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The Demand for Cost Allocations: The Case of
Incentive Contracts Versus Fixed-Price Contracts

Susan I. Cohen
Martin Loeb

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The Demand for Cost Allocations: The Case of Incentive Contracts
Versus Fixed-Price Contracts

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ABSTRACT

The question of why firms allocate costs for internal reporting has been brought to the forefront of accounting research. This cost allocation literature focuses on finding settings in which cost allocations arise as a part of optimal contracting, under conditions of asymmetric information and divergence of preferences. This paper presents a setting in which cost allocations are part of the optimal contract between the government and a firm supplying goods to the government. We provide conditions under which an incentive contract (based on fully allocated costs) dominates a fixed-fee contract (in which no allocation takes place) in the context of a bidding model for government procurement. Thus, we provide an economic rationale for cost allocation.

I. INTRODUCTION

The question of why firms allocate costs for internal reporting has been brought to the forefront of accounting research by Zimmerman [1979], and has been further addressed by Demski [1981], Magee [1988], and Cohen and Loeb [forthcoming 1988], among others. This cost allocation literature focuses on finding settings in which cost allocations arise as part of optimal contracting, under conditions of asymmetric information and divergence of preferences within a firm. The purpose of this paper is to present a setting in which cost allocations are part of the optimal contract between the government and a firm supplying goods to the government.

The necessity of determining actual full costs for the purpose of contracting between firms or between the government and a firm is often given as a primary reason for allocating indirect costs [e.g., Horngren and Foster, 1987, pp. 411-412]. Moriarity [1981, pp. 8-9] points out that while text books often give this justification, no rationale is provided for why such contracts are optimal in the first place. We provide conditions under which an incentive contract (based on fully allocated costs) dominates a fixed-fee contract (in which no allocation takes place) in the context of a bidding model for government procurement. Thus, we provide an economic rationale for cost allocation.

Government procurement policy has large efficiency and distributional effects on the allocation of the nation's resources. Procurement policy has, therefore, drawn the attention of academicians in accounting [Wright and Bedingfield, 1985; Greer and Liao, 1987] and

economics [Cummins 1977; Laffont and Tirole 1987; McAfee and McMillan 1986, 1987].

Four common forms of contracts used by the government are the cost-plus contract, the fixed-price contract, the cost-plus incentive contract and the fixed-price incentive contract. With a cost-plus contract the government agrees to pay the contractor full costs plus a fixed fee (or fixed percentage of actual costs). Using a fixed-price contract the supplier agrees to deliver goods for an amount agreed upon before actual costs are realized. The supplier, therefore, fully absorbs any variance between actual and estimated costs. A cost-plus incentive contract is characterized by the government agreeing to pay a fixed fee and to share in any deviations between actual and estimated costs. Finally, with a fixed-price incentive contract the government agrees to a certain price (full estimated costs plus a fee) and to share in any deviation between actual and estimated costs with the added stipulation that total payments are to be capped at some predetermined level. Fixed-price incentive contract is thus a misnomer; such contracts are just cost-plus incentive contracts with a ceiling on total payments. Following McAfee and McMillan [1986, p. 328], hereafter referred to as MM, we assume that the payment cap is not binding. We thus do not distinguish between a fixed-price incentive contract and a cost-plus incentive contract, and refer to both as incentive contracts.

The enforcement of a fixed price contract does not depend on the government and the supplier agreeing on a method of allocating indirect costs. However, if the government contract is not the supplier's sole source of revenue, the problem of allocating indirect costs arises in all

cost-based forms of contracts. We define indirect costs as those costs that are not separable in the output produced for the government contract and the output produced for sale in outside markets.

MM have analyzed fixed-price contracts, cost-plus contracts, and incentive contracts in a principal-agent framework in which indirect costs are not explicitly modeled. They demonstrate (p. 328) that the government is always better off with a fixed-price or an incentive contract rather than a cost-plus contract for the case of two or more bidders. MM (pp.328-332) go on to demonstrate that for a finite number of bidders greater than one, if bidders expect rivals to face different costs, the government will be strictly better off with an incentive contract rather than a fixed-price contract. MM show that incentive contracts help stimulate bidding, and thereby help alleviate the adverse selection problem associated with finding the lowest cost producer. This bidding-competition effect of incentive contracts is sufficient to yield the dominance of incentive contracts over fixed-price contracts, even when bidders are all risk-neutral, and thus when there is no benefit to risk-sharing per se. When bidders are risk-neutral the degree of risk sharing is determined solely by a tradeoff between the benefits of stimulating competition and the benefits of providing incentives for the winning bidder to expend effort to subsequently reduce costs. That is, the cost share is chosen to balance the problems of adverse selection versus moral hazard.

Cohen and Loeb [1988] use a generalization of the MM model with risk-neutral bidders to show that incentive contracts requiring an allocation of indirect costs may result in the dominance of fixed-price

contracts over incentive contracts. This result is due to the fact that cost allocations may result in an inefficient cross subsidization of products. The fact that cost allocations may pervert the desirable properties of incentive contracts may not come as a surprise, given the well-known arbitrary nature of cost allocations [Thomas, 1969, 1974]. In this paper we use the same generalized model to show conditions under which incentive contracts dominate fixed-price contracts even when the presence of indirect costs are explicitly recognized. That is, we show conditions under which there is a demand for cost allocation.

In the next section, we introduce the problem of indirect costs by examining the special case where there is only one possible supplier, so that bidding does not take place. We show that because of indirect costs the government cannot obtain an efficient (first-best) solution, even if the government could perfectly audit costs ex post. Section III contains our model for the case where there are many possible suppliers and bidding takes place. In Section IV, we present conditions under which a fixed price contract dominates an incentive contract, and give a proof of our result. We conclude with a brief summary in Section V.

II. SOLE SOURCING, INDIRECT COSTS, AND AUDITING

Consider the case where the government purchases G units of a customized product from a single supplier without putting the contract up for bid.¹ The government contractor is assumed to sell another physically distinct output, Q , in the private sector. In this section we assume that no moral hazard problem exists, and that the contractor knows with certainty the demand for the outside product Q and the costs of production. If the government shared the contractor's information, the government should, from society's viewpoint, pay the supplier the economic (opportunity) cost of producing the G units. We will see that even if the government could costlessly and perfectly audit *ex post* production costs, the government would be unable to measure economic costs when there are indirect costs.

Suppose the revenue curve for the outside product is given by $R(Q)$, where $R(\cdot)$ is continuously twice differentiable, weakly concave, and $R'(0) > 0$. Let $C_T(Q,G)$ be the firm's total cost of producing both the government and private outputs. The cost curve $C_T(Q,G)$ represents total production costs (including opportunity cost of capital), and is composed of direct costs with respect to Q , $C_Q(Q)$, direct costs with respect to G , $C_G(G)$, plus indirect costs, $C_I(Q,G)$. Thus:

$$C_T(Q,G) = C_Q(Q) + C_G(G) + C_I(Q,G). \quad (1)$$

$C_Q(Q)$ is assumed to be twice continuously differentiable, strictly increasing and strictly convex; $C_I(Q,G)$ is assumed to be twice

continuously differentiable and nondecreasing in Q . Indirect costs include purely fixed costs so that $C_Q(0) = C_G(0) = 0$.

Define Q^* to be quantity to be sold by the firm in the private sector in the absence of the government contract, and define $\hat{Q}(G)$ to be the quantity to be sold when the firm receives the government contract. Thus:

$$Q^* \equiv \operatorname{argmax} R(Q) - C_T(Q, 0) \quad (2)$$

and

$$\hat{Q}(G) \equiv \operatorname{argmax} R(Q) - C_T(Q, G). \quad (3)$$

The supplier's incremental production costs of satisfying the government contract may be written as $I(G)$, where:

$$I(G) = C_T(\hat{Q}(G), G) - C_T(Q^*, 0). \quad (4)$$

Finally, the supplier's opportunity cost of satisfying the government contract may be written as $OC(G)$, where:

$$\begin{aligned} OC(G) &= [R(Q^*) - C_T(Q^*, 0)] - [R(\hat{Q}(G)) - C_T(\hat{Q}(G), G)] \\ &= R(Q^*) - R(\hat{Q}(G)) + I(G) \end{aligned} \quad (5)$$

If there were no indirect costs, then there would be no divergence between production costs (as defined above) and opportunity cost. To see

this, note that when $C_I(Q,G)$ is identically zero, $Q^* = \hat{Q}(G)$, so that $OC(G) = I(G)$.

In the absence of indirect costs, if the government could perfectly audit ex post direct costs $C_G(G)$, the government could pay the contractor the opportunity cost of fulfilling the contract and efficiency could be achieved. This conclusion follows from the fact that $OC(G) = I(G) = C_G(G)$ when there are no indirect costs.

When indirect costs are present, it will generally be the case that even perfect and costless auditing of all ex post costs will not be sufficient for the government to measure opportunity cost and reach the (first-best) efficient solution. To see this, note that in general $Q^* \neq \hat{Q}(G)$, so that for the government to measure opportunity cost, the government must determine the actual revenue the firm generates from outside sales, $R(\hat{Q}(G))$, the revenue the firm would have generated from outside sales in the absence of the government contract, $R(Q^*)$, the production cost that would have obtained in the absence of the government contract, $C_T(Q^*,0)$, as well as actual realized production costs, $C_T(\hat{Q}(G),G)$. Since cost and demand conditions may change from period to period, it would not be possible for the government to determine opportunity cost by measuring only realized costs and revenues.

III. MULTIPLE POTENTIAL SUPPLIERS

Suppose there are $n \geq 2$ firms that have the capability of producing the G units of the customized product that the government wants to purchase. The potential suppliers, the agents, are assumed to be risk-neutral and are indexed by $i=1,2,\dots,n$. As in the previous section, we suppose that each agent currently produces and sells another product in the private sector, and let Q be a measure of this output.

Each agent is assumed to face the same direct costs, $C_Q(Q)$ and $C_G(G)$, but to face different indirect costs, $C_{Ii}(Q,G)$. Indirect costs are parameterized by a productivity parameter, $V_i \in [V_*, V^*]$. Agent i knows the value of the parameter V_i . The principal (the government) and all other agents $j \neq i$ know that V_i represents an independent draw from the distribution $F(V)$, with density function $f(V)$, but they do not know the value of V_i . We thus follow MM in using an independent-private-values model [Milgrom and Weber 1982] in which agents differ only with respect to a single parameter. In addition to the adverse selection problem of selecting the lowest cost producer, a moral hazard problem exists once the contract is awarded because each agent can further reduce indirect costs by supplying unobservable effort, e . The agent's cost of supplying effort is represented by a twice continuously differentiable, strictly increasing, and convex function, $h(e)$. Agent i 's indirect costs are given by:

$$C_{Ii}(Q,G) = (V_i + W - e)C_I(Q,G) + h(e), \quad (6)$$

where $C_I(Q,G)$ has the properties detailed in Section II, and W is a random cost factor that is common to all agents, with $E[W] = 0$. Since the agents are all assumed to be risk-neutral, without loss of generality we can deal solely with the expected values. For given values of G, Q , and e , the i th agent's expected costs may be written as:

$$C_Q(Q) + C_G(G) + (V_i - e)C_I(Q,G) + h(e). \quad (7)$$

The government is assumed to award the contract for a fixed quantity G of (customized) goods by means of a first-price, sealed-bid auction. The winning bid is represented by b , and $a \in [0,1)$ represents the cost share ratio. The contract calls for the government to pay the successful bidder:

$$P = b + a(C^*(Q,G) - b), \quad (8)$$

where $C^*(Q,G)$ is the bidder's observable cost of meeting the contract, defined below. A fixed-price contract is specified by setting $a = 0$; an incentive contract is specified by setting $a > 0$.

For an incentive contract to be enforceable, the government and bidders must agree on a definition of observable cost. Suppose that the government can perfectly audit realized (ex post) direct costs, $C_G(G)$, and realized (ex post) total indirect costs, $(V - e)C_I(Q,G)$. We further assume that auditing costs are known to be the same regardless of which firm wins the contract. Thus, without loss of generality, we may assume these auditing costs are zero. Given that both the government and the

supplier can measure actual direct and indirect costs, defining observable costs reduces to defining an indirect cost allocation between government output and other output. Suppose the bidders and the government agree to use the cost allocation rule $k(Q,G)$ that assigns a share of indirect costs to the government contract. Cost allocations are defined to have the following characteristics:

$$(i) \quad 0 \leq k(Q,G) \leq 1;$$

and either

- (iia) A. $k(Q,0) = 0$ for all positive Q , and
- B. $k(0,G) = 1$ for all positive G , and
- C. $k(Q,G)$ is non-increasing in Q , for all $G > 0$;

or

$$(iib) \quad k(Q,G) = K, \text{ a constant.}$$

Indirect costs are often allocated on the basis of usage as measured by physical units, hours of direct labor, dollars of direct labor, or dollars of direct material. Such cost allocations may be represented by $k(Q,G) = G/(G+Q)$. Clearly, this allocation is very sensitive to the way in which output is measured. Thus, the indirect costs to be allocated to a contract for 100 jet fighter planes depends on whether output is to be

measured in terms of planes, labor hours, labor dollars, dollars of direct materials or some other measure.

For a given rule cost allocation rule, $k(G,Q)$, observable costs may now be expressed as:

$$C^*(Q,G) = C_G(G) + k(Q,G)(V - e)C_I(Q,G) \quad (9)$$

Note that $h(e)$ may not be charged to the project, since by definition, the cost of effort is not observable. Recall from the previous section that indirect costs will generally cause observable cost to differ from opportunity cost because (i) observable cost does not represent incremental production cost, and (ii) each bidder would change the output produced for non-government customers in response to being awarded the contract. Hence, revenues from sale of the outside good, as well as production costs of that good, change when a firm is awarded the contract.

As in the previous section, we let $R(Q)$ represent an agent's revenue curve from sale of its private sector product, and we assume all agents face the same revenue curve. We allow for the possibility that all bidders sell their outside products in a perfectly competitive market. However, each agent may sell her (his) product in different outside markets. We assume agents face common revenue functions so that we may use an independent-private-values model, and, thereby focus on how indirect costs affect the bidding for the government contracts. In addition, we assume that $R'(Q) \geq C'_Q(Q)$ for all Q ; that is, in the absence of indirect costs the optimal quantity of the private sector output is

indeterminant. In this case, the indirect costs have a major impact on the choices made by the agents.

Before we present an agent's bidding optimization problem, we introduce some additional notation. Let $P^*(V_i)$ be the i th bidder's profits given that the i th bidder is not successful in securing the government contract. Then:

$$P^*(V_i) = \max_{Q,e} R(Q) - C_Q(Q) - (V_i - e)C_I(Q,0) - h(e). \quad (10)$$

With the cost allocation rule $k(Q,G)$ and the incentive share ratio a , the i th bidder's profits, if successful in winning the contract, are:

$$\begin{aligned} \hat{P}(b_i, V_i; a) = \max_{Q,e} \{ & R(Q) - C_Q(Q) - C_G(G) - (V_i - e)C_I(Q,G) \\ & + b_i + a[C^*(Q,G) - b] - h(e) \}. \end{aligned} \quad (11)$$

Define

$$\begin{aligned} P(V_i; a) = \max_{Q,e} \{ & R(Q) - C_Q(Q) - (1 - a)C_G(G) \\ & - (1 - ak(Q,G))(V_i - e)C_I(Q,G) - h(e) \}. \end{aligned} \quad (12)$$

Denoting the solution to (12) by $(Q(V_i, G; a), e(V_i, G; a))$, and using (9) and (12), we can rewrite (11) as:

$$\hat{P}(b_i, V_i; a) = (1 - a)b_i + P(V_i, a). \quad (13)$$

The economic cost of fulfilling the contract can now be defined as:

$$C(V_i; a) = P^*(V_i) - P(V_i; a). \quad (14)$$

Notice that we do not require that $C(V_i; a)$ be positive for all V_i .

Combining (13) and (14), we can write the incremental profit for the winning bidder as:

$$\hat{P}(b_i, V_i; a) - P^*(V_i) = (1 - a)b_i - C(V_i; a). \quad (15)$$

Our model reduces to the MM model with risk-neutral bidders for the case when there are no indirect costs. Then, $C_I(Q, G)$ can be taken as identically equal to one (or any constant) for any positive G , and $C_I(Q, 0) = 0$. The expression in (14) can then be easily shown to equal $(1 - a)[V_i + C_G(G) - e(V_i, G; a)] - h(e(V_i, G; a))$, where $e(\cdot)$ is the effort chosen by the agent when the cost parameter is V_i , the government wishes G produced, and the cost share is a . The quantity $V_i + C_G(G)$ is referred to as the opportunity cost for the i th agent by MM, so that (14) is still the expected cost of the contract to a bidder gross of the cost of effort.

The i th agent selects a bid b_i to maximize the probability of winning times the expression in (15). Since the contract is awarded via a first-price sealed bid auction, the probability of agent i winning equals the probability that $b_i < b_j$ for all $j \neq i$. A bid function $B(V_i; a)$ is a symmetric Nash equilibrium bidding strategy, if when

$b_j = B(V_j; a)$ for all $j \neq i$, then $b_i = B(V_i; a)$ is the optimal bid for agent i . Since the b_j 's are all independent draws from the distribution of V , the probability that agent i is the winning bidder is $[1 - F(B^{-1}(b))]^{(n-1)}$. This function $B(\cdot)$ is the solution to the following maximization problem, where the subscripts have been deleted:

$$\underset{b}{\text{MAX}} [1 - F(B^{-1}(b))]^{(n-1)} [(1 - a)b - C(V; a)]. \quad (16)$$

It is straightforward to verify that the bid function $B(V; a)$ satisfies the first-order differential equation:

$$s(V)[(1 - a)B(V; a) - C(V; a)] + (1 - a)B'(V; a) = 0 \quad (17)$$

where $s(V) = -(n-1)f(V)/(1 - F(V))$. Let

$$S(V) = \int_{V_*}^V s(t) dt \quad (18)$$

Then, $(1 - a)B(V; a)$ can be solved for as

$$(1 - a)B(V; a) = -\exp(S(V)) \int_V^{V^*} s(t) \exp(S(t)) C(t; a) dt \quad (19)$$

We first demonstrate that the contract is awarded to the most efficient firm, i.e. we show that $B(V; a)$ is increasing in V . From (17), and the fact that $s(V) < 0$ for all V , we see that the sign of $B'(V; a)$ is the same as the net benefit of the contract to the winning bidder

(equation (15)). First, it is clear that this net benefit must not be negative in equilibrium. Second, if this net benefit were equal to zero in equilibrium for some V , the value of equation (16) would then be zero. The bidder's position could then be improved (i.e. the value of the maximand in (16) could be made strictly positive) by increasing the bid slightly, which would make the probability of winning smaller, but still positive and the bidder would then have strictly positive expected benefit. It thus could not have been the case that the original bid was an equilibrium bid. Hence, it must be the case that the net benefit of winning the contract is strictly positive in equilibrium. Thus, we have $B'(V;a)$ strictly positive and bids increasing in V .

In equilibrium, the observed costs $C^*(\cdot)$ can be written as a function of V and the cost share parameter a . For notational simplicity, we continue to use $C^*(\cdot)$ and define

$$C^*(V;a) \equiv C_G(G) + k(Q(V,G;a),G) \cdot (V - e(V,G;a)) \cdot C_I(Q(V,G;a),G). \quad (20)$$

Now that we have shown that the most efficient (lowest cost) bidder wins the contract, we can compare fixed price contracts with incentive contracts. In the following section, we provide sufficient conditions for the government (principal) to be strictly better off with an incentive contract which uses a cost allocation than with a fixed-price contract which avoids cost allocations.

IV. THE DEMAND FOR COST ALLOCATIONS IN OPTIMAL CONTRACTS

The government wishes to minimize its ex ante expenditures on the contract. In order to show conditions under which the government is better off with a contract that calls for an allocation of indirect costs, we show conditions under which an incentive contract dominates a fixed-price contract. This will be sufficient to demonstrate the desirability of a cost allocation based incentive contract, since an incentive contract based on no cost allocation, i.e., $k(Q,G) \equiv 0$, is always dominated by a fixed-price contract (Theorem 1, Cohen and Loeb, 1988).²

If a firm with cost parameter $V \in [V_*, V^*]$ is the winning bidder, the government's expenditure will be

$$(1 - a)B(V;a) + aC^*(V;a). \quad (21)$$

The ex ante expected expenditure for the government for a given cost share, a , is the expected value of (21), where the expectation is taken with respect to the distribution of the minimum of n independent draws from the distribution of V ; for ease of exposition, we denote this expectation by $E_{\text{MIN } V}[\cdot]$.

We wish to show conditions under which the expected expenditure is minimized at a positive cost share, a^* . Suppose expression (21) is decreasing in a at $a = 0$, for all V . Then there exists an $\hat{a} > 0$ such that

$$E_{\text{MIN } V}[(1 - \hat{a}) B(V;\hat{a}) + \hat{a}C^*(V;\hat{a})] < E_{\text{MIN } V}[B(V;0)], \quad (22)$$

which means that $a = 0$ cannot be ex ante optimal. Therefore, to show the demand for cost allocations, it is sufficient to show conditions under which the derivative of expression (21) with respect to a is negative at $a = 0$.

Letting single primes and double primes denote first and second derivatives with respect to Q , we can now state our major result.

THEOREM: If (i) $R''(Q) - C_Q''(Q) \leq 0$ for all Q , and

(ii) $[k(Q,G)C_I(Q,G)]/C_I'(Q,G)$ is decreasing in Q ,

then incentive contracts dominate fixed-price contracts.

Proof:

To demonstrate our result, we show that the derivative of the government's total expenditure is negative for all V at $a = 0$. Using (19), the definition of $(1 - a)B(V;a)$, it is simple to verify that:

$$\frac{d[(1 - a)B(V;a) + aC^*(V;a)]}{da} = -\exp(S(V)) \int_V^{V^*} s(t)\exp(S(t)) \frac{dC(t;a)}{da} dt + C^*(V;a) + a \frac{dC^*(t;a)}{da}. \quad (23)$$

From the definitions of $C(t;a)$ and $C^*(t;a)$, we have the fact that

$$\frac{dC(t;a)}{da} = -C^*(t;a). \quad (24)$$

Substituting into the integral on the right hand side of (23), and integrating by parts, we get

$$\begin{aligned}
 & -\exp(S(V)) \int_V^{V^*} s(t) \exp(S(T)) \frac{dC(t;a)}{da} dt = \\
 & \exp(S(V)) \int_V^{V^*} s(t) \exp(S(T)) C^*(t;a) dt = \\
 & -C^*(V;a) - \int_V^{V^*} \exp(S(T)) \frac{dC^*(t;a)}{dt} dt
 \end{aligned} \tag{25}$$

Substituting the final expression in (25) for the first expression on the right-hand-side of (23) gives us:

$$\frac{d[(1-a)B(V;a) + aC^*(V;a)]}{da} = - \int_V^{V^*} \exp(S(t)) \frac{dC^*(t;a)}{dt} dt + a \frac{dC^*(V;a)}{da}. \tag{26}$$

For the right-hand-side of (26) negative at $a = 0$, it is sufficient that

$$\frac{dC^*(V;0)}{dV} > 0 \quad \text{for all } V. \tag{27}$$

Differentiating $C^*(V;0)$ with respect to V , we get

$$\frac{dC^*(V;0)}{dV} = \frac{dQ}{dV} (V - e) [k' C_I + k C_I'] + (1 - \frac{de}{dV}) k C_I. \tag{28}$$

When $a = 0$, the first order conditions for the winning agent's maximization problem are:

$$R' - C'_Q - (V - e)C'_I = 0$$

and (29)

$$C_I - h' = 0.$$

Using the second order conditions, and Cramer's rule we can solve for dQ/dV and de/dV . Denoting the Hessian matrix by H , we have:

$$\frac{dQ}{dV} = \frac{-h''}{|H|} C'_I \quad \text{and} \quad \frac{de}{dv} = \frac{-(C'_I)^2}{|H|}. \quad (30)$$

Substituting (30) and the definition of $|H|$ into the right-hand side of (28), and collecting terms, we get

$$\begin{aligned} \frac{dC^*(V;0)}{dV} = \frac{-h''}{|H|} [C'_I(V - e)(k'C_I + kC'_I) + \\ kC_I(R'' - C''_Q - (V - e)C''_I)]. \end{aligned} \quad (31)$$

Since $|H| > 0$ from the second order conditions, and R is concave and C_Q is convex by assumption, a sufficient condition for the right-hand side of (31) positive is

$$C'_I(k'C_I + kC'_I) - kC_IC''_I < 0 \quad \text{for all } Q. \quad (32)$$

Q.E.D.

We now present an example that meets conditions of the Theorem, thus, demonstrating that the Theorem does not hold vacuously. Let:

$$(1) R(Q) = pQ,$$

$$(2) C_Q(Q) = vQ, \text{ where } p \text{ and } v \text{ are constants such that } p > v > 0,$$

$$(3) k(Q,G) = G/(Q+G), \text{ and}$$

$$(4) C_I(Q,G) = (Q+G)^2 + F, \text{ where } F \text{ is a positive constant.}$$

It is straight-forward to verify that this example does indeed satisfy the sufficient conditions given in the Theorem. Note that in order for the conditions to be satisfied, it is necessary for there to be some purely fixed costs, i.e., $F > 0$.

If we restrict the cost allocation rule to be of the familiar form $k = G/(G+Q)$, we can gain further insight into the nature of the indirect costs that will yield a positive demand for the cost allocation.

Consider the following:

COROLLARY: Suppose that $R(Q)$ and $C_Q(Q)$ satisfy condition (i) of the Theorem, and suppose $k = G/(Q+G)$. If $C_I(Q,G)$ is weakly convex in Q , then having average indirect costs being greater than marginal indirect costs is a sufficient condition for incentive contracts to dominate fixed price contracts.

Proof:

We will show that condition (ii) of the Theorem will hold. For notational ease, we will suppress the arguments of the C_I function for this proof.

Suppose average indirect costs are greater than marginal indirect costs (hereafter referred to as $AC > MC$), then:

$$C_I' < C_I/(Q+G). \quad (33)$$

Since $C_I' \geq 0$, (33) implies:

$$- [C_I \cdot C_I'/(Q+G)] + (C_I')^2 < 0. \quad (34)$$

By assumption $C_I'' \geq 0$, so that (34) is sufficient for:

$$[G/(Q+G)] \cdot [-C_I \cdot C_I'/(Q+G) + (C_I')^2 - C_I C_I''] < 0. \quad (35)$$

Condition (35) is equivalent to condition (ii) of Theorem.³

Q.E.D.

Note that the convex function $C_I(Q,G) = (Q+G)^2 + F$ given above satisfies condition (ii) of the Theorem, but not the $AC > MC$ condition of the corollary. Thus, the $AC > MC$ condition is not a necessary condition. An example of a cost curve that meets the corollary's conditions is $C_I(Q,G) = QG + F$, for $F > G^2$.

A necessary condition for both indirect cost curves $QG + F$ and $(Q+G)^2 + F$ to satisfy condition (ii) of the Theorem is that the fixed cost component F is strictly positive. Note, that if indirect costs consisted only of pure fixed costs, we cannot show that incentive contracts strictly dominate fixed-price contracts. In fact, if indirect

costs consisted only of fixed costs, as one would expect fixed-price contracts would dominate incentive contracts, and there would be no demand for cost allocations. (This follows from (35) in the proof of Theorem 2 of Cohen and Loeb (1988).) This leads us to believe that for cost allocation incentive contracts to dominate fixed-price contracts, the indirect cost function should have both a fixed component and a component that alters the total marginal cost of outside output, Q ; i.e., there must be both fixed and marginal effects.

V. SUMMARY

Two common types of government procurement contracts are fixed-price contracts and incentive contracts. The enforcement of incentive contracts depends upon an allocation of indirect costs between government and non-government work, while it is not necessary to allocate indirect costs in order to enforce fixed-price contracts. In this paper we have shown conditions under which incentive contracts dominate fixed-price contracts. That is, we have shown conditions under which there is a positive demand for cost allocations.

We have not tried to determine an optimal cost allocation for an incentive contract. Rather, we have found market and cost environments in which any reasonable cost allocation will be preferred to no allocation (i.e., an incentive contract is preferred to a fixed-price contract). Also note, that we have restricted our examination of contracts to linear incentive contracts, since these are the form of contracts used in government procurement. Even if one could find a non-linear incentive contract that is superior to linear incentive contracts, that would not change our basic result that there exist conditions whereby cost allocations are useful in contracting.

REFERENCES

- Coddington, E. A., *An Introduction to Ordinary Differential Equations*, Prentice Hall (1961), Englewood Cliffs, N.J..
- Cohen, S. I., and Loeb, M., "Improving Performance Through Cost Allocations," *Contemporary Accounting Research*, (forthcoming, Vol. 5, No. 1, Fall 1988).
- Cohen, S. I., and Loeb, M., "On the Optimality of Incentive Contracts in the Presence of Joint Costs," Working Paper # 1445 (March 1988), Bureau of Economic and Business Research, University of Illinois at Urbana-Champaign.
- Cummins, J. M., "Incentive Contracting for National Defense: A Problem of Optimal Risk Sharing," *Bell Journal of Economics*, vol. 8 (1977), pp. 169-185.
- Demski, J., "Cost Allocation Games," in *Joint Cost Allocations*, ed. S. Morilarity, University of Oklahoma (1981), Norman, O.K., pp. 142-173.
- Greer, W. R. Jr., and Liao, S. S., "Weapon Pricing Models for Defense Acquisition Policy," *Journal of Accounting and Public Policy*, vol. 6 (1987), pp. 271-284.
- Horngren, C. T., and Foster, G., *Cost Accounting: A Managerial Emphasis*, 6th edition, Prentice Hall (1987), Englewood Cliffs, N.J..
- Laffont, J.-J. and Tirole, J., "Auctioning Incentive Contracts," *Journal of Political Economy*, vol. 95 (1987), pp. 921-940.
- McAfee, R. P., and J. McMillan, J., "Competition for Agency Contracts," *Rand Journal of Economics*, vol. 18 (1987), pp. 296-307.
- McAfee, R.P., and McMillan, J., "Bidding for Contracts: A Principal-Agent Analysis," *Rand Journal of Economics*, vol 17, no. 3 (1986), pp. 326-338.
- Milgrom, P. R. and Weber, R. J., "A Theory of Auctions and Competitive Bidding," *Econometrica*, vol. 50 (1982), pp. 1089-1122.
- Morilarity, S., "Some Rationales for Cost Allocations," in *Joint Cost Allocations*, ed. S. Morilarity, University of Oklahoma (1981), Norman, O.K., pp. 8-13.
- Thomas, A. L., *The Allocation Problem in Financial Accounting*, Studies in Accounting Research No. 3 (American Accounting Association, 1969).
- Thomas, A. L., *The Allocation Problem: Part Two*, Studies in Accounting Research No. 9 (American Accounting Association, 1974).

Wright, H., and Bedingfield, J., *Government Contract Accounting*, 2nd edition, Federal Publications (1985), Washington, D.C..

Zimmerman, J. L., " The Costs and Benefits of Cost Allocations," *The Accounting Review*, vol. 54 (July 1979), pp. 504-521.

FOOTNOTES

1. Greer and Liao [1987, p. 276] note that: "Due to high startup costs and low quantity requirements, most major weapon systems are acquired using sole source contracts."

2. If $k(Q, G) = 0$, then observed costs will be constant with respect to V ; i.e., $C^*(V; a)$ will be equal to $C_G(G)$. It is straightforward to verify that Theorem 1 of Cohen and Loeb [1988] holds when observed costs are generalized to be non-increasing instead of decreasing.

3. It is easy to verify that if C_I is weakly concave, average indirect cost greater than marginal indirect cost will be a necessary condition for (ii). Since the Theorem provides sufficient, but not necessary conditions for incentive contracts to dominate fixed-price contracts, (33) will not be a necessary condition for incentive contracts to dominate fixed-price contracts.

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