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AN ADJUSTMENT PROCEDURE FOR PREDICTING SYSTEMATIC RISK

Abstract

This paper looks at the currently available beta adjustment techniques and suggests a multiple root-linear model to adjust for the regression tendency of betas. Our empirical investigation indicates that cross-sectional betas are not normally distributed but their distribution tends to normal after a square-root transformation. The multivariate normality observed among betas after the transformation, makes the functional form of our model correct. Also, we observe that the disturbance term of the multiple root-linear model passes the tests for normality and homoscedasticity. These findings make the ordinary least squares estimates unbiased and efficient. Finally, the mean square errors are found to be lower when our adjustment procedure is used vis-a-vis the existing procedures.

AN ADJUSTMENT PROCEDURE FOR PREDICTING SYSTEMATIC RISK

Estimation of systematic risk is one of the important aspects of investment analysis and has attracted the attention of many researchers. In spite of substantial contributions in the recent past there still remains room for improvement in the methodologies currently available to forecast systematic risk. This paper is concerned with an improved method of predicting systematic risk for individual securities.

The central model in most of the research pertaining to systematic risk has been the single index model:

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{R}_{mt} + \tilde{\varepsilon}_{it} \qquad (1)$$

where $\tilde{R}_{it}$ and $\tilde{R}_{mt}$ are, respectively, the random return on security i and the corresponding random market return in period t, α_i and β_i are the regression parameters[1] appropriate to security i, and $\tilde{\varepsilon}_{it}$ is the random disturbance term with $E(\tilde{\varepsilon}) = 0$. One of the many assumptions that model (1) is based on is that β_i is constant over time.[2] In his seminal paper Blume [3] observed that β_i was changing over time and there was a regular pattern in the movement which he termed as the regression tendency of beta coefficients. Blume's empirical finding supported the hypothesis that over time betas appear to take less extreme values and exhibit a tendency towards the mean value. This would mean that the historical betas would be poor estimators of the future betas. To account for this tendency Blume [3] suggested a regression adjustment procedure. Alternatively, Vasicek [12] proposed a Bayesian adjustment technique where the adjusted beta is the weighted average of the historical beta and the mean of the cross-sectional betas.[3]

Let us suppose that we are interested in predicting the betas in period t and let β_{t-1} be the historical beta in period (t-1). The various adjustment procedures described above can be put in the following framework:

$$\hat{\beta}_{ti} = d_{1i} + d_{2i}\beta_{t-1i} \tag{2}$$

where $\hat{\beta}_{ti}$ denotes the predicted value of beta for security i in period t.

(i) Unadjusted betas are obtained by substituting $d_{1i} = 0$ and $d_{2i} = 1$ in (2).

(ii) Let $\hat{\delta}_1$ and $\hat{\delta}_2$ be the ordinary least squares (OLS) estimates obtained by estimating the linear regression model,

$$\beta_{t-1} = \delta_1 + \delta_2\beta_{t-2} + u \tag{3}$$

where u is the disturbance term. Now, the adjustment procedure suggested by Blume is obtained by substituting $d_{1i} = \hat{\delta}_1$ and $d_{2i} = \hat{\delta}_2$ in (2). The forecasting model can be written as

$$\hat{\beta}_{ti} = \bar{\beta}_{t-1} + \hat{\delta}_2(\beta_{t-1i} - \bar{\beta}_{t-2}) \tag{4}$$

where $\bar{\beta}_t$ denotes the cross-sectional mean value in period t. Blume's hypothesis is clearly reflected in (4). In each period beta shifts towards the mean and the amount of shift is proportional to the distance of beta from the past mean value.[4]

(iii) The adjustment technique suggested by Vasicek involves substituting $d_{1i} = \bar{\beta}_{t-1} w_i$ and $d_{2i} = 1 - w_i$ in (2) with

$$w_i = \frac{\bar{s}_{bi}^{2}}{\bar{s}_b^{2} + s_{bi}^{2}} \qquad (5)$$

where $\bar{s}_b^{2}$ is the estimated variance of the cross-sectional betas in period (t-1) and s_{bi}^{2} is the estimated variance of β_{t-1i}.

Here we should note that for Vasicek's adjustment the coefficients d_{1i} and d_{2i} in (2) are different for each security, whereas MLPFS[5] and Blume's adjustment techniques use the same coefficients for all the securities. The primary purpose of all these approaches is the same - to shrink the beta values towards their mean. While in Blume's method the amount of shrinkage depends only on how far the historical beta is away from the average value, Vasicek's method, in addition, takes into account the precision of the historical betas. Blume ([4], p. 789) justifies the regression line (3) by assuming that β_{t-1} and β_{t-2} are bivariate normal. Vasicek's Bayesian estimation uses a normal prior for beta. We test the validity of these normality assumptions in the next section.

Now let us look at an implication of the hypothesis that low betas exhibit an increasing tendency and high betas exhibit a decreasing tendency over time. If we consider the scatter diagram in the $(\beta_{t-2}, \beta_{t-1})$ plane, observations corresponding to low values of β_{t-2} should in general lie above the line $\beta_{t-1} = \beta_{t-2}$ and observations corresponding to high values of β_{t-2} should in general lie below the line.[6] Equation (3) implies that the extreme values of beta which are on either side of but equidistant from the mean will shift by equal amounts. This may not always be true. Actually Blume's ([3], p. 8) evidence shows that the regression tendency towards the mean is stronger for lower values of beta than for higher values. This suggests that rather than a linear regression, a nonlinear relationship such as

$$\beta_{t-1} = \delta_1' \beta_{t-2}^{\delta_2} e^u \tag{6}$$

where $\delta_1', \delta_2 > 0$, may, perhaps capture the regression tendency better. Equation (6) leads to a log-linear regression

$$\log \beta_{t-1} = \delta_1 + \delta_2 \log \beta_{t-2} + u \tag{7}$$

where $\delta_1 = \log \delta_1'$ and u is now the additive disturbance term. Both the linear and the log-linear regressions are special cases of the following general regression model:

$$\beta_{t-1}(\lambda) = \delta_1 + \delta_2 \beta_{t-2}(\lambda) + u \tag{8}$$

where $\beta(\lambda) = \frac{\beta^\lambda - 1}{\lambda}$ is the Box-Cox [5] transformation. When $\lambda = 1$ we obtain the linear regression model and when $\lambda \to 0$ equation (8) reduces to the log-linear model. The Box-Cox transformation might, in addition to correcting any functional misspecification present in model (3), push the distribution of transformed variables towards normality. In section II we try to find a suitable data transformation under the Box-Cox framework. Another way to generalize the model (3) would be to include some extra variables. A natural step is to include betas from an additional lag period; for example, we may consider

$$\beta_{t-1} = \delta_1 + \delta_2 \beta_{t-2} + \delta_3 \beta_{t-3} + u \tag{9}$$

where we would expect $\delta_2 > \delta_3$. We discuss this multiple regression tendency and look at some empirical evidence in Section III.

No matter how good a model is to explain β_{t-1}, its usefulness lies in predicting β_t, the future betas. Earlier studies by Klemkosky and Martin [10], and Eubank and Zumwalt [7] indicate that, taking mean square error (MSE) as a criterion, there is very little difference between Blume's and Vasicek's adjustment techniques and they, in general, outperform MLPFS technique and unadjusted betas. Results of a comparative study on the performance of the adjustment procedures suggested by us vis-a-vis Blume's and Vasicek's techniques are reported in Section IV. In the last section we summarize our findings and make some concluding remarks.

I. Sampling Properties of Beta

Monthly observations on $\tilde{R}_i$ and $\tilde{R}_m$ were obtained from the CRSP tapes.[7] The time span considered was from July 1948 through June 1983 and this was divided into seven non-overlapping estimation periods of sixty months each. Using model (1) beta coefficients for individual securities were estimated in all the seven periods and some selected sample statistics for these estimates have been reported in Table I.

As expected the means of the cross-sectional betas are all very close to unity and the standard deviations are around 0.45.[8] The last three columns of Table I are concerned with the normality of betas. It can be seen that the empirical distributions of betas are positively skewed and often platykurtic. The following test statistic which combines the skewness and kurtosis measures was used for testing normality of betas:

$$N\left[\frac{(\text{skewness})^2}{6} + \frac{(\text{kurtosis} - 3)^2}{24}\right]$$

where N is the sample size (see Bera and Jarque [1]).[9] The values of the statistic have been reported in the last column. The null hypothesis that betas are normally distributed, is overwhelmingly rejected in all the seven periods.[10] Since the betas are not univariate normal in any of the periods, the possibility of bivariate normality of betas from any two consecutive periods can be ruled out. These empirical findings cast some doubt on the validity of both Blume's and Vasicek's procedures.

II. Simple and Nonlinear Regression Tendency

In order to decide the nature of the regression tendency, we considered the equations (3), (7), and (8). We refer to these three as the 'linear', 'log-linear', and 'Box-Cox' models respectively. We estimated these three models for t = 3,4,5,6 and 7. One of the important results we obtained was that the estimates of lambdas from the Box-Cox regressions were around 0.5. This could imply that a Box-Cox model with $\lambda = 0.5$, i.e.,

$$\sqrt{\beta_{t-1}} = \delta_1 + \delta_2 \sqrt{\beta_{t-2}} + u \tag{10}$$

would be more appropriate.[11] We refer to equation (10) as the 'root-linear' model. This model was also estimated for the above mentioned values of t. The results of the various regressions have been reported in Table II.[12] The coefficient of determination (R^2) values are not very high. It must be noted that the R^2 values for different models are not comparable since the dependent variables are different. In the case of the linear model the t-statistics on slope and the intercept are highly significant. However, the regression coefficients vary drastically from one estimation period to another indicating instability in the regression tendency. This is true for all the four models. Next we looked at the normality of the disturbance term. While normality was rejected in all the regressions with the linear and the log-linear models, it was accepted in all but one of the regressions with the root-linear model. This implies that the OLS estimates are efficient when we use the root-linear model, assuming that the functional form is well specified and there is no heteroscedasticity. Also, these results led us to the conjecture that even though betas were not normally distributed, the square-root transformation could probably make them normal while the log-transformation would not.[13] To verify this we applied the normality test on log-betas and root-betas. The outcomes have been reported in Table III. The numbers are quite striking. The log-transformation makes the distribution more skewed but negatively, and the moderate platykurtosis changes to strong leptokurtosis. As a result the problem of non-normality becomes more acute. With the square-root transformation, on the other hand, the values of skewness and kurtosis change in such a way that the normality hypothesis can be accepted at 10% and 1% significance levels, in four out of seven periods. Also, the values of the test statistic in the remaining three periods are not much above the critical value. Given these results it may not be irrational to conclude that betas follow a 'root-normal' distribution, i.e., square root of the variable is normal.[14] Of course this does not mean that $\sqrt{\beta_{t-1}}$ and $\sqrt{\beta_{t-2}}$ in equation (10) are jointly normally distributed.[15]

III. Multiple Regression Tendency

By examining the betas over the seven time periods, we noticed that the regression tendencies were rather fuzzy. For example, the tendency to decrease or to increase persisted even after crossing the cross-sectional mean values. Often the extreme beta values did not move towards the grand mean in the next period but did so in the period after. Such empirical observations implied that the tendency from period (t-2) to (t-1) was not the best estimate of the tendency from period (t-1) to (t). An alternative possibility is to include more 'lags' in the model. This has an intuitive appeal since the regression tendency could be spread over more than just the previous and current periods. To test this hypothesis we decided to work with model (9), which we refer to as the 'multiple linear' (ML) model. Also, given the success of the 'root-linear' model, we considered the 'multiple root-linear' (MRL) model:

$$\sqrt{\beta_{t-1}} = \delta_1 + \delta_2 \sqrt{\beta_{t-2}} + \delta_3 \sqrt{\beta_{t-3}} + u \qquad (11)$$

Results from the regressions using (9) and (11) are given in Table IV. The R^2 values for the two regressions are low and are not directly comparable to those in Table II since the sample sizes are different. In both the multiple regressions, the slope coefficients and the intercepts are highly significant. We expected the coefficient on lag 1 (δ_2) to be greater than the coefficient on lag 2 (δ_3), since we felt that information from the immediate past should have more bearing on the current betas than information from the distant past. Empirically this is found to be true in all the periods for both the regression models.[16]

To evaluate the two models, we tested for normality and homoscedasticity[17] of the disturbance terms. The values of the test statistics are in Table IV. Both the models pass the test for homoscedasticity in three out of four cases with the MRL model being marginally better than the ML model. The fundamental difference between the two models is revealed by the normality test statistic. For the MRL model the normality of the disturbance term is accepted in all the four estimation periods at 5% level of significance; however, for the ML model we strongly reject the normality hypothesis in all four periods.

To conclude, among all the models that we have considered so far, the MRL model captures the regression tendency best. Of course this does not necessarily mean that betas adjusted on the basis of the MRL model would be the best in forecasting. A comparative study is needed to evaluate the performance of different adjustment procedures and this we do in the next section.

IV. Performance of Different Adjustment Procedures

For this study we considered the unadjusted betas and the betas obtained using Vasicek, Blume, log-linear, root-linear, ML and MRL adjustment techniques.[18] The criterion used for comparison was Mean Square Error (MSE) given by

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (A_i - P_i)^2$$

where A_is are the actual estimates of beta in period t, P_is are the predicted values of beta using a particular technique, and N is the number of securities for which predictions were made. Following Klemkosky and Martin [10] we decomposed the MSE into three components, namely, bias, inefficiency, and random error. The results have been reported in Table V. The ML model outperforms Blume's technique in all the periods and the MRL model does the same in all but one period. Also, the MRL model performs better than the simple root-linear model in all the periods. However, in terms of MSE there is very little to choose between the MRL and the ML models. Both these models outperform Vasicek's adjustment and as expected all the adjusted betas do better than the unadjusted betas.

Given these results and the acceptance of normality of the disturbance term in model (11), the choice of MRL model over ML model seemed reasonable. To further justify this choice we decided to examine the trivariate normality of $\sqrt{\beta_{t-1}}$, $\sqrt{\beta_{t-2}}$, and $\sqrt{\beta_{t-3}}$ [see footnote 15]. The test was performed for both betas and root-betas using the test statistic developed by Bera and John [2].[19] The results are in Table VI. As anticipated we rejected the joint normality of betas (recall that the univariate normality had been rejected in all time periods). In the case of root-betas we rejected the joint normality hypothesis only in two periods. Even in these two cases the test statistic was much smaller than that obtained for the actual betas. To summarize, model (11) has a well justified functional form with a disturbance term that is both normal and homoscedastic. Therefore, the OLS estimates will be unbiased and efficient. Moreover, this model leads to a reduction in MSE. Hence, the adjustment technique using the multiple root-linear model is definitely a major improvement over currently available techniques.

V. Summary and Conclusions

In this paper we have provided empirical evidence that betas obtained from the single index model are not normally distributed, but become normal after the square-root transformation. This finding suggests that the adjustment techniques proposed by Blume, and Vasicek may not always be appropriate. On the other hand, the joint normality of the root betas, observed in several periods, together with the normality and homoscedasticity of the disturbance term shows that the multiple root-linear model is quite suitable. Also, with mean square error as a criterion the adjustment technique suggested by us performs better than the other techniques. Our model can be extended by including some firm-specific variables. Also, further investigation is necessary to determine the optimal lag-length.

FOOTNOTES

[1]The parameter β_i, called beta, measures the systematic risk of security i and is defined as $Cov(\tilde{R}_i, \tilde{R}_m)/Var(\tilde{R}_m)$.

[2]Many attempts have been made to relax this assumption. Fabozzi and Francis [8], Sunder [11], and others have tried to account for the variations of beta over time by treating it as a random coefficient.

[3]Some straightforward ways to adjust beta towards the average value are also available. One such technique used by Merrill Lynch, Pierce, Fenner & Smith, Inc. (MLPFS) is a particular case of Vasicek's Bayesian estimate, where the variances of historical betas are all assumed to be equal.

[4]It can be observed that if the average beta increases over two periods then equation (4) assumes that this trend will persist in the next period also. This may not be realistic. Elton and Gruber [6] suggest that by applying a mean-correction to the betas obtained using Blume's adjustment the forecasts could be improved.

[5]MLPFS beta estimates are easily obtained from equation (4) by putting $\bar{\beta}_{t-1} = \bar{\beta}_{t-2} = 1$, i.e.,

$$\hat{\beta}_{ti} = 1 + \hat{\delta}_2(\beta_{t-1i} - 1).$$

Therefore, the performances of MLPFS and Blume's techniques can be expected to be close whenever the average value of betas is close to unity.

[6] The 'high' and 'low' values are relative to the mean.

[7] For $\tilde{R}_m$ we have used the value weighted return series including ordinary dividends.

[8] These statistics are similar to those reported by Blume [3], although his time span was different (July 1926 through June 1968) and the length of each estimation period in his study was seven years.

[9] The test statistic is derived using the Lagrange multiplier test principle with Pearson family of distributions as alternatives, and this test has very good power compared to other tests of normality. Under the normality hypothesis, the statistic is asymptotically distributed as central χ^2 (Chi-square) with 2 degrees of freedom. Given our large sample sizes (see column 2 of Table I) we can safely apply this test. The asymptotic critical values at 10%, 5% and 1% significance levels are, respectively, 4.61, 5.99 and 9.21.

[10] Here we should note that because of the large sample size the test will have very high power and consequently a slight departure from normality will result in the rejection of the normality hypothesis.

[11] We are thankful to Paul Newbold for pointing this out.

[12] The number of cases is less than those reported in Table I since the common set of companies between any two periods was less than the number of companies in either period.

[13] Note that the disturbance term is a linear combination of the dependent and indpendent variables.

[14]This at first sight may appear questionable since the root-transformation is defined only for positive betas and only positive roots have been considered. Empirically, about 7500 betas were estimated and of these only 13 were found to be negative. Also, the empirical mean and standard deviation of root-betas were approximately 1.03 and 0.22, respectively. For a normal random variable with the above mean and standard deviation, the probability of it being negative is less than 8.3×10^{-7}.

[15]If $\sqrt{\beta_{t-1}}$ and $\sqrt{\beta_{t-2}}$ are bivariate normal then $E(\sqrt{\beta_{t-1}} | \sqrt{\beta_{t-2}}) = \delta_1 + \delta_2 \sqrt{\beta_{t-2}}$ with $E(u) = 0$ in (10). This will ensure unbiasedness of the OLS estimates. In addition, if u is normally distributed and homoscedastic, estimates will be efficient. In section IV we test the joint normality of the root-betas.

[16]It may be interesting to investigate the problem of determining the 'lag-length'.

[17]To test for homoscedasticity we used White's [13] test since it does not assume any specific form of heteroscedasticity. The test statistic is calculated as $N \cdot R^2$ where N is the sample size and R^2 is the coefficient of determination obtained by regressing the square of the residuals on all second order products and cross-products of the original regressors. Under homoscedasticity, the test statistic is asymptotically distributed as central χ^2 with 5 degrees of freedom. The critical values at 10%, 5% and 1% significance levels are 9.24, 11.07 and 15.09, respectively.

[18]It should be noted that the log-linear and the root-linear models would forecast log β_t and $\sqrt{\beta_t}$, respectively, and not β_t. As pointed out by Granger and Newbold [9], the residual variance was taken into account by us in obtaining the predicted values of β_t from the forecasts of log β_t and $\sqrt{\beta_t}$.

[19]The test statistic is a generalization of the test noted in footnote 9 and is derived using multivariate Pearson family of distributions. Under the hypothesis of multivariate normality the test statistic is asymptotically distributed as central χ^2 with 6 degrees of freedom. The critical values for this test are 10.64, 12.59, and 16.81 at 10%, 5% and 1% significance levels, respectively.

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Table I

Summary Statistics of Beta Coefficients

Period	Number of Companies	Mean	Standard Deviation	Skewness	Kurtosis	Normality Test Statistics
7/48 - 6/53	910	1.114	0.441	0.236	2.697	11.928
7/53 - 6/58	938	0.924	0.434	0.325	2.701	20.007
7/58 - 6/63	923	1.044	0.377	0.563	3.244	51.050
7/63 - 6/68	956	1.169	0.494	0.395	2.824	26.094
7/68 - 6/73	1056	1.235	0.478	0.486	2.845	42.628
7/73 - 6/78	1255	1.160	0.412	0.590	3.375	80.164
7/78 - 6/83	1208	1.096	0.465	0.353	2.836	26.442

Table II

Results from Simple Regressions

	Number of Companies	R^2	Slope	T-Value	Intercept	T-Value	Normality Test Statistic	Lambda
(Periods 1 and 2)								
Linear	826	0.4060	0.61428	23.732	0.25493	8.274	11.9743	
Log-linear	826	0.3854	0.73431	22.729	-0.20224	-13.279	380.9569	
Root-linear	826	0.4091	0.67788	23.885	0.24098	8.077	57.9667	
Box-Cox	826	0.4106	0.65528	23.958	-0.15030	-12.408		0.68
(Periods 2 and 3)								
Linear	820	0.1765	0.36060	13.239	0.70314	25.105	113.6054	
Log-linear	820	0.1895	0.29343	13.831	0.03092	2.419	87.3352	
Root-linear	820	0.1909	0.33833	13.892	0.68615	29.180	9.8535	
Box-Cox	820	0.1918	0.33344	13.935	0.04674	3.891		0.43
(Periods 3 and 4)								
Linear	752	0.2402	0.63619	15.399	0.48521	10.665	34.1079	
Log-linear	752	0.2308	0.61132	15.002	0.04919	3.096	74.7675	
Root-linear	752	0.2460	0.63018	15.643	0.41271	10.070	0.1739	
Box-Cox	752	0.2463	0.63241	15.656	0.09354	6.240		0.61
(Periods 4 and 5)								
Linear	810	0.3494	0.56290	20.832	0.54048	15.908	46.6408	
Log-linear	810	0.3371	0.50811	20.271	0.07143	5.854	122.5901	
Root-linear	810	0.3520	0.54402	20.951	0.49916	17.865	0.9673	
Box-Cox	810	0.3520	0.54402	20.951	0.08636	6.917		0.50
(Periods 5 and 6)								
Linear	900	0.2525	0.39491	17.418	0.62473	21.073	34.1631	
Log-linear	900	0.2664	0.44022	18.057	-0.01012	-0.966	41.2577	
Root-linear	900	0.2621	0.42262	17.862	0.57904	22.155	0.4503	
Box-Cox	900	0.2634	0.42675	17.918	0.00071	0.066		0.41

Table III

Normality Test of Transformed Betas

Period	Log Transformation			Root Transformation		
	Skewness	Kurtosis	Test Statistic	Skewness	Kurtosis	Test Statistic
7/48 - 6/53	-0.998	4.333	218.434	-0.308	2.816	15.671
7/53 - 6/58	-0.904	3.585	141.133	-0.239	2.505	18.506
7/58 - 6/63	-0.577	3.793	75.400	0.044	2.937	0.450
7/63 - 6/68	-0.827	3.741	130.844	-0.162	2.702	7.719
7/68 - 6/73	-0.672	4.018	125.077	-0.013	2.744	2.913
7/73 - 6/78	-0.477	3.373	54.867	0.080	2.915	1.716
7/78 - 6/83	-0.855	3.670	169.774	-0.210	2.690	13.716

Table IV

Results from Multiple Regressions

	Number of Companies	R^2	Slope 1 (T-Value)	Slope 2 (T-Value)	Intercept (T-Value)	Test Statistic Homoscedasticity	Test Statistic Normality
(Periods 1, 2 and 3)							
Multiple Linear	717	0.1760	0.26492 (6.702)	0.13097 (3.421)	0.65720 (18.421)	14.9853	75.5904
Multiple Root-linear	717	0.1809	0.25402 (7.104)	0.11417 (3.011)	0.65217 (21.174)	18.5703	5.4309
(Periods 2, 3 and 4)							
Multiple Linear	664	0.2923	0.52416 (10.773)	0.24562 (5.942)	0.36237 (7.414)	5.9760	38.8388
Multiple Root-linear	664	0.3058	0.49646 (10.381)	0.24748 (6.754)	0.31021 (7.113)	12.6824	0.7498
(Periods 3, 4 and 5)							
Multiple Linear	640	0.3537	0.49740 (14.594)	0.13828 (3.044)	0.45639 (9.954)	14.2080	40.5092
Multiple Root-linear	640	0.3510	0.47718 (14.457)	0.12225 (2.854)	0.43987 (11.088)	6.2720	1.3340
(Periods 4, 5 and 6)							
Multiple Linear	691	0.2834	0.29027 (8.997)	0.16970 (5.580)	0.54193 (15.683)	24.1850	34.5790
Multiple Root-linear	691	0.2944	0.30287 (9.057)	0.18114 (5.976)	0.51173 (16.666)	6.5645	5.4836

Table V

Mean Square Errors of Adjusted and Unadjusted Beta Coefficients

	Unadjusted	Vasicek	Blume	Log-linear	Root-linear	Multiple Linear	Multiple Root-linear
(Period 4)							
Mean Square Error (MSE)	.19998	.18681	.18921	.19198	.19014	.18727	.18771
Portion of MSE due to:							
Bias	.01018	.01437	.00280	.00149	.00229	.00582	.00509
Inefficiency	.01565	.00026	.01227	.01492	.01368	.01611	.01734
Random Error	.17415	.17219	.17415	.17556	.17417	.16533	.16527
(Period 5)							
Mean Square Error (MSE)	.18324	.15119	.13822	.14334	.13887	.13677	.13854
Portion of MSE due to:							
Bias	.00084	.00345	.00199	.00305	.00168	.00211	.00218
Inefficiency	.04800	.00936	.00183	.00472	.00241	.00157	.00177
Random Error	.13440	.13838	.13440	.13557	.13477	.13309	.13458
(Period 6)							
Mean Square Error (MSE)	.18555	.13774	.12119	.12205	.12026	.11793	.11548
Portion of MSE due to:							
Bias	.00995	.00646	.01578	.01897	.01646	.01499	.01489
Inefficiency	.07582	.03162	.00564	.00367	.00434	.00667	.00453
Random error	.09978	.09965	.09978	.09941	.09947	.09628	.09605
(Period 7)							
Mean Square Error (MSE)	.18475	.15917	.14725	.14668	.14675	.13002	.12984
Portion of MSE due to:							
Bias	.00545	.00448	.00117	.00165	.00129	.00155	.00178
Inefficiency	.03445	.01070	.00124	.00046	.00089	.00459	.00436
Random Error	.14484	.14398	.14484	.14458	.14457	.12388	.12370

Table VI

Multivariate Normality Test Statistics

Period	Beta	Root-beta
1, 2 and 3	90.71251	31.88872
2, 3 and 4	130.48173	21.18911
3, 4 and 5	106.57415	14.30298
4, 5 and 6	124.25677	11.01903
5, 6 and 7	131.86308	7.35194

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