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Local Income Redistribution:
A Club-Theoretic Analysis

Jan K. Brueckner

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Local Income Redistribution: A Club-Theoretic Analysis

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Abstract

This paper analyses optimal income redistribution in a club model where interclub transfers are ruled out (redistribution must proceed on a local level in heterogeneous clubs). This assumption induces a trade-off between equity and efficiency in that the efficiency loss associated with club heterogeneity must be borne in order for redistribution to occur. The optimal club structure depends on society's evaluation of this trade-off.



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Local Income Redistribution: A Club-Theoretic Analysis

by

Jan K. Brueckner

1. Introduction

The standard view in public finance is that income redistribution is best carried out at the national level. Proponents of this view (see Oates (1972), for example) claim that nothing is gained by decentralizing the redistributive function, and that consumer mobility will in any case subvert local redistribution as the rich flee (and the poor flood into) jurisdictions attempting it.¹ In a recent revival of interest in the redistribution issue, Pauly (1973) and Brown and Oates (1985) challenged the orthodox view by arguing that redistribution decisions should be decentralized when consumer altruism has limited spatial scope (with the rich caring only about the welfare of the poor in their own community).

Rather than extending recent developments, the present paper returns to the standard framework and explores an issue that was not adequately treated in the earlier literature: the inefficiency of local redistribution. The paper's key assumption is that intercommunity transfers are not possible, perhaps as a result of the weakness of the central government. This assumption means that income redistribution, if it is to occur, must proceed entirely on the local level. The analysis in the paper is devoted to characterizing optimal redistribution in the resulting second-best framework. Optimality is judged on the basis of a standard social welfare function (the interdependent-utilities assumption of recent work is dropped).

Analysis of local redistribution is interesting for several reasons. First, the problem is conceptually intriguing since it involves a trade-off between equity and efficiency (this is discussed further below). Second, even though local redistribution is infeasible in a frictionless world (making the analysis purely academic), actual frictions leave considerable room for such policies. The wide variation in property tax liabilities within a typical community, for example, shows that redistribution via unequal sharing of local public good costs is commonplace in today's economy and of appreciable magnitude. This suggests that an understanding of the economics of local redistribution could be valuable in today's setting. The analysis may also provide a good picture of the redistribution options open to pre-modern economies, where central-government weakness ruled out a national redistribution policy and restricted mobility made local redistribution feasible.²

The paper's analytical framework is drawn from the theory of clubs. In the standard club model, as originally proposed by Buchanan (1965) and refined by Berglas (1976b) and Berglas and Pines (1981), the optimal club (or community) structure does not depend on the nature of redistribution. Homogeneous clubs are formed to promote efficiency in public consumption while society's equity goals are met by appropriate transfers between clubs. Equity and efficiency considerations are, by contrast, no longer separable when interclub transfers are ruled out, as in the present analysis. Since income redistribution now requires that different types of individuals be mixed in heterogeneous clubs, the pursuit of equity entails an efficiency loss. This loss arises because public consumption in mixed clubs cannot be tailored to suit individual preferences.

The analysis in the paper formalizes this equity-efficiency trade-off and shows that if society has a strong taste for redistribution, the efficiency loss from mixed clubs will be worth bearing. Otherwise, a homogeneous-club structure based on the original distribution of income may be optimal. The paper also explores the effect of various parameter changes on the optimal club structure. In addition to the contributions mentioned above, the analysis complements the work of Berglas (1976a, 1984), who showed that mixed clubs can be optimal for reasons unrelated to income redistribution.³

2. Redistribution with Interclub Transfers

It is useful to begin by reviewing the income redistribution problem when interclub transfers are allowed. For simplicity, assume that there are two types of individuals in the economy, a and b. Let θ denote the proportion of the total population N composed of type-a individuals (the economy has θN a-types $(1-\theta)N$ b-types). Let the exogenous incomes for the two types be I_a and I_b , and let $U(x_a, z)$ and $V(x_b, z)$ denote their well-behaved utility functions, with x_a and x_b giving the consumption levels of a composite private good x and z denoting consumption of a congested public good. $C(z, n)$ denotes the cost in terms of x of providing a public consumption level z to a club of n people.⁴ Costs are increasing and convex in z ($C_z > 0$, $C_{zz} \geq 0$) and the assumption that the public good is congested further implies that $C_n > 0$. An additional assumption is that for any $z > 0$, average cost $C(z, n)/n$ is a U-shaped function of n (this guarantees the existence of a positive and finite optimal club size).

The economy's utility-possibility frontier is derived under the requirement of horizontal equity (identical utilities for identical people), with the utility level of the a-types maximized subject to the requirement that the b-types achieve a given utility and that the relevant resource constraint is satisfied. While the economy's resource constraint depends on how the population is organized into clubs, Berglas and Pines (1981) prove that any organization involving mixed clubs is dominated by one with homogeneous clubs.⁵ With homogeneous clubs, the resource constraint becomes

$$\begin{aligned} \theta N I_a + (1-\theta) N I_b &= \theta N x_a + (\theta N/n_a) C(z_a, n_a) \\ &+ (1-\theta) N x_b + ((1-\theta) N/n_b) C(z_b, n_b), \end{aligned} \quad (1)$$

where n_a and n_b denote the populations of the type a and type b clubs and z_a and z_b denote their public good levels. The LHS of (1) is total income in the economy, while the terms on the RHS involving x_a and x_b add up to total x consumption. The remaining terms give the total cost of public good provision in all of the economy's clubs. As written, (1) reflects the standard practice of ignoring the integer problem. The difficulty is that while the number of clubs is necessarily an integer, the expressions $\theta N/n_a$ and $(1-\theta) N/n_b$, which give the number of clubs of each type in (1), need not be integer-valued. As long as the group populations are large relative to the optimal club sizes, this problem is inconsequential.⁶

The utility frontier with interclub transfers (hereafter the "homogeneous-club" frontier) is generated by maximizing $U(x_a, z_a)$ subject to (1) and the constraint $V(x_b, z_b) = v$ for all feasible values of v .

The necessary conditions for the type-a clubs are the Samuelson condition $n_a U_z / U_x = C_z^a$ and $C_n^a = C^a / n_a$, which states that n_a is chosen to minimize per capita public good cost (the a-superscript indicates that C is evaluated at (z_a, n_a)). Analogous conditions hold for the type-b clubs.

The curve GG' in Figure 1 represents the homogeneous-club frontier.⁷ Of special interest is the "no-redistribution" point, denoted H, which is characterized by the absence of interclub transfers. At this point, each club consumes resources exactly equal to the exogenous income of its members, or

$$n_k I_k = n_k x_k + C(z_k, n_k), \quad k = a, b. \quad (2)$$

Letting v^* and u^* denote the utilities associated with the no-redistribution point, it is easily seen that for $v > v^*$, redistribution must flow toward the b-types, while for $v < v^*$, redistribution must flow toward the a-types. In other words,

$$n_b I_b > n_b x_b + C(z_b, n_b) \text{ as } v < v^*, \quad (3)$$

with the reverse inequality holding for the a-types.

The final ingredient in the choice problem is a social welfare function, which under the horizontal equity requirement can be written $W(u, v)$. As usual, the social welfare maximum corresponds to a point of tangency between the utility frontier and an indifference curve of W , as shown in Figure 1. The direction of redistribution depends on the properties of W , on the features of the individual utility functions and the public good cost function, and on the incomes and relative sizes of the groups.

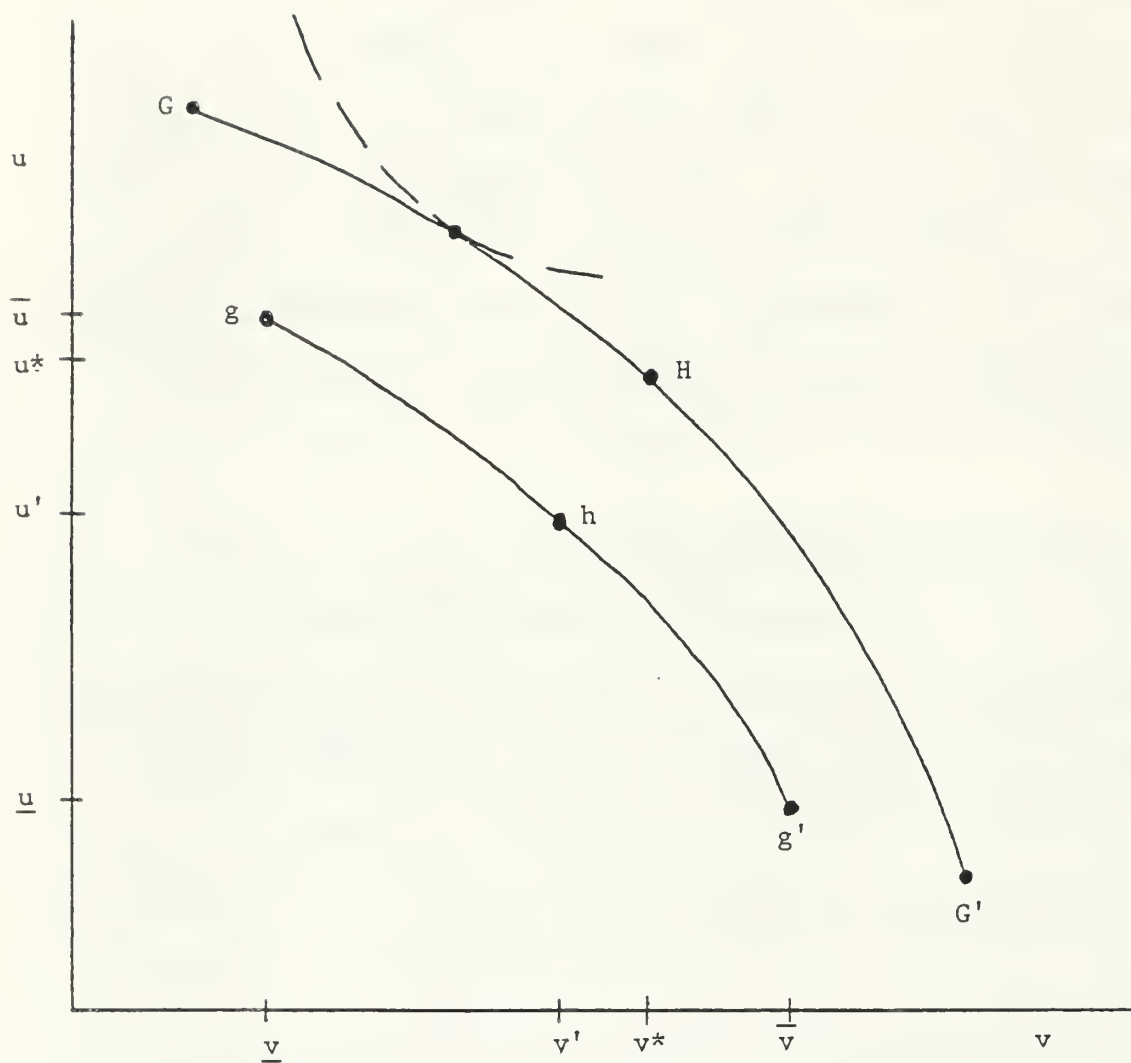


Figure 1.

3. The Mixed-Club Utility Frontier

When interclub transfers are ruled out, all points on the homogeneous-club frontier other than the no-redistribution point H become inadmissible. A given group's utility can be raised above its level at point H only through redistribution within a mixed club. To characterize the resulting opportunity locus, it is convenient to first derive the mixed-club utility frontier.

Under the requirement of horizontal equity, admissible mixed-club configurations must afford identical utilities to individuals of a given type. While configurations of non-identical clubs need not violate this principle, it will become clear below that such arrangements are dominated by a configuration of identical clubs where types a and b are mixed in the same proportion as in the general population. Letting n denote the population of a representative mixed club and z denote its public good level, the club resource constraint is⁸

$$\theta n I_a + (1-\theta) n I_b = \theta n x_a + (1-\theta) n x_b + C(z, n). \quad (4)$$

Note that the club is composed of θn a-types and $(1-\theta)n$ b-types, reflecting their proportions in the population. To derive the mixed-club utility frontier, $U(x_a, z)$ is again maximized subject to (4) and $V(x_b, z) = v$ for all feasible values of v . Additional restrictions on the problem are the nonnegativity constraints $x_a \geq 0$ and $x_b \geq 0$, which were superfluous in the earlier analysis. To see why these constraints are needed in the mixed-club problem, note that the highest feasible value of v in the problem results from setting $x_a = 0$ in (4) and choosing x_b and z to maximize $V(x_b, z)$ (let \bar{x}_b and \bar{z}_b denote the maximizing values and $\bar{v} \equiv V(\bar{x}_b, \bar{z}_b)$). Since the nonnegativity constraint

on x_a will be binding when v is at or near this maximum, it cannot be ignored (a parallel argument applies to the constraint $x_b \geq 0$).

When x_a and x_b are both positive, the necessary conditions for a mixed-club optimum include the Samuelson condition $\theta nU_z/U_x + (1-\theta)nV_z/V_x = C_z$ and the per-capita cost-minimization condition $C_n = C/n$. While the latter condition still holds when x_a or x_b is zero, the Samuelson condition is replaced when $x_a = 0$ by

$$\frac{\theta nU_z}{U_x + \mu} + \frac{(1-\theta)nV_z}{V_x} = C_z, \quad (5)$$

where $\mu \geq 0$ is the Lagrange multiplier associated with the constraint $x_a \geq 0$. Eq. (5) shows that the sum of the marginal rates of substitution exceeds C_z when the nonnegativity constraint on x_a is binding, a conclusion which also holds when the constraint $x_b \geq 0$ is binding.^{9, 10}

For later reference, it will be useful to establish the properties of the mixed-club frontier in the transferable-utility case, where the utility functions are given by $x_a + t(z)$ and $x_b + s(z)$. First, it is easy to see that the frontier is linear in this case over the range where x_a and x_b are positive. This follows because in general, the slope of the frontier when $x_b > 0$ is given by

$$-\lambda = -\frac{1-\theta}{\theta} \frac{U_x + \mu}{V_x}, \quad (6)$$

where λ is the multiplier associated with the type-b utility constraint. When $x_a > 0$ (so that $\mu = 0$) and utility is transferable, the slope expression (6) becomes $-(1-\theta)/\theta$, establishing linearity. A further

implication of (6) is that the frontier's absolute slope exceeds $(1-\theta)/\theta$ when the nonnegativity constraint on x_a is binding, with the curve becoming vertical as the lower endpoint is reached (manipulation of (5) shows that strict concavity in fact holds over this range). By symmetry, the frontier flattens over the range where $x_b = 0$, becoming horizontal at its upper endpoint (concavity again holds).

Several additional properties of the mixed-club frontier can be established. First, since mixed clubs are inefficient, the mixed-club frontier must lie below the homogeneous-club frontier. Formally, this can be seen by noting that the constraint (4) is equivalent to the constraint (1) together with the side conditions $z_a = z_b$ and $n_a = n_b$. Since the mixed-club optimization problem thus has a smaller opportunity set, it follows that for a given v , the maximal value of u is no larger than in the homogeneous-club problem. In the normal case where optimal homogeneous clubs are different across types, u will be strictly lower in the mixed-club problem.

An additional observation is that since the benefits of public consumption are always available in a mixed club regardless of the extent of redistribution, each group's minimum utility level on the mixed-club frontier exceeds its minimum utility level on the homogeneous-club frontier. In particular, the minimum type-a utility $\underline{u} \equiv U(0, \bar{z}_b)$ exceeds the utility $U(0,0)$ at the lower endpoint G' of the homogeneous-club frontier (the latter utility results from complete expropriation of the a 's). Similarly, $\underline{v} \equiv V(0, \bar{z}_a) > V(0,0)$ (\bar{z}_a and \bar{z}_b are defined analogously).

A further question concerns the relation between the highest feasible mixed-club utilities (\bar{v} and \bar{u}) and the utilities at point H (v^* and u^*). The answer is that at least one of the inequalities $\bar{v} > v^*$ and $\bar{u} > u^*$ must hold. This fact is established by focusing on the b's and showing that while $\bar{v} > v^*$ must hold if $I_b \leq I_a$, the relationship is ambiguous otherwise. To see the first claim, note that when $I_b \leq I_a$ holds, total club income on the LHS of (4) exceeds or equals income in a homogeneous type-b club (nI_b). Since type-b expenditure is higher in a homogeneous club for given x_b , z , and n ($nx_b + C$ versus $(1-\theta)nx_b + C$ (recall $x_a = 0$)), it follows that the homogeneous-club constraint lies strictly below the mixed-club constraint, implying $\bar{v} > v^*$. If $I_b > I_a$ holds instead, then a repetition of the preceding argument gives $\bar{u} > u^*$. However, since mixed-club income is now lower than nI_b , \bar{v} and v^* cannot be compared in general (instead of dominating the homogeneous-club constraint, (4) with $x_a = 0$ now intersects it). It is worth noting, however, that $\bar{v} > v^*$ will still hold provided an additional requirement is satisfied. This can be seen by rewriting (4) with $x_a = 0$ as

$$\frac{\theta(nI_a - C(z, n))}{1-\theta} + nI_b = nx_b + C(z, n). \quad (7)$$

If the a's can afford to operate an optimal type-b club, so that $n_b^* I_a - C(z_b^*, n_b^*) > 0$, then it follows that such a club is also affordable under (7), implying $\bar{v} > v^*$. Summarizing the above discussion, Figure 1 shows the location of the mixed-club frontier, denoted gg' . The case where $\bar{v} > v^*$ and $\bar{u} > u^*$ both hold is illustrated.

Consider next the location of the no-redistribution point on the frontier, which satisfies $I_k = x_k + C/n$, $k = a, b$ (each individual's income exactly covers his x consumption plus his share of public good costs). It is easily seen that this point (denoted h in Figure 1) lies to the southwest of H (its coordinates are v' and u').¹¹ Unlike in the homogeneous-club problem, location relative to the no-redistribution point is not a perfect indicator of resource flows. In other words, in contrast to (3), it is not necessarily true that $I_b > x_b + C(z,n)/n$ as $v < v'$ (a v above v' need not imply consumption in excess of income). It is easy to show, however, that the value of a group's consumption must exceed its income when utility exceeds the point- H level. To see this, suppose there exist points on the mixed-club frontier where u exceeds u^* . Since u^* is the highest utility achievable subject to the constraint $I_a = x_a + C(z,n)/n$, a consumption bundle yielding $u > u^*$ must satisfy $I_a < x_a + C/n$ (which in turn implies $I_b > x_b + C/n$). The reverse statements hold at points with $v > v^*$. Since this type of argument cannot be made when $u < u^*$ and $v < v^*$, the relationship between income and the value of consumption in this range is ambiguous.¹²

4. Restribution Without Interclub Transfers

With the above background, it is possible to consider the full income redistribution problem in the absence of interclub transfers. Feasible outcomes consist of all points on the mixed-club frontier together with the single point on the homogeneous club frontier where no transfers occur--the no-redistribution point H . Referring to Figure 1, it is clear that since H Pareto-dominates all points to the south

and west on the mixed-club frontier, such points are not candidates for the optimum. As a result, the opportunity locus for the problem consists of H together with the portions of the mixed-club frontier satisfying $u > u^*$ and $v > v^*$, as shown in Figure 2 (the case where \bar{u} and \bar{v} both exceed the point-H utilities is again illustrated).

Figure 2 shows that the location of the optimum depends on the shape of the social welfare function's indifference curves. Point H is optimal for an indifference map containing the solid curve, while point f (involving redistribution in favor of the a's) is optimal when the indifference map includes the dotted curve. It is clear that for redistribution to be optimal in the situation shown in Figure 2, W's indifference curves must either be steep or relatively flat, indicating that social preferences place a heavy weight on one group's welfare. The intuitive reason for this outcome is that when the a's and the b's are mixed to achieve redistribution, an efficiency loss results since public consumption can no longer be tailored to suit individual tastes. For society to tolerate this loss, the social welfare function must exhibit a strong preference for redistribution, heavily favoring one group.

To make this notion more precise, suppose that the social welfare function can be written $W(u,v,\alpha)$, where α is a weighting parameter lying in the interval $[\underline{\alpha}, \bar{\alpha}]$. Assume that $W_{u\alpha} > 0$ and $W_{v\alpha} < 0$, so that an increase in α shifts preferences in the a's favor. Under these assumptions, the slope expression W_v/W_u is a decreasing function of α , indicating that indifference curves become flatter as α increases. The

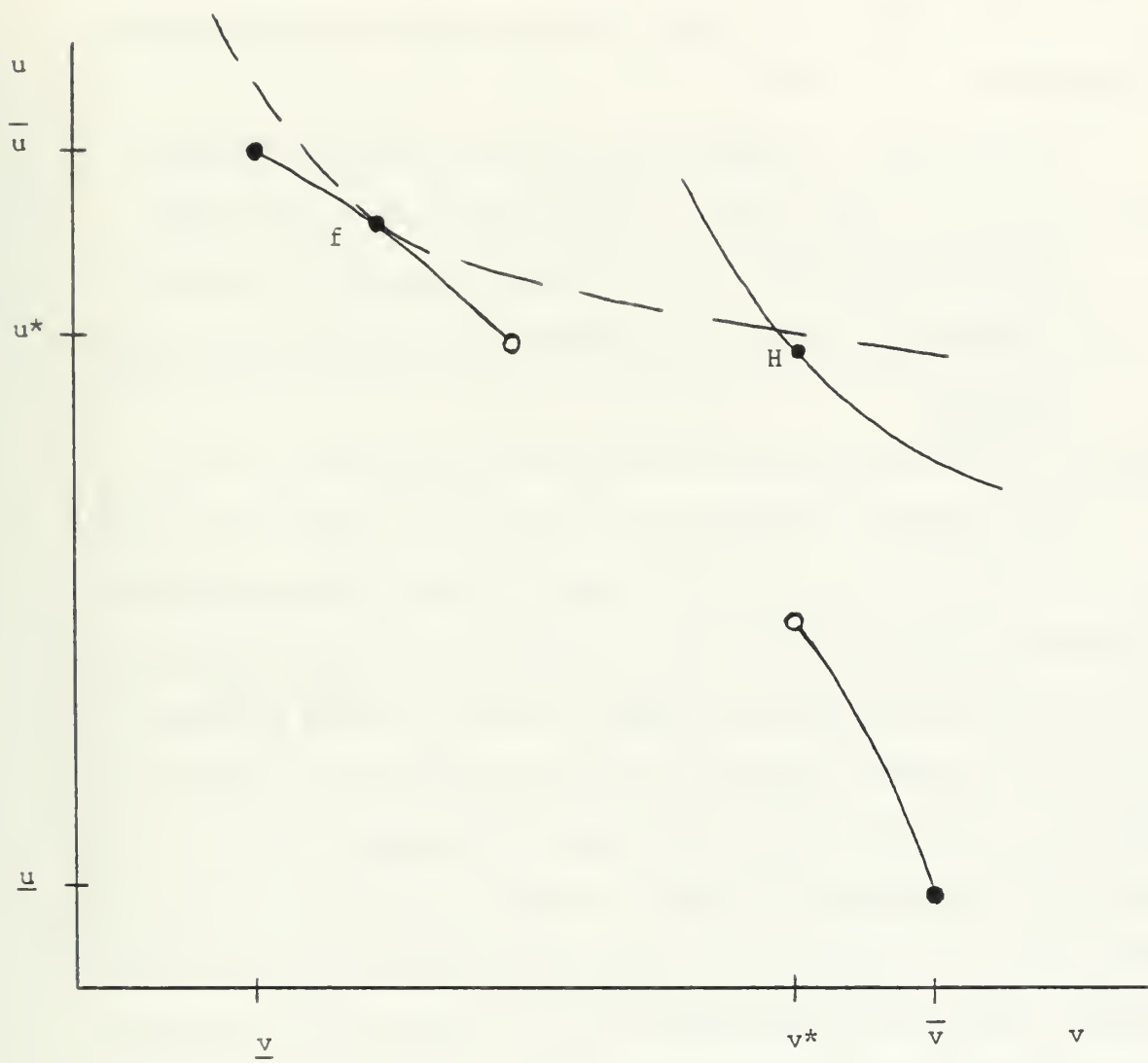


Figure 2.

curves are assumed to be horizontal ($W_v \equiv 0$) when $\alpha = \bar{\alpha}$ and vertical ($W_u \equiv 0$) when $\alpha = \underline{\alpha}$. Given these assumptions, the following result can be established:

Proposition 1: Suppose that point H is the unique optimum for $\alpha = \hat{\alpha}$. Then there exist positive numbers δ and ϵ such that H is optimal for all α in the interval $[\hat{\alpha} - \epsilon, \hat{\alpha} + \delta]$ while redistribution is optimal for all α lying outside this interval.

To prove this proposition, first consider the case where $\bar{v} > v^*$ and $\bar{u} > u^*$. Let $v_1(\alpha)$ and $u_1(\alpha)$ denote the values of v and u which maximize $W(u, v, \alpha)$ along the upper arm of the opportunity locus (extended to include the lower endpoint). Let $F_1(\alpha) \equiv W(u_1(\alpha), v_1(\alpha), \alpha) - W(u^*, v^*, \alpha)$ denote the utility differential between the upper arm and point H. Since H is the unique optimum when $\alpha = \hat{\alpha}$, it follows that $F_1(\hat{\alpha}) < 0$. Furthermore, since W 's indifference curves are horizontal when $\alpha = \bar{\alpha}$ and since $\bar{u} > u^*$, the inequality $F_1(\bar{\alpha}) > 0$ holds. Application of the envelope theorem yields in addition $F_1'(\alpha) = W_\alpha(u_1(\alpha), v_1(\alpha), \alpha) - W_\alpha(u^*, v^*, \alpha) > 0$, where the inequality follows because $u_1(\alpha) \geq u^*$ and $v_1(\alpha) < v^*$ while $W_{u\alpha} > 0$ and $W_{v\alpha} < 0$. Together, the above facts imply that there exists a $\delta > 0$ such that point H is optimal ($F_1(\alpha) \leq 0$) for $\hat{\alpha} \leq \alpha \leq \hat{\alpha} + \delta$ while redistribution in favor of the a's is optimal ($F_1(\alpha) > 0$) for $\hat{\alpha} + \delta < \alpha \leq \bar{\alpha}$.¹³ Reversing this argument for the lower arm of the opportunity locus proves that there exists an $\epsilon > 0$ such that point H is optimal for $\hat{\alpha} - \epsilon \leq \alpha \leq \hat{\alpha}$ while redistribution in favor of the b's is preferred for $\underline{\alpha} \leq \alpha < \hat{\alpha} - \epsilon$. Finally note that if $\bar{u} \leq u^*$, then redistribution in favor of the a's is never preferred and $\delta = \bar{\alpha} - \hat{\alpha}$

($\bar{v} \leq v^*$ similarly implies $\epsilon = \hat{\alpha} - \underline{\alpha}$). It should be realized that although Figure 2 may suggest the contrary, it is conceivable that under certain conditions point H is never optimal for any value of α . As a result, the assumption that $\hat{\alpha}$ exists is not inconsequential.

While Proposition 1 shows that a discrete change in α is required to move the optimum away from the no-redistribution point, Figure 1 shows that any change in α , no matter how small, will achieve this outcome when interclub transfers are allowed. The reason, of course, is that when such transfers are possible, there is no efficiency loss from redistribution. As a result, the slightest change in social preferences is sufficient to bring it about.

It is interesting to consider a special case where the optimum can be determined directly. First, suppose that the social welfare function assumes the Benthamite form $\theta Nu + (1-\theta)Nv$. Assume in addition that utility is transferable, as discussed above. Since W's indifference curves are linear with slope $-(1-\theta)/\theta$ while the mixed-club frontier consists of an intermediate linear segment with this same slope together with concave extremities, it follows immediately that point H is optimal. Redistribution, of course, can be made optimal in this example by altering W's welfare weights in favor of one group.

Changes in the parameters I_a , I_b , and θ can alter the solution to the welfare maximization problem. While the effect of an increase in either I_a or I_b is not clearcut,¹⁴ the impact of a change in the distribution of income that leaves total income unchanged is straightforward. The mixed-club utility frontier is unaffected by such a change since $\theta I_a + (1-\theta)I_b$ is constant. However, point H moves uphill along

the homogeneous-club frontier (which itself remains fixed) as I_a rises and I_b falls (moving downhill in the reverse case). Inspection of Figure 2 shows that such a movement will eventually make H preferred to f when f is initially optimal. More generally, Figure 1 shows that suitable adjustment of the income distribution can make point H Pareto-superior to any point on the mixed-club frontier. These observations suggest

Proposition 2: For any social welfare function, redistribution favoring a given group will become suboptimal as the distribution of income shifts in its favor (and total income remains fixed).

The reason for this result is that as the original income distribution becomes more equitable from the perspective of a given W, society will be less inclined to tolerate the efficiency loss of mixed-club redistribution. In contrast to Proposition 2, Figure 1 shows that the social optimum is insensitive to the distribution of income when inter-club transfers are allowed. Resource flows, however, are affected, with transfers to (away from) a given group shrinking (growing) as the income distribution shifts in its favor.

The first step in analyzing the effect of a change in the type-a proportion θ is to deduce the impact on the opportunity locus. First, the no-redistribution point is unaffected. The impact on the mixed-club frontier is found by applying the envelope theorem and differentiating the Lagrangean expression for the mixed-club problem. This yields

$$\frac{\partial u}{\partial \theta} = \gamma n[(I_a - x_a) - (I_b - x_b)], \quad (8)$$

where γ is the positive multiplier associated with the resource constraint (4). When $I_a - x_a < C/n$ holds, $I_b - x_b > C/n$ and $\partial u / \partial \theta$ from (6) is negative. In other words, if resources are being transferred from the b's to the a's (yielding type-a consumption in excess of income), then the a's are hurt by an increase in θ , which shrinks the relative size of the expropriated group. Recalling from section 3 that $I_a < x_a + C/n$ holds at points on the mixed-club frontier where $u > u^*$, it follows that the upper arm of the opportunity locus shifts downward as θ increases. The reverse argument shows that the lower arm shifts upward as θ increases. These movements are in fact part of an overall rotation of the mixed-club frontier in response to the higher θ (the axis is the no-redistribution point h in Figure 1). Recalling that the relation between I_a and $x_a + C/n$ may be perverse near the redistribution point, the rotation near that point may be opposite to that occurring at the extremes of the frontier.¹⁵

A change in θ will typically shift the indifference map of the social welfare function, with a higher θ flattening the indifference curves (this follows from differentiating W_v / W_u with respect to θ under the assumptions $W_{u\theta} > 0$, $W_{v\theta} < 0$). Since both the indifference curves and the arms of the opportunity locus rotate as θ increases, the impact on the optimum is unclear. If, however, the degree of rotation is similar and if no redistribution is initially optimal, it is likely that this outcome remains optimal as θ increases. This is true, for example, in the special case considered above, where no redistribution is optimal regardless of the value of θ .¹⁶

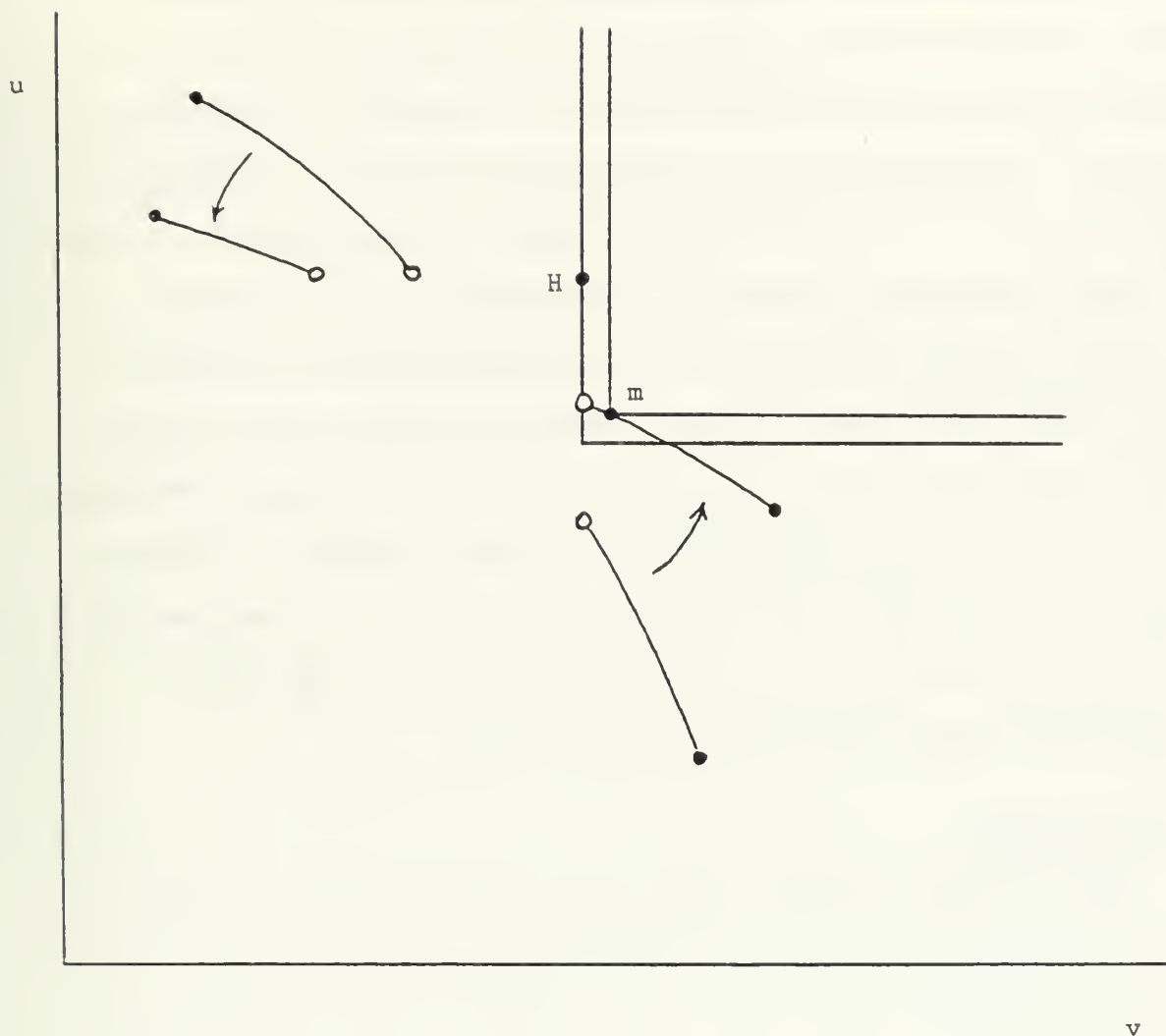
In the one admissible case where indifference curves do not depend on θ , a strong statement can be made:

Proposition 3: Suppose $u^* \neq v^*$ and that $\bar{v} > v^*$ and $\bar{u} > u^*$ hold for all θ . Then if the social welfare function is Rawlsian, redistribution is optimal at either high or low values of θ .

This claim is illustrated in Figure 3, which shows the rotating opportunity locus together with the right-angled indifference curves of the Rawlsian welfare function. While point H is initially optimal, the redistribution point m is optimal following the increase in θ . Note that if H had been located at the corner of an indifference curve ($u^* = v^*$), then redistribution would never be optimal for any θ . Note also that if H had been located below instead of above the 45° line, a decline in θ (yielding a clockwise rotation of the upper and lower arms) would have been required to make redistribution optimal.

When interclub redistribution is allowed, the story is somewhat different. A calculation analogous to (6) shows that the homogeneous-club frontier rotates counterclockwise around the no-redistribution point when θ increases. As in the earlier discussion, this movement combined with the changing slope of W's indifference curves yields an ambiguous impact on the location of the optimum in the general case. In the Rawlsian case, however, inspection of Figure 1 shows that the direction of redistribution is unaffected by an increase in θ , in contrast to Proposition 3.

While Proposition 3 applies to an extreme case, it seems safe to make the generalization that redistribution will be optimal for high



v

Figure 3.

or low values of θ when the social welfare function exhibits strong (though not necessarily infinite) aversion to inequality. In this case, indifference curves will be nearly right-angled, and the rotation accompanying an increase in θ will be slight. To gain an intuitive understanding of the role played by aversion to inequality, note first since an increase in θ enlarges the a-group relative to the b-group, the type-b utility can be raised above v^* with a smaller sacrifice from each type-a individual (the lower arm of the opportunity locus shifts up). For this reduced efficiency loss to elicit redistribution in the b's favor, the fact that there are now fewer b's and more a's must have little impact on welfare judgments. This will only be the case under social welfare functions with a strong aversion to inequality, where the utility differential between the groups carries more weight than their relative sizes.¹⁷

5. Conclusion

This paper has analyzed optimal income redistribution in a club model where prohibition of interclub transfers induces a trade-off between equity and efficiency. A main lesson of the analysis is that if society's equity standards are unbalanced, heavily favoring one group, then the efficiency loss from mixed-club redistribution will be worth bearing.

As discussed in the introduction, the model's planning solutions are directly relevant only in the special case of a pre-modern economy, where the central government is weak (precluding intercommunity transfers)

and mobility costs are high (allowing some scope for local redistribution). The analysis, however, has contemporary policy implications. In particular, the discussion shows that given the redistributive power of modern central governments, no redistribution whatsoever should occur at the local level, in contrast to current practice. Ideally, consumers should be segregated into homogeneous communities where public spending and individual preferences are perfectly matched, with equity goals achieved via intercommunity transfers. While it is perhaps excessive to expect implementation of this arrangement, the analysis clearly identifies it as ideal.

Footnotes

¹For a more recent summary of this position, see Wildasin (1985).

²Brown and Oates (1985) point out that disparity among local re-distribution policies under the British Poor Laws in the 1600's led to considerable illegal migration of the poor (along with subsequent deportment to their original jurisdictions). This suggests that not all premodern societies were characterized by low mobility.

³Berglas' 1976a paper showed that mixed clubs will be desirable when there is complementarity between groups in production. His 1984 paper showed that with multiple public services, there may again be gains from mixing individuals.

⁴For simplicity, it is assumed that the rate-of-use of the public facility is not a choice variable of the consumer, in contrast to the models of Berglas and Pines (1981) and Berglas (1984).

⁵Although their argument applied to a model where the rate-of-use of the public facility is variable, the proof can be adapted to the present model.

⁶In this case, the "extra" individuals (those filling the fractional club) can be dispersed among a large number of clubs, yielding a solution virtually indistinguishable from one where the population divides perfectly into an integer number of clubs. See Scotchmer (1985) for an analysis of decentralization of optimal club structures when the integer problem is not ignored.

⁷The slope $\partial u/\partial v$ of the frontier equals $-\lambda$, where λ is the positive Lagrange multiplier associated with the type-b utility constraint. While the curvature of the frontier is indeterminate in general, it is shown as convex.

⁸The integer problem is again ignored. The relationship between this formulation and that of Berglas (1976a) should also be noted. The income term on the right of (4) is replaced in Berglas by the club's output of the private good $F(\theta n, (1-\theta)n)$, which depends on inputs of both types of labor. While Berglas chooses θ optimally, initially ignoring the overall population constraint, such an exercise is inappropriate in the present problem since the linearity of (4) in θ yields corner solutions.

⁹When $x_b = 0$, (5) is replaced by the condition

$$\frac{\theta n U_z}{U_x} + \frac{(1-\theta)n V_z}{V_x} \left(1 - \frac{\eta \theta}{(1-\theta)U_x}\right) = C_z,$$

where η is the multiplier associated with the constraint $x_b \geq 0$.

¹⁰The behavior of x_b and z over the range where $x_a = 0$ can be analyzed by eliminating x_b in V using (4). Differentiation then shows that dV/dz has the sign of $(1-\theta)nV_z/V_x - C_z$, which is negative by (5) when μ is finite. This establishes that as v increases over the range where $x_a = 0$, z must be falling and x_b rising. When the consumption levels reach the values x_b and z_b where the condition $(1-\theta)nV_z/V_x - C_z$ holds, v will have achieved its maximum value \bar{v} ((5) shows that μ must be infinite at this point). Finally, note that as v rises and z declines over the range where $x_a = 0$, $u = U(0, z)$ falls. By symmetry, the reverse of the above discussion applies when $x_b = 0$.

¹¹The reason is that for each group, both allocations lie on the same constraint (recall (2)) while the mixed-club allocation satisfies a condition (the mixed-club Samuelson condition) different from the one which guarantees maximum utility subject to that constraint (the homogeneous-club Samuelson condition). As a result, utilities at point h are lower for both groups.

¹²Some insight into this ambiguity can be gained by considering the derivative $d(x_b + C(z, n)/n)/dv$. A positive value for this derivative, for example, would indicate that type- b consumption exceeds income for all $v > v'$. Inverting the utility constraint to write x_b as $x_b(z, v)$, with $\partial x_b/\partial z = -V_z/V_x$ and $\partial x_b/\partial v = 1/V_x$, the above derivative equals

$$\left(\frac{C_z}{n} - \frac{V_z}{V_x}\right) \frac{\partial z}{\partial v} + \frac{1}{V_x} = \theta \left(\frac{U_z}{U_x} - \frac{V_z}{V_x}\right) \frac{\partial z}{\partial v} + \frac{1}{V_x}$$

(the equality follows from the Samuelson condition). Since the difference between the MRS's can have either sign and since the $\partial z/\partial v$ is also ambiguous (it depends on how the MRS's change when x varies holding z fixed), the sign of the above expression is indeterminate. In particular, it is possible for the derivative to be negative at the no-redistribution point, implying that type- b consumption initially falls below income as v rises above v' . The previous argument shows that any such effect must have been reversed by the time v reaches v^* .

¹³Note that since the lower arm utility differential $F_2(\alpha)$ (defined analogously to $F_1(\alpha)$) starts out negative at $\hat{\alpha}$ and is decreasing in α , redistribution in favor of the b 's cannot be optimal for $\alpha > \hat{\alpha}$.

¹⁴When I_a increases, for example, the no-redistribution point moves straight upward and the mixed-club frontier also shifts up. The impact on the optimum is not determinate.

¹⁵ It should be noted that the above rotation ultimately leads to a horizontal frontier as θ reaches unity (its height is u^*). Similarly, a vertical frontier at v^* results when θ falls to zero.

¹⁶ At this point, the undesirability of mixed-club structures with non-identical clubs can be demonstrated. The procedure is to ask whether the utility outcomes along the upper arm of the opportunity locus can be improved upon by shifting to a non-identical club structure while maintaining horizontal equity (the same argument applies to the lower arm). First, since $\partial u / \partial \theta$ is negative on the upper arm of the locus, it follows that decreasing the type-a proportion in a club below the population proportion raises type-a utility (club population is adjusted optimally and type-b utility is held fixed). While creating such clubs is desirable, they require the existence of other clubs with type-a proportions above θ . Since these clubs (whose populations are again chosen optimally) yield a lower type-a utility than the original club, the requirement of horizontal equity is violated. By suitably adjusting club populations as the proportions diverge from θ , equality of type-a utilities conceivable could be preserved. However, since the utility of type-a's in an optimal-size club where their proportion is high is lower than in the original club, the same conclusion must hold when the former club's population is not optimal. It follows that if horizontal equity can be preserved as clubs become dissimilar, the type-a utility must fall. While the above argument implicitly assumes that type-a utility remains above u^* (so that u is decreasing in the type-a proportion), if club reorganization drives u below u^* in some club and horizontal equity is preserved, the outcome is clearly inferior to the original mixed-club structure.

¹⁷ Since indifference curves show some curvature in this case, the location of H relative to the 45° line is not an issue. Regardless of H 's location, redistribution will be optimal for both high and low values of θ (this can be seen from a diagram like Figure 3).

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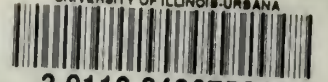
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