S
25a	3530	3.46	1.73	45.4	"	120	S	0.456	0.582	0.303	0.74	S
b	2500	"	"	"	"	130	S	0.455	0.610	0.442	0.97	S
26a	3140	4.25	2.13	43.8	"	189	S	0.485	0.659	0.473	1.10	S
b	2990	"	"	"	"	178	S	0.488	0.665	0.464	1.07	S
27a	3100	2.72	1.36	45.7	None	156	S	0.433	0.545	0.479	1.11	S
b	3320	"	"	"	"	160	S	0.429	0.538	0.464	1.10	S
28a	3380	3.46	1.73	45.4	"	136	S	0.458	0.596	0.350	0.84	S
b	3250	"	--	"	"	153	S	0.519	0.519	0.470	1.11	S
29a	3150	4.25	2.13	43.8	"	175	S	0.485	0.659	0.437	1.02	S
b	3620	"	"	"	"	196	S	0.480	0.645	0.435	1.07	S

RANGE OF TEST VARIABLES FOR SIMPLE-SPAN RECTANGULAR BEAMS
WITH STIRRUPS AND UNDER ONE OR TWO SYMMETRICAL CONCENTRATED LOADS

Test Series	Table No.	No. of Beams	No. of Shear Fail.	No. of Flex. Fail.	f'_c psi	d in.	a/d	p o/o	p' o/o	α deg.	r o/o	f_{yw} ksi
Richart (2)*												
Series 1910	17	6	-	6	2030-3570	10	2.4	1.40-1.56	-	45;90	0.35;0.52	54.5;93.3
Series 1913	18	9	1	8	1380-2180	15	2.67	1.47	-	45	0.17-1.39	40[x]
Series 1922	19	6	-	6	3689-4124	21	1.71	2.33	-	90	1.38-1.40	39.6-42.9
Slater, Lord, Zipprodt (15)	20	4	1	3	3000-5960	32.75 16.9	1.74 3.37	2.33-2.50	p'=p	90	0.23-0.88	70
Slater, Lyse (16)	21	30	1+	28	1210-5060	4.1-12.2	2.95-8.78	2.1-4.7	-	90;20	0.42-0.85	73.4
Thompson, Hubbard, Fehrer(9)	22	3	3	-	2570	12	1.67	2.0	-	90	0.29	45[x]
Johnston, Cox(17)	23	20	10	10	3190	12	3.00	0.39-0.87	-	90	0.10	45[x]
Moretto (4)	24	40	26	14	2320-5060	18.25 19.50	1.75 1.64	3.98 1.86	0.50 0.47	90;67.5; 45	0.28-1.12	46.0-55.0
Clark (5)	25	50	43	7	2000-6900	15.37 12.37	1.17-2.54	1.63-3.42	-	90	0.34-1.22	48.0
Gaston (11)	26	9	-	9	2120-5900	9.23-10.72	3.36-3.90	0.62-7.22	-	90	0.28-1.83	45[x]
Moody (12)	27	2	2	-	3250;3680	21	1.52	4.25	2.13	90	0.52;0.95	44.0;47.3
Total		179	87+	91								

* Numbers in parentheses refer to corresponding entries in Bibliography.
\+ One additional beam failed because of insufficient anchorage of stirrups.
x Assumed values.

TABLE 17

TESTS BY RICHART, SERIES 1910 (2)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: b = 8; d = 10; a = 24; a/d = 2.4; L = 72; L' = 78

Loading: 2 equal loads at 1/3-points

Tension Reinforcement: Monolith, ovoid, and corrugated bars

Concrete Strength: Tests on 6-in. cubes; reduced to cyl. strength by $f'_c = 0.75\ f'_{cu}$

Age at Test: From 60 to 70 days

* Bent-up bars not included in web reinforcement.

Beam	f'_c psi	Tension Reinf. No., Size	p o/o	f_y ksi	Web Reinf.	α deg.	r o/o	f_{yw} ksi	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$
					Stirrups as Web Reinforcement								
282.1	2420	2-3/4"	1.40		1/4" round	45	0.35	54.5	191	32.0	T	29.3	1.09
2	3570	monolith	"	37.7	loops	"	"	"	"	32.0	T	36.1	0.89
3	2410		"			"	"	"	"	33.8	T	29.3	1.15
281.1	2670	3-11/16"	1.56	40.0	3/16" round	90	0.52	93.3	485	40.0	T	32.4	1.24
2	2320	ovoid	"		loops	"	"	"	"	36.4	T	29.8	1.22
3	2030		"			"	"	"	"	36.7	T	27.4	1.34
					Both Stirrups and Bent-up Bars Used as Web Reinforcement								
281.5	2570	2-11/16"	1.48	37.6	3/16" round	90	0.34*	99.4	339*	41.0	T	31.0	1.32
6	2570	and 1-5/8"	"		loops and	"	"	"	"	37.8	DT,B	30.9	1.22
7	2030	ovoid	"		1-5/8"	"	"	"	"	40.0	T	26.9	1.49
284.1	2420	4-5/8"	1.50	63.3	3/16" round	45	0.25*	63.7	159*	52.8	T	30.1	1.76
2	2560	corr.	"		stirr. and	and	"	"	"	49.3	T	31.1	1.58
3	2410		"		2-5/8"	90	"	"	"	47.5	DT	30.0	1.58
284.5	2570	4-5/8"	1.50		1/4" sq.	45	0.56*	55.6	311*	54.5	T	31.1	1.75
6	2900	corr.	"		stirr. and	and	"	"	"	50.0	T	33.4	1.50
7	2030		"	64.8	2-5/8"	90	"	"	"	51.0	T	27.0	1.88

TABLE 18

TESTS BY RICHART, SERIES 1913 (2)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: b = 8; d = 15; D = 17; a = 40; a/d = 2.67; L = 120; L' = 126

Loading: 2 equal loads at 1/3-points

Tension Reinforcement: Four 3/4-in. plain round bars, p = 0.0147, f_y = 36,300 psi

End Anchorage: Hooks

Web Reinforcement: 1/4, 3/8, and 1/2-in. plain round bars, f_{yw} = 40,000 psi assumed

Concrete Strength: Tests on 6-in. cubes; reduced to cyl. strength by $f'_c = 0.75\, f'_{cu}$

Age at Test: From 80 to 238 days

Beam	f'_c psi	α deg.	r o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$
303.1	2180	45	0.17	68	38.7	DT	37.8	1.02	0.90
304.1	1910	"	0.35	140	39.5	T	34.8	1.13	
2	1680	"	0.35	140	40.2	T	32.0	1.26	
305.1	1820	"	0.78	312	38.0	T	33.6	1.13	
2	1380	"	0.78	312	40.0	T	28.6	1.40	
306.1	1840	"	0.78	312	40.0	T	33.4	1.20	
2	1910	"	0.78	312	35.7	T	34.8	1.02	
307.2	1760	"	1.39	556	40.7	T	33.0	1.23	
308.1	2030	"	0.82	328	44.0	T	36.2	1.22	

TABLE 19

TESTS BY RICHART, SERIES 1922 (2)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: b = 8; d = 21; D = 24; a = 36; a/d = 1.71; L = 108; L* = 120

Loading: 2 equal loads at 1/3-points

Tension Reinforcement: Four 1 1/8-in. corrugated round bars; p = 0.0233; f_y = 52,400 psi

End Anchorage: Hooks

Web Reinforcement: Plain round vertical stirrups

Age at Test: Around 60 days

Beam	f'_c psi	Stirr. Size in.	s in.	r o/o	f_{yw} ksi	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$
				Stirrups as Web Reinforcement						
23.1	4124	3/8	4	1.38	42.9	592	212.5	T	136.6	1.56
2	3689	"	"	"	"	"	216.4	T	130.2	1.66
24.1	4106	1/2	7	1.40	40.1	561	218.5	T	136.4	1.60
2	3790	"	"	"	"	"	216.0	T	132.0	1.64
25.1	3788	5/8	11	1.39	39.6	550	227.3	T	132.2	1.72
2	4041	"	"	"	"	"	221.2	T	135.4	1.63
				Bent-Up Bars As Web Reinforcement						
29.1	3931	2-1 1/8"		0.96	52.4	503	223.4	T	134.1	1.67
2	4203	Bent-Up		"	"	"	211.7	T	137.6	1.54

TESTS BY SLATER, LORD, AND ZIPPRODT, 1926 (15)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: a = 57; L = 114; L' = 128; D = 36 (18 for Beam 61)

Loading: One load at midspan

Flexural Reinforcement: Equal tension and compression reinforcement; 1 1/4-in. round plain bars; f_y = about 55,000 psi; some bars, not known which, had much lower yield strength.

End Anchorage: Hooks

Web Reinforcement: 1/2 and 3/8-in. plain round vertical bars; f_{yw} = about 70,000 psi

Age at Test: About 60 days

Beam	f'_c psi	b in.	d in.	a/d	p=p' o/o	t	Stirr. Size in.	s in.	r o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$
43	4880	12.0	32.75	1.74	2.50	0.901	1/2	2	0.82	574	496.2	T	415.6	1.19	
48	3000	12.1	"	"	2.48	"	1/2	2	0.81	567	496.2	T	347.6	1.43	
50	5960	12.1	"	"	2.48	"	3/8	1	0.88	616	540.0	T	429.6	1.26	
61	3600	11.8	16.9	3.37	2.33	0.867	3/8	4	0.23	161	121.3	DT	99.2	1.22	0.92

TABLE 21

TESTS BY SLATER AND LYSE, 1930 (16)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: a = 36; L = 114; L' = 132; a/d = 2.95 to 8.78

Loading: Two equal loads

Tension Reinforcement: Rail-steel bars, f_y = from 59,300 to 63,000 psi

End Anchorage: Hooks

Web Reinforcement: 3/8-in. stirrups, f_{yw} = 73,400 psi

Age at Test: 28 days

Beam	f'_c psi	b in.	d in.	p o/o	α deg.	r o/o	rf_{yw} psi	Krf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$
1A	1210	8.2	10.2	2.1	90	0.854	627		33.0	C	15.3	2.15
B	1520	"	"	"	"	"	"		32.0	C	17.9	1.79
C	1450	"	"	"	"	"	"		36.2	C	17.3	2.09
2A	2530	"	10.3	2.8	"	"	"		47.4	C	27.9	1.70
B	2940	"	"	"	"	"	"		40.0	C	30.5	1.31
C	2910	"	"	"	"	"	"		46.8	C	30.3	1.54
3A	4020	"	"	3.7	"	"	"		58.6	C	39.1	1.50
B	4200	"	"	"	"	"	"		64.8	C	39.8	1.63
C	4000	"	"	"	"	"	"		66.6	C	39.0	1.71
4A	4670	"	10.1	4.7	"	"	"		74.5	C	43.0	1.73
B	4660	"	"	"	"	"	"		71.1	C	42.9	1.66
C	5060	"	"	"	"	"	"		79.0	C	43.9	1.80
6A	2490	"	14.2	3.0	"	"	"		92.5	DT*	53.7	1.73
B	2600	"	"	"	"	"	"		106.6	C	55.3	1.92
C	2670	"	"	"	"	"	"		92.0	C	56.1	1.64
7A	2800	8.3	12.2	2.8	"	0.843	619		69.3	C	42.1	1.65
B	2860	"	"	"	"	"	"		63.9	C	42.6	1.50
C	3200	"	"	"	"	"	"		71.3	C	45.4	1.57
8A	3020	8.1	8.0	3.1	20	0.421	309	135	25.8	C	19.1	1.35
B	2650	"	"	"	"	"	"	"	31.7	DT	17.7	1.79
C	2600	"	"	"	"	"	"	"	33.4	C	17.5	1.91
9A	3120	7.9	5.9	3.2	"	0.432	317	139	15.3	C	10.4	1.47
B	2670	"	"	"	"	"	"	"	18.6	C	9.5	1.96
C	2900	"	"	"	"	"	"	"	16.6	C	10.0	1.66
10A	3040	8.0	4.1	3.0	"	0.427	315	138	6.8	C	4.9	1.37
B	2750	"	"	"	"	"	"	"	7.7	C	4.6	1.65
C	2660	"	"	"	"	"	"	"	6.2	C	4.5	1.37
1A-A	3730	"	"	4.0	"	"	"	"	9.8	C	6.0	1.63
B	3900	"	"	"	"	"	"	"	10.3	C	6.1	1.68
C	3800	"	"	"	"	"	"	"	9.0	C	6.0	1.49

* Stirrups too short.

TABLE 22

TESTS BY THOMPSON, HUBBARD, AND FEHRER, 1938 (9)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: b = 8; d = 12; a = 20; a/d = 1.67; L = 60; L' = 74

Loading: 2 equal loads at 1/3-points

Tension Reinforcement: 4 - 7/8 in. round bars, structural grade; deformed (?) bars

End Anchorage: Hooks

Web Reinforcement: 1/4-in. round vertical stirrups at 3.5 in.; r = 0.0029; f_{yw} = 45,000 psi assumed

Age at Test: 28 days

Beam	f'_c psi	p o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$
IC 1	2570	2.0	129	97.0	S	74.8	1.30	1.03
2	"	"	"	88.0	S	"	1.18	0.94
3	"	"	"	98.0	S	"	1.31	1.04

TABLE 23

TESTS BY JOHNSTON AND COX, 1939 (17)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: b = 12, d = 12; D = 13.3; a = 36; a/d = 3.00; L = 108; L' = 120

Loading: Two equal loads at 1/3-points

Tension Reinforcement: Hard grade deformed and sq. twisted bars

End Anchorage: Hooked

Concrete Strength: Average concrete strength reported

Web Reinforcement: Vertical 1/4-in. deformed stirrups at 8 in.; inter. grade, f_{yw} = 45,000 psi assumed

Age at Test: 28 days

Beam	f'_c psi	p o/o	f_y ksi	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$
A1 I	3190	0.451	62.2	47	30.6	T	30.9	0.99	0.90
II	"	"	"	"	30.9	T	"	1.00	0.91
A2 I	"	0.39	59.2	"	28.7	T	29.0	0.99	0.90
II	"	"	"	"	28.4	T	"	0.98	0.90
A3 I	"	0.41	60.3	"	28.3	T	29.6	0.96	0.88
II	"	"	"	"	28.2	T	"	0.95	0.87
B1 I	"	0.78	59.2	"	45.2	DT	38.9	1.16	1.06
II	"	"	"	"	45.3	T	"	1.16	1.06
B2 I	"	0.82	60.3	"	44.5	T	40.2	1.11	1.01
II	"	"	"	"	45.2	DT	"	1.12	1.02
B3 I	"	0.83	63.2	"	52.8	T	39.9	1.32	1.21
II	"	"	"	"	52.9	DT	"	1.32	1.21
I1 I	"	0.82	58.6	"	46.0	DT	39.7	1.16	1.06
II	"	"	"	"	45.5	DT	"	1.15	1.05
T1 I	"	0.87	58.4	"	54.2	DT	40.7	1.33	1.22
II	"	"	"	"	54.3	T	"	1.33	1.22
T2 I	"	0.81	61.8	"	47.1	DT	39.5	1.19	1.09
II	"	"	"	"	45.7	DT	"	1.16	1.06
T3 I	"	0.78	64.4	"	42.8	DT	38.9	1.10	1.01
II	"	"	"	"	45.5	DT	"	1.17	1.07

TABLE 24

TESTS BY MORETTO, 1945 (4)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: b = 5.5; d = 18.25; D = 21; a = 32; a/d = 1.75; L = 96; L' = 120

Loading: 2 equal loads at 1/3-points

Tension Reinforcement: Four 1-in. square deformed bars; p = 0.0398; f_y = 48,000 psi

End Anchorage: Hooks

Compression Reinforcement: Two 1/2-in. square deformed bars; t = 0.932; p' = 0.005

Web Reinforcement: 1/4-in. plain bars, 3/8 in. and 1/2-in. deformed bars; s = 6.5 in.

Age of Test: 28 days

Beam	f'_c psi	α deg.	r o/o	f_{yw} ksi	rf_{yw}	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$
1V 1/4 1	3880	90	0.28	55.0	154	116.4	DT	96.4	1.21	0.93
2	3040	"	"	"	"	116.8	DT	85.3	1.37	1.05
2V 1/4 1	4680	"	"	"	"	135.8	DT	103.5	1.31	1.00
2	4930	"	"	"	"	134.4	DT	104.9	1.28	0.98
1I 1/4 1	4020	67.5	"	"	"	126.9	DT	97.8	1.30	0.99
2	3570	"	"	"	"	115.0	DT	92.8	1.24	0.95
2I 1/4 1	4840	"	"	"	"	144.0	DT	104.4	1.38	1.06
2	4560	"	"	"	"	120.0	DT	102.7	1.17	0.89
1D 1/4 1	3360	45	"	"	"	115.3	DT	90.0	1.28	0.98
2	3590	"	"	"	"	121.8	DT	92.9	1.31	1.00
2D 1/4 1	3370	"	"	"	"	142.3	DT	90.0	1.58	1.21
2	3540	"	"	"	"	138.8	DT	92.5	1.50	1.15
1V 3/8 1	2780	90	0.615	47.9	295	142.9	DT	81.2	1.76	1.11
2	3450	"	"	"	"	150.8	DT	91.2	1.65	1.04
2V 3/8 1	4260	"	"	"	"	148.3	DT	100.0	1.48	0.93
2	3980	"	"	"	"	139.0	DT	97.6	1.42	0.89

TESTS BY MORETTO, 1945 (4)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Beam	f'_c psi	α deg.	r o/o	f_{yw} ksi	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$
1I 3/8 1	3780	67.5	0.615	47.9	295	162.5	DT	95.4	1.70	1.07
2	3370	"	"	"	"	155.5	DT	90.0	1.73	1.09
2I 3/8 1	4210	"	"	"	"	171.0	DT	99.9	1.71	1.08
2	4160	"	"	"	"	165.0	DT	99.2	1.66	1.04
1D 3/8 1	2940	45	"	"	"	132.0	DT	83.9	1.57	0.99
2	2770	"	"	"	"	127.5	DT	81.0	1.57	0.99
2D 3/8 1	3790	"	"	"	"	139.9	DT	95.2	1.47	0.92
2	3580	"	"	"	"	147.4	DT	93.0	1.58	0.99
1V 1/2 1	3740	90	1.12	50.7	568	157.0	C	94.8	1.66	0.78
2	3590	"	"	"	"	157.0	C	92.9	1.69	0.79
2V 1/2 1	5060	"	"	"	"	188.8	C	105.6	1.79	0.84
2	4570	"	"	"	"	184.0	C	102.6	1.79	0.84
1I 1/2 1	3070	67.5	"	"	"	177.5	C	85.8	2.07	0.97
2	3110	"	"	"	"	178.0	C	86.3	2.06	0.96
2I 1/2 1	4080	"	"	"	"	196.3	C	98.3	2.00	0.94
2	4340	"	"	"	"	196.1	C	100.8	1.95	0.91
1D 1/2 1	3090	45	"	"	"	165.0	C	86.2	1.91	0.89
2	2320	"	"	"	"	145.0	C	72.9	1.99	0.93
2D 1/2 1	3330	"	"	"	"	171.5	C	89.5	1.92	0.90
2	3660	"	"	"	"	180.2	C	93.8	1.92	0.90
Series 1a										
b = 5.5 in.; d = 19.5 in.; a = 32 in.; a/d = 1.64; p = 0.0186; p' = 0.0047; t = 0.936										
1aV 1/4 1	3650	90	0.28	46.0	129	105.2	DT	82.5	1.28	1.02
2	3430	"	"	"	"	107.0	DT	80.0	1.34	1.07
1aV 3/8 1	3385	"	0.615	52.0	320	115.6	T,DT	79.7	1.45	0.88
2	3290	"	"	"	"	119.1	T,DT	78.6	1.52	0.93

TABLE 25

TESTS BY CLARK, 1951 (5)
SIMPLE-SPAN RETANGULAR BEAMS WITH STIRRUPS

Loading: 2 equal symmetrical loads at various positions on beam
Tension Reinforcement: Deformed bars
End Anchorage: 1/2 by 8-in. plates 1/4-in. thick welded to the end of bars
Web Reinforcement: 3/8-in. vertical deformed bars; f_{yw} = 48,020 psi
Age at Test: 28 to 30 days; beams kept moist until the day prior to testing

Beam	f'_c psi	p o/o	a in.	a/d	s in.	r o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$	Ratio $\frac{P_{test}}{P_f}$
8 by 18 in. Beams; Span = 72 in.; d = 15.37; f_y = 46,500 psi													
A1-1	3575	3.10	36	2.54	7.2	0.38	182	100.0	DT	75.9	1.32	0.97	0.83
2	3430	"	"	"	"	"	"	94.0	DT	74.5	1.26	0.92	0.78
3	3395	"	"	"	"	"	"	100.0	DT	74.0	1.35	0.99	0.84
4	3590	"	"	"	"	"	"	110.0	DT	76.0	1.44	1.06	0.91
B1-1	3388	3.10	30	1.95	7.5	0.37	178	125.4	DT	88.9	1.41	1.04	0.88
2	3680	"	"	"	"	"	"	115.4	DT	92.3	1.25	0.92	0.79
3	3435	"	"	"	"	"	"	128.1	DT	89.2	1.43	1.05	0.89
4	3380	"	"	"	"	"	"	120.6	DT	88.6	1.36	1.00	0.84
5	3570	"	"	"	"	"	"	108.6	DT	91.0	1.19	0.88	0.75
B2-1	3370	3.10	30	1.95	3.75	0.73	351	135.4	DT	88.4	1.53	0.90	0.95
2	3820	"	"	"	"	"	"	144.9	DT	94.0	1.54	0.90	0.98
3	3615	"	"	"	"	"	"	150.6	DT	91.5	1.64	0.96	1.03
B6-1	6110	3.10	30	1.95	7.5	0.37	178	170.6	DT	106.3	1.60	1.18	1.06
C1-1	3720	2.07	24	1.56	8.0	0.34	163	124.9	DT	100.9	1.24	0.94	0.95
2	3820	"	"	"	"	"	"	139.9	DT	101.9	1.37	1.03	1.05

TABLE 25 (CONT'D)

Beam	f'_c psi	p o/o	a in.	a/d	s in.	r o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$	Ratio $\frac{P_{test}}{P_f}$
C1-3	3475	2.07	24	1.56	8.0	0.34	163	110.6	DT	97.6	1.13	0.85	0.85
4	4210	"	"	"	"	"	"	128.6	DT	106.3	1.21	0.91	0.96
C2-1	3430	2.07	24	1.56	4.0	0.69	331	130.4	DT	97.0	1.34	0.81	1.00*
2	3625	"	"	"	"	"	"	135.4	DT	99.6	1.36	0.82	1.03*
3	3500	"	"	"	"	"	"	145.6	T	97.8	1.49	0.90	1.11*
4	3910	"	"	"	"	"	"	129.6	DT	100.5	1.29	0.78	0.98*
C3-1	2040	2.07	24	1.56	8.0	0.34	163	100.6	DT	71.6	1.40	1.06	0.87
2	2000	"	"	"	"	"	"	90.1	DT	70.9	1.27	0.96	0.78
3	2020	"	"	"	"	"	"	84.6	DT	71.3	1.19	0.90	0.73
C4-1	3550	3.10	24	1.56	8.0	0.34	163	139.1	DT	113.5	1.23	0.93	0.86
C6-2	6560	3.10	24	1.56	8.0	0.34	163	190.6	DT	132.2	1.44	1.09	0.94
3	6480	"	"	"	"	"	"	195.6	DT	132.1	1.48	1.12	0.97
4	6900	"	"	"	"	"	"	192.7	DT	130.4	1.48	1.12	0.95
8 by 18 in. Beams; Span = 72 in.; d = 15.37 in.; f_y = 48,630 psi													
D1-1	3800	1.63	18	1.17	6.0	0.46	221	135.4	DT	124.1	1.09	0.76	0.91
2	3790	"	"	"	"	"	"	160.4	T	121.0	1.32	0.92	1.08*
3	3560	"	"	"	"	"	"	115.4	DT	120.4	0.96	0.67	0.78
D2-1	3480	1.63	18	1.17	4.5	0.61	293	130.4	DT	119.0	1.10	0.69	0.89
2	3755	"	"	"	"	"	"	140.4	DT	123.4	1.14	0.72	0.94
3	3595	"	"	"	"	"	"	150.4	T	120.9	1.24	0.78	1.01*
4	3550	"	"	"	"	"	"	150.6	T	120.3	1.25	0.79	1.02*

TABLE 25 (CONT'D)

Beam	f'_c psi	p o/o	a in.	a/d	s in.	r o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$	Ratio $\frac{P_{test}}{P_f}$
D3-1	4090	2.44	18	1.17	3.0	0.92	442	177.6	DT	148.5	1.20	0.64	0.83
D4-1	3350	1.63	18	1.17	2.25	1.22	586	140.4	DT	116.8	1.20	0.55	0.96
6 by 15 in. Beams; Span = 96 in.; d = 12.37; f_y = 46,500 psi													
D1-6	4010	3.42	24	1.94	8.0	0.46	221	78.6	DT	60.3	1.30	0.90	0.81
7	4060	"	"	"	"	"	"	80.6	DT	60.5	1.33	0.92	0.82
8	4030	"	"	"	"	"	"	83.6	DT	60.4	1.38	0.96	0.86
6 by 15 in. Beam; Span = 115 in.; d = 12.37 in.; f_y = 46,500 psi													
E1-2	4375	3.42	25	2.02	5.0	0.73	351	99.7	DT	60.0	1.66	0.98	1.04
6 by 15 in. Beams; Span = 120 in.; d = 12.37 in.; f_y = 46,500 psi													
D2-6	4280	3.42	30	2.43	6.0	0.61	293	75.7	DT	49.5	1.53	0.96	0.96
7	4120	"	"	"	"	"	"	70.7	DT	48.8	1.45	0.91	0.90
8	3790	"	"	"	"	"	"	75.7	DT	46.9	1.61	1.02	0.98
D4-1	3970	3.42	30	2.43	7.5	0.49	235	75.7	DT	46.0	1.58	1.07	0.97
2	3720	"	"	"	"	"	"	70.7	DT	46.6	1.52	1.03	0.92
3	3200	"	"	"	"	"	"	74.2	DT	43.2	1.72	1.17	1.02
D5-1	4020	3.42	30	2.43	10.0	0.37	178	65.7	DT	48.2	1.36	1.00	0.84
2	4210	"	"	"	"	"	"	70.7	DT	49.3	1.44	1.06	0.90
3	3930	"	"	"	"	"	"	70.7	DT	47.8	1.48	1.09	0.91

* Considered tension failure.

TABLE 26

TESTS BY GASTON, 1952 (11)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: b = 6; D = 12; a = 36; a/d = 3.36 to 3.90; L = 108; L' = 120

Loading: 2 equal loads at 1/3-points

Tension Reinforcement: Deformed bars

End Anchorage: None, straight bars

Web Reinforcement: 1/4 and 3/8-in. vertical deformed stirrups, f_{yw} = 45,000 psi assumed

Age at Test: Around 30 days

Beam	f'_c psi	d in.	p o/o	f_y ksi	r o/o	rf_{yw} psi	M_{test} kip-ft	Mode of Fail.	M_s Eq. 18 kip-ft	Ratio $\frac{M_{test}}{M_s}$
T1Lb	2520	10.72	0.62	46.0	0.28	126	20.2	T	18.7	1.08
T2La	2120	10.65	0.97	40.4	0.42	189	24.2	T	20.0	1.21
T4Lb	2810	10.44	2.52	43.3	0.92	414	47.8	T	32.4	1.48
T5L	2500	10.37	3.22	40.2	0.92	414	53.9	T	32.3	1.67
T11L	2900	9.23	7.22	45.3	1.83	824	67.6	C	35.4	1.91
T1Ha	5880	10.58	1.38	44.2	1.05	473	35.1	T	34.9	1.01
T2H	5400	10.44	2.52	45.6	1.05	473	53.9	T	42.3	1.27
T3H	5920	9.52	4.20	43.2	1.83	824	67.7	T	42.4	1.60
T5H	5900	9.23	7.22	40.6	1.83	824	86.3	T	46.8	1.85

TABLE 27

TESTS BY MOODY, SERIES III, 1953 (12)
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

Dimensions: b = 7; d = 21; D = 24; a = 32; a/d = 1.52; L = 96; L' = 120

Loading: 2 equal loads at 1/3-points

Flexural reinforcement: No. 11 deformed bars, f_y = 43,800 psi, t = 0.914

End Anchorage: Hooks

Web Reinforcement: Vertical stirrups

Age at Test: 28 days

Beam	f'_c psi	p o/o	p' o/o	Web Reinf.	s in.	r o/o	f_{yw} ksi	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$
30	3680	4.25	2.13	No. 3	6	0.52	47.3	246	215	S	179.5	1.20	0.80
31	3250	"	"	No. 4	"	0.95	44.0	418	228	S	169.8	1.34	0.73

TABLE 20

TESTS BY RICHART, SERIES 1917 (2)
SIMPLE-SPAN RECTANGULAR BEAMS WITH BENT-UP BARS

Dimensions: b = 8.1; d = 10; D = 12; a = 48; a/d = 4.8; L = 114; L' = 120

Loading: 2 equal loads

Tension Reinforcement: Plain round bars, hooked; f_y = 37,500 to 45,700 psi

Web Reinforcement: Bent-Up bars, hooked

Concrete Strength: f'_c = 4770 psi for a zone 4 in. deep and 54 in. long at top center of each beam; f'_c = 3040 to 3770 for the remainder

Age at Test: About 60 days

Beam	p o/o	x* in.	α deg.	r o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$
16B6.1	3.65	9.6	45	0.56	210	40.2	T	30.3	1.33	0.94
6.2	"	"	"	"	"	40.0	T	30.3	1.32	0.93
16B7.1	3.64	9.6	45	0.56	"	42.2	T	30.3	1.39	0.98
7.2	"	"	"	"	"	40.8	T	30.3	1.35	0.95
16B8.1	3.60	9.6	45	0.80	320	40.8	T	30.1	1.36	0.83
8.2	"	"	"	"	"	40.0	T	30.1	1.33	0.81
16B9.1	3.65	12.0	45	0.80	370	41.9	T	30.3	1.38	
9.2	"	"	"	"	"	41.5	T	30.3	1.37	
16B10.1	3.55	16.8	45	3.28	1490	36.3	T	30.0	1.21	
10.2	"	"	"	"	"	31.5	T	30.3	1.05	
16B18.1	3.62	16	28	1.96	900	45.5	T,DT	30.2	1.51	
18.2	"	"	"	"	"	40.9	T	30.2	1.36	
16B19.1	3.63	16	45	1.29	590	37.7	T	30.2	1.25	
19.2	"	"	"	"	"	41.1	T	30.2	1.36	

* Distance from load to first bent-up bar.

TABLE 29

TESTS BY RICHART, SERIES 1911 (2)
SIMPLE-SPAN RECTANGULAR BEAMS WITH BENT-UP BARS

Dimensions: b = 8; d = 10; D = 12; a = 24; a/d = 2.4; L = 72; L* = 78

Loading: 2 equal loads at 1/3-points

Tension Reinforcement: Three 3/4-in. plain round bars, hooked
$p = 0.0165$; f_y = about 38,000 psi

Web Reinforcement: One 3/4-in. round bar bent up, α = about 27 deg.

Concrete Strength: Tests on 6 by 8 by 40-in. control beams; reduced to cylinder strength by $f_c' = 6.7 f_r$

Age at Test: Around 60 days

Beam	f_c' psi	r* o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	P_s Eq. 18 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$
292.1	1760	0.50	190	30.7	DT	25.5	1.20	0.87
2	"	"	"	28.9	DT	25.5	1.14	0.83
3	"	"	"	29.8	DT	25.5	1.17	0.85

* r computed as $r = \frac{A_w}{b\ a\ \sin 27^\circ}$.

TABLE 30

AMOUNT OF WEB REINFORCEMENT REQUIRED TO PREVENT SHEAR FAILURES
NORMAL ACI BEAMS WITHOUT COMPRESSION REINFORCEMENT

f'_c psi	p o/o ACI*	p/f'_c 10^{-5} in^2/lb	n Eq. 16	k Eq. 14	$\frac{M_s}{bd^2 f'_c}$ Eq. 18
2000	0.91	0.46	10.0	0.345	0.166
2500	1.13	0.45	9.0	0.351	0.160
3000	1.36	0.45	8.3	0.376	0.164
3750	1.72	0.46	7.7	0.398	0.160

* Steel percentages as given by ACI Code balanced design requirements for f_s = 20,000 psi and $f_c = 0.45\ f'_c$.

f'_c	q Eq. 31	$\frac{M_f}{bd^2 f'_c}$ Eq. 29	Ratio M_f/M_s	rf_{yw} psi Eq. 26	r(o/o) computed for f_{yw}(ksi)= 40	45	50
			f_y = 40,000 psi				
2000	0.182	0.169	1.02	11	0.03	0.02	0.02
2500	0.181	0.167	1.04	22	0.06	0.05	0.04
3000	0.181	0.167	1.02	11	0.03	0.02	0.02
3750	0.184	0.168	1.05	25	0.06	0.06	0.05
			f_y = 45,000 psi				
2000	0.205	0.188	1.14	69	0.17	0.15	0.14
2500	0.203	0.186	1.16	81	0.20	0.18	0.16
3000	0.204	0.186	1.14	69	0.17	0.15	0.14
3750	0.207	0.187	1.17	84	0.21	0.19	0.17
			f_y = 50,000 psi				
2000	0.228	0.207	1.25	126	0.32	0.28	0.25
2500	0.226	0.205	1.28	140	0.35	0.31	0.28
3000	0.227	0.205	1.25	126	0.32	0.28	0.25
3750	0.229	0.205	1.28	141	0.35	0.31	0.28

RANGE OF TEST VARIABLES FOR SIMPLE-SPAN T-BEAMS
UNDER TWO SYMMETRICAL CONCENTRATED LOADS

Test Series	Table No.	No. of Beams	No. of Shear Fail.	f' psi	A_s in^2	Reinf. in Flange	b in.	b' in.	d in.	e in.	a/d	F_t
				Beams Without Web Reinforcement								
Bach, Graf Heft 10(18)*	32	5	5?	2510-2690	3.9	None	19.7	7.9	13.9	3.9	2.83	0.815
Braune Myers(19)	33	2	2	1700	1.56	Yes	42	6	10.9	4.25	3.30	0.55
Richart Ser.1922(2)	34	2	2	3570;3610	3.91	None	20	8	21	6	1.71	0.76
Thompson Ferguson(20)	35	15	15	2540-3500	0.88	None	19;22	4.25	4.5-7	1.5;2.13	4.0-6.22	0.58-0.65
Ferguson Thompson(21)	36	15	15	3960-6580	1.58	None	17	4;7	8.25	1.5	3.39	0.65-0.75
		39	39?									
				Beams With Web Reinforcement								
Bach, Graf Heft 10(18)	32	55	50?	2650	3.9	None	19.7	7.9	13.9	3.9	2.83	0.82
Heft 12(22)	37	48	-	2580	3.9	None	19.7	7.9	13.4	3.9	2.94	0.80
Braune Myers(19)	33	8	-	1700	2.34;3.51	Yes	42	6	10.0-10.9	4.25	3.3-3.6	0.59;0.62
Richart Ser.1922(2)	34	6	-	3799-4346	3.91	None	20	8	21	6	1.71	0.76
Graf Heft 67(23)	38	8	6	1370-1540	6.77	Yes	53.2	9.9	21.3	3.9	2.77	0.63
		125	56									

* Numbers in parentheses refer to corresponding entries in the Bibliography.

TABLE 32

TESTS BY BACH AND GRAF, HEFT 10, 1911 (18)
SIMPLE-SPAN T-BEAMS UNDER TWO SYMMETRICAL CONCENTRATED LOADS

Dimensions: b = 19.7; b' = 7.9; D = 15.7; d = 13.9; e = 3.9; a = 39.4; a/d = 2.83; L = 118.1; L' = 133.9
Loading: 2 equal loads at 1/3-points
Tension Reinforcement: Two 1.57-in. plain round bars; A_s = 3.90 in^2; f_y = 43,600 psi
Anchorage: Hooks
Web Reinforcement: Plain round vertical stirrups
Reinforcement in Flange: None
Concrete Strength: Average f'_{cu} = 3530 psi, f'_c = 0.75 f'_{cu} = 2650 psi; variation from -8 to +12 percent
Age at Test: About 45 days
Number of Beams: 3 companion specimens in each group; 2 in groups c and d

Beams With Web Reinforcement											
Group	Size W.R. in.	s in.	r o/o	f_{yw} ksi	rf_{yw} psi	P_{test} kips	F_t	P_s Eq. 35 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$	Mode of Fail.
b	0.51	3.35	1.56	37.8	580	94.4	0.82	60.6	1.56	0.73	B,S ?
c	0.51	5.51	0.95	38.6	370	86.0	"	"	1.42	0.82	B,S ?
d	0.28	5.51	0.27	40.2	109	77.2	"	"	1.27	1.04	S
8	0.39	7.87	0.39	41.0	160	80.0	"	"	1.32	1.00	S
9	0.28	7.87	0.19	43.8	83	72.0	"	"	1.19	1.03	S
10	0.20	7.87	0.10	48.2	48	65.8	"	"	1.08	0.99	S
11	0.39	5.91	0.52	41.0	213	82.9	"	"	1.37	0.96	S
12	0.28	5.91	0.26	43.8	114	79.4	"	"	1.31	1.06	S
13	0.20	5.91	0.13	48.2	63	72.4	"	"	1.19	1.05	S
15	0.39	3.94	0.79	41.0	324	94.1	"	"	1.55	0.94	S
16	0.28	3.94	0.39	43.8	171	88.2	"	"	1.45	1.08	S
17	0.20	3.94	0.20	48.2	96	80.1	"	"	1.32	1.10	S
18	0.20	1.97	0.39	48.2	188	89.3	"	"	1.47	1.07	S
14	0.39	5.91	0.52	41.0	213	82.2	"	"	1.36	0.96	S
19	0.28	5.91	0.26	43.8	114	71.9	"	"	1.19	0.97	S

TABLE 32 (CONT'D)

TESTS BY BACH AND GRAF, HEFT 10, 1911 (18)
SIMPLE-SPAN T-BEAMS UNDER TWO SYMMETRICAL CONCENTRATED LOADS

Beams With Web Reinforcement

Group	Size W.R. in.	s in.	r o/o	f_{yw} ksi	rf_{yw} psi	P_{test} kips	F_t	P_s Eq. 35 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$	Mode of Fail.
20	0.28	5.91	0.26	43.8	114	74.4	0.82	60.6	1.23	1.00	S
21	.79 by .08	5.91	0.53	57.5	305	87.5	"	"	1.44	0.90	S
22	.79 by .08	5.91	0.39	52.2	141	77.2	"	"	1.27	0.99	S
23	0.28	5.91	0.26	43.8	114	67.6	"	"	1.12	0.92	S

Beams Without Web Reinforcement

Beam	f'_c psi	P_{test} kips	A_c in^2	F_t	$\frac{M_{test}}{A_c d f'_c F_t}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
e-330	2650?	57.3	89.9	0.815	0.418	0.93	S,B
331	2650?	48.5	"	"	0.354	0.79	S,B
7-441	2570	52.9	"	"	0.398	0.88	S,B
442	2510	52.9	"	"	0.408	0.89	S,B
444	2690	57.3	"	"	0.412	0.92	S,B

TESTS BY BRAUNE AND MYERS, 1917 (19)
SIMPLE-SPAN T-BEAMS UNDER TWO SYMMETRICAL CONCENTRATED LOADS

Dimensions: $b = 42$; $b' = 6$; $D = 12$; $e = 4.25$, $a = 36$; $L = 108$; $L' = 120$

Loading: Two equal loads at 1/3-points

Tension Reinforcement: 5/8-in. square twisted bars; f_y = 40,320 and 66,350 psi for the two bars tested

Reinforcement in Flange: Four 3/8-in. round long. bars; 3/8-in. square transverse bars at 8 in.; all beams except I

Web Reinforcement: 1/4-in. square plain bars and bent-up bars

Concrete Strength: Average f'_{cu} = 2270 psi; $f'_c = 0.75\ f'_{cu}$ = 1700 psi

Age at Test: 90 days

Beams Without Web Reinforcement

Beam	p* o/o	d in.	a/d	P_{test} kips	Mode of Fail.	A_c in^2	F_t	$\frac{M_{test}}{A_c d f'_c F_t}$	Ratio $\frac{M_{test}}{M_s}$
I-1	0.34	10.9	3.30	33.4	S	109	0.53	0.562	1.14
2	"	"	"	28.8	S	"	"	0.484	0.98

Beams With Web Reinforcement

Beam	p** o/o	d in.	a/d	Web Reinf.	α deg.	r o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	F_t	P_s Eq. 35 kips	Ratio $\frac{P_{test}}{P_s}$
II-1	0.51	10.9	3.30	Stirr.	90	0.52	350	92.0	T	0.59	39.1	2.36
2	"	"	"		"	"	"	86.0	T	"	"	2.20
III-1	0.51	10.9	3.30	3B-up		?	?	97.4	T	0.59	39.1	2.49
2	"	"	"	Bars				95.4	T	"	"	2.44
IV-1	0.91	10.0	3.60	Stirr.	90	2.36	1580	129.6	T	0.62	43.7	2.97
2	"	"	"	+B-up	45	"	"	139.2	T	"	"	3.19
V-1	0.91	10.0	3.60	Stirr.	90	2.82	1870	139.2	T	0.62	43.7	3.19
2	"	"	"	+B-up	45	"	"	139.2	T	"	"	3.19

* Bars not hooked.

** Bars hooked.

TABLE 34

TESTS BY RICHART, SERIES 1922 (2)
SIMPLE-SPAN T-BEAMS UNDER TWO SYMMETRICAL CONCENTRATED LOADS

Dimensions: b = 20; b' = 8; D = 24; d = 21, e = 6, a = 36, a/d = 1.71
L = 108; L' = 120

Loading: 2 equal loads at 1/3-points

Tension Reinforcement: Four 1 1/8-in. corrugated round bars, p = 0.0093, f_y = 52,400 psi

Anchorage: Hooks

Web Reinforcement: Plain round vertical stirrups

Reinforcement in Flange: None

Age at Test: About 60 days

Beams Without Web Reinforcement

Beam	f'_c psi	P_{test} kips	Mode of Fail.	A_c in.^{2}	F_t	$\frac{M_{test}}{A_c d f'_c F_t}$	Ratio $\frac{M_{test}}{M_s}$	Mode of Fail.
2210.1	3610	180.3	DT	125.0	0.76	0.451	1.10	S
2210.2	3570	167.2	DT	125.4	"	0.421	1.03	S

Beams With Web Reinforcement

Beam	f'_c psi	Size W.R. in.	s in.	r o/o	f_{yw} ksi	rf_{yw} psi	P_{test} kips	Mode of Fail.	F_t	P_s Eq.35 kips	Ratio $\frac{P_{test}}{P_s}$
226.1	4037	3/8	4	1.38	42.9	592	259.5	T	0.76	172.7	1.50
2	4331	"	"	"	"	"	245.5	T	"	178.1	1.38
227.1	3799	1/2	7	1.40	40.1	561	258.5	T	"	167.5	1.54
2	4346	"	"	"	"	"	265.8	T	"	178.3	1.49
228.1	4058	5/8	11	1.39	39.6	550	261.4	T	"	173.0	1.51
2	4152	"	"	"	"	"	257.2	T	"	174.8	1.47

TABLE 35

TESTS BY THOMPSON AND FERGUSON, 1950 (20)
SIMPLE-SPAN T-BEAMS UNDER TWO SYMMETRICAL CONCENTRATED LOADS

Dimensions: a = 28; L = 84; L' = 96
Loading: Two equal loads at 1/3-points
Tension Reinforcement: Two 3/4-in. round deformed bars, inter. grade, A_s = 0.88 in^2
End Anchorage: Welded anchorage plate
Web Reinforcement: None
Reinforcement in Flange: None
Shoulders: Width = b"; depth from top of beam = e" = 0
Age at Test: 28 days
Series H-B and K-B: Beams with B-tile considered in analysis; comp. strength of B-tile = 4160 psi; 5/8-in. layer of tile included in the overall dimensions of beams
Mode of Failure: All beams failed in shear

Beam	f'_c psi	P_{test} kips	A_c in.^{2}	F_t	$\frac{M_{test}}{A_c d f'_c F_t}$	Ratio $\frac{M_{test}}{M_s}$
b=19;b'=4.25;b"=0;d=7;D=7.5;e=1.5;e"=0;a/d=4.0;d/e=4.67;b/b'=4.47						
N-1	3000	10.68	30.67	0.65	0.357	0.82
2	2990	10.76	30.67	"	0.361	0.83
3	2540	9.66	30.97	"	0.378	0.83
b=22;b'=4.25;b"=0;d=4.5;D=5.5;e=1.5;e"=0;a/d=6.22;d/e=3.00;b/b'=5.18						
G-4	3320	6.30	30.99	0.58	0.326	0.78
5	3150	7.10	31.28	"	0.383	0.90
6	3170	7.90	31.19	"	0.425	1.00
b=19;b'=4.25;b"=7;d=6.25;D=7.5;e=1.5;e"=3.5;a/d=4.48;d/e=4.17;b/b'=4.47						
L-1	3150	12.30	30.95	0.61	0.463	1.08
2	3280	13.40	30.81	"	0.487	1.15
3	3220	12.30	30.88	"	0.454	1.07
b=22;b'=4.25;b"=0;d=4.5;D=5.5;e=2.13;e"=0;a/d=6.22;d/e=2.11;b/b'=5.18						
HB-2	3270	9.14	31.09	0.60	0.470	1.11
5	3150	9.14	31.19	"	0.487	1.13
8	3020	8.90	31.48	"	0.489	1.12
b=19;b'=4.25;b"=8.25;d=6.25;D=7.5;e=2.13;e"=4.13;a/d=4.48;d/e=2.93;b/b'=4.47						
KB-1	3340	13.78	34.44	0.62	0.435	1.03
4	3350	12.25	34.44	"	0.385	0.92
7	3500	14.76	34.20	"	0.447	1.08

TABLE 36

TESTS BY FERGUSON AND THOMPSON, 1953 (21)
SIMPLE-SPAN T-BEAMS UNDER TWO SYMMETRICAL CONCENTRATED LOADS

Dimensions: b = 17; d = 8.25; D = 9.5; e = 1.5; a = 28; a/d = 3.39; L = 64; L'=72
Loading: Two equal loads at 1/3-points
Tension Reinforcement: Two No. 8 deformed bars, rail steel; $A_s = 1.58 \text{ in}^2$
End Anchorage: Welded steel block at each end
Web Reinforcement: None
Reinforcement in Flange: None
Age at Test: Around 28 days
Shoulders: Width = b", depth from top of beam = e"
Mode of Failure: All beams failed in shear

Beam	f'_c psi	b' in.	b" in.	e" in.	d/e	b/b'	P_{test} kips	A_c in.^{2}	F_t	$\frac{M_{test}}{A_c d f'_c F_t}$	Ratio $\frac{M_{test}}{M_s}$
A 1	4310	4	--	--	5.5	4.25	13.06	31.38	0.65	0.254	0.68
2	3960	"	--	--	"	"	12.12	31.54	"	0.256	0.65
3	5090	"	--	--	"	"	15.12	31.10	"	0.252	0.74
4	5070	"	--	--	"	"	14.22	31.10	"	0.238	0.70
5	6580	"	--	--	"	"	15.22	30.74	"	0.198	0.72
6	5610	"	--	--	"	"	16.00	30.58	"	0.246	0.78
D 1	4540	7	--	--	5.5	2.43	21.90	35.16	0.75	0.312	0.85
2	4290	"	--	--	"	"	23.40	35.37	"	0.350	0.93
B 1	5175	4	7	4	5.5	4.25	15.94	34.81	0.65	0.231	0.69
2	4860	"	"	"	"	"	14.20	35.02	"	0.218	0.62
3	5800	"	"	"	"	"	17.72	34.60	"	0.231	0.75
4	6290	"	"	"	"	"	19.72	34.39	"	0.238	0.83
5	5950	"	"	"	"	"	17.22	34.46	"	0.220	0.73
C 1	4860	4	7	6	5.5	4.25	19.74	35.02	0.67	0.295	0.84
2	4860	"	"	"	"	"	17.44	35.02	"	0.261	0.75

TESTS BY BACH AND GRAF, HEFT 12, 1911 (22)
SIMPLE-SPAN T-BEAMS WITH BENT-UP BARS UNDER TWO SYMMETRICAL CONCENTRATED LOADS

Dimensions: b = 19.7; b' = 7.9; D = 15.7; d = around 13.4; e = 3.9; a = 39.4; a/d = 2.94; L = 118.1;
L' = 133.9
Loading: 2 equal loads at 1/3-points
Tension Reinforcement: From 4 to 7 plain round bars; A_s = about 3.9 in^2; f_y = about 47,000 psi
Anchorage: Only hooked bars included
Web Reinforcement: Bent-up bars
Reinforcement in Flange: None
Concrete Strength: Average f'_{cu} = 3440 psi; $f'_c = 0.75 f'_{cu}$ = 2580 psi; Variation from -8.3 to +7.0 percent
Age at Test: Around 45 days
Number of Beams: Three companion specimens in each group
Calculated Quantities: F_t = 0.80; P_s = 56.8 kips (from Eq. 35)
* Cr = crushing at hooks.

Group	P_{test} kips	Ratio $\frac{P_{test}}{P_s}$	No. of B-up Layers	α deg.	No. and Area of Bent-up Bars No-(in^2)	No. and Area Straight Bars No-(in^2)	Spacing Between Bends, From Load (in.)	r o/o	rf_{yw} psi	Mode* of Fail.
25	76.0	1.34	1	18	2-1.91	2-1.92	0			Cr
29	92.6	1.63	"	45	3-1.79	3-2.10	10.8			Cr
50	81.9	1.44	"	"	3-1.77	3-2.00	14.8			Cr
31	83.8	1.48	2	45	2-1.78;2-1.78	1-0.35	2.0-25.6	1.15	540	Cr
34	86.7	1.53	"	"	2-1.78;2-1.78	1-0.35	0-27.6	1.15	540	Cr
33	92.6	1.63	"	"	2-1.78;2-1.78	1-0.35	2.0-25.6	1.15	540	Cr
47	105.8	1.86	"	"	2-1.78;2-1.78	1-0.35	3.9-17.3	1.50	710	T
45	86.9	1.53	"	30	2-1.78;2-1.78	1-0.35	2.0-10.8	3.52	1650	Cr
46	95.2	1.68	"	"	2-1.78;2-1.78	1-0.35	2.0-21.7	1.90	890	Cr
36	101.4	1.79	3	45	1-0.89;2-0.88;2-0.88	1-1.25	0-12.8-9.8	1.24	580	T,Cr
38	109.1	1.92	"	"	1-0.89;2-0.88;1-0.88	2-1.25	0-12.8-8.5	1.24	580	T
48	100.7	1.77	"	"	1-0.89;2-0.88;2-0.88	1-1.25	3.9-10.8-7.9	1.08	510	T,Cr
49	107.3	1.89	"	"	1-0.89;2-0.88;1-0.95	2-1.18	3.9-10.8-6.5	1.08	510	T
43	90.0	1.58	"	30	1-0.89;2-0.88;2-0.88	1-1.25	2.0-14.4-9.9	1.37	640	Cr
44	99.9	1.76	"	"	1-0.89;2-0.88;1-0.95	2-1.18	2.0-14.0-10.0	1.34	630	Cr
40	100.3	1.77	5	45	Five-1-0.54	1-1.25	1.2-8.3-8.1-5.9-8.5	1.02	480	T,Cr
42	104.3	1.84	"	"	Five-1-0.54	2-1.25	1.0-8.5-8.1-5.9-8.5	1.02	480	T

TESTS BY GRAF, HEFT 67, 1931 (23)
SIMPLE-SPAN T-BEAMS UNDER TWO SYMMETRICAL CONCENTRATED LOADS

Dimensions: b = 53.2, b' = 9.9, D = 23.6, d = 21.3, e = 3.9, a = 59.1, a/d = 2.77, L = 177.2; L' = 205

Loading: 2 equal loads at 1/3-points

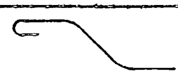
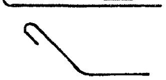
Tension Reinforcement: Ten 0.866-in. plain round bars; A_s = 6.77 in^2; f_y = 46,000 psi; all bars hooked

Reinforcement in Flange: Four 0.28-in. long. plain round bars; 0.28-in. transverse bars at 4.5 in., under loads at 2.5 in.; f_y = 48,000 psi

Web Reinforcement: Five long. bars bent up at 45 degrees, s = about 10.2 in.; 0.28-in. vert. stirrups at 7.1 in.

Concrete Strength: Tests on 7.9-in. cubes; reduced to cyl. strength by $f'_c = 0.75\ f'_{cu}$

Age at Test: Around 30 days

Beam	f'_c psi	Type of Bent-up Bars	r o/o	rf_{yw} psi	P_{test} kips	Mode of Fail.	F_t	P_s Eq. 35 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_{sw}}$	Ratio $\frac{P_{test}}{P_f}$
III 6-1	1540		1.33	638	231	T	0.63	85	2.72	1.20	1.11
2	"		"	"	220	T	"	85	2.59	1.14	"
III 7-1	1480		"	"	170	S,Cr*	"	82	2.07	0.91	0.82
2	"		"	"	165	S,Cr	"	82	2.01	0.88	"
III 8-1	1370		"	"	209	S,Cr	"	76	2.75	1.20	0.94
2	"		"	"	176	S,Cr	"	76	2.32	1.02	"
III 9-1	1410		"	"	209	S,Cr	"	78	2.68	1.18	0.96
2	"		"	"	182	S,Cr	"	78	2.33	1.02	"

* Cr = Crushing at hooks.

TESTS BY RICHART AND LARSEN, SERIES 1917 (25)
RESTRAINED BEAMS WITH BENT-UP BARS

Dimensions: b = 8; d = 15

Spans: f = 32 in.; g = 48 in.; h = 48 in.; L = 216 in.

Loading: $P_1 = P/4$; $P_2 = P/4$, M_A = 8P in-k; M_B = 4P in-k;

Long. Reinforcement: Eight 5/8-in. round plain bars at support, from 4 to 8 bars at midspan (See Fig. 19), f_y = about 37,600 psi; p = about 1.95 o/o

Web Reinforcement: 3/8-in. round plain vertical stirrups, f_{yw} = 45,100 psi

Age at Test: 60 days

Beam	f'_c	Bent-up Bars					Stirrups			P_{test}	Mode of Fail.	P_s Eq. 18	Ratio	Ratio
		No. of Bars		α	s_b	s	No.	s_s	s				$\frac{P_{test}}{P_s}$	$\frac{P_{test}}{P_t}$
	psi	Total	Layers	deg.	in.	in.		in.	in.	kips		kips		
380.1	3060	--	--	--	--	--	--	--	--	102.8	DT	127.3	0.81	0.71
2	3665	--	--	--	--	--	--	--	--	104.0	DT	139.6	0.75	0.70
400.1	3158	4	1	22	8	--	--	--	--	151.0	DT	129.5	1.17	1.04
2	3165	"	"	"	"	--	--	--	--	149.7	DT	129.8	1.15	1.03
382.1	3315	5	2	22	8	12	--	--	--	175.5	T,DT	133.2	1.32	1.19
2	2748	"	"	"	"	"	--	--	--	183.7	T,Cr,DT	120.0	1.53	1.27
386.1	2870	6	3	32.5	8	8	--	--	--	188.2	T,Cr	123.1	1.53	1.31
2	3525	"	"	"	"	"	--	--	--	188.0	T,Cr	136.9	1.37	1.27
391.1	2892	6	3	32.5	12	8	--	--	--	187.8	T	123.7	1.52	1.31
2	3495	"	"	"	"	"	--	--	--	172.0	T,DT	136.4	1.26	1.17
392.1	2818	6	3	32.5	16	8	--	--	--	146.4	DT	121.9	1.20	1.02
2	2795	"	"	"	"	"	--	--	--	176.4	T,DT	121.2	1.45	1.23
383.1	3082	6	3	32.5	8	12	--	--	--	183.2	T	128.0	1.43	1.26
2	2950	"	"	"	"	"	--	--	--	181.5	T,Cr,DT	125.3	1.45	1.26
385.1	2985	5	2	32.5	8	12	--	--	--	176.3	T,Cr	125.4	1.41	1.23
2	3362	"	"	"	"	"	--	--	--	190.0	T	133.5	1.42	1.31
387.1	3398	4	2	32.5	8	16	--	--	--	182.3	T,Cr,DT	133.9	1.36	1.26
2	2965	"	"	"	"	"	--	--	--	168.4	T,DT	124.8	1.35	1.12

TABLE 39 (CONT'D)

TESTS BY RICHART AND LARSEN, SERIES 1917 (23)
RESTRAINED BEAMS WITH BENT-UP BARS

Beam	f'_c	Bent-up Bars: No. of Bars Total	Bent-up Bars: No. of Bars Layers	Bent-up Bars: α	Bent-up Bars: s_b	Bent-up Bars: s	Stirrups: No.	Stirrups: s_s	Stirrups: s	P_{test}	Mode of Fail.	P_s Eq. 18	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_f}$
	psi			deg.	in.	in.		in.	in.	kips		kips		
388.1	3260	4	1	32.5	14	--	--	--	--	173.8	T,B,Cr	131.2	1.32	1.20
2	2970	"	"	"	"	--	--	--	--	143.2	B,DT	125.0	1.15	1.04
389.1	3210	4	1	32.5	14	--	2	4	7	174.0	T,DT	131.4	1.32	1.19
2	3102	"	"	"	"	--	"	"	"	169.0	T	128.8	1.31	1.16
390.1	2905	8	4	45	8	8	--	--	--	181.2	T,Cr	124.9	1.45	1.24
2	2735	"	"	"	"	"	--	--	--	186.0	T,Cr	121.0	1.53	1.26
393.1	3155	6	3	45	8	8	--	--	--	164.0	T,Cr	129.4	1.27	1.06
2	2325	"	"	"	"	"	--	--	--	170.0	T,DT	108.5	1.56	1.23
394.1	3145	6	3	45	12	8	--	--	--	172.3	T,Cr	129.6	1.33	1.14
2	3355	"	"	"	"	"	--	--	--	185.4	T,Cr,DT	134.1	1.38	1.26
395.1	3120	6	3	45	16	8	--	--	--	180.6	T,Cr,DT	129.2	1.40	1.24
2	3015	"	"	"	"	"	--	--	--	167.0	T,Cr,DT	126.6	1.32	1.15
384.1	3080	6	3	45	8	12	--	--	--	176.7	T,Cr,DT	128.7	1.37	1.20
2	3442	"	"	"	"	"	--	--	--	178.6	T,Cr,DT	136.3	1.31	1.28
399.1	3352	5	2	45	12	12	--	--	--	176.1	T,Cr,DT	134.0	1.31	1.22
2	2810	"	"	"	"	"	--	--	--	184.9	T,Cr	122.1	1.51	1.29
396.1	3410	6	2	45	24	3	5	8	8	182.6	T,Cr	135.3	1.35	1.24
2	3362	"	"	"	24	10	"	"	"	165.0	T,Cr	134.1	1.23	1.12
397.1	3235	6	2	45	24	3	5	12	8	192.5	T,Cr	132.1	1.46	1.20
2	2682	"	"	"	"	"	"	"	"	179.6	T,Cr	119.2	1.50	1.27
398.1	2682	6	2	45	24	3	5	16	8	168.0	DT	117.9	1.42	1.19
2	2990	"	"	"	"	"	"	"	"	173.1	T,DT	125.6	1.38	1.19
381.1	3070	6	2	45	24	3	--	--	--	165.0	Cr,DT	127.7	1.29	1.14
2	3385	"	"	"	"	"	--	--	--	124.1	DT	134.3	0.92	0.88

TABLE 40a

TESTS BY MOODY, SERIES I, 1953 (12)
RESTRAINED BEAMS WITHOUT WEB REINFORCEMENT

Critical Section: Inner Loadpoint
Calculation of Ultimate Moment: Eq. (43); $T_A/T_B = 1$; $p_o = 2p$
Loading: See Fig. 20; $M_B = 5.33P$

Dimensions: b = 7; d = 12; t = 0.729; g/d = 2.67

Beam	f'_c psi	p o/o	P_{test} kips	k	A	$\frac{M_{test}}{bd^2 f'_c kA}$	Ratio $\frac{M_{test}}{M_s}$
1950 Series							
1 a	2510	2.86	77	0.623	0.578	0.451	0.98
b	2800	"	87	0.615	0.579	0.462	1.04
2 a	2370	3.76	75	0.673	0.571	0.436	0.94
b	2730	"	95	0.664	0.572	0.484	1.08
c	3790	"	94	0.641	0.575	0.356	0.89
3 a	2290	4.76	89	0.715	0.565	0.509	1.09
b	2970	"	101	0.696	0.568	0.455	1.04
4 a	4320	2.86	100	0.587	0.582	0.359	0.95
b	4040	"	89	0.591	0.581	0.340	0.88
5 a	4060	3.76	120	0.631	0.577	0.429	1.11
b	4040	"	110	0.631	0.577	0.396	1.02
6 a	4550	4.76	115	0.672	0.571	0.349	0.96
b	3570	"	120	0.685	0.570	0.455	1.11
7 a	4790	2.86	115	0.578	0.583	0.377	1.06
b	5000	"	100	0.580	0.583	0.313	0.91
8 a	4790	3.76	145	0.629	0.577	0.441	1.25
b	4690	"	110	0.630	0.577	0.341	0.95
9 a	5270	4.76	130	0.664	0.572	0.350	1.05
b	4650	"	130	0.670	0.572	0.386	1.07
1952 Series							
I g	4430	0.95	90	0.405	0.603	0.440	1.18
h	3540	1.47	89	0.486	0.591	0.462	1.12
i	3320	2.10	99	0.550	0.587	0.488	1.16
j	4850	1.47	105	0.469	0.596	0.409	1.16
k	3860	2.10	109	0.541	0.588	0.469	1.18
l	5100	1.47	107	0.467	0.596	0.399	1.17
m	4390	2.10	105	0.534	0.589	0.402	1.08
n	5240	1.47	118	0.466	0.596	0.429	1.28
o	5050	2.10	128	0.527	0.590	0.431	1.26
p	5970	2.86	133	0.572	0.584	0.353	1.17
q	4880	3.76	130	0.628	0.578	0.388	1.11
r	5930	4.76	140	0.659	0.574	0.330	1.09

TABLE 40b

TESTS BY MOODY, SERIES II AND IV, 1953 (12)
RESTRAINED BEAMS WITHOUT WEB REINFORCEMENT

Critical Section: Inner load point

Calculation of Ultimate Moment: Eq. (43); $T_A/T_B = 1$; $p_o = 2p$

Loading: See Fig. 20

Beam	f'_c psi	p o/o	P_{test} kips	k	A	$\frac{M_{test}}{bd^2 f'_c kA}$	Ratio $\frac{M_{test}}{M_s}$
			Series II;b=7;d=21;t=0.845;M_B=5.33P;g/d=1.52				
17a	2650	2.15	188	0.572	0.518	0.414	0.92
b	3000	"	170	0.561	0.519	0.337	0.78
18a	2170	2.72	220	0.626	0.510	0.523	1.11
b	2700	"	180	0.609	0.513	0.364	0.77
19a	3030	3.46	241	0.642	0.508	0.422	0.97
b	3240	"	219	0.637	0.508	0.361	0.85
20a	2890	4.25	235	0.679	0.502	0.412	0.94
b	2960	"	249	0.678	0.502	0.427	0.98
IIa	3820	0.54	130	0.322	0.547	0.324	0.81
b	3720	0.84	145	0.395	0.540	0.316	0.78
c	4040	1.20	168	0.446	0.534	0.302	0.78
d	3440	1.63	210	0.506	0.527	0.396	0.95
			Series IV; b=7;d=12;t=0.729;M_B=6.86P;g/d=4.0				
IVg	3390	0.95	63	0.419	0.601	0.502	1.20
h	3750	1.47	70	0.483	0.595	0.442	1.10
i	3490	2.10	68	0.548	0.587	0.412	1.00
j	3600	2.86	83	0.598	0.581	0.451	1.10
k	3630	3.76	88	0.644	0.576	0.444	1.09
l	3920	4.76	81	0.679	0.571	0.363	0.92

TABLE 40c

TESTS BY MOODY, SERIES VI and V; 1953 (12)
RESTRAINED BEAMS WITHOUT WEB REINFORCEMENT

Critical Section: Support

Calculation of Ultimate Moment: Eq. (18)

Loading: See Fig. 20

Beam	f'_c psi	p o/o	P_{test} kips	k	k+np'	$\frac{M_{test}}{bd^2 f'_c (k+np')}$	Ratio $\frac{M_{test}}{M_s}$
Series VI; b=7;d=11.75;t=0.817;p'=0.5p;M_A=6.4P;g/d=2.73							
VI a	4090	0.95	77	0.300	0.335	0.372	0.96
b	4160	1.47	129	0.351	0.406	0.505	1.31
c	3580	2.10	110	0.401	0.483	0.421	1.03
d	3900	2.86	118	0.435	0.543	0.369	0.93
e	4120	3.76	128	0.467	0.606	0.339	0.88
f	5570	2.10	140	0.383	0.455	0.365	1.14
g	5530	2.86	130	0.422	0.519	0.300	0.94
h	5300	3.76	155	0.457	0.586	0.330	0.99
i	6020	4.76	146	0.483	0.641	0.251	0.84
Series V; b=7;d=11.75;p'=0;M_A=5.33P;g/d=2.73							
V b	3770	1.47	64.0	0.379	--	0.247	0.62
d	3600	2.86	76.5	0.484	--	0.242	0.59
f	3380	4.76	74.5	0.574	--	0.212	0.51

TESTS BY MOODY, SERIES I, 1953 (12)
RESTRAINED BEAMS WITH WEB REINFORCEMENT

Critical Section: Inner load point
Calculation of Ultimate Moment: Eqs. (43), (28); $T_A/T_B = 1$; $p_o = 2p$
Loading: See Fig. 20; $M_B = 5.33$ P

Web Reinforcement: Stirrups of inter. grade deformed bars
Dimensions: b = 7; d = 12; t = 0.729; g/d = 2.67

Beam	f'_c psi	p o/o	Web Reinforcement Size No.	s in.	r o/o	f_{yw} ksi	rf_{yw} psi	P_{test} kips	k	A	M_s Eq. 43 k-in.	Ratio $\frac{M_{test}}{M_s}$	Ratio $\frac{M_{test}}{M_{sw}}$
Vertical Stirrups; 1950 Series													
10 a	3070	4.76	3	6	0.52	47.3	246	163	0.694	0.568	527	1.65	1.10
b	2810	"	"	"	"	"	"	138	0.700	0.568	500	1.47	0.99
11 a	3560	"	4	"	0.95	44.0	418	190	0.685	0.570	574	1.76	0.96
b	3180	"	"	"	"	"	"	174	0.692	0.569	539	1.72	0.94
12 a	4000	"	5	"	1.47	41.2	606	190	0.678	0.571	609	1.66	0.75
b	3220	"	"	"	"	"	"	159	0.691	0.569	542	1.57	0.71
Vertical stirrups; 1952 Series													
I s	3470	4.76	5	5	1.72	47.6	819	220	0.686	0.570	566	2.07	0.79
t	3700	"	"	4	2.14	47.6	1019	240	0.682	0.570	586	2.18	0.72
u	3740	2.86	3	6	0.52	53.8	280	160	0.596	0.582	525	1.62	1.04
v	3580	"	3	4.5	0.70	53.8	377	170	0.599	0.581	514	1.76	1.00
w	4210	"	4	6	0.95	45.8	435	180	0.589	0.582	554	1.73	0.93
x	3830	"	5	6	1.47	47.6	700	217	0.594	0.582	531	2.18	0.91
y	4790	4.76	3	6	0.52	53.8	280	220	0.669	0.578	661	1.77	1.13
z	4850	"	3	4.5	0.70	53.8	377	222	0.668	0.578	664	1.78	1.01
α	5070	"	4	6	0.95	45.8	435	260	0.666	0.573	667	2.08	1.11
β	5130	"	5	6	1.47	47.6	700	279	0.665	0.573	668	2.23	0.93

TABLE 41a (CONT'D)

TESTS BY MOODY, SERIES I, 1953 (12)
RESTRAINED BEAMS WITH WEB REINFORCEMENT

Beam	f'_c psi	p o/o	Web Reinforcement Size No.	s in.	r o/o	f_{yw} ksi	rf_{yw} psi	P_{test} kips	k	A	M_s Eq. 43 k-in.	Ratio $\frac{M_{test}}{M_s}$	Ratio $\frac{M_{test}}{M_{sw}}$
45-degree inclined stirrups													
13 a	3460	4.76	3	6	0.74	47.3	350	185	0.687	0.570	566	1.74	1.02
b	2860	"	"	"	"	"	"	170	0.699	0.568	505	1.79	1.05
14 a	3510	"	4	"	1.35	44.0	594	250	0.686	0.570	570	2.33	1.06
b	3600	"	"	"	"	"	"	240	0.684	0.570	577	2.22	1.01
15 b	3710	"	5	"	2.09	41.2	861	304	0.683	0.570	587	2.76	1.01
T-Beams; Vertical stirrups; b=23, b'=7, d=11.75, e=4; t=0.839 Critical Section: Support; Eqs. (43),(28); T_A/T_B=0.5; p_o=0.0714													
16 a	3610	4.76	5	6	1.47	41.2	606	271	0.636	0.675	611	2.36	1.07
b	3240	"	"	"	"	"	"	282	0.643	0.675	576	2.61	1.18

TABLE 41b

TESTS BY MOODY, SERIES IV AND II, 1953 (12)
RESTRAINED BEAMS WITH WEB REINFORCEMENT

Critical Section: Inner Load Point

Calculation of Ultimate Moment: Eqs. (43), (28); $T_A/T_B = 1$; $p_o = 2p$

Loading: See Fig. 20

Web Reinforcement: Vertical stirrups of inter. grade deformed bars

Beam	f'_c	p	Web Reinforcement					P_{test}	k	A	M_s	Ratio	Ratio
			Size	s	r	f_{yw}	rf_{yw}				Eq. 43	$\frac{M_{test}}{M_s}$	$\frac{M_{test}}{M_{sw}}$
	psi	o/o	No.	in.	o/o	ksi	psi	kips			k-in.		
Series IV; b=7;d=12;t=0.729;M_B=6.86P;g/d=4.0													
IV m	2860	4.76	3	6	0.52	53.8	280	146	0.698	0.568	504	1.99	1.28
n	3710	"	4	"	0.95	45.8	435	198	0.682	0.570	586	2.32	1.24
o	3420	"	5	"	1.47	47.6	700	218	0.687	0.570	562	2.66	1.11
Series II; b=7;d=21;t=0.845;M_B=5.33P;g/d=1.52													
21 a	3560	2.72	3	6	0.52	47.3	246	310	0.590	0.516	1373	1.20	0.80
b	3640	"	"	"	"	"	"	283	0.589	0.516	1387	1.09	0.73
22 a	3000	"	4	"	0.95	44.0	418	300	0.602	0.514	1247	1.28	0.70
b	2710	"	"	"	"	"	"	290	0.608	0.513	1169	1.32	0.72
23 a	3230	"	5	"	1.47	41.2	606	300	0.596	0.515	1301	1.23	0.56
b	3160	"	"	"	"	"	"	350	0.598	0.514	1284	1.45	0.66
II e	3420	"	"	5	1.72	43.5	748	390	0.593	0.516	1344	1.55	0.62
f	3330	"	"	4	2.14	"	931	340	0.595	0.515	1323	1.37	0.48

TABLE 42

TESTS BY GRAF, HEFT 80, 1935 (26)
SHEAR-PROPER TYPE OF FAILURES

(a) Small Rectangular beams

Dimensions: b = 7.9 in.; D = 11.8 in.; See Fig. 24.

Loading: See Fig. 24.

Reinforcement: 0.39-in. plain round bars; f_y = 49,000 psi; some bars bent as indicated below

Concrete Strength: Tests on 7.9-in. cubes; $f'_c = 0.75 f'_{cu}$ assumed

Age at Test: 14 days

Arrangement of Reinforcement

Group	Tot. No. of Reinf. Bars	No. of Bars at A-A Hor.	No. of Bars at A-A Bent.	α deg.	$A_s(1+\sin\alpha)$ in.^{2}	p_t Eq. 46 o/o
a	--	--	--	--	--	--
b	6	6	--	--	0.73	0.79
c	14	14	--	--	1.70	1.83
d	14	7	7	16	1.94	2.09
e	14	10	4	45	2.05	2.20
f	12	6	6	16	1.66	1.79
g	12	8	4	60	1.88	2.02

Physical Properties of Beams and Test Results

Beam	f'_c psi	y in.	x in.	p_t Eq. 46 o/o	P_{test} kips	v_{test} psi	v_c Eq. 47 psi	Ratio $\frac{v_{test}}{v_c}$
1 a	1590	6.7	1	--	88.2	474	499	0.95
b	"	"	"	0.79	110.2	593	667	0.89
c	"	"	"	1.83	154.3	829	889	0.93
d	"	"	"	2.09	154.3	829	944	0.88
e	"	"	"	2.20	154.3	829	968	0.86
2 a	1500	9.8	--	--	88.2	474	482	0.98
b	"	"	--	0.79	125.7	676	650	1.04
c	"	"	--	1.83	167.6	900	872	1.03
d	"	"	--	2.09	165.3	889	927	0.96
e	"	"	--	2.20	185.2	996	951	1.05
3 a	1600	9.8	1	--	77.2	415	501	0.83
b	"	"	"	0.79	132.3	711	669	1.06
c	"	"	"	1.83	158.7	853	891	0.96
d	"	"	"	2.09	165.3	889	946	0.94
e	"	"	"	2.20	176.4	949	970	0.98
4 a	2080	11.8	-	--	88.2	474	591	0.80
f	"	"	-	1.79	209.4	1126	972	1.16
g	"	"	-	2.02	231.5	1245	1021	1.22
5 a	1930	11.8	1	--	77.2	415	563	0.74
f	"	"	"	1.79	183.0	984	944	1.04
g	"	"	"	2.02	207.2	1114	993	1.12

TESTS BY GRAF, HEFT 80, 1935 (26)

(b) Large T-Beams

Dimensions: b = 49.2; b' = 9.8; D = 22.8; d = 21.4; e = 3.1; L = 137.8; L' = 161.4; x = 2.0
Loading: One load 11.8 in. from end support; see Fig. 24.
Tension Reinforcement: 0.63 and 0.71-in. plain round bars, hooked; f_y = 62,000 and 53,400 psi, resp.
Web Reinforcement: Bent-up bars and 0.24-in. round vert. stirrups at 7.9 in.
Reinforcement in Flange: Four 0.28-in. round horiz. bars; 0.28-in. round transverse bars at 4.9 in.
Concrete Strength: Tests on 7.9-in. cubes, $f'_c = 0.75\ f'_{cu}$ assumed
Age at Test: From 12 to 23 days

Beam	f'_c psi	Reinf. Bars at A-A Horiz.	Bent	α deg.	$A_s(1+\sin\alpha)$ in^2	p_t Eq. 46 o/o	P_{test} kips	V_{test} kips	v_{test} psi	v_c Eq. 47 psi	Ratio $\frac{v_{test}}{v_c}$
1246	1560	3-0.71 2-0.63	--	--	1.80	0.80	172	157	699	664	1.05
1247	1550	3-0.71 2-0.63	1-0.71 1-0.63	45	3.01	1.34	185	169	753	777	0.97
1270	1780	2-0.71 1-0.63	5-0.71 1-0.63	45	4.99	2.22	247	226	1004	1008	1.00
1271	1640	3-0.71	3-0.71 2-0.63	62.7	4.59	2.04	231	212	942	943	1.00
1272	3040	3-0.71	3-0.71 2-0.63	62.7	4.59	2.04	296	271	1206	1206	1.00

TABLE 45

OTHER SHEAR-PROPER TYPE OF FAILURES

Invest.	Beam	f'_c psi	b in.	D in.	p_t Eq. 46 o/o	rf_{yw} psi	a/d	x in.	x/D	P_{test} kips	f_w at Fail. o/o of f_{yw}	Ratio $\frac{P_{test}}{P_{sw}}$	Ratio $\frac{P_{test}}{P_f}$	V_{test} kips	v_{test} psi	v_c Eq. 47 psi	Ratio $\frac{v_{test}}{v_c}$
Clark	D1-1	3800	8	18	1.39	460	1.17	14.5	0.81	135.4		0.76	0.91	67.7	470	1210	0.39
	3	3560	"	"	"	"	"	"	"	115.4		0.67	0.78	57.7	401	1165	0.34
	D2-1	3480	"	"	1.39	610	"	"	"	130.4		0.69	0.89	65.2	453	1150	0.39
	2	3755	"	"	"	"	"	"	"	140.4		0.72	0.94	70.2	487	1202	0.41
	D3-1	4090	"	"	2.08	920	"	"	"	177.6		0.64	0.83	88.8	617	1412	0.44
	D4-1	3350	"	"	1.39	1220	"	"	"	140.4		0.55	0.96	70.2	487	1126	0.43
Moody Series III	30	3680	7	24	5.57	250	1.52	24	1.00	215	83	0.80	0.67	107.5	640	2081	0.31
	31	3250	"	"	"	420	"	"	"	228	67	0.73	0.72	114.0	679	2000	0.34
Moody Series II	21a	3560	7	24	4.76	250		24	1.00	310	100	0.82		103.3	615	1883	0.33
	b	3640	"	"	"	"		"	"	283	"	0.74		94.3	561	1898	0.30
	22a	3000	"	"	"	420		"	"	300	80	0.71		100.0	595	1778	0.33
	b	2710	"	"	"	"		"	"	290	"	0.73		96.7	576	1723	0.33
	23a	3250	"	"	"	610		"	"	300	55	0.57		100.0	595	1825	0.33
	b	3160	"	"	"	"		"	"	350	"	0.66		116.7	695	1808	0.38
	IIe	3420	"	"	"	750		"	"	390		0.62		130.0	774	1857	0.42
	f	3330	"	"	"	930		"	"	340		0.48		113.3	674	1840	0.37

TESTS BY GRAF, HEFT 67, SERIES II, 1931 (23)
SIMPLE-SPAN T-BEAMS UNDER ONE UNSYMMETRICAL CONCENTRATED LOAD

Dimensions: b = 49.2; b' = 9.8; D = 22.8; d = 20.7; e = 3.15; L = 212.6; L' = 240.2; a/d = 2.09 for short segment, 8.18 for long segment

Loading: One concentrated load 43.3 in. from support

Tension Reinforcement: Ten 0.87-in. round plain bars at load; hooked, f_y = about 46,000 psi

Reinforcement in Flange: Four 0.28-in. long. bars; 0.28-in. transverse bars at 4.9 in.; f_y = about 48,000 psi

Web Reinforcement: Bent-up bars and 0.28-in. vertical stirrups

Concrete Strength: Tests on 7.9-in. cubes, $f'_c = 0.75\ f'_{cu}$ = 1370 psi

Age at Test: 27 to 35 days

							Short Segment			Long Segment			
Group	Beam	P_{test} kips	F_t	P_s Eq. 35 kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_f}$	r o/o	rf_{yw} psi	$\frac{P_{test}}{P_{sw}}$	r o/o	rf_{yw} psi	$\frac{P_{test}}{P_{sw}}$	Mode of Fail.
1	1203	165.3	0.64	51.6	3.20	1.10	1.37	650	1.39	0.50	230	2.18	T
	1205	165.3	"	"	3.20	1.10	"	"	1.39	"	"	2.18	T
2	1204	132.3	"	"	2.56	0.88	"	"	1.11	0.03	20	2.44	S
	1206	121.3	"	"	2.35	0.81	"	"	1.02	"	"	2.24	S

TABLE 47

TESTS BY GRAF, HEFT 67, SERIES I, 1931 (23)
SIMPLE-SPAN T-BEAMS UNDER THREE CONCENTRATED LOADS

Dimensions: b = 49.2; b' = 9.8; D = 23.2; d = 20.8; e = 3.54; L = 212.6; L' = 240.2; M/Vd = 8.52 at midspan

Loading: Three equal and symmetrical concentrated loads, at midspan and at 35.4 in. from supports

Tension Reinforcement: Eleven 0.87-in. round plain bars at midspan, hooked, f_y = about 46,000 psi

Reinforcement in Flange: Four 0.28-in. long. bars; 0.28-in. transverse bars at 4.9 in.; f_y = about 48,000 psi

Web Reinforcement: Bent-up bars and 0.24-in. vertical stirrups

Concrete Strength: Tests on 7.9-in. cubes; $f'_c = 0.75 f'_{cu}$ = 1490 psi

Age at Test: 26 to 41 days

Group	Beam	P_{test} kips	F_t	M_s Eq. 35 in-k	At First Load rf_{yw} psi	At First Load $\frac{M_{test}}{M_s}$	At First Load $\frac{M_{test}}{M_{sw}}$	At Midspan rf_{yw} psi	At Midspan $\frac{M_{test}}{M_s}$	At Midspan $\frac{M_{test}}{M_{sw}}$	At Midspan $\frac{M_{test}}{M_f}$	Mode of Fail.
1	1197	209	0.64	2118	560	1.75	0.83	230	2.92	2.00	1.07	T at
	1200	218	"	"	"	1.83	0.86	"	3.04	2.08	1.11	Midspan
2	1198	198	"	"	560	1.66	0.78	20	2.77	2.65	1.01	T-S at
	1201	209	"	"	"	1.75	0.83	"	2.92	2.79	1.07	Midspan
3	1199	172	"	"	560	1.43	0.67	20	2.39	2.28	0.88	S at
	1202	187	"	"	"	1.56	0.74	"	2.60	2.49	0.95	Midspan

TABLE 46

TESTS BY BACH AND GRAF, HEFT 48, 1921 (27)
SIMPLE-SPAN T-BEAMS UNDER SIXTEEN EQUAL CONCENTRATED LOADS

Dimensions: b = 47.2, b' = 9.8; D = 27.6; d = 25.2; e = 3.94; L = 212.6; L' = 244.1

Loading: 16 equal and symmetrical concentrated loads. See Figs. 26-29.

Tension Reinforcement: Round plain bars, hooked

Reinforcement in Flange: Two 0.28-in. long. bars, 0.28-in. transverse bars at 3.9 in.

Web Reinforcement: Bent-up bars and 0.28-in. round vertical stirrups

Concrete Strength: Tests on 7.9-in. cubes, $f'_c = 0.75\ f'_{cu}$

Age at Test: 42 to 48 days

Beam	f'_c psi	A_s in^2	f_y ksi	Size of Bent-Up Bars in.	P_{test} kips	M_{test} in-k	F_t	M_s Eq. 35 in-k	Ratio $\frac{M_{test}}{M_s}$	M_f in-k	Ratio $\frac{M_{test}}{M_f}$	Mode of Fail.
1024	3230	5.79	50.5	None	105.8	2859	0.57	4210	0.68	7031	0.41	B?
1026	3250	5.73	50.5	0.98	262.3	6973	"	4210	1.65	6987	1.00	T
1025	3050	5.78	51.2	0.71	264.6	7031	"	4140	1.69	7204	0.98	T
1031	2750	5.63	49.8	0.59	211.6	5625	"	3750	1.50	6857	0.82	S
1032	2750	5.87	50.5	0.98	202.8	5390	"	3793	1.42	7119	0.76	S

TABLE 47

TESTS BY BACH AND GRAF, HEFT 20, 1912 (28)
SIMPLE-SPAN T-BEAMS UNDER EIGHT EQUAL CONCENTRATED LOADS

Number of Beams: Three in each group
Dimensions: $b = 23.6$; $b' = 7.9$; $D = 15.7$; $d = 13.6$; $e = 3.94$;
$L = 157.5$, $L' = 173.2$
Loading: 8 equal and symmetrical concentrated loads. See Fig. 31.
Tension Reinforcement: Round plain bars, numerous sizes from 0.39 to 1.57 in. in diam.; average $f_y = 46{,}000$ psi
Reinforcement in Flange: None
Web Reinforcement: Bent-up bars and 0.28-in. plain round vertical stirrups; $f_{yw} = 58{,}300$ psi
Concrete Strength: Tests on 11.8-in. cubes; $f'_c = 0.75\ f'_{cu} = 2490$ psi ± 7.7 percent
Age at Test: Around 45 days
Computed Quantities: $F_t = 0.77$; $M_s = 1259$ in-k; $P_s = 57.5$ kips, average $M_f = 2135$ in-k, $P_f = 97.5$ kips

Group	No. of Reinf. Bars	A_s in^2	Anch. Long. Bars	No. of B-Up Bars	α deg.	P_{test} kips	Ratio $\frac{P_{test}}{P_s}$	Ratio $\frac{P_{test}}{P_f}$	Mode of Fail.
51	2	3.90	None	-	--	47.0	0.82	0.48	B
52*	"	3.90	"	-	--	67.6	1.17	0.69	B
53	"	3.90	Hooks	-	--	51.5	0.90	0.53	B
54*	"	3.96	"	-	--	94.0	1.63	0.96	B
55	4	3.81	"	2	13	73.5	1.28	0.75	B
56*	"	3.80	"	"	"	100.5	1.75	1.03	T
57	6	3.86	None	4	45	90.5	1.57	0.93	B
58	"	3.86	Hooks	"	"	95.5	1.66	0.98	T
59	"	3.91	None	"	"	86.1	1.50	0.88	B
60	"	3.91	Hooks	"	"	95.5	1.66	0.98	T
61	7	3.92	None	5	"	85.2	1.48	0.87	B
62	"	3.94	Hooks	"	"	99.6	1.73	1.02	T
63	"	3.90	None	"	"	90.3	1.57	0.93	B
64*	"	3.90	Hooks	"	"	106.4	1.85	1.09	T
65	"	3.94	"	"	30	100.0	1.74	1.02	T
66	"	3.91	"	"	"	102.0	1.77	1.05	T

* Vertical stirrups along the entire span.

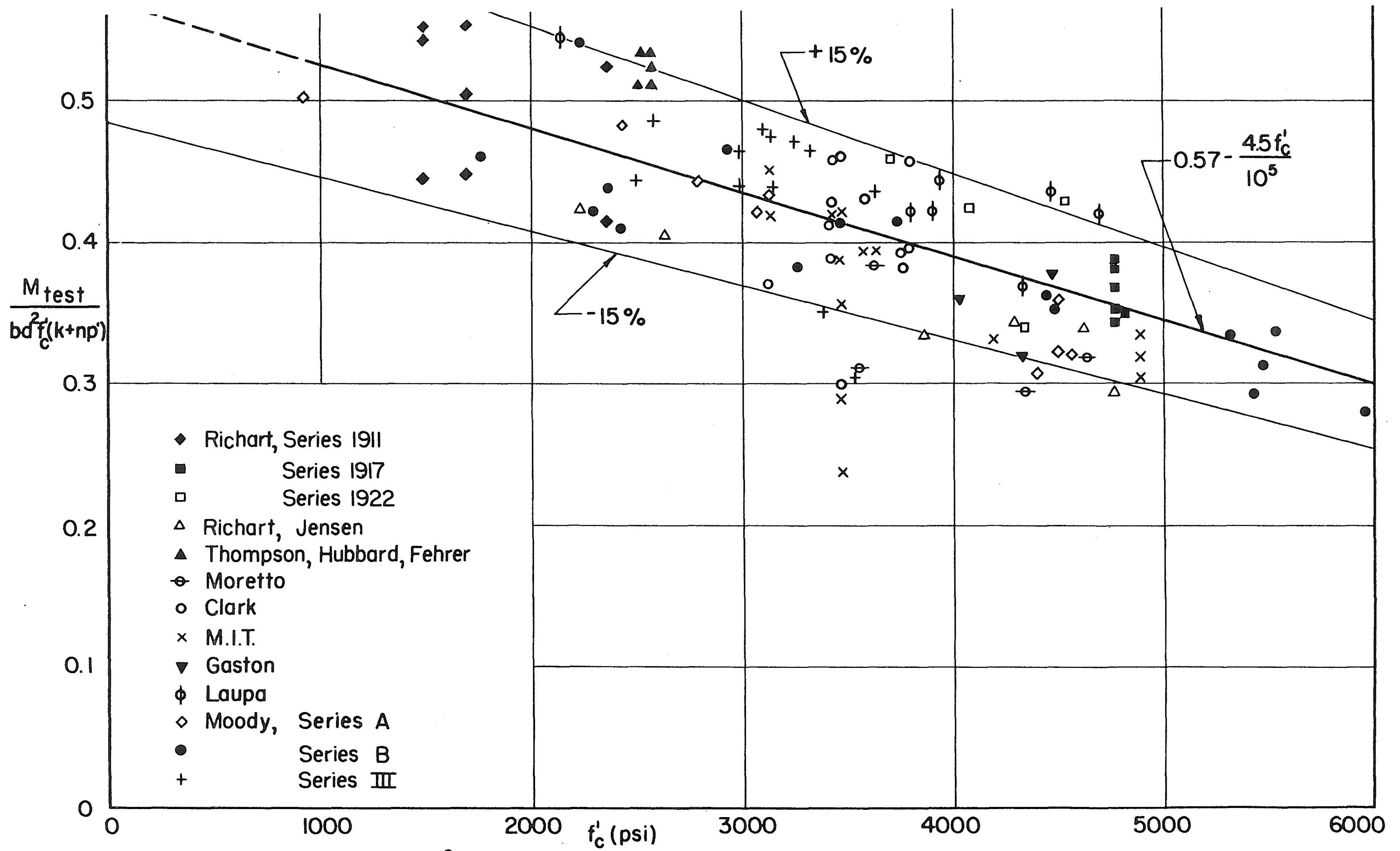


FIG. 1a $M_{test} / bd^2 f'_c (k+np')$ VERSUS CONCRETE STRENGTH

SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

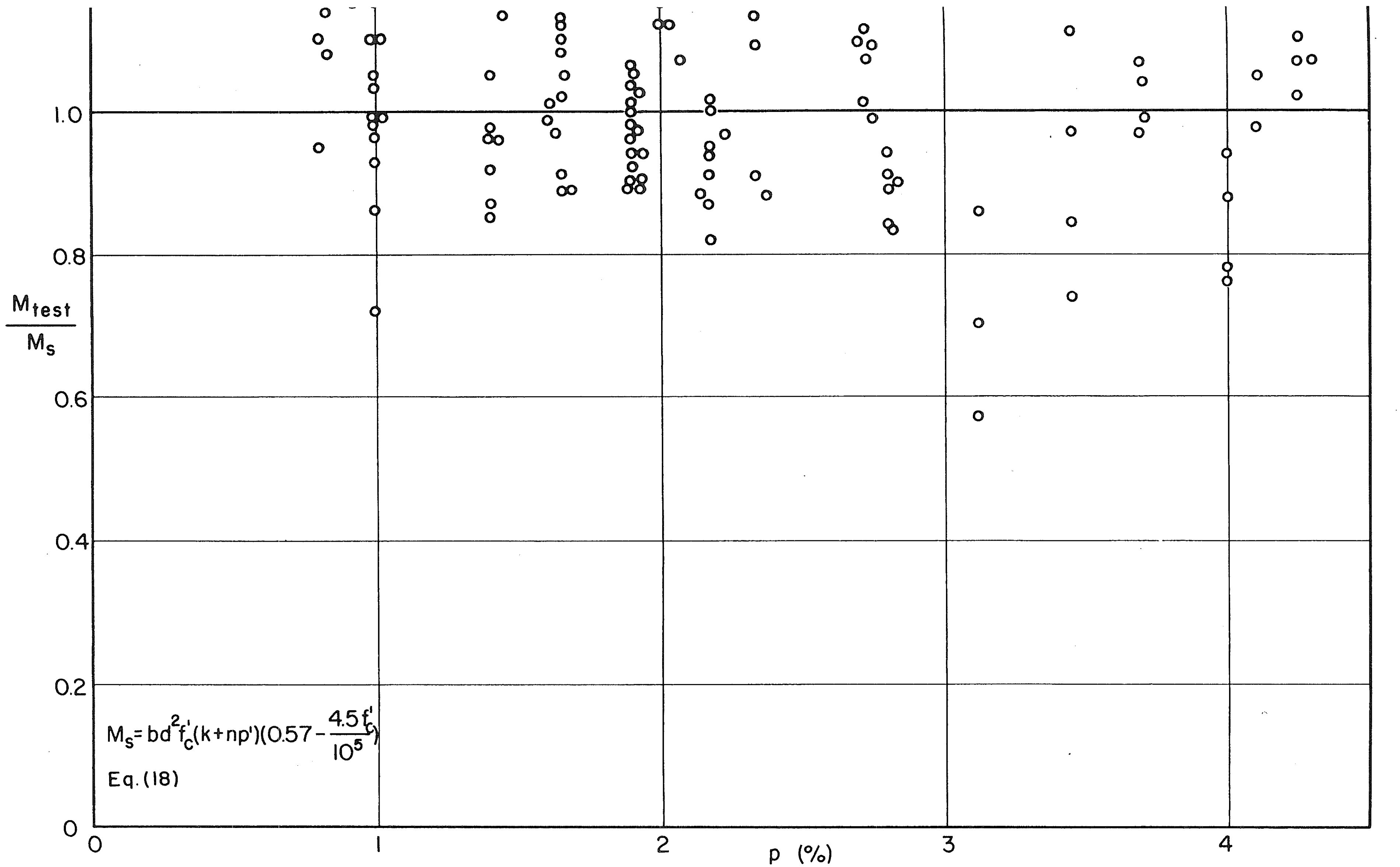


FIG. 1b

M_{test} / M_s VERSUS p

SIMPLE-SPAN RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT

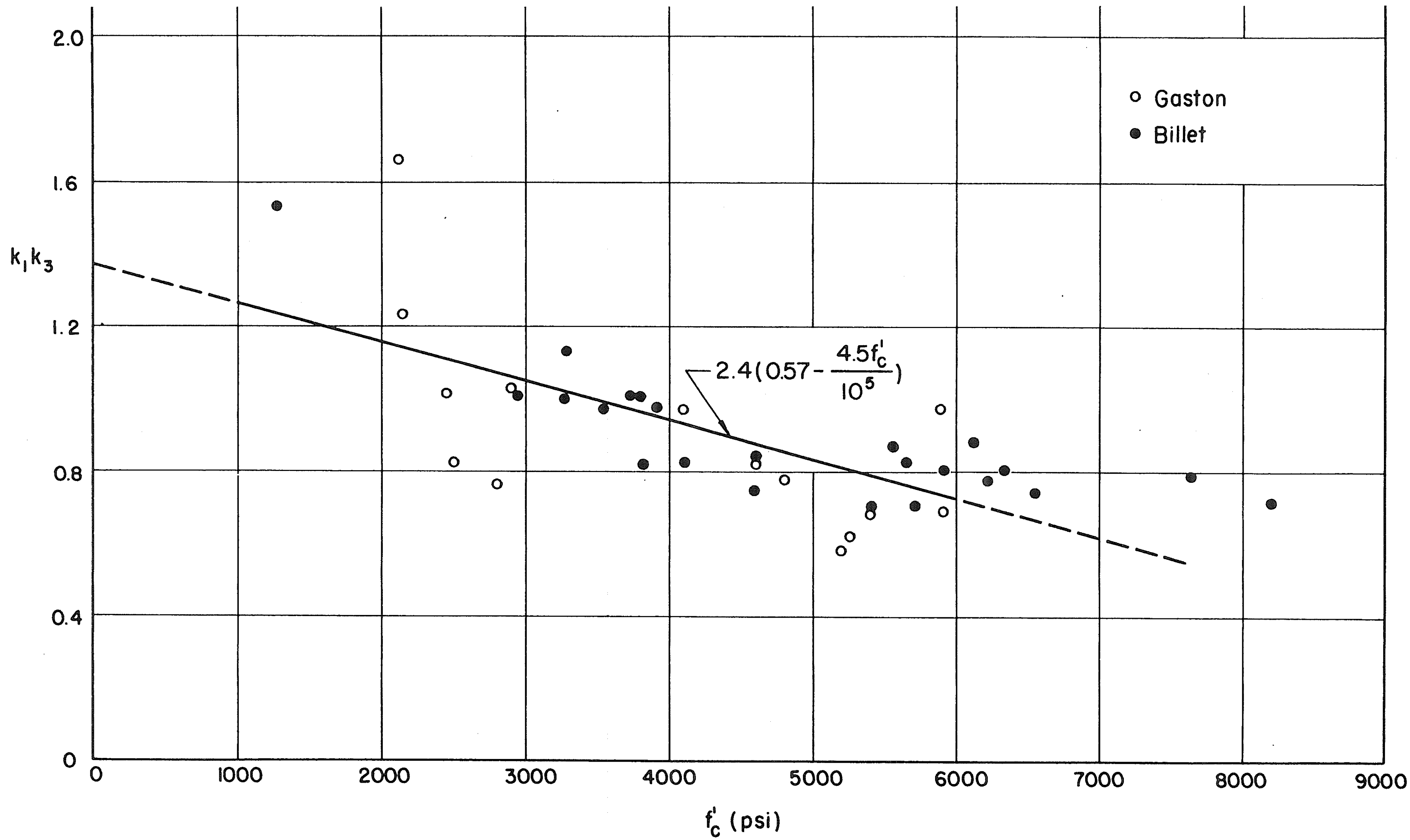


FIG. 2 k_1k_3 VERSUS CONCRETE STRENGTH

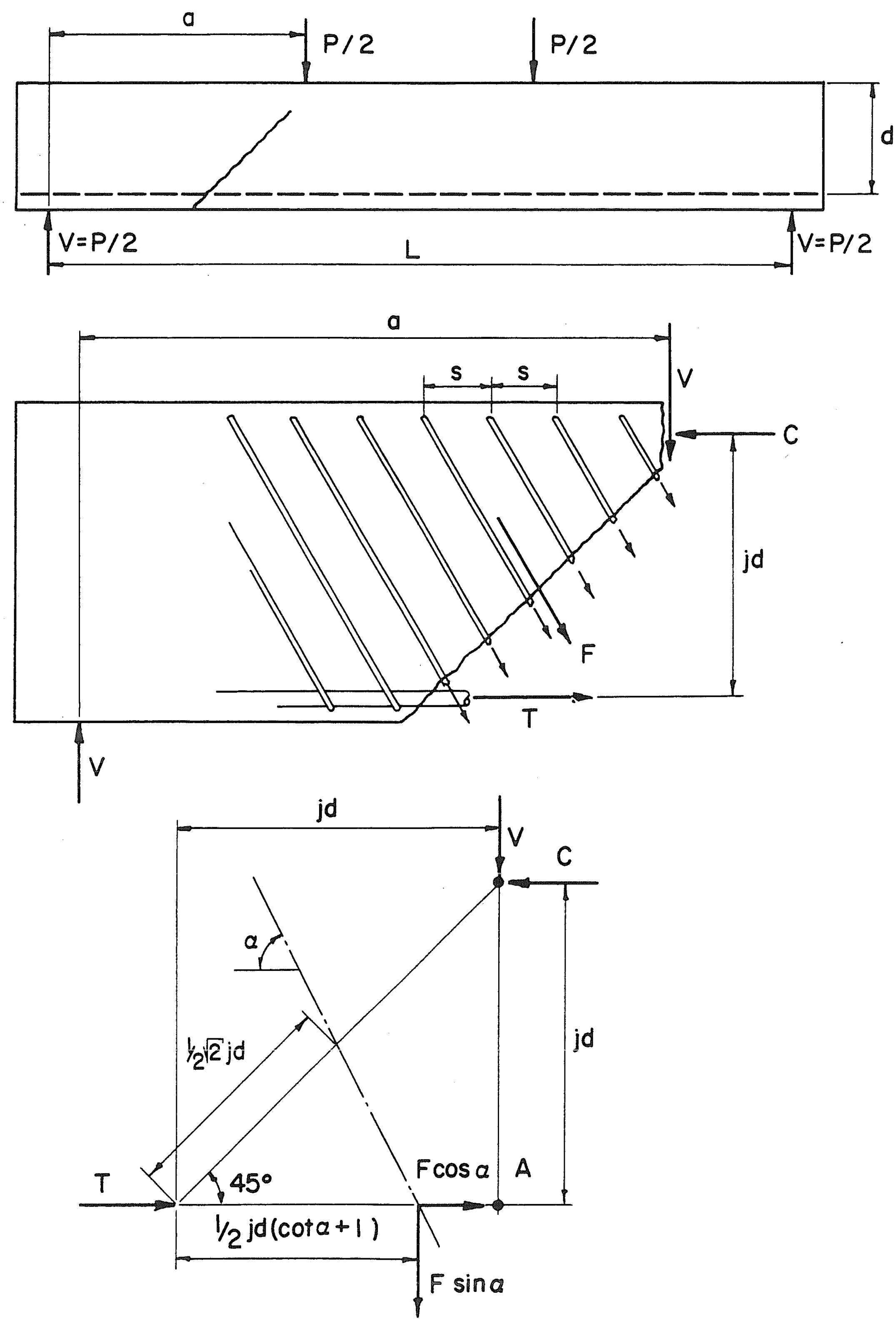


FIG. 3 INTERNAL FORCES AT SECTION OF DIAGONAL CRACK

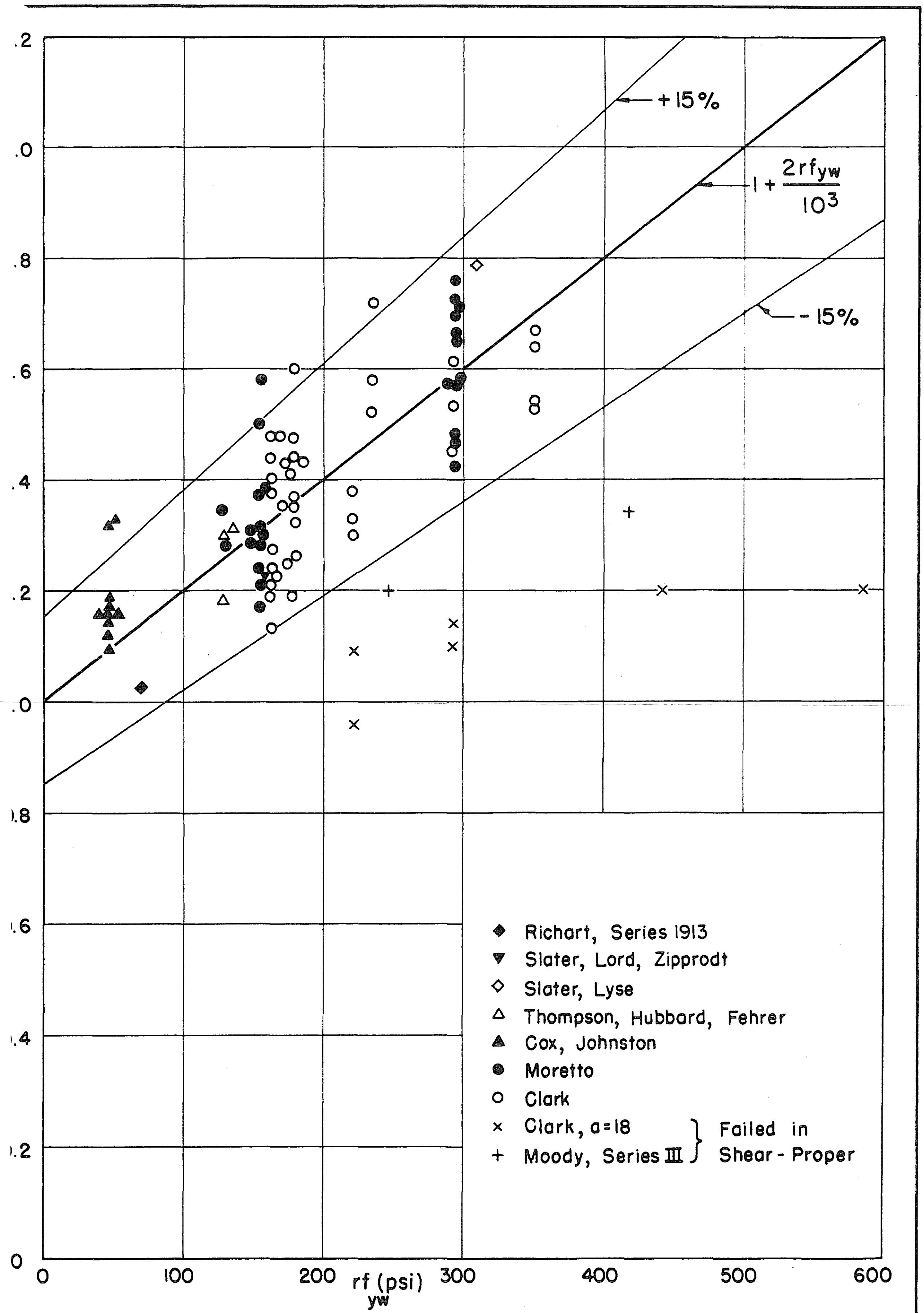


FIG. 4 P_{test}/P_s VERSUS rf_{yw} FOR SHEAR FAILURES SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

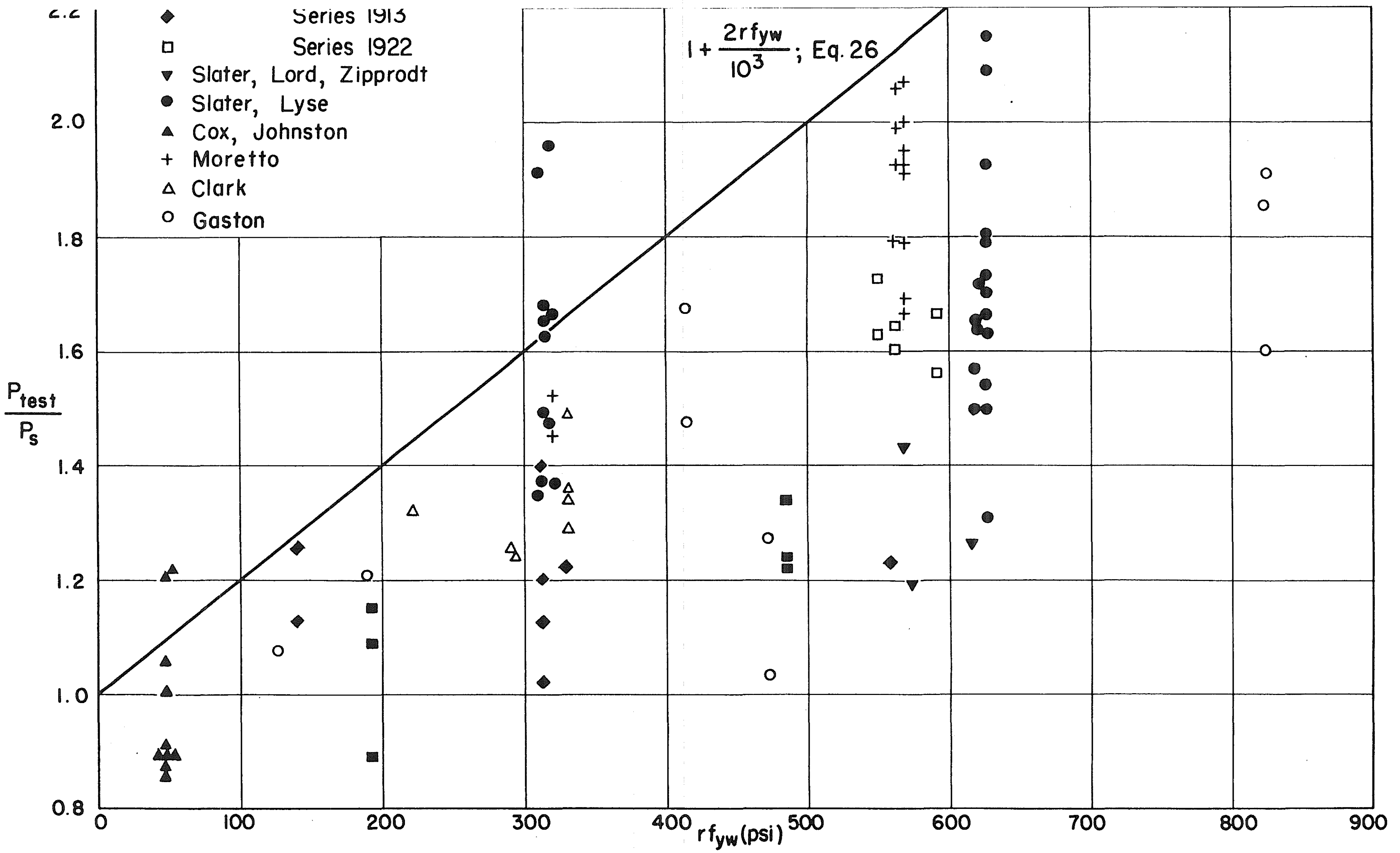


FIG. 5

P_{test}/P_s VERSUS rf_{yw} FOR FLEXURAL FAILURES
SIMPLE-SPAN RECTANGULAR BEAMS WITH STIRRUPS

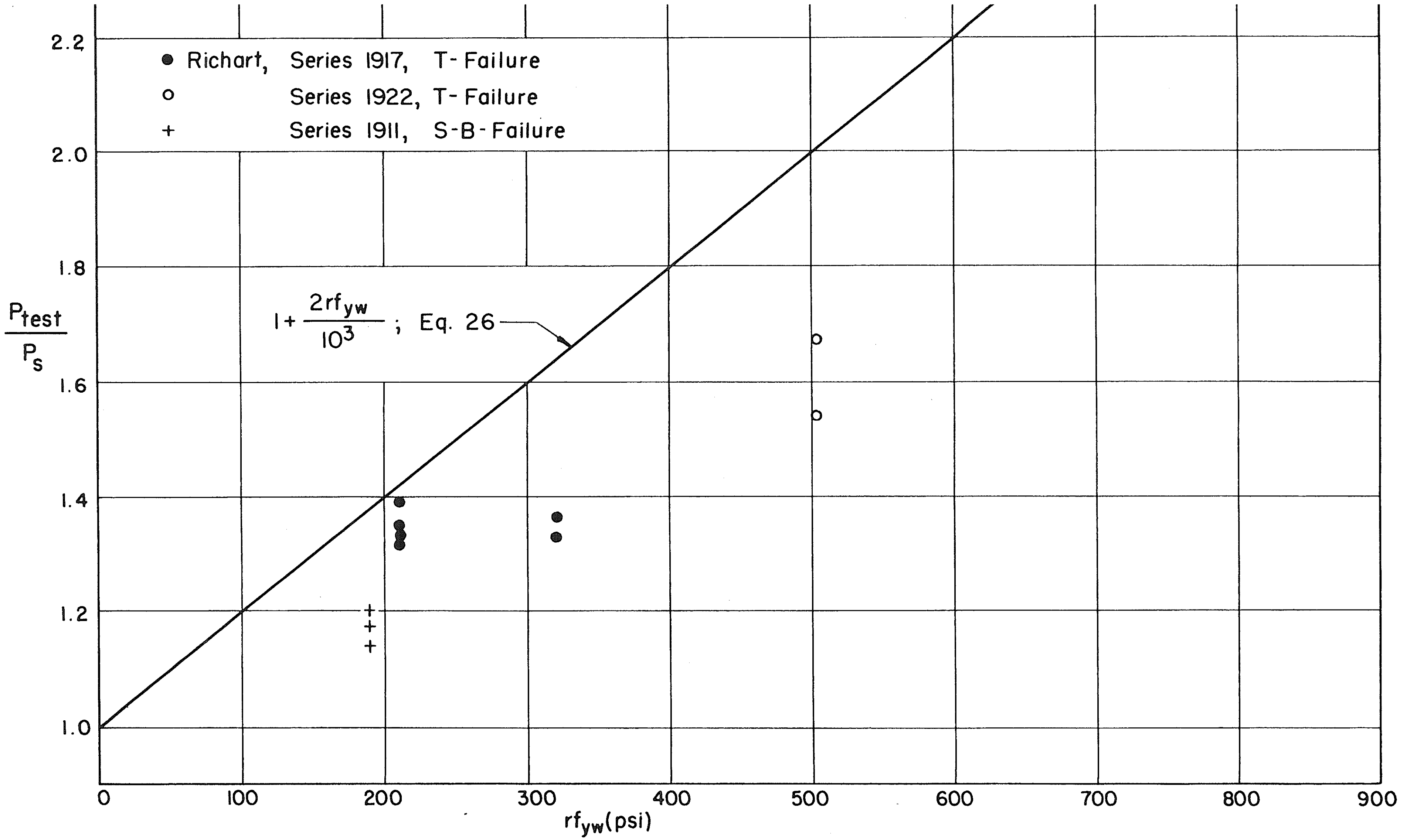


FIG. 6

P_{test}/P_s VERSUS rf_{yw}

SIMPLE-SPAN RECTANGULAR BEAMS WITH BENT-UP BARS

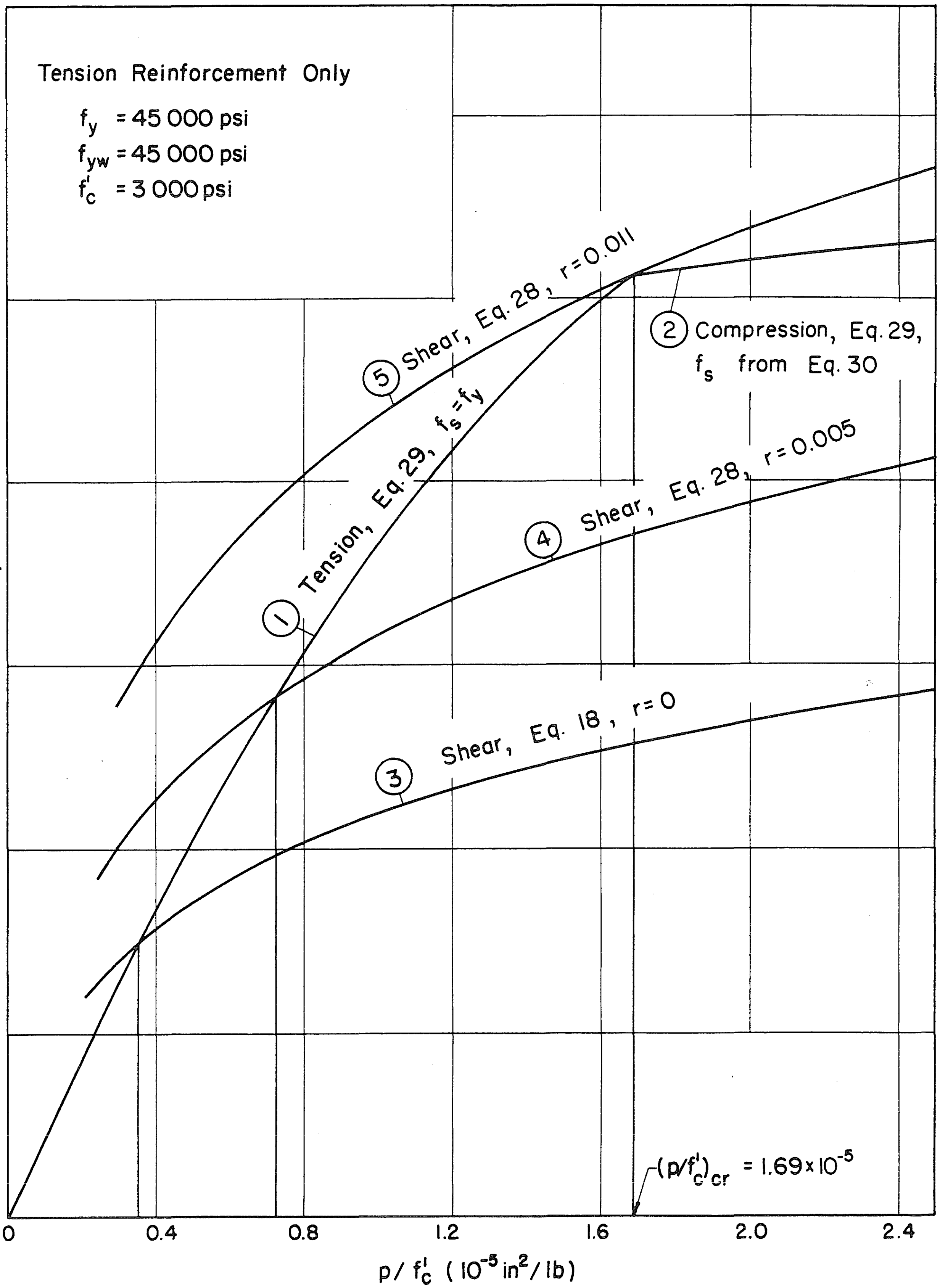


FIG. 7 $M/bd^2f'_c$ vs. p/f'_c FOR FLEXURAL AND SHEAR FAILURES

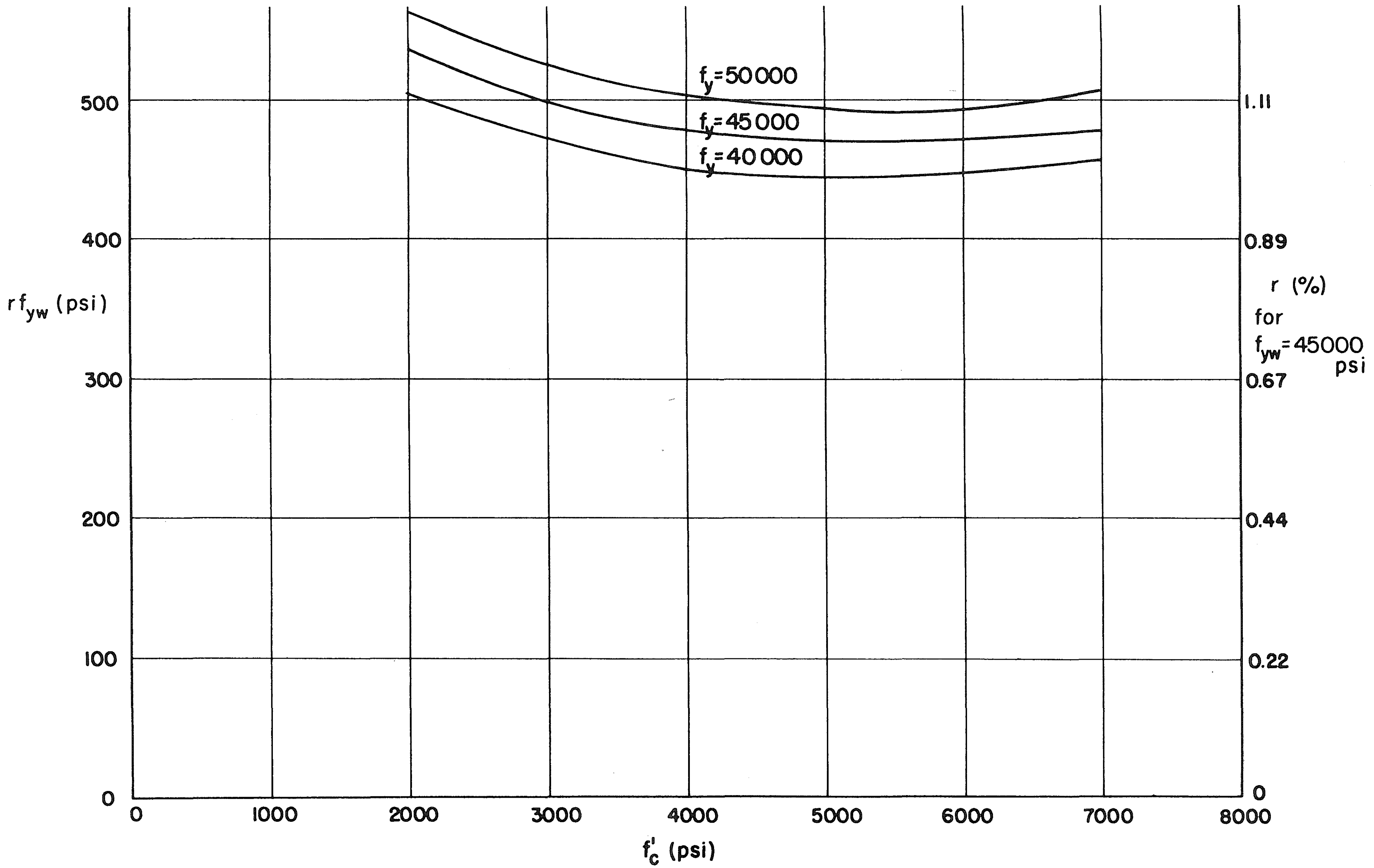


FIG. 8 MAXIMUM USEFUL AMOUNT OF WEB REINFORCEMENT VERSUS f'_c AND f_y

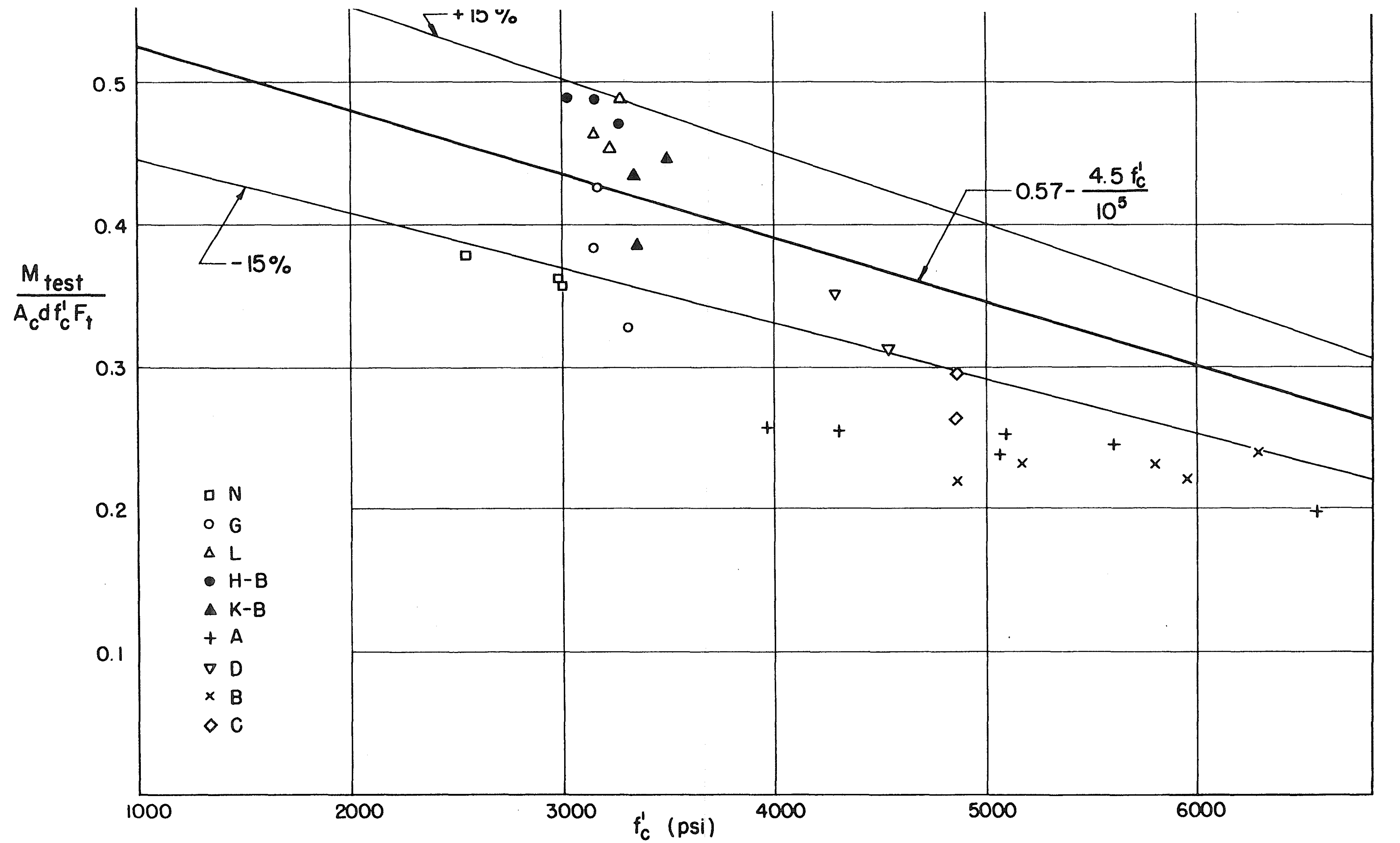


FIG. 9 TESTS BY FERGUSON AND THOMPSON
SIMPLE-SPAN T-BEAMS WITHOUT WEB REINFORCEMENT

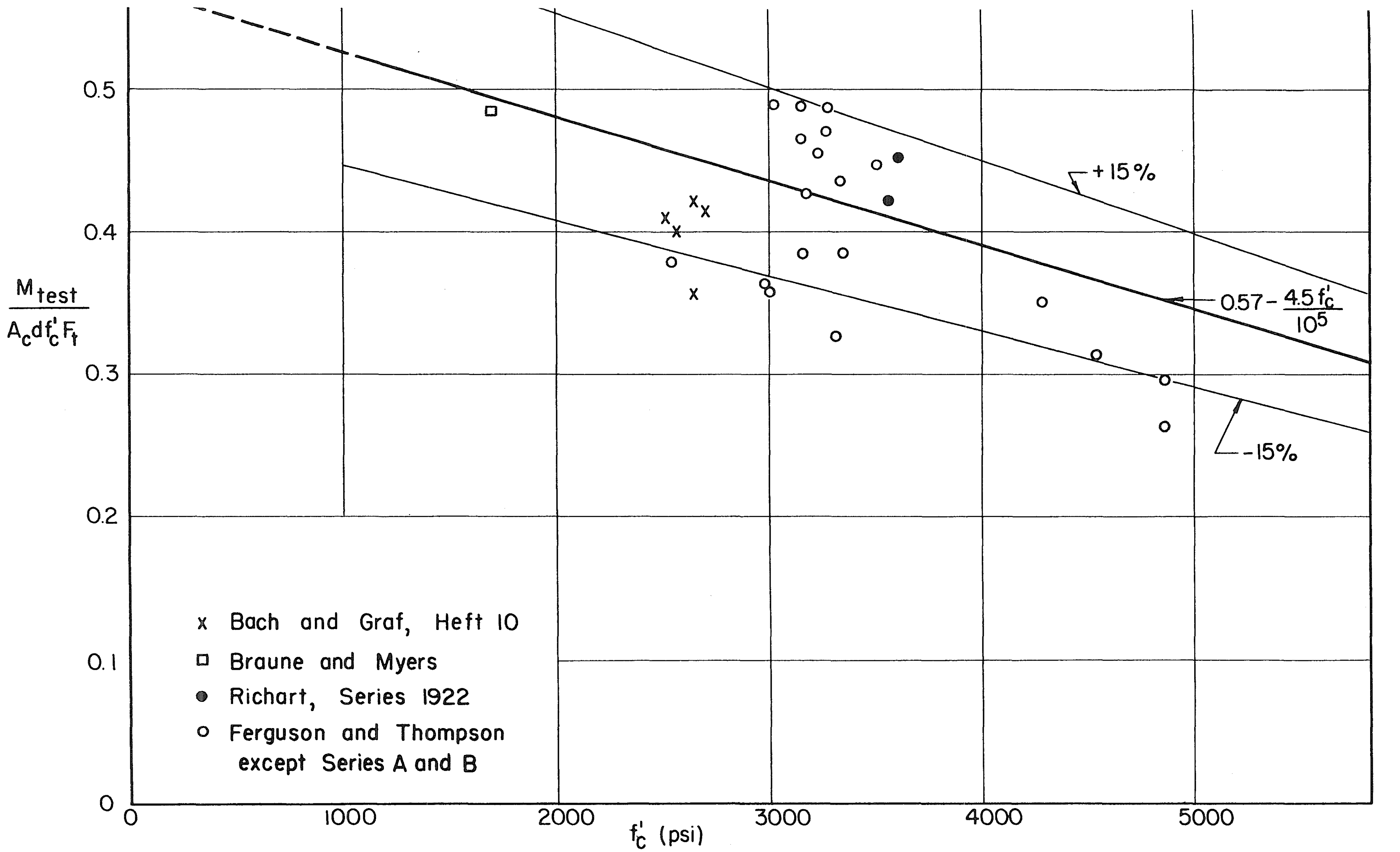


FIG. 10 $M_{test}/A_c d f'_c F_t$ VERSUS CONCRETE STRENGTH
SIMPLE-SPAN T-BEAMS WITHOUT WEB REINFORCEMENT

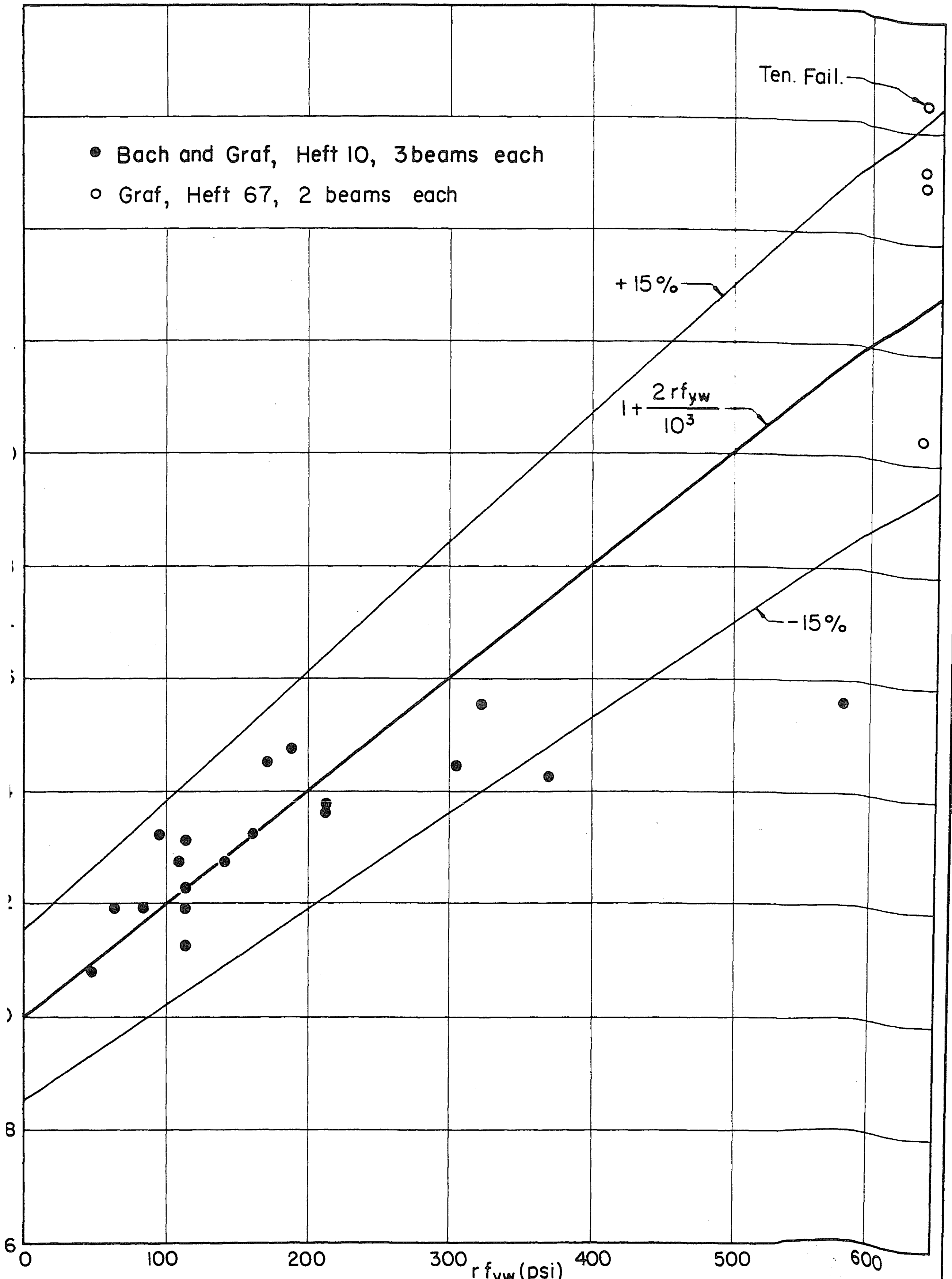


FIG. 11 P_{test} / P_s VERSUS $r f_{yw}$

SIMPLE-SPAN T-BEAMS WITH WEB REINFORCEMENT

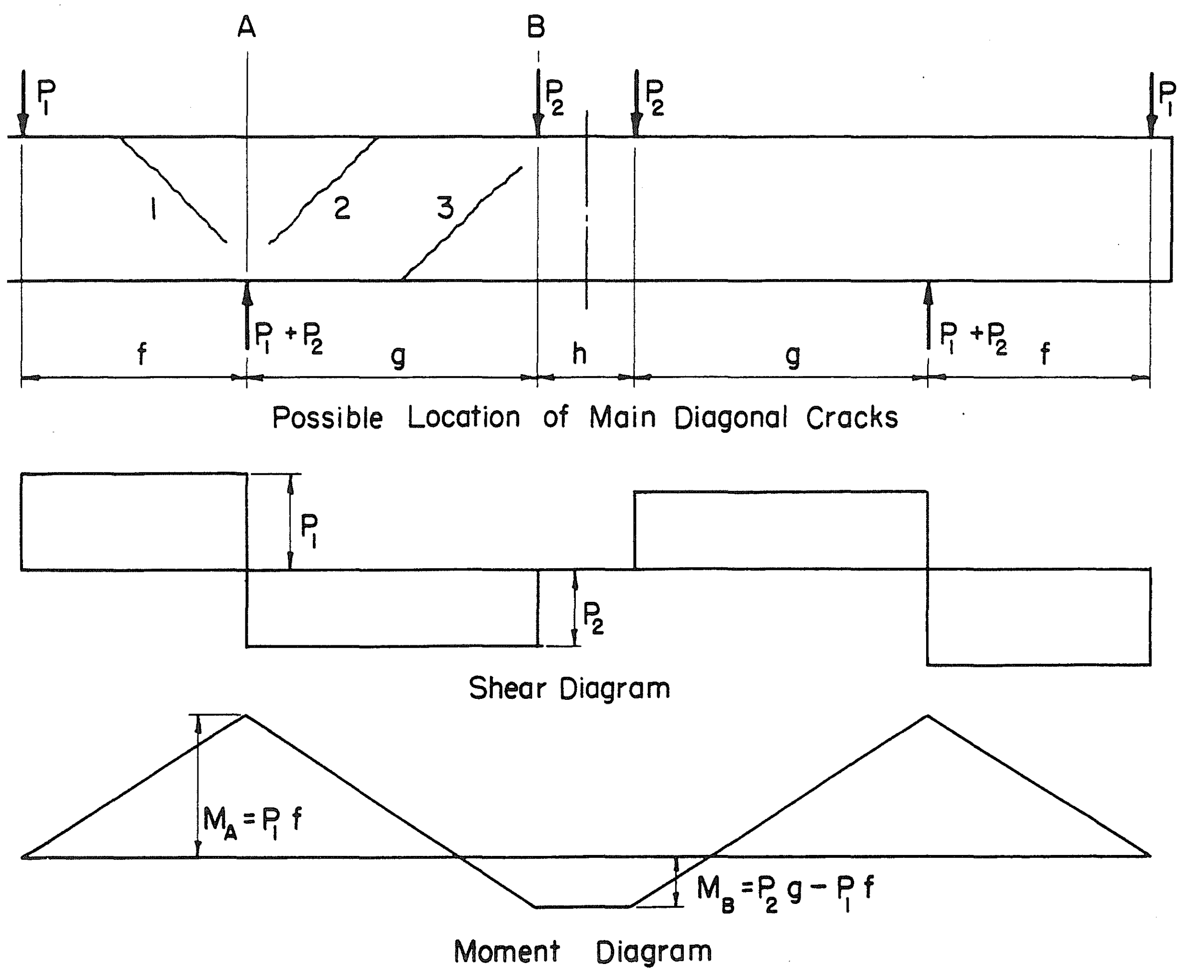


FIG. 12 RESTRAINED BEAM UNDER SYMMETRICAL CONCENTRATED LOADS

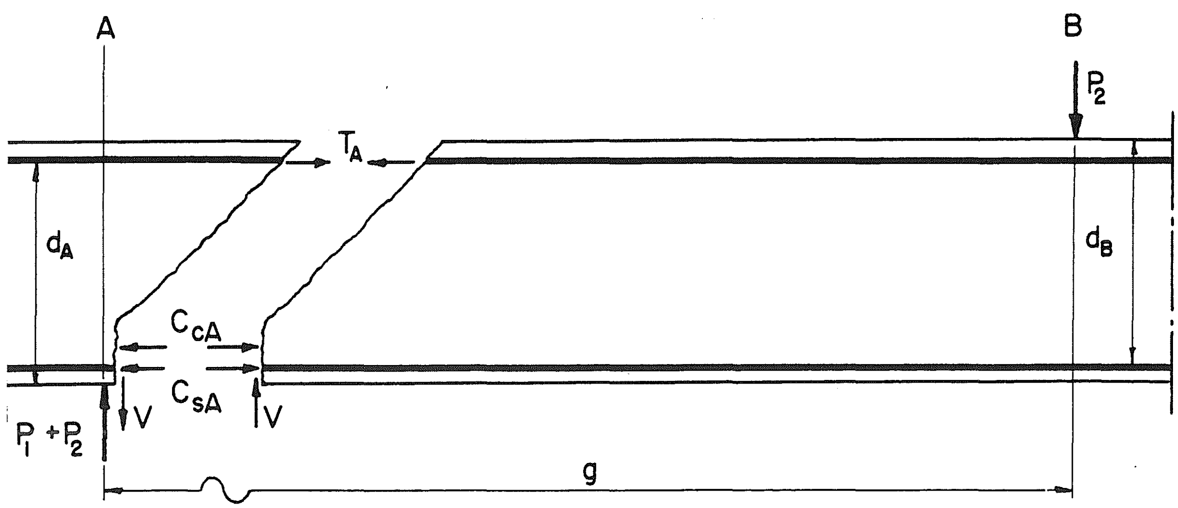


FIG. 13 CONTINUOUS TOP AND BOTTOM REINFORCEMENT RESTRAINED BEAM WITH NO BOND FAILURE

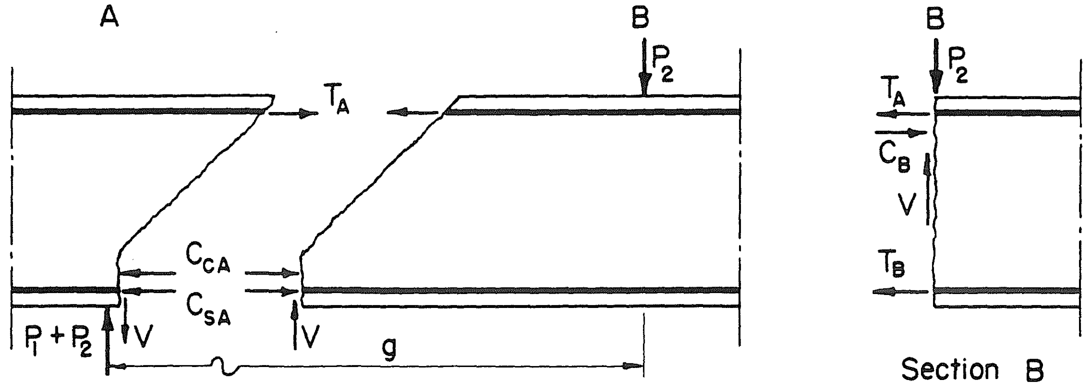


FIG. 14 CONTINUOUS TOP AND BOTTOM REINFORCEMENT BOND DESTROYED IN RESTRAINED BEAM WITH ONE CRACK

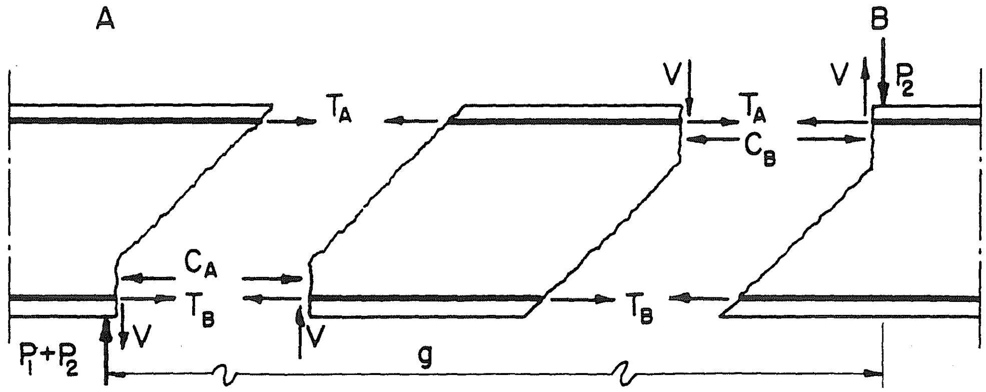


FIG. 15 CONTINUOUS TOP AND BOTTOM REINFORCEMENT BOND DESTROYED IN RESTRAINED BEAM WITH TWO CRACKS

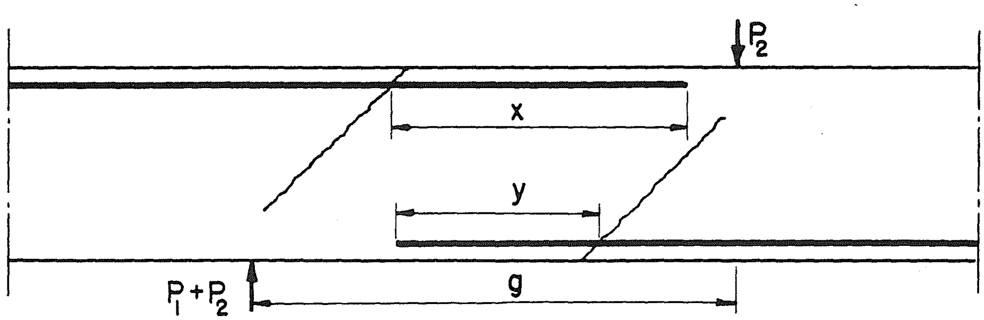


FIG. 16a STRAIGHT BARS CUT OFF BEYOND POINT OF CONTRAFLEXURE RESTRAINED BEAMS

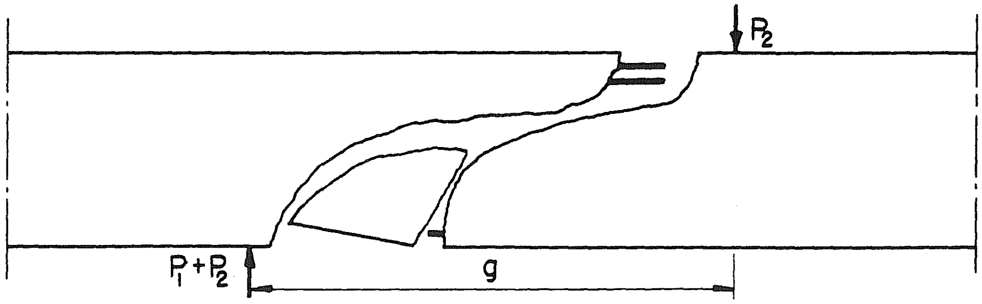


FIG. 16b STRIPPING TYPE OF BOND FAILURE RESTRAINED BEAMS

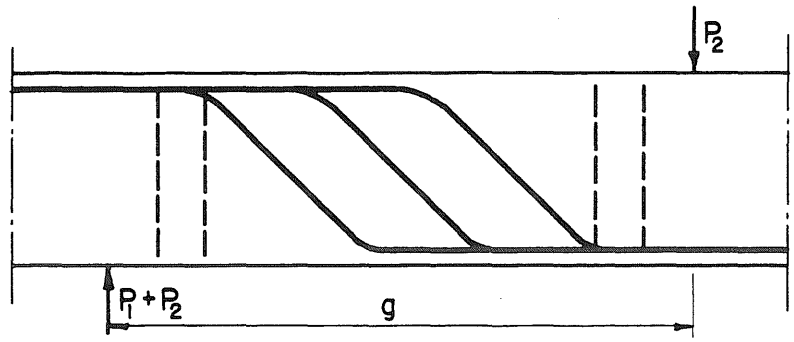


FIG. 17 RESTRAINED BEAM WITH ALL BARS BENT UP

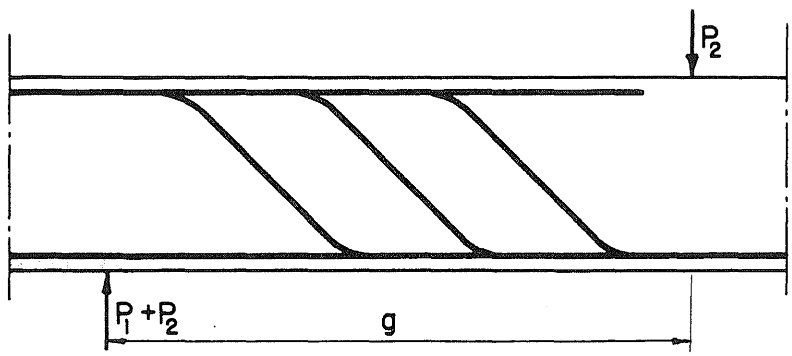


FIG. 18 RESTRAINED BEAM WITH BENT-UP AND STRAIGHT BARS

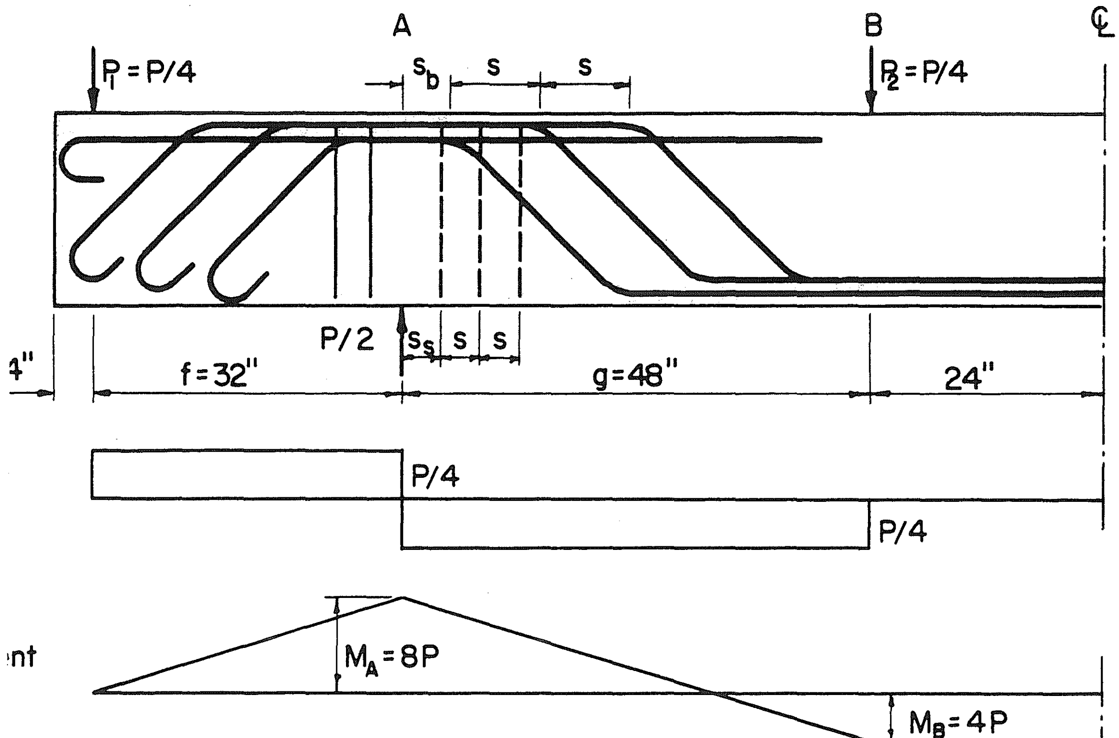


FIG. 19 TYPICAL RESTRAINED BEAM OF RICHART AND LARSEN

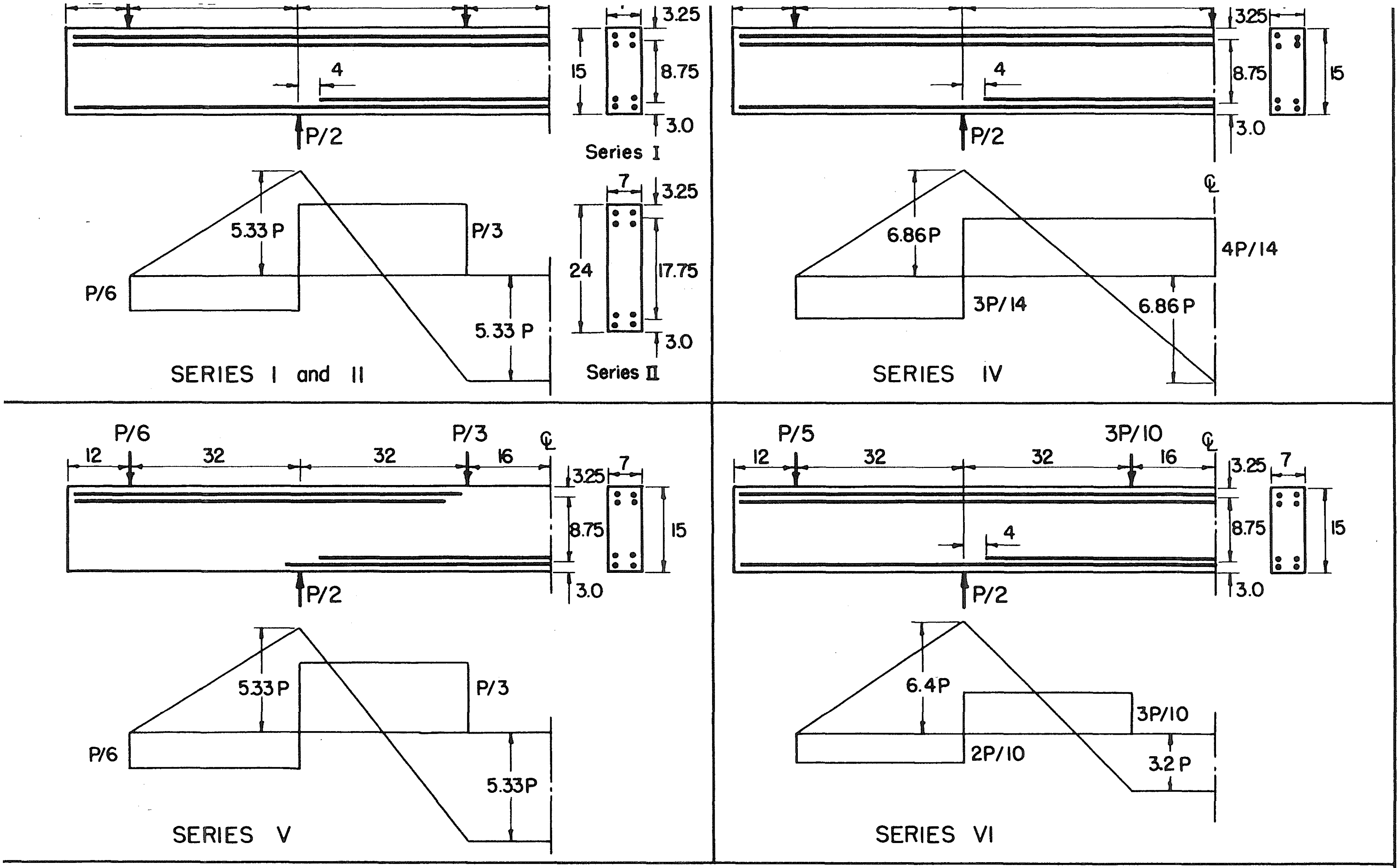


FIG. 20 RESTRAINED BEAMS OF MOODY

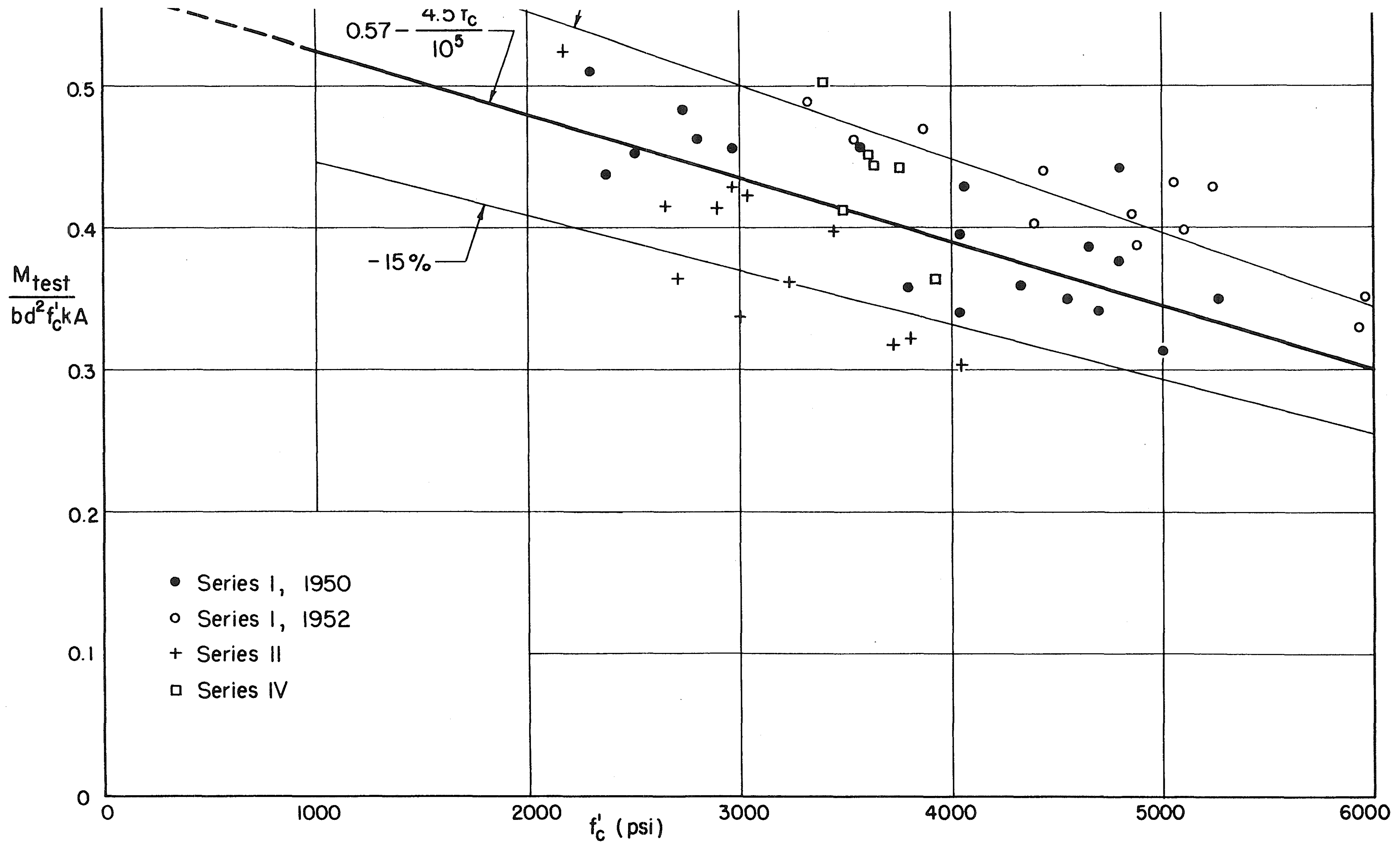


FIG. 21a BEAMS OF MOODY, SERIES I, II, AND IV

RESTRAINED BEAMS WITHOUT WEB REINFORCEMENT

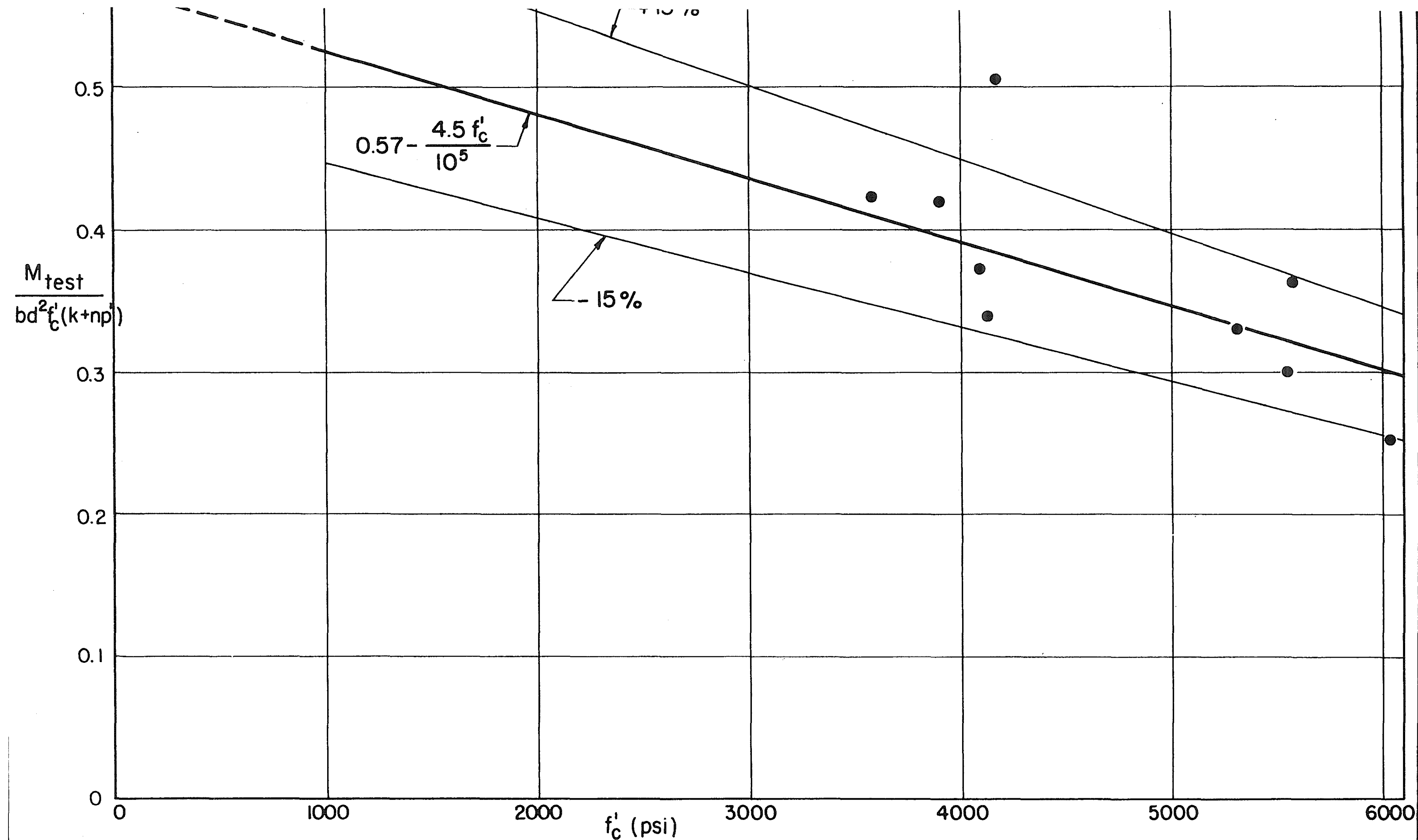


FIG. 21b BEAMS OF MOODY, SERIES VI

RESTRAINED BEAMS WITHOUT WEB REINFORCEMENT

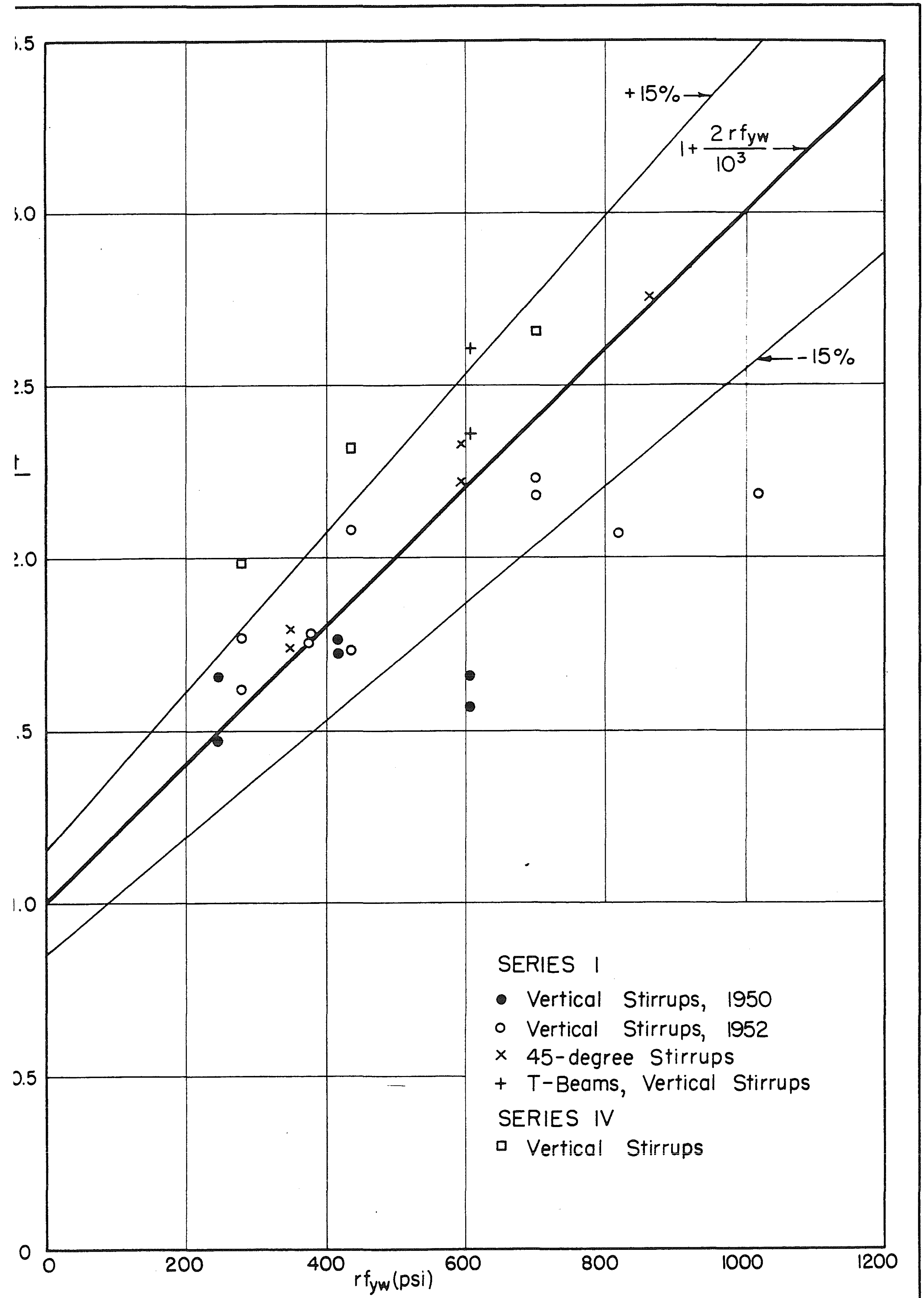


FIG. 22 BEAMS OF MOODY, SERIES I AND IV
RESTRAINED BEAMS WITH WEB REINFORCEMENT

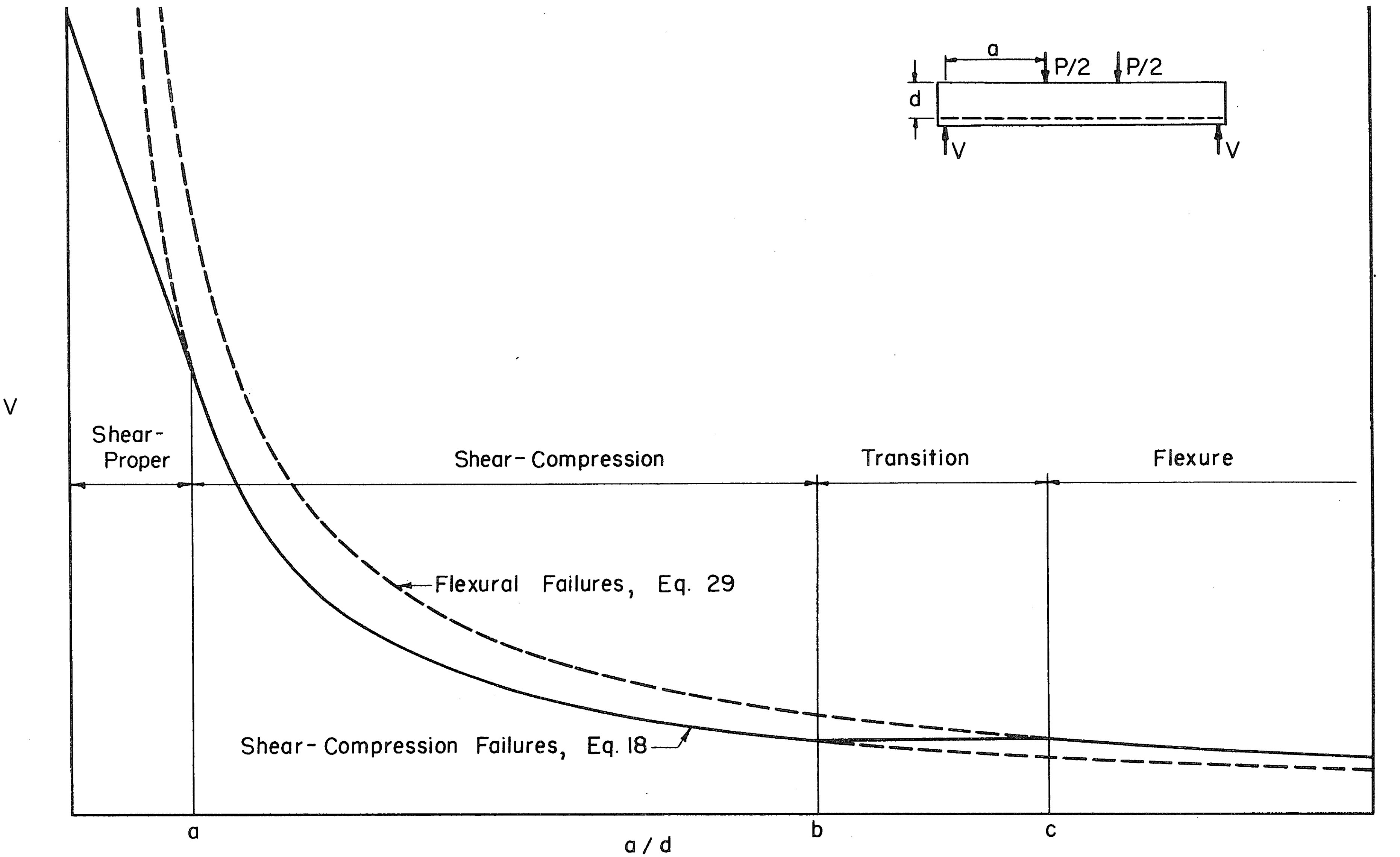


FIG. 23 SHEAR FORCE V VERSUS a/d

POSSIBLE MODES OF SHEAR FAILURE FOR SIMPLE-SPAN BEAMS

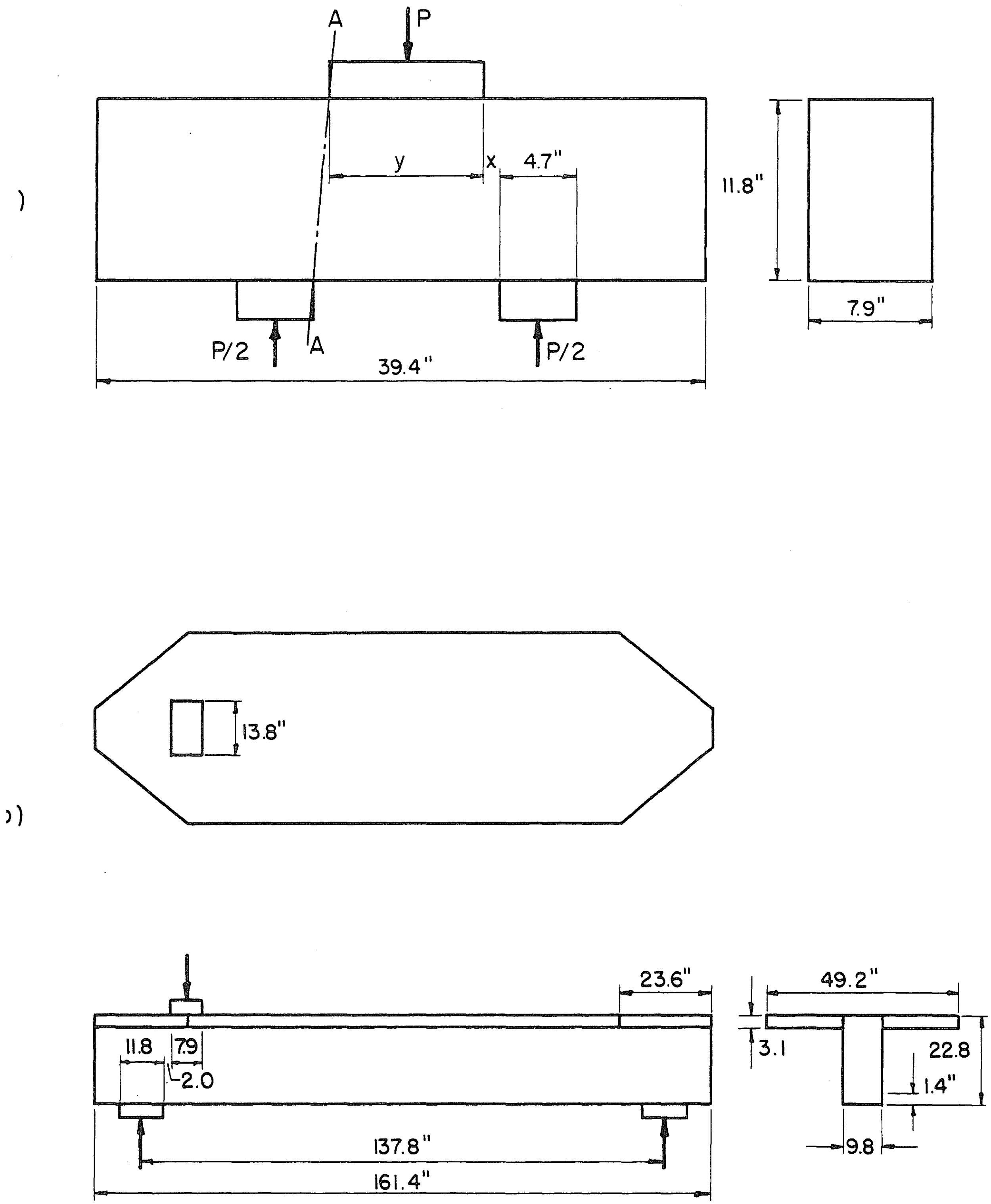


FIG. 24 BEAMS OF GRAF, HEFT 80

SHEAR-PROPER TYPE OF FAILURES

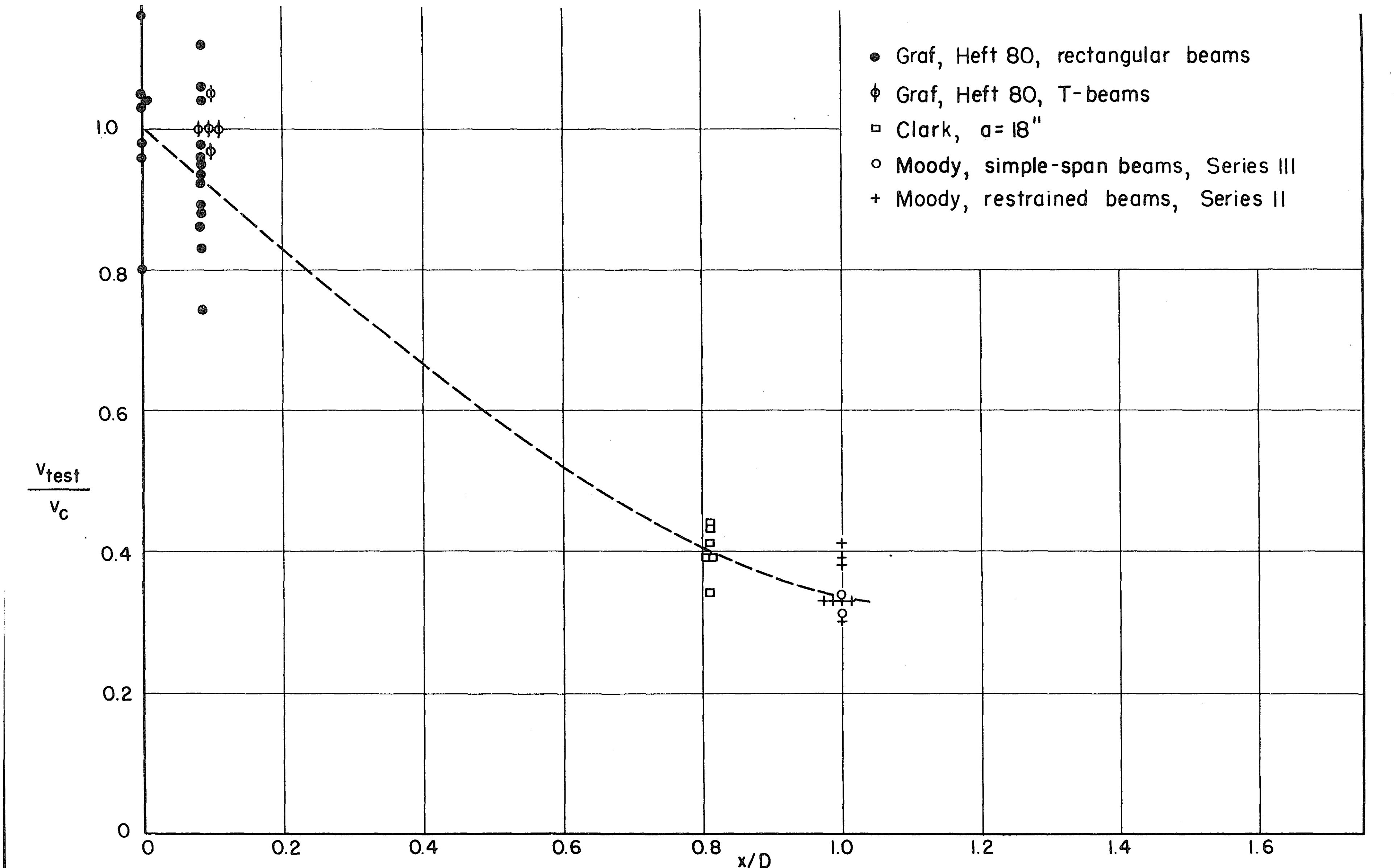


FIG. 25 NOMINAL SHEARING STRESS RATIO v_{test}/v_c VERSUS x/D
SHEAR-PROPER TYPE OF FAILURES

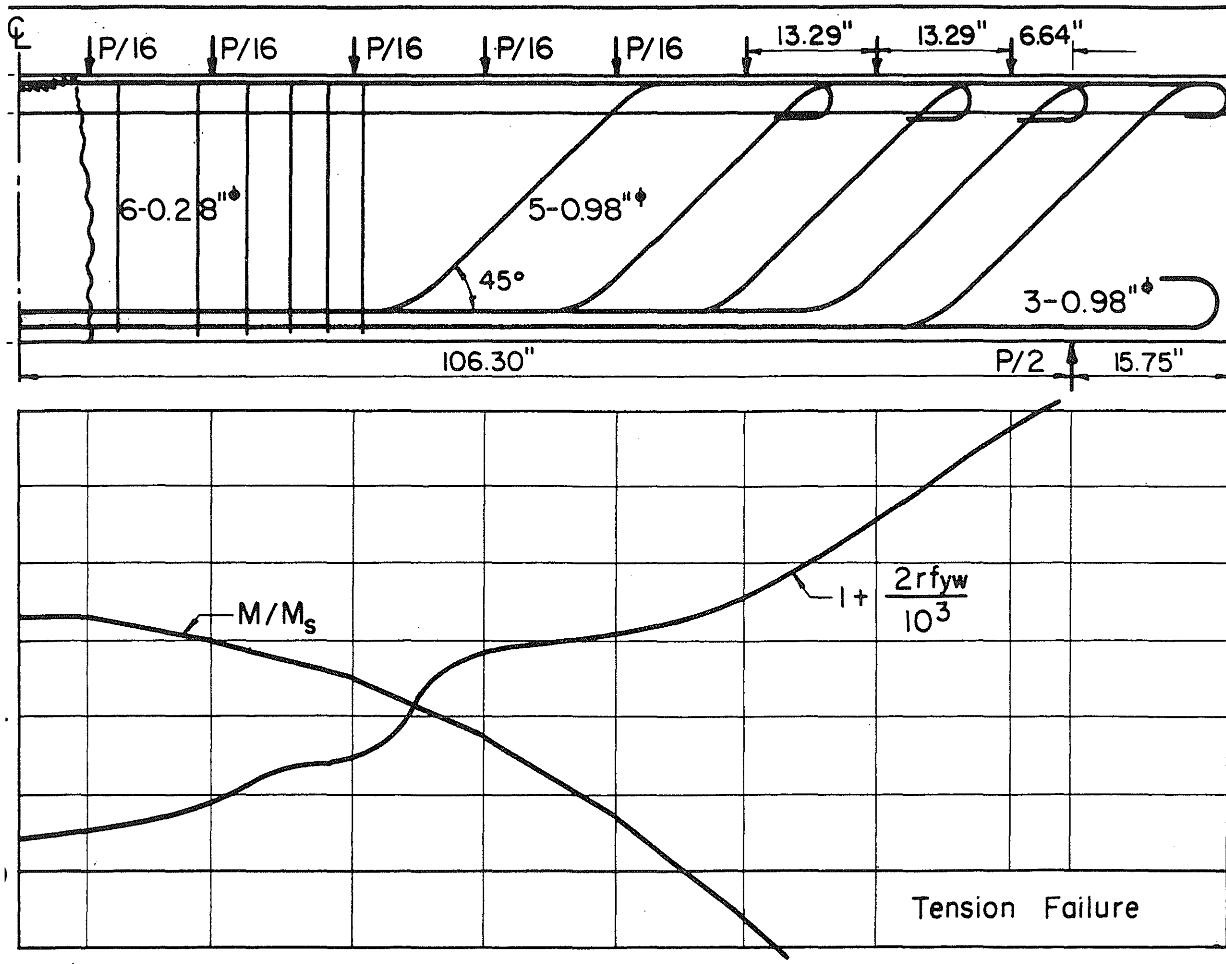


FIG. 26 BEAM 1026 OF BACH AND GRAF, HEFT 48

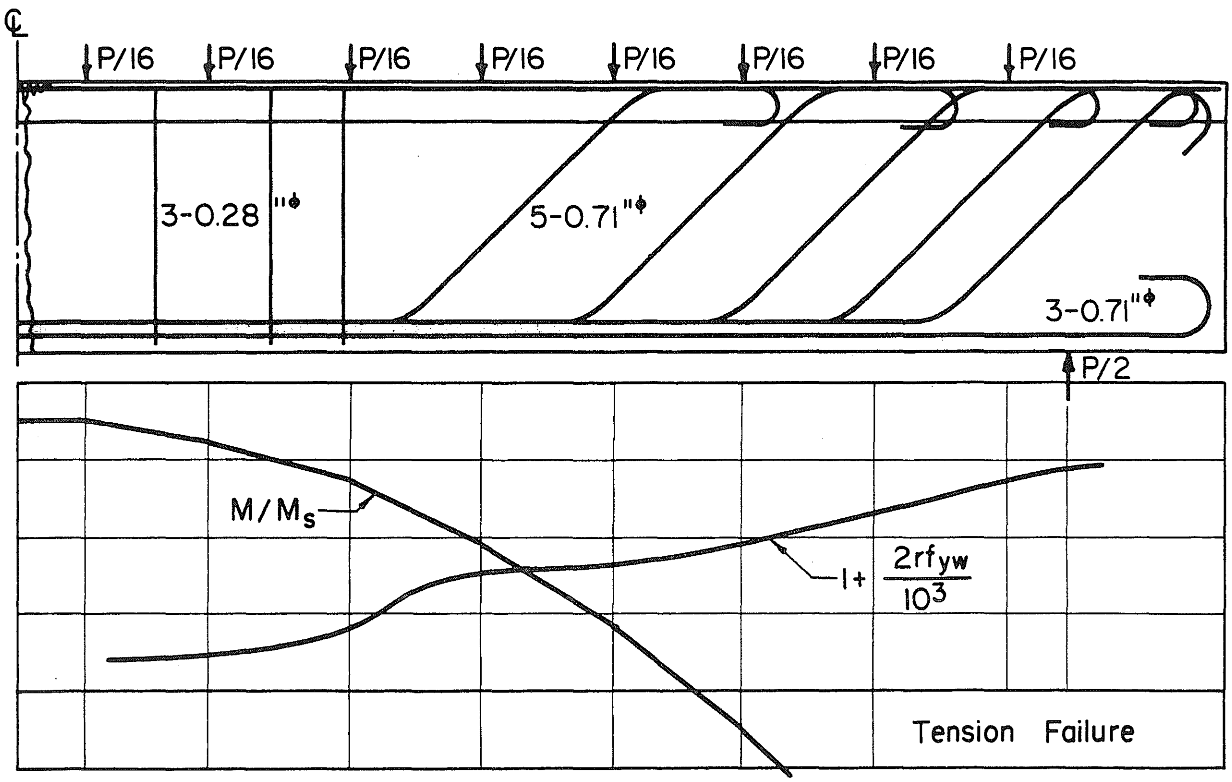


FIG. 27 BEAM 1025 OF BACH AND GRAF, HEFT 48

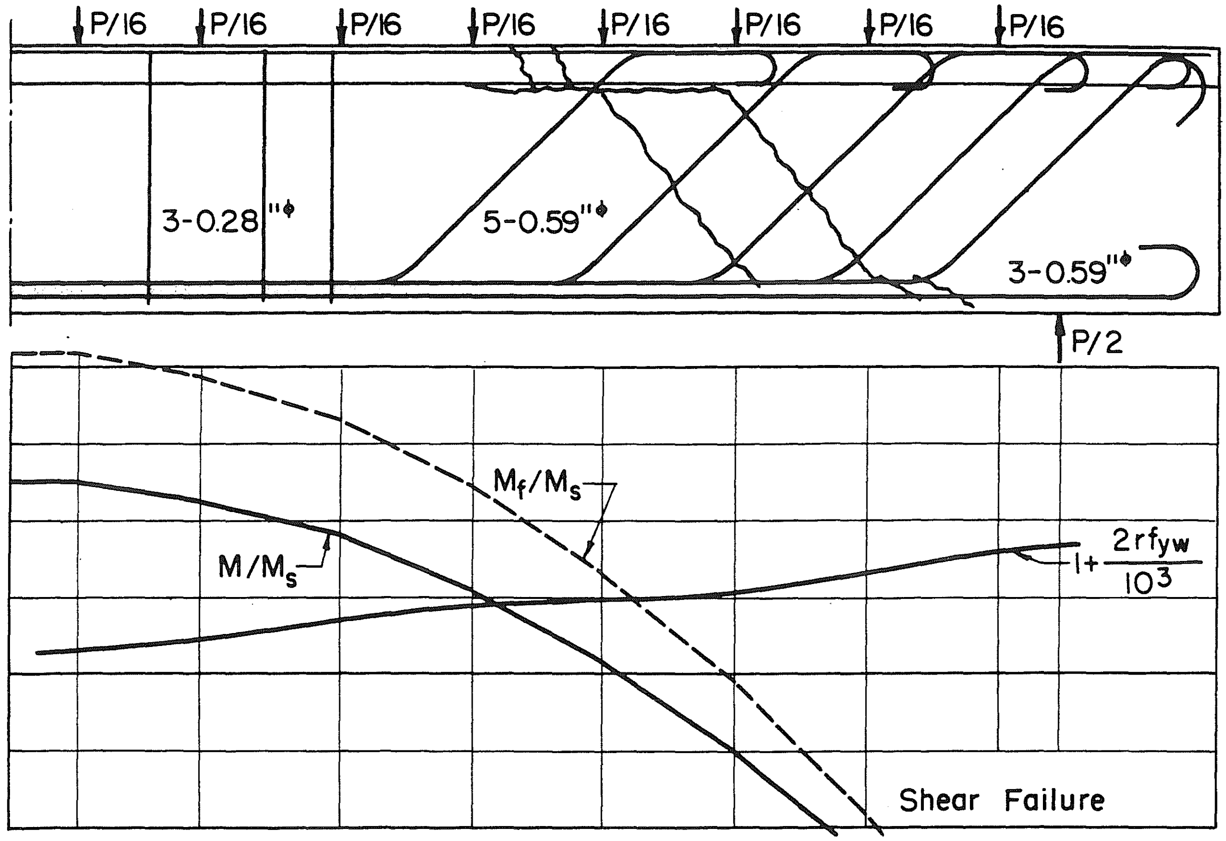


FIG. 28 BEAM 1031 OF BACH AND GRAF, HEFT 48

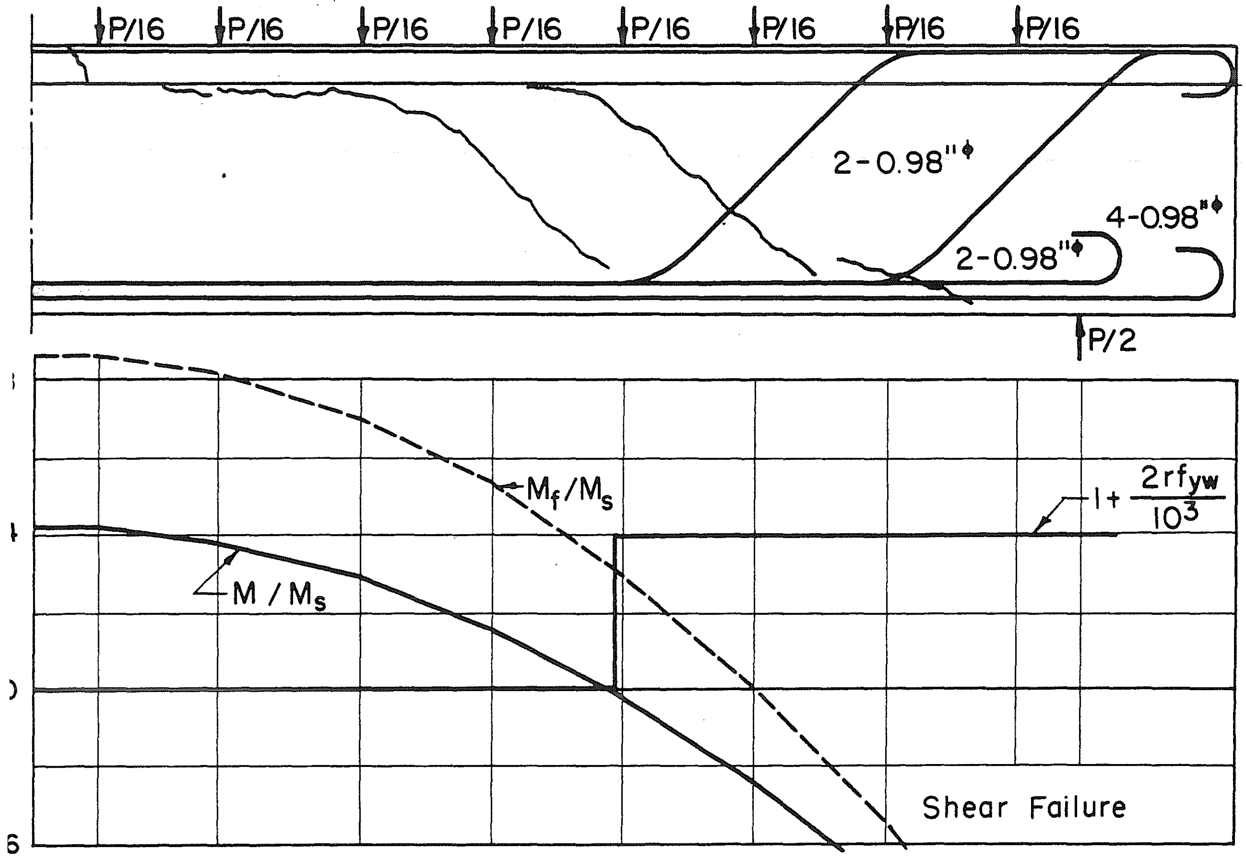


FIG. 29 BEAM 1032 OF BACH AND GRAF, HEFT 48

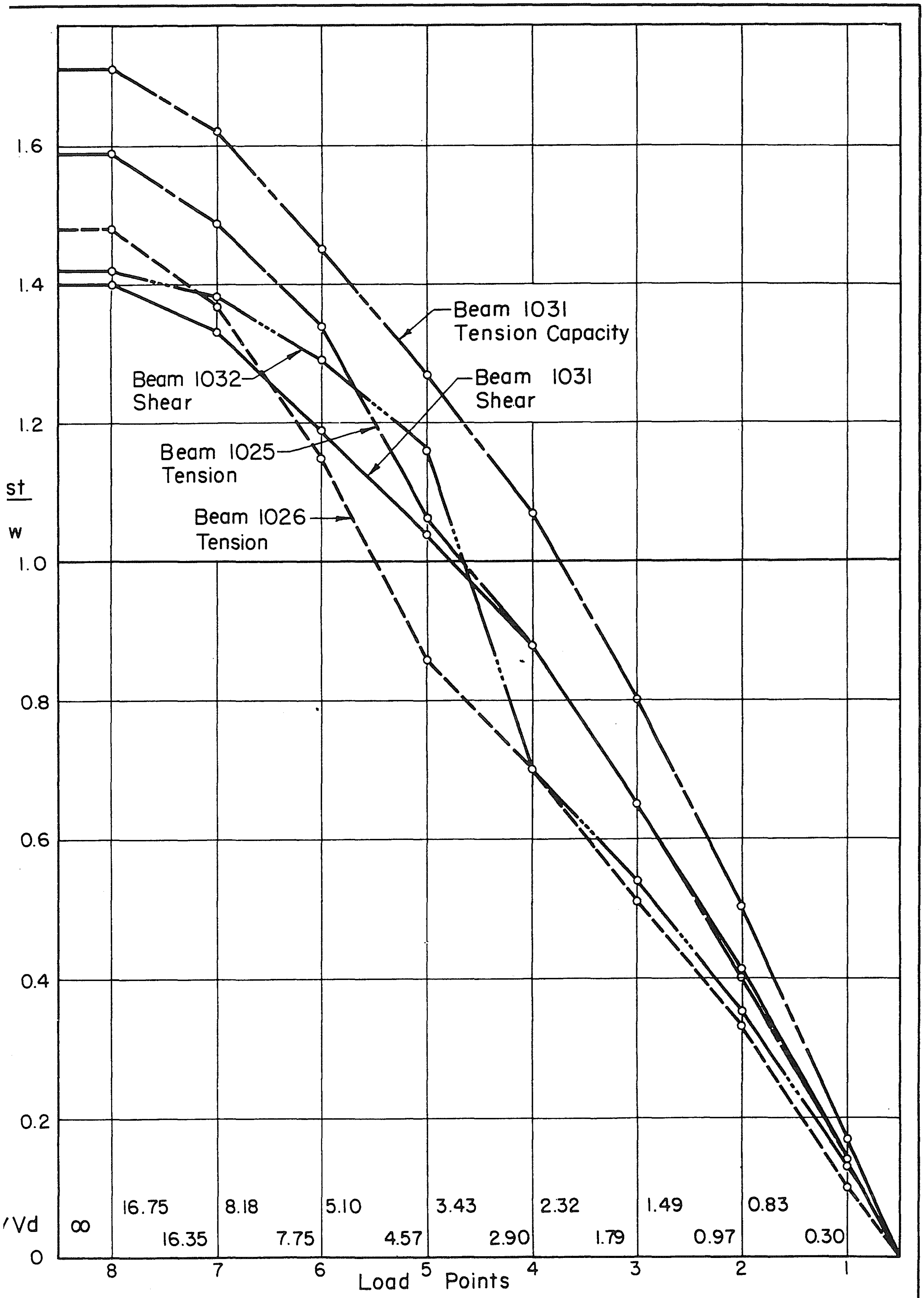


FIG. 30 M_{test}/M_{sw} VERSUS M/Vd

T-BEAMS OF HEFT 48 UNDER SIXTEEN CONCENTRATED LOADS

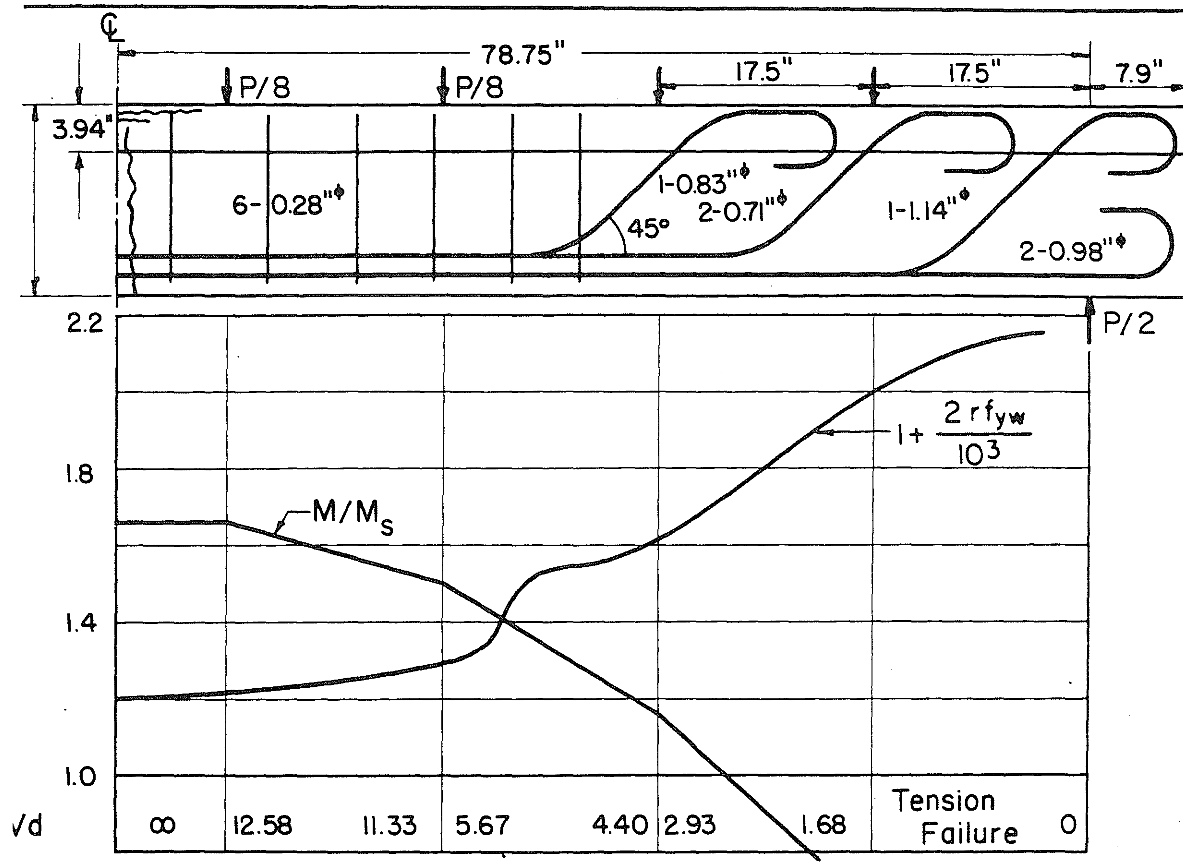


FIG. 31 BEAMS 60 OF BACH AND GRAF, HEFT 20

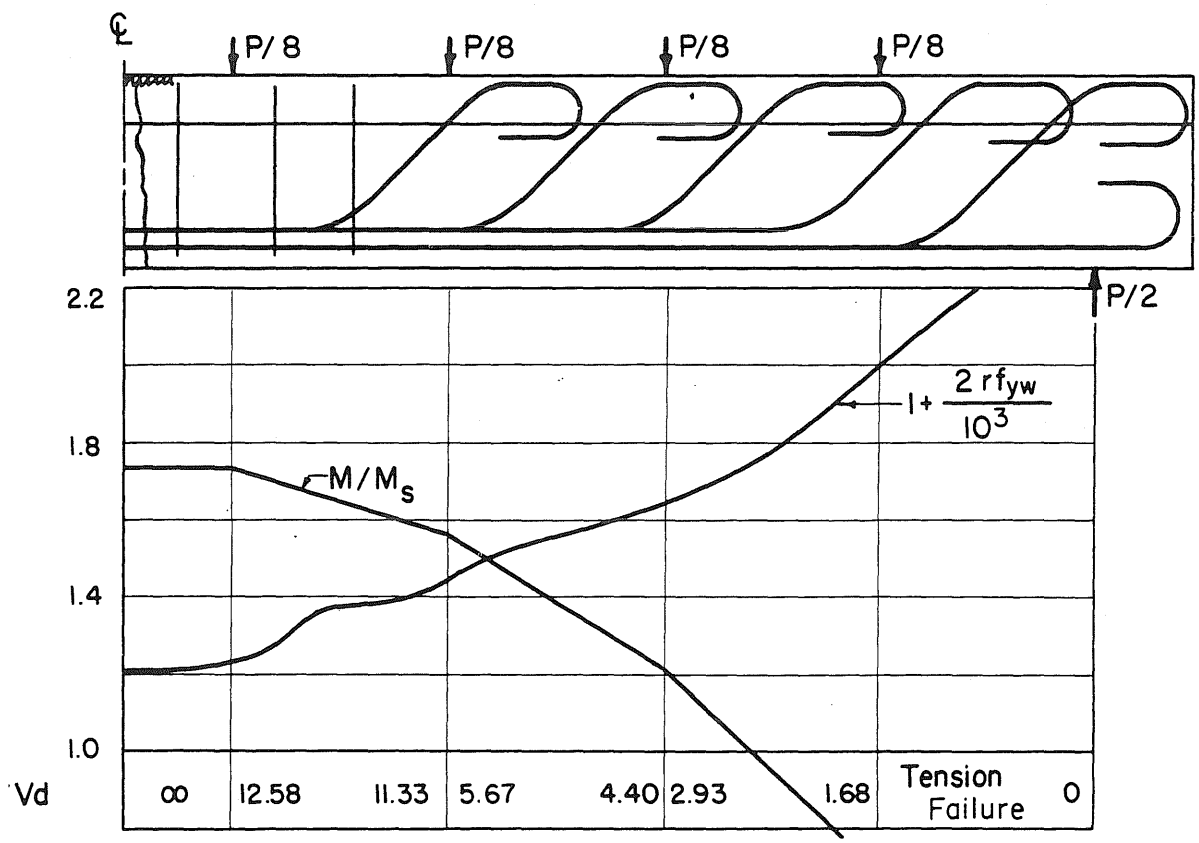


FIG. 32 BEAMS 62 OF BACH AND GRAF, HEFT 20