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Sharecropping in Dual Agrarian Economies: A Synthesis

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ABSTRACT

By building a model of sharecropping in Less Developed Economies which incorporates both monitoring costs and (dual) family labour supply this paper builds a model whose predictions are consistent with most of the stylised facts about developing agriculture.

SHARECROPPING IN DUAL AGRARIAN ECONOMIES: A SYNTHESIS

For almost two decades sharecropping has been an institution in search of a generally agreed upon theoretical basis.[1] Despite the considerable theoretical and empirical work that has been done on this issue there does not appear to be a consensus model of sharecropping.[2] In view of the fact that share leasing prevails in countries as different in their general economic structure as Bangladesh and the United States it may perhaps be too much to expect that one model can satisfactorily explain all known facts. In this paper, we therefore limit ourselves to agrarian economies which display dual labour markets (Bangladesh would be a good example). By incorporating two important features of sharecropping that are observable in such economies our model succeeds in generating comparative static predictions that are corroborated by most of the known evidence on this issue.

The two assumptions which have been frequently recognized in the descriptive accounts of such economies, yet inadequately explored in the theoretical literature are, first, the existence of a dual labour market, and secondly, the fact that sharecroppers are not always landless peasants but rather are themselves small landowners.[3] The importance of imperfect labour markets has been stressed even by those who are poles apart on the question of the efficiency of sharecropping, while the propertied status of sharecropper should be evident from the fact that, given the non-existence of markets for draught-animals, the only way to do qualify for tenantship is to own bullocks;[4] and also--possibly no less important--it is a means of establishing credibility with landlords as regards their ability as cultivators. We have omitted the complications of uncertainty from our model. This is done with a view to focusing our attention to the ramifications of two important elements that we have introduced in our model. Besides, those who accept the recent results of Newbery and Stiglitz (1979), i.e., sharecropping offers no extra risk-spreading opportunities where there is only output uncertainty, will consider this as a justified omission in the context of our model.

By modelling a peasant economy in which the above empirical regularities are emphasized we obtain implications that are consistent with the following facts. First, labour intensity and output per acre are higher on land owned by the sharecropper than on land sharecropped for the landlord. This supports the Marshallian misallocation paradigm and is reflected in the empirical work of Bell (1977), Hossain (1978) and others for South Asia. Secondly, there appears to be a positive correlation between the real agricultural wage and the incidence of sharecropping, as noted by Bardhan and Srinivasan (1971). Thirdly, the incidence of sharecropping is directly related to the extent of labour-market distortion, a phenomenon pointed out by both Hossain (1978) and Bardhan (1979). Fourthly, there is a negative correlation between the elasticity of substitution in production and the incidence of sharecropping (See, for example, Rao(1971)). Fifthly, there appears to be a negative relationship between the incidence of sharetenancy and the measure of labour-intensity in production. On a cross-section basis, this would suggest that the more labour intensive crops are likely to be grown under sharetenancy (Bardhan (1977)). And on a time-series basis, this would imply that labour-saving technical progress will diminish the role of sharetenancy (Day (1967)). Sixthly, as the cost of monitoring wage-labor increases, so does the incidence of tenancy. This finding has been borne out by the empirical studies of Lucas (1982) and Bogue (1959).[5] Finally, there appears to be a negative correlation between the supply of sharecropped land and the amount of owned land by the tenant (Hossain (1978)).

The organization of the paper is as follows. Section I sets out the basic model. Section II analyzes the static properties of the solution. Section III turns to the comparative static results we have obtained and points out their consistency with the known facts. The Appendix contains proofs of various results that are only stated in the text.

I. THE MODEL

A. The Assumptions and the Structure:

Consider a simple agrarian economy in which there are two classes of agents, landlords (l) and sharetenants (s), and three factors of production, land, labor and bullocks. The tenant has a small amount of land owned by himself (k^{so})[6]. In addition, he rents in some land (k^{ss}) on a fixed share-rental rate (r). The landlord is assumed to have one unit of land at his disposal, of which he retains a given amount $(k^{\ell o})$ and leases out the remainder for sharecropping. For simplicity, we shall further assume that each agent has a given endowment of draught animals (bullocks) whose services (b) have an upper-bound of unity. Taking the lead of Bliss and Stern (1982), we shall assume that the quantity of bullock services required for cultivation is an increasing function of the amount of land cultivated, i.e., $b = b(k)$, with $\partial b/\partial k > 0$. Identical production functions are assumed for all types of production--owner-cultivation by the landlord on the retained land, tenant-cultivation of the owned-land and the sharecropped land--and are of the following form:

$$(1) \quad q = \underline{f}(k, n, b(k)) \equiv f(k, n)$$

where q, k and n are respectively the output, the land-input and the labor-input. Assuming $\underline{f}$ is concave and linear homogeneous in all inputs (i.e., we have assumed constant returns to scale), then f is also concave and linear homogeneous in (k, n) as long as $k < \overline{k}$, where $\overline{k}$ is the upper limit of k defined by the farmer's bullock constraint. (See Bliss and Stern (1982), chapter 6 for an empirical discussion of this issue.)

We shall, as noted earlier, assume that all tenants are identical in managerial and entrepreneurial skills, possesion of bullocks, land, etc. The landlord is assumed to divide his sharecropped land among m such tenants:

(2) $k^{\ell o} = 1 - mk^{ss}$

The implicit assumption embodied in equation (2) is that the share-tenant rents land from one landlord.[7] This is obviously a restrictive assumption; the analysis could be extended to contracting with a limited number of landlords, thereby complicating the analysis without, however, adding much by way of insight. If we had global constant returns to scale, the actual value of m would have been indeterminate. However, in our case, there is a lower bound for m in view of the bullock constraint on the tenant. That is, if k^{ss} is very large, the tenants' bullock-constraint may be violated. With this caveat, the specific value of m is irrelevant to our analytical results.[8]

The tenant is assumed to divide his work-effort into three activities: work on the owned land (n^{so}), work on the sharecropped land (n^{ss}) and work for wages (n^{sh}). We assume (following Bardhan and Srinivasan (1971), Lucas (1979) and others) that the sharetenant (implying the family) can work outside the farm for wages. This assumption is in accord with the South-Asian experience. On the other hand, the landlord is assumed to work only on self-retained land. (The same was assumed in Bardhan and Srinivasan (1971); for empirical evidence, see Bardhan and Rudra (1980)). We assume that there is a given agricultural wage rate (w) per unit of work-effort. Given the above assumptions, we can express the income of the sharetenant (y^s) and of the landlord (y^ℓ) as follows:

(3a) $y^s = f^o(k^{so}, n^{so}) + (1-r)\, f^s(k^{ss}, n^{ss}) + wn^{sh}$

Production from owned land	Income from share-cropped land	Wage income

(3b) $y^\ell = f^\ell(k^{\ell o}, n^{\ell h} + n^{\ell o}) + mr\, f^s(k^{ss}, n^{ss}) - wn^{\ell h}$

Production from self-cultivation	Income from share rented land	Wage bill

Finally, we shall assume that landlords and tenants have identical tastes and can be represented by a twice-differentiable, strictly quasi-concave utility function:

(4) $u^i = u\,(y^i, e^i) \qquad i = 1, s$

with $u_1 > 0$, $u_2 < 0$.

The above posits that utility is a function of income (y) and work-effort (e). In the case of the landlord, we will assume that his work-effort has two components. The first component comes from working on self-cultivated land along with hired hands (n^{1o}) and the second component consists of effort expended on monitoring of the work-effort of the hired hands in the self-cultivated land (n^{1m}).[9] For simplicity, (like Lucas (1979)), we will assume that the monitoring effort of the landlord (n^{1m}) is a monotonically increasing function of the amount of hired wage labour (n^{1h}). More specifically, we will assume that the total work-effort of the landlord can be written as:

$$(5a)\quad e^{l} = n^{lo} + n^{lm} = n^{lo} + \phi(n^{lh};\mu) \text{ with } \phi' \equiv \partial\phi/\partial n^{lh} > 0$$

The item μ is a shift parameter which increases the monitoring cost and we shall endow it with an economic interpretation in a later section. On the other hand, in the case of the tenant, it consists of the work-effort devoted to the owned land (n^{so}), that to the sharecropped land (n^{ss}) and finally that made available for hire (n^{sh}). However, we assume that the tenant distinguishes between hired labour and family labour, and charges a lower reservation price for family labour as compared to the market wage rate.[10] We embody this distortion of the labour market by assuming that the tenants' "real cost" of one unit of market labour is different from that of one unit of family labour, with a given conversion rate ($\lambda > 1$) betewen the two.[11] Thus,

$$(5b)\quad e^{s} = n^{so} + n^{ss} + \lambda n^{sh} \qquad \lambda > 1.$$

Note that the distortions that we have introduced in (5a) and (5b) give rise to a certain kind of duality in the labour market: whereas the implicit wage level to the sharecropper is lower than the existing market wage rate (because of the lower reservation-wage level for family labour), the implicit wage level faced by the landlord is higher than the market wage rate because of the presence of a positive monitoring cost. This duality is crucial to the story of sharecropping told here.[12]

B. The Tenant Equilibrium

In this model, we shall assume the tenant maximizes his utility by allocating his effort among alternative employment possibilities,

n^{so}, n^{ss} and n^{sh}. The optimality problem of the tenant can be stated as

$$\underset{\{n^{so},\, n^{ss},\, n^{sh}\}}{\text{maximize}} \quad u^s\,(y^s,\, n^{so} + n^{ss} + \lambda n^{sh})$$

$$\text{subject to } y^s = f^o\,(k^{so},\, n^{so}) + (1 - r)\, f^s\,(k^{ss},\, n^{ss}) + wn^{sh}.$$

The first-order conditions for optimality are

$$(6a)\quad u_1^s\, f_2^o + u_2^s = 0$$

$$(6b)\quad u_1^s\,(1-r)\, f_2^s + u_2^s = 0$$

$$(6c)\quad u_1^s\, w + u_2^s\, \lambda = 0$$

where $u_1 \equiv \partial u^s/\partial y^s$, $f_2^o \equiv \partial f^o/\partial n^{so}$ etc. The above first-order conditions carry the obvious marginal interpretations and hence will not be elaborated here. Note that in the above the tenant takes the share-contract terms (k^{ss}, r) and the wage rate (w) as the given parameters. Further, it should be noted that the above does not assume that the landlord has power over n^{ss}, as is posited in much of the existing literature in this area, e.g., Cheung (1969). It will be noted that the disutility of effort is used essentially in the above formulation, since a utility function involving only y^s would lead to an unbounded solution because we have imposed no constraint explicitly on $n^{so} + n^{ss} + \lambda n^{sh}$, the total amount of labour that can be supplied by a family.

Given the concavity-convexity assumptions, the above equations can be solved to derive the supply function of effort to various uses:[13]

(7a) $n^{ss} = n_*^{ss}(k^{ss}, r, w; \lambda, k^{so})$

(7b) $n^{so} = n_*^{so}(k^{ss}, r, w; \lambda, k^{so})$

(7c) $n^{sh} = n_*^{sh}(k^{ss}, r, w; \lambda, k^{so})$

The effects of change in various parameters on the equilibrium values are summarized in Table I. In view of the indeterminacies that afflict so many comparative statics results it is worth emphasizing the determinate sign of every entry.

TABLE I

	∂k^{ss}	∂w	∂r	∂k^{so}	$\partial \lambda$
∂n_*^{ss}	+	-	-	0	+
∂n_*^{so}	0	-	0	+	+
∂n_*^{sh}	-	+	+	-	-

For subsequent reference, also note that

(7d) $\partial n_*^{ss}/\partial k^{ss} = - f_{21}^{s}/f_{22}^{s} = n_*^{ss}/k^{ss} > 0$

(7e) $\partial n_*^{ss}/\partial r = f_{2}^{s}/(1-r)\ f_{22}^{s} < 0.$

where the last equality in 7(d) is due to constant returns to scale.

C. The Landlord Equilibrium

The choices available to the landlord in this model are with regard to the amount of labour he puts in ($n^{\ell o}$), the amount of labour he wishes to hire in ($n^{\ell h}$) and also the share-contract terms (k^{ss} and r). Unlike Cheung and many others, we will not require that the landlord can enforce a minimum labour-intensity requirement. Rather, we will assume that the landlord can influence the effort of the tenant only indirectly by changing the share contract terms. Like Lucas, we will assume that the landlord makes his optimality decisions subject to the reaction function of the tenant--assuming that the landlord can perceive the supply of effort embodied in equation (7a). In this model, like that of Lucas (1979), the landlord incurs monitoring costs for hired labour if he opts for self-cultivation; on the other hand, if he opts for share-tenancy, there is the disincentive effect, part or all of which is swamped by the lower real costs of family labour. The above noted distortions are crucial to the existence of sharecropping in the present model.

Now the optimizing problem of the landlord can be stated as:

$$\underset{\{n^{\ell o},\ n^{\ell h},\ k^{ss},\ r\}}{\text{maximize}} \quad u^{\ell}\ (y^{\ell},\ n^{\ell o} + \phi(n^{\ell h}))$$

$$\text{subject to } y^{\ell} = f^{\ell}(1-mk^{ss},\ n^{\ell h} + n^{\ell o}) + mr\ f^{s}(k^{ss},\ n^{ss}) - wn^{\ell h}$$

The first-order conditions for a maximum are

$$(8a)\quad u_1^{\ell}\ f_2^{\ell} + u_2^{\ell} = 0$$

(8b) $u_1^{\ell} (f_2^{\ell} - w) + u_2^{\ell} \phi' = 0$

(8c) $u_1^{\ell} [-mf_1^{\ell} + mr \{f_1^s + f_2^s (\partial n^{ss}/\partial k^{ss})\}] = 0$

(8d) $u_1^{\ell} [mf^s + rm f_2^s (\partial n^{ss}/\partial r)] = 0$

The choices of k^{ss} and r are made on the assumption that the landlord can correctly perceive the tenants' supply of effort (n_*^{ss}).[14]

II. The General Equilibrium Solution

The present model has seven equilibrium conditions, equations (6) and (8), in eight variables (n^{so}, n^{ss}, n^{sh}, n^{lo}, n^{lh}, k^{ss}, r, w). In keeping with the surplus-labor assumption, if the real wage level is fixed, i.e., $w = \overline{w}$, then the number of equations will equal the number of variables. As our primary interest is not in the mathematical questions involved in the existence of a solution, but rather in the qualitative implications of equilibrium, we proceed on the assumption that an equilibrium exists.[15]

Given the above specification, by rearranging equations (6) and (8), we can derive

(9) $\overline{w} = (1-r) \lambda f_2^s = \lambda f_2^o = f_2^{\ell} + (u_1^{\ell}/u_2^{\ell}) \phi'$

From the above, it is obvious that $(1-r) f_2^s = f_2^o$, which implies that the tenant devotes more labour inputs to the owned land vis-a-vis the rented land because of a share-tax on sharecropping. However, if one compares labour allocation between the landlord cultivated land and the tenant cultivated sharecropped land, the traditional Marshallian inefficiency does not necessarily follow. Three sets of distortions seem to be intermingled in the system: While the distortion introduced by sharecropping is somewhat corrected by the lower "real" costs of family labour, the advantages of self-cultivation by landlords is also somewhat negated by the presence of monitoring costs in addition to the market wage rate.

From the above equation, it is very difficult to infer whether the labour-intensity on the sharecropped land will differ from that of the landlord operated land.

The land allocation rule, as implied by (8c), is given by

$$\text{(9b)}\quad f_1^{\ell} = r\,\{f_1^s + f_2^s\,(\partial n_*^{ss}/\partial k^{ss})\}$$

From Table I, we know $(\partial n_*^{ss}/\partial k^{ss}) > 0$. Thus, from the above one cannot immediately infer whether the marginal productivity of land under sharecropping is more than the land under cultivation by the landlord. We shall show in a moment that the question of productive efficiency (in the second-best sense) is crucially linked to the elasticity of substitution in production.

<u>Proposition I</u>: With production functions obeying constant returns to scale, the (sharecropping) general equilibrium solution would imply $r \lesseqgtr \alpha$ as $\sigma \gtreqless 1$, where α is the share of land in the output under general competitive conditions and σ is the elasticity of substitution in production.

<u>Proof</u>: From equation (8d), we get $f^s + f_2^s r\,(\partial n_*^{ss}/\partial r) = 0$. Now substituting from (7c) the value of $(\partial n_*^{ss}/\partial r)$ into the above further rearranging, we get $f_{22}^s\,(1-r)\,f^s + r(f_2^s)^2 = 0$. Noting that f_{22}^s

$= f^s_{21}$ (k^{ss}/n^{ss}) and also by further rearranging, one can reduce the above equation to $(1-r)/r = f^s_2\ f^s_1/f^s_{21} \cdot f^s.\ (f^s_2\ n^{ss}/f^s_1/k^{ss})$. Remembering that $\sigma \equiv f^s_2\ f^s_1/f^s_{21}\ f^s$, $\alpha \equiv f^s_1 k^{ss}/f^s$, one can reduce the above equation to $(1-r)/r = \sigma(1-\alpha)/\alpha)$ which, on further manipulations, will yield $r = \pi\alpha$ where $(\pi)^{-1} \equiv [\sigma(1-\alpha) + \alpha]$. Utilizing this, the proposition immediately follows.

The above proposition shows that if the production function is Cobb-Douglas, a widely used production function for agriculture, then the fraction of sharetenancy output going to the landlord for the use of his land equals the share of output going to land under a standard, purely competitive economy.

Proposition II: With production functions subject to constant returns to scale, the sharecropping general equilibrium solution implies that $f^\ell_1 \lessgtr f^s_1$ as $\sigma \gtrless 1$.

Proof: Equation (9b) implies $f^\ell_1 = r\ \{f^s_1 + f^s_2\ (\partial n^{ss}_*/\partial k^{ss})\}$. From equation (7d), we know $\partial n^{ss}_*/\partial k^{ss} = n^{ss}_*/k^{ss}$, which, on substitution, will yield: $f^\ell_1 = r\ [f^s_1 + f^s_2\ (n^{ss}_*/k^{ss})]$. Since f^s is subject to constant returns to scale, the above equation can be rewritten as $f^\ell_1 = r\ (f^s/k^{ss})$. Now utilizing proposition I, we can write $f^\ell_1 = \pi f^s_1$ from which the above proposition immediately follows.

Note that if the production function is Cobb-Douglas, then land allocation under sharetenancy would be efficient, i.e., the marginal productivity of land under sharetenancy and under owner cultivation by the landlord will be equal.

Proposition III: With constant returns to scale production functions, under the sharecropping equilibrium, $f^\ell_2 \gtrless f^s_2$ as $\sigma \gtrless 1$.

Proof: Since f_1^{ℓ} and f_1^{s} are homogeneous of degree zero, then f_1^{ℓ} and f_1^{s} can be expressed as a function of land labor ratios. Further, it can be easily shown that f_1^{ℓ} and f_1^{s} are decreasing (and identical) functions of land-labor ratios and therefore, $f_1^{\ell} \lesseqgtr f_1^{s}$ would imply that $k^{\ell}/(n^{\ell h} + n^{\ell o}) \gtreqless k^{ss}/n^{ss}$. Noting that since f_2^{ℓ} and f_2^{s} are increasing functions of land-labor ratios, $(k^{\ell o}/n^{\ell h} + n^{\ell o}) \gtreqless (k^{ss}/n^{ss})$ would imply that $f_2^{\ell} \gtreqless f_2^{s}$. Therefore, it follows $f_1^{\ell} \lesseqgtr f_1^{s}$ implies that $f_2^{\ell} \gtreqless f_2^{s}$. But proposition II states that $f_1^{\ell} \lesseqgtr f_1^{s}$ if $\sigma \gtreqless 1$. Thus, $f_2^{\ell} \gtreqless f_2^{s}$ if $\sigma \gtreqless 1$. Q.E.D.

Some remarks are in order. First, the above propositions, taken together, seem to resolve a paradox exposed by Reid and elaborated upon by Lucas: the Marshallian tradition of sharecropping would require that the marginal products of land and labour would both be greater than those under competitive wage and rental markets. However, like Lucas and others, we have shown that under constant returns to scale both the above cannot be true and the paradox is thereby eliminated.

Secondly, as noted by Bell (1977), who made the most thorough empirical investigation in this area (followed by Hossain (1977) and others who replicated the same test for other areas), there is a significant difference in both input intensity and output per acre between the sharecropped and owned land of the tenant, the latter being more efficiently cultivated than the former, thus supporting the Marshallian position. (However, a comparison of labour-intensity between the landlord retained land and the sharecropper cultivated land may not yield significant difference because of labour market distortions).

The prediction of our model seems to be consistent with the findings of Bell, Hossain and others in South Asia.

Thirdly, the above results also show that if the production function is Cobb-Douglas, irrespective of the precise functional form of the utility function, the equilibrium share of land under sharecropping is equal to the imputed share of land under a competitive structure. This is also true with respect to labour and the economy would be achieving second-best optimality (first-best optimality is not achievable because of labour-market distortions). However, if the elasticity of substitution is less than unity, then the equilibrium share rental of land (the market rental rate) would be greater than the imputed competitive output-share of land. Similar results were also obtained by Newbery and Stiglitz (1979) who argued that the above result "perhaps provides an explanation of the small share of labour in most sharecropping arrangements. Traditional sharecropping arrangements have involved shares of workers of between one half and two-thirds, while the share of labour under more modern conditions of productions appears to be considerably greater" (p. 321).

Fourthly, the above result can be further interpreted to imply way that, as long as the elasticity of substitution in production, σ, lies in the interval $[0,1)$, the return to the landlord from sharecropping is likely to be greater than that from an alternative fixed rental system. The reverse is, of course, true for σ lying in the open interval $(1,\infty)$. A curious vindication of the result has been provided by Rao (1979), who noted that in India, crops for which there is very little scope for decision making, for product as well as factor substitution, are sharecropped; whereas those with greater possibilities of substitutability are cultivated under the fixed-rental system. As an illustration, he noted that crops like rice where there is 'limited scope for allocative decision' is sharecropped; but crops like tobacco, chillies and sugar cane, where there is greater 'scope' are cultivated under the fixed-rental system.

Finally, the above analysis perhaps provides some clues why sharecropping persists in the face of the incentives problem. Even if sharecropping is inefficient (vis-a-vis owner-cultivation of the tenant), the landlord cannot hope to organize the cultivation of land any better because of pervasive labour-market distortions. This also seems to explain why landlords engage in both self-cultivation and sharecropping simultaneously.

III. COMPARATIVE STATICS

We now describe some comparative static results derivable from the present exercise and check their consistency against the empirical evidence accumulated in this area.

The equilibrium conditions of the landlord described by equations (8a) - (8d), on substituting of the values of $(\partial n_*^{ss}/\partial k^{ss})$ and $(\partial n_*^{ss}/\partial r)$ and on further simplification, can be reduced to the following:

(10a) $f_2^{\ell} - x^{\ell} = 0$

(10b) $f_2^{\ell} - (w + x^{\ell}\phi') = 0$

(10c) $f_1^{\ell} - \pi f_1^{s} = 0$

where $x^1 \equiv -u_2^{\ell}/u_1^{\ell}$ and $\pi \equiv 1/[\sigma(1-\alpha) + \alpha]$. Totally differentiating the above equations, one can find that $(\partial k^{ss}/\partial w)$ is in general indeterminate, a result earlier noted by Newbery (1975). The reason is essentially that an increase in the wage rate gives rise to two opposite effects: on the one hand, it will increase the cost of self-cultivation by the landlord; but on the other hand, it will decrease the labor-input supplied by the tenant on the sharecropped land. While the former will act as an inducement for sharecropping, the latter will act as a repellent. However, our results shows that for σ in the range (0,1), i.e., the range in which sharecropping is practised at least in the context of the present model,[16]

(a) $\partial k^{ss}/\partial w > 0$.

(See the Appendix for derivation.) This is consistent with the evidence from India, where, on the basis of cross-section data, Bardhan and Srinivasan (1971) found that a positive association exists between the wage rate and the incidence of sharecropping.

(b) $(\partial k^{ss}/\partial \lambda) > 0$;

as the degree of distortion in the labour market increases, the incidence of sharecropping increases. A possible measure of this distortion in the labour market may be provided by the rate of unemployment in the economy. The above result would imply that the larger the extent of unemployment facing landless households, the larger the extent of sharetenancy. This was verified for India with cross-section (regional data) by Bardhan (1979). Furthermore, Hossain (1978) found that for Bangladesh the supply of sharecropped land is (ceteris paribus) positively related to the size of the farmer household. Besides, casual empiricism confirms that over-populated countries do have a higher incidence of sharecropping.

(c) $\frac{\partial k^{ss}}{\partial k^{so}} < 0$

implying that the supply of sharecropped land is negatively related to the amount of family-owned land. This result will follow--if not for any other reason--because of the bullock constraint of the tenant household. If families of different sizes do have differential transactions costs for outside employment, this result will be further reinforced. This result was empirically verified for Bangladesh by Hossain (1978).

(d) $\frac{\partial k^{ss}}{\partial \sigma} < 0$;

i.e., as the elasticity of substitution in production increases, the incidence of sharecropping decreases. Rao (1971) found that in India crops with low elasticity of substitution in production seem to be grown under sharecropping, which supports the above prediction.

(e) $\frac{\partial k^{ss}}{\partial \mu} > 0$

The result implies that as the cost of monitoring the wage-labor increases, so does the incidence of tenancy (see, Lucas (1979)). Indirect evidence for this finding has been provided by empirical studies of South Asia (and, curiously enough, also of the U.S.), where it has been observed that absentee landlords--who presumably have to incur higher monitoring costs to employ wage labor--usually prefer some form of tenancy to wage cultivation (see, for example, Lucas (1982), and Bogue (1959)).

(f) $\frac{\partial k^{ss}}{\partial \alpha} < 0$

This implies that the higher the labour-intensity of crop production, the higher the percentage of area under tenancy. Alternatively, on a time series basis, this would suggest that if there is a labour-saving technical progress, the incidence of sharecropping will diminish. Evidence for the former is provided in Bardhan (1977) and Bardhan (1979) while on the latter, the best source is Day (who records how sharecropping had its demise in the U.S. with the introduction of labour-saving techniques in agriculture).

CONCLUSION

One reason why sharecropping has appeared to provide such a disparate series of results is the inherent complexity of the issues. A typical peasant for example can, hypothetically, choose between wage labour, sharecropping and farming (i.e., "renting" in the European sense). The typical landlord has all these options, i.e., he treats his labour supply like that of a peasant and, in addition, he has to decide which system to adopt for the cultivation of his own land. A general model encompassing all these choices becomes unwieldy and analytically intractable. How then should the model be simplified? In this paper we have introduced various restrictions on peasant and landlord behaviour which, we believe, are well justified in the descriptive accounts of sharecropping. The landlord, for example, was assumed not to offer any wage labour, nor is the tenant assumed to be able to obtain as much land as he wished to sharecrop. By incorporating such assumptions we have built a model that is at once simple and yet provides closer agreement with the known facts of sharecropping than any other extant model.[17]

Footnotes

1. A survey of this literature can be found in Quibria and Rashid, (1984).

2. For an interesting and emphatic critique of the empirical assumptions made in many models, see Bell and Zusman, (1976).

3. One important precursor is Mazumdar (1975). However, his model involves maximization by one party alone and does not address itself to the wide set of questions we pose in the present exercise.

4. This view has received a recent empirical support in the important study of Bliss and Stern (1982). Based on an intensive study of Palanpur, an Indian village, they argue that "a would-be tenant had to be 'qualified' if his desire to lease-in land was to count seriously. To be qualified, the tenant must have the means to cultivate, notably he has to own bullocks... This is not surprising in view of our observation...that it is nearly impossible to obtain the use of bullock-services through the market or even, with the exception of rather marginal cultivation, through exchange arrangement." (pp. 128-29)

5. We must add that though our model has had considerable success in explaining sharecropping in South Asia, there is much more variation in historical and geographical terms than is captured in the present paper. For an interesting paper in this regard, see Pearce (1983), as well as several other papers in the same issue of the Journal of Peasant Studies.

6. Bell and Zusman have argued, quite emphatically, that any analysis which seeks to explain the rental rate "must" take into account the fact that the tenant possesses land of his own (p. 579). But interestingly enough, later in the theoretical formulation, "to make matters simple" they assume that tenants are landless.

7. This was assumed by many including Bardhan and Srinivasan, Lucas and others. The Indian evidence, cited by Bell, indicates that the overwhelming majority of the tenants lease land from one or two landlords only.

8. This result contrasts with Braverman and Srinivasan (1981) where m is an important control variable for the landlord. The difference arises from their somewhat peculiar assumption (which is neither supported empirically nor by a priori tenant utility maximization) that the tenant cannot take advantage of the labor market; and the landlord by varying the land size (or the tenant number) can nail

him down to the utility-equivalent level of the wage laborer. Another important difference of the Braverman-Srinivasan model with the present one is also worth noting: Whereas in the Braverman-Srinivasan world, the tenants and wage laborers are drawn from the same pool, in the present analysis they are assumed to have come from two different pools. Furthermore, whereas in Braverman and Srinivasan the tenant and the landless wage laborer enjoy the same utility level in equilibrium, in the present case this is not true.

9. A special case of the present model is provided by the instance where the landlord does not involve himself in any physical work; his effort on the self-retained land--whatever he decides to make--is devoted exclusively to monitoring of hired hands. This special case is applicable to a sizeable segment of the landlord class in South Asia. However, this would not change any of the qualitative results of the paper. We would have to simply replace equation (5a) by the following modified form:

(5a') $e^{l} = \phi(n^{lh};\mu)$ with $\phi' > 0$.

10. This is in the spirit of Sen, Bardhan (1973), Mazumder (1975) and others. For an empirical study, see Hossain (1977). Hossain made a comparative study of the market and (imputed) family wage rates for Bangladesh and found that the former was much higher than the latter. It may be noted that such an assumption of duality can also help explain a much-noted "stylized fact" of the peasant agriculture: the inverse relationship between the farm size and the output per acre (traceable to higher labour-intensities of smaller farmers vis-a-vis bigger farmers).

11. The economic basis of duality--though it has been the subject of good deal of recent discussion--has been far from settled. While some economists (notably A. K. Sen) would argue its justification in terms of subjective costs like alienation of working as a wage laborer, others would put it in terms of objective costs like search costs (measured in time) associated with market employment. In either case, however, the fact remains that $\lambda > 1$. In the text, we have--basically for reasons of simplicity--assumed that λ is fixed for cultivators of all sizes, i.e., the search and transactions costs for outside employment are the same across the board. A recent study by Huq (1984) in rural Bangladesh tends to cast doubt on the presumption: he finds that the landed class has lower transactions cost, and has a better chance of outside employment for its surplus labor, than the landless. Besides, as landownership increases, the household can find employment for its excess labor either because of better social connections or it can possibly organize non-farm activities more easily. Given the above, we elected to explore the implication of an alternative assumption that

(5b') $\lambda = \lambda(k^{so})$, with $\lambda > 1$ and $\lambda'(k^{so}) < 0$.

The assumption has the interesting implication that, even within the tenant class, the same sort of size-productivity inverse relationship will exist (because of differential transactions costs). However, the direct empirical evidence in this regard is yet too fragmentary to arrive at a firm conclusion.

12. In emphasizing the importance of these labour-market distortions in the emergence and persistence of sharecropping, at least in South Asia context Abhijit Sen (1981) went so far as to argue, "Sharecropping could be seen as an institutional response whereby rich owners could capture some of the 'cheapness' of 'poor peasants' family labour and could save on supervision. Thus, despite its disincentive effects, sharecropping was not necessarily unattractive to large owners, given the factors causing the inverse relationships between farm size and labour use" (p. 327). However, though these aspects of labour distortions have received adequate attention in the empirical literature, it has most surprisingly been ignored in the theoretical literature.

13. The continuity of the labour supply functions defined by (7a)-(7c) can be ensured if the maximum to the tenant optimizing problem is a unique one. In the present case, this can be shown as follows. For a given set of data $\{k^{ss}, k^{so}, w\}$, the income function of the tenant can be written as $y^s = \psi(n^{so}, n^{ss}, n^{sh})$ (see the equation (3a) in the text). The income function is a concave function of its arguments, given the concavity of the production function. Now the set $V = \{(y^s, n^{so}, n^{ss}, n^{sh}) | y^s \leq \psi (n^{ss}, n^{so}, n^{sh})\}$ is a convex set, which follows from the concavity of ψ. Now the utility function $u^s(y^s, n^{so} + n^{ss} + \lambda n^{sh})$ is defined to be a strictly quasi-concave function. Then it is relatively straight-forward to prove that a quasi-concave function u^s defined over a convex set V defines a unique maximum. Therefore, the tenant solution $(y^s_*, n^{so}_*, n^{ss}_*, n^{sh}_*)$ represents a unique maximum. This in turn ensures the continuity of the supply functions.

14. Carrying through the same line of argument as resorted in footnote (13), one can show that, given our assumptions, the landlord solution is a unique one.

15. Given our assumptions of a certain world and constant returns to scale, the reader might wonder whether we also run into the Bell-Braverman (1980) existence problem. The Bell-Braverman model however does not include any of the "imperfections" included in our mode; dual family labour for tenants and monitoring costs for the landlord. Of course, if both these effects are very weak then the Bell-Braverman results can be modified to apply; however, this is not very interesting. We assume throughout that labour market "imperfections" are sufficiently strong to make such models worth separate analysis.

16. σ has been parametrised by assuming a C.E.S. production function. See the Appendix for details.

17. Although we have not concerned ourselves with land-reform issues in this paper, it should be clear that because tenants are more productive on their own land than on sharecropped land, any measures that distribute land to the tiller will increase static efficiency. If the social welfare function is egalitarian e.g., Benthamite, then this will be a further reason for land reform.

References

P. K. Bardhan, "Size, Productivity and Returns to Scale: An Analysis of Farm Level Data in Indian Agriculture," J. Pol. Econ., Nov.-Dec. 1973, 81, 1370-1386.

P. K. Bardhan, "Variations in Forms of Tenancy in a Peasant Economy," J. Development Econ., June 1977, 4, 105-18.

P. K. Bardhan, "Agricultural Development and Land Tenancy in a Peasant Economy: A Theoretical and Empirical Analysis," Amer. J. Agri. Econ., Feb. 1979, 61, 48-57.

__________, and A. Rudra, "Terms and Conditions of Sharecropping Contracts: An Analysis of Village Survey Data in India," J. Dev. Stud., April 1980, 16, 287-302.

__________, and T. N. Srinivasan, "Cropsharing Tenancy in Agriculture: A Theoretical and Empirical Analysis," Amer. Econ. Rev., March 1971, 61, 48-64.

C. Bell, "Alternative Theories of Sharecropping: Some Tests and Evidence from Northeast India," J. Dev. Stud., July 1977, 13, 317-346.

C. Bell and A. Braverman, "On the Non-Existence of 'Marshallian' Sharecropping Contracts," Indian Econ. Rev., July 1980, 15, 201-203.

__________, and P. Zusman, "A Bargaining Theoretic Approach to Cropsharing Contracts," Amer. Econ. Rev., Sept. 1976, 66, 578-588.

C. T. Bliss, and N. H. Stern, Palanpur - Studies in the Economy of a North Indian Village, Oxford, Oxford University Press, 1982.

M. B. Bogue, Patterns From the Sod, Arno Press, New York, 1959.

A. Braverman, and T. N. Srinivasan, "Credit and Sharecropping in Agrarian Societies," J. Dev. Econ., Dec. 1981, 289-312.

S. N. S. Cheung, The Theory of Sharetenantcy, Chicago, Chicago University Press, 1969.

R. H. Day, "The Economics of Technological Change and the Demise of the Sharecropper," Amer. Econ. Rev., June 1967, 57, 427-449.

M. Hossain, "Agrarian Structure and Land Productivity: An Analysis of Farm Level Data in Bangladesh Agriculture," unpublished doctoral dissertation, University of Cambridge, 1977.

__________, "Factors Affecting Tenancy: The Case of Bangladesh Agriculture," Bangladesh Dev. Stud., Summer 1978, 2, 139-163.

S. Huq, "Patterns and Distribution of Rural Opportunities: A Comparative Study of Two Bangladesh Villages," unpublished doctoral dissertation, Boston University, 1984.

R. E. B. Lucas, "Sharing, Monitoring and Incentives: Marshallian Misallocation Reassessed," J. Pol. Econ., June 1979, 87, 501-521.

R. E. B. Lucas, "Labor in Four Districts of India," Boston University mimeo, November 1982.

A. Marshall, Principles of Economics (8th ed.), London, MacMillan, 1956.

D. Mazumdar, "Size of Farm and Productivity: A Problem of Indian Peasant Agriculture," Economica, May 1965, 32, 161-173.

__________, "The Theory of Sharecropping and Labor Market Dualism," Economica, August 1975, 42, 261-71.

D. M. G. Newbery, "The Choice of Rental Contract in Peasant Agriculture," in L. Reynolds (ed.), Agriculture in Development Theory, New Haven, Yale University Press.

D. M. G. Newbery and J. E. Stiglitz, "Sharecropping, Risksharing and the Importance of Imperfect Information," in J. A. Roumasset et. al. eds., Risk, Uncertainty and Agricultural Development, New York, Agriculture Development Council, 1979.

R. Pearce, "Sharecropping: Towards a Marxist View," Journal of Peasant Studies, Vol. 10, Nos. 2 & 3 (Jan./April 1983), edited by T. J. Byres.

M. G. Quibria and S. Rashid, "The Puzzle of Sharecropping: A Survey of Theories," World Development, February 1984, forthcoming.

C. H. H. Rao, "Uncertainty, Entrepreneurship and Sharecropping in India," J. Polit. Econ., May/June 1971, 79, 578-95.

Abhijit K. Sen, "Market Failure and Control of Labor Power: Towards an Explanation of 'Structure' and Change in Indian Agriculture: Part 2," Cambridge J. of Econ., 1981, 5, 327-350.

A. K. Sen, Employment, Technology and Development, Oxford, Oxford University Press, 1975.

Appendix

The total differentials of equations (10a) - (10c) provide us with:

$$\begin{bmatrix} a & b & -mf_{21}^{\ell} \\ a+(1-\phi')\frac{\partial x^{\ell}}{\partial n^{\ell o}} & b+(1-\phi')\frac{\partial x^{\ell}}{\partial n^{\ell h}} & -mf_{21}^{\ell} \\ f_{21}^{\ell} & f_{21}^{\ell} & -(mf_{11}^{\ell}+\pi f_{11}^{s}) \end{bmatrix} \begin{bmatrix} dn^{\ell o} \\ dn^{\ell h} \\ dk^{ss} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial x^{\ell}}{\partial w}\, dw \\ dw + \phi' \left(\frac{\partial x^{\ell}}{\partial w}\right) dw + x^{\ell}\phi'_{\mu} d\mu \\ f_1^s \frac{\partial \pi}{\partial \sigma} \cdot d\sigma + \pi f_{12}^s \frac{\partial n^s}{\partial \lambda} \cdot d\lambda + f_1^s \frac{\partial \pi}{\partial \alpha} \cdot d\alpha + \pi f_{12}^s \frac{\partial n^s}{\partial w} \cdot dw \end{bmatrix}$$

where $a = f_{22}^{\ell} - \frac{\partial x^{\ell}}{\partial n^{\ell o}}$ and $b = f_{22}^{\ell} - \frac{\partial x^{\ell}}{\partial n^{\ell h}}$.

Note that, in taking these differentials, we have assumed, for simplicity, that $\phi'' = 0$. An alternative assumption would not add much insight, but serve to clutter up the already messy algebra. Further note that,

$$\frac{\partial x^{\ell}}{\partial n^{\ell h}} = \frac{-[(f_2^{\ell} - w)p + \phi' q]}{(u_1^{\ell})^2}; \quad \frac{\partial x^{\ell}}{\partial n^{\ell o}} = \frac{-[f_2^{\ell} p + q]}{(u_1^{\ell})^2}$$

$$\frac{\partial x^{\ell}}{\partial w} = \frac{n^{\ell h} p}{(u_1^{\ell})^2} \text{ and } \frac{\partial x^{\ell}}{\partial k^{ss}} = 0,$$

where $p \equiv u_1^\ell u_{21}^\ell - u_2^\ell u_{11}^\ell$ and $q \equiv u_1^\ell u_{22}^\ell - u_2^\ell u_{12}^\ell$.

Given that both income and leisure are normal goods, $p < 0$ and $q < 0$.

This further implies that

$$\frac{\partial x^\ell}{\partial n^{\ell h}} > 0,\ \frac{\partial x^\ell}{\partial n^{\ell o}} > 0 \text{ and } \frac{\partial x^\ell}{\partial w} < 0.$$

Define $z \equiv \frac{\partial x^\ell}{\partial n^{\ell h}} - \frac{\partial x^\ell}{\partial n^{\ell o}}$

Unpon substitution, we get,

$$z = \frac{wp + (1-\phi')q}{(u_1^\ell)^2},\ \text{provided } (1-\phi') > 0.$$

However, from (10a) and (10b), we find that $(1-\phi') = \frac{w}{x^\ell}$ hence, $(1-\phi') > 0$.

Let D denote the determinant of the matrix on the LHS and D_{ij} the cofactor of the element in the i^{th} row and j^{th} column.

$$D_{31} = f_{12}^\ell(1-\phi')z;\ D_{32} = -f_{12}^\ell z;\ D_{33} = (1-\phi')f_{22}^\ell z;\ D > 0.$$

We now have:

$$\frac{\partial k^{ss}}{\partial w} = D^{-1}\{-f_{12}^\ell z + \pi f_{12}^\ell \frac{\partial n^{ss}}{\partial w} \cdot (1-\phi')f_{22}^\ell z\}$$

Since $\pi f_{12}^s \frac{\partial n^{ss}}{\partial w} = \frac{-\pi(1-\alpha)f_1^s}{\alpha w} = \frac{-(1-\alpha)f_1^\ell}{\alpha w}$ from (10c) and, by CRS, $f_{22}^\ell = -f_{21}^\ell \cdot \frac{k^\ell}{n^{\ell h}+n^{\ell o}}$.

Upon substitution we get

$$\frac{\partial k^{ss}}{\partial w} = D^{-1}\{-f_{12}^\ell z + \frac{(1-\alpha)}{\alpha} f_{12}^\ell z \frac{(1-\phi')}{w} \frac{f_1^\ell k^\ell}{(n^{\ell h}+n^{\ell o})}\}$$

From (10a) and (10b), $\frac{(1-\phi')}{w} = \frac{1}{f_2^\ell}$. Whence,

$$\frac{\partial k^{ss}}{\partial w} = D^{-1} f_{12}^\ell z\{-1 + \frac{1-\alpha}{\alpha} \cdot \frac{f_1^\ell}{f_2^\ell} \frac{k^\ell}{n^{\ell h}+n^{\ell o}}\}$$

Since $\frac{f_1^{\ell}k^{\ell}}{f_2^{\ell}(n^{\ell h}+n^{\ell o})}$ is the relative factor share on the landlords retained land, which we denote by $\frac{\alpha^*}{(1-\alpha^*)}$, we get:

$$\frac{\partial k^{ss}}{\partial w} = D^{-1}f_{12}^{\ell}w\{-1 + \frac{(1-\alpha)}{\alpha}\frac{\alpha^*}{(1-\alpha^*)}\}$$

We will need the following lemma:

Lemma 1: For a CES Production Function, $\frac{(1-\alpha^*)\alpha^*}{(1-\alpha^*)\alpha} \gtrless 1$ as $\sigma \gtrless 1$.

Proof: $\frac{\alpha^*}{1-\alpha^*} = f_1^{\ell} \cdot \frac{k^{\ell}}{f_2^{\ell}(n^{\ell h}+n^{\ell o})} = \frac{\pi f_1^s k^{\ell}}{f^{\ell} - k^{\ell}f_1^{\ell}}$

$$= \frac{\pi f_1^s k^{\ell}}{f^{\ell} - \pi f_1^s k^{\ell}}, \text{ from (10c) and CRS.}$$

Rearranging: $\frac{\alpha^*}{1-\alpha^*} = \frac{\pi f_1^s k^s (k^{\ell}/k^s)}{f_1^{\ell} - \pi f_1^s k^s (k^{\ell}/k^s)}$

$$= \frac{(\pi f^s k_1^s)/f^s}{(f^{\ell}/f^s)/(k^{\ell}/k^s) - \pi(f_1^s k^s/f^s)}$$

$$= \frac{\pi\alpha}{(f^{\ell}/f^s)/(k^{\ell}/k^s) - \pi\alpha}$$

By the CES property, $\frac{f^{\ell}}{f^s} / \frac{k^{\ell}}{k^s} = \left(\frac{f_1^{\ell}}{f_1^s}\right)^{\sigma} = \pi^{\sigma}$, from (10c).

Hence, $\frac{\alpha^*}{1-\alpha^*} = \frac{\pi\alpha}{\pi^{\sigma} - \pi\alpha} = \frac{\alpha}{\pi^{\sigma-1} - \alpha}$

Therefore, $\frac{(1-\alpha)\alpha^*}{(1-\alpha^*)\alpha} = \frac{1-\alpha}{\pi^{\sigma-1} - \alpha} \gtrless 1$ according as, $(1-\alpha) \gtrless \pi^{\sigma-1} - \alpha$, or as, $1 \gtrless \pi$, i.e., $\sigma \gtrless 1$.

Q.E.D.

From Lemma 1,

$$\frac{\partial k^{ss}}{\partial w} = D^{-1}f_{12}^{\ell} z\{-1 + \frac{(1-\alpha)\alpha^*}{(1-\alpha^*)\alpha}\} \lessgtr 0 \text{ as } \sigma \gtrless 1.$$

Since sharecropping is most likely to exist for σ in (0,1), we finally get,

(a) $\frac{\partial k^{ss}}{\partial w} > 0.$

From our system of total differentials,

(b) $$\frac{\partial k^{ss}}{\partial \lambda} = D^{-1}\left\{D_{33}\pi f^{s}_{12} \frac{\partial n^{ss}}{\partial \lambda}\right\}$$

As $\frac{\partial n^{ss}}{\partial \lambda} > 0$, $D_{33} > 0$, we have $\frac{\partial k^{ss}}{\partial \lambda} > 0$.

(c) $$\frac{\partial k^{ss}}{\partial \alpha} = D^{-1}\left\{f^{s}_{1} \frac{\partial \pi}{\partial \alpha} D_{33}\right\}$$

As $\frac{\partial \pi}{\partial \alpha} = \frac{-(1-\sigma)}{\pi^2} < 0$ for σ in (0,1), the relevant range of share-cropping, we have $\frac{\partial k^{ss}}{\partial \alpha} < 0$.

(d) $$\frac{\partial k^{ss}}{\partial \sigma} = D^{-1}\left\{f^{s}_{1} \frac{\partial \pi}{\partial \sigma} D_{33}\right\}$$

As $\frac{\partial \pi}{\partial \sigma} = \frac{-(1-\alpha)}{\pi^2} < 0$, it follows that $\frac{\partial k^{ss}}{\partial \sigma} < 0$.

(e) $$\frac{\partial k^{ss}}{\partial \mu} = D^{-1}\left\{D_{32}\ x^{\ell}\phi'_{\mu}\right\}$$

Assuming that $\phi'_{\mu} > 0$, i.e., the shift parameter increases the (marginal) cost of monitoring, since $D_{32} > 0$ and $D > 0$, we get $\frac{\partial k^{ss}}{\partial \mu} > 0$.

Finally, note that the optimising calculus of the landlord implies that $\frac{\partial k^{ss}}{\partial k^{so}} = 0$. In other words, in the present formulation, the landlords optimising k^{ss} is independent of the tenants own holding, k^{so}. Now note that the bullock capacity of the tenant sets the constraint $k^{so} + k^{ss} \leqslant \overline{k}$. This implies that $\frac{\partial k^{ss}}{\partial k^{so}} \leqslant -1 < 0$. In other words, there is likely to be an ex post negative relationship between tenant land-holding and the availability of sharecropped land to him, even if the landlord does not discriminate among tenants on the basis of land-holdings. However, the landlord will discriminate when tenants have different labor transactions costs (as was noted in footnote 11) and tenant optimisation will imply that $\frac{\partial n^{ss}}{\partial k^{so}} < 0$. For example, suppose $\lambda = \lambda(k^{so})$ with $\lambda' < 0$ and $\lambda > 1$. In this context, $\frac{\partial n^{ss}}{\partial k^{so}} < 0$ and $\frac{\partial x^{\ell}}{\partial k^{so}} = pf_2^s \frac{\partial n^{ss}}{\partial k^{so}} \cdot \frac{1}{(u_1^{\ell})^2} > 0$, instead of being zero. The corresponding comparative static result now becomes, $\frac{\partial kss}{\partial k^{so}} = D^{-1}\{D_{31} \frac{\partial x\ell}{\partial k^{so}} + D_{32}\ \phi' \frac{\partial x\ell}{\partial k^{so}}\}$,

or (f) $$\frac{\partial k^{ss}}{\partial k^{so}} = D^{-1}\{\phi' z f_{12}^{\ell} \frac{\partial x^{\ell}}{\partial k^{so}}\} < 0.$$