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Use of Three Stock Index Futures in Hedging Decisions

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Abstract

In this study, Working, Johnson and Rutledge hedge models are used to show how three stock index futures can be used to do hedging decisions. These three hedge models led to optimal hedging positions in the index futures markedly different from the consistent one to one short hedge, and in some cases called for hedging behavior considered speculative, with long positions in both the futures and the index portfolio, or a short position in the futures greater than the value of the underlying index portfolio.

A. Introduction

The inception of trading in stock index futures has created a new opportunity for portfolio managers by providing a new means to adjust the risk and return of a portfolio to desired levels. By adjusting the proportion of his futures position to the value of his portfolio, the portfolio manager can theoretically attain any risk-return combination he desires. As an alternative, the new futures markets provide a cheap and readily available opportunity to achieve a desired position along a risk-return continuum.

In this paper we examine four models of hedging behavior applied to stock index futures which capture a wide spectrum of concern for risk and return: the traditional one to one hedge; a variance minimizing model first formulated by Johnson (1960), associated with the low-risk portion of the risk-return spectrum; a basis arbitrage model first suggested by Working (1953), in which the hedger attempts to use relative movements in the spot and futures markets to improve return while retaining the risk-minimizing framework of the traditional hedge; and a utility maximization model devised by Rutledge (1972), in which mean return is maximized subject to a constraint on variance of return.

Unlike general capital equilibrium models (Black, 1976; Dusak, 1973), the models examined here do not abstract completely from the particular nature of futures trading. The role of hedging is considered as a separate activity; the risk-spreading opportunities in the capital markets do not subordinate the hedging problem as one of mere diversification across available assets (Stoll, 1979).

Under each model, an optimal hedge ratio or decision rule is estimated, and measures of the effectiveness of the hedges are devised. The effectiveness of the estimated hedge ratios is evaluated according to their own criterion and also according to the other three optimization criteria. While it is expected that each optimal hedge estimate will perform best in its own criterion, it is possible that one particular optimal hedge may perform adequately under all four criteria. Thus, it may be possible to substitute a less complex decision rule or optimal hedge while attaining "satisfactory" results under a more complex model. In addition, the behavior of the optimal hedge ratios are compared across the three exchanges (Kansas City, New York and Chicago) to ascertain whether there are general differences in the distribution of an optimal hedge estimate due to differences in stock index contract specifications. Further, three different maturities of contract will be used to construct optimal hedge ratio estimates--a short, intermediate and long maturity--to ascertain whether there are general differences in results over contract maturity. In order to facilitate comparisons with other empirical results derived for commodities and with each other, hedges will be calculated for the same (calendar-time) one month intervals in each of the contracts.

In Section B, the various optimization models are developed. In Section C, the empirical estimates are calculated and discussed, and Section D contains the conclusions.

B. Optimization Models

A large number of hedging models for commodity futures have been developed, each emphasizing a particular aspect of the hedger's problem.

The traditional rationale for hedging has been that hedging reduces the price risk of holding a commodity.

The constant equal and opposite hedge strategy (the classic) assumes implicitly that the hedger is unskilled or uninterested in forming expectations on the movements of spot prices, and that he derives his profits solely from subjecting the commodity to a process of transformation (storage or production of another commodity (Ward and Schimkat, 1979)). Thus, this hedge has been viewed as a sort of insurance (Samuelson, 1973), and the criteria of its effectiveness is related to risk elimination, however defined.

An alternative approach to hedging received its first definition in Working (1953). Working expanded the possible uses of futures contracts in merchandising activities to include, among others, the possibility of improved return through selective hedging. Here, Working emphasized the return maximization aspect of hedging, in which positions in the futures and the commodity markets were determined simultaneously in order to capture increased return arising from relative movements in the spot and futures prices. Working derived this alternative use through an examination of the year-to-year constancy of the relation between the size of the "spot premium" (or in more modern terms, the basis--cash price less futures price at a point in time) and the gain or loss from subsequent storage with hedging in wheat futures. Working found that a large "negative basis" (cash price less than futures price) was likely to be followed by a large positive change in the basis (basis widens) and a large positive basis by a large negative change in the basis (basis narrows). A short position in the futures (of magnitude equal to the commodity held) should be undertaken selectively, then, only if the basis were "sufficiently narrow" to allow the hedger to believe that the basis change would be positive, a change favorable to the hedger.

The Working, or basis arbitrage hedge, then, involves an automatic hedge/no hedge decision using the size of the current basis as the deciding factor. Because it is automatic (both the decision to hedge and the hedging commitment are determinant), the Working strategy, if it results in improved profit, can be easily implemented as an improvement over the classic strategy.

The measure of effectiveness used for the Working strategy is the increase in gross profit over the classic one-to-one hedge, where profit is defined as:

$$(1) \quad \pi = (S_2 - S_1) - H(F_2 - F_1)$$

where H = the hedge ratio

S_1, S_2 = beginning and ending spot prices

F_1, F_2 = beginning and ending futures prices

Early research defined the risk involved in hedging in a number of ways (Howell and Watson, 1938; Howell, 1948; Yamey, 1953; Graf, 1953). It was not until the inception of modern portfolio theory that definitions of risk and return in terms of mean and variance of return were applied to the hedging problem. Thus, Johnson's (1960) early treatment of the hedging problem retained the objective of risk minimization but defined risk as the variance of return on a two-asset hedged portfolio.

Johnson's model treats the hedger as essentially infinitely risk averse, and defines risk in terms of the variance of the total position of the hedger in the spot and futures market. The variance of the return on a hedged portfolio is minimized; the minimum variance hedge ratio is expressed in terms of expectations regarding the variances and covariance of price changes in the spot and futures markets. Thus, Johnson's model differs from Working's in that the objective is to minimize risk and the position is defined in terms of absolute rather than relative price changes.

The Johnson minimum variance hedge ratio between the dollar amount invested in futures and spot (X_f^*, X_s) is:

$$(2) \quad \frac{X_f^*}{X_S} = - \frac{Cov(S\Delta F\Delta)}{\sigma_{F\Delta}^2} = H$$

where $Cov(S\Delta F\Delta)$ = the covariance of spot, futures price changes

$\sigma_{F\Delta}^2$ = variance of futures price changes

The minimum variance hedge, then, is defined as the coefficient of the regression spot price changes on futures price changes.

Johnson also developed a measure of the effectiveness of the hedged position in terms of the reduction in variance of the hedged over the variance of the unhedged position:

$$(3) \quad e = 1 - \frac{V(H)}{V(U)}$$

where $V_U(U)$ = variance of unhedged spot = $X_S^2 \sigma_{S\Delta}^2$, $\sigma_{S\Delta}^2$ = variance of spot price changes,

$V_H(H)$ = variance of hedged spot

or substituting the minimum variance X_f^*:

$$(4)\quad e = 1 - \frac{X_S^2\ \sigma_{S\Delta}^2\ (1 - \rho^2)}{X_S^2\ \sigma_{S\Delta}^2} = \rho^2$$

where ρ^2 = the squared simple correlation coefficient of spot, futures price changes

Thus, Johnson's measure of effectiveness is the squared simple correlation coefficient of spot price changes to futures price changes, or in this case, with one estimated parameter, the R^2 of the regression of spot price change on futures price change.

Finally utility of return hedge models were quantified and extended by a number of researchers (Rutledge, 1972; Peck, 1975; Holthausen, 1979; Feder, Just and Schmitz, 1977; Rolfo, 1980; Anderson and Danthine, 1980, 1981). In particular, Rutledge formulated the hedging problem mathematically as a constrained optimization problem in which expected return of the hedged position was maximized subject to a series of constraints on risk (variance of the position), storage capacity, and minimum inventory holdings.

Dropping both the capacity and convenience yield constraints (which here are inappropriate for a stock portfolio holding decision), the utility maximizing optimal hedge ratio is:

$$(5)\quad H = \frac{\sigma_{SB\Delta}\ (\mu_S - c) + \sigma_{S\Delta}^2(\mu_{B\Delta} - c)}{(\mu_S - c)(\sigma_{B\Delta}^2 + \sigma_{SB\Delta}) + (\mu_{B\Delta} - c)(\sigma_S^2 - \sigma_{SB\Delta})}$$

where $\sigma_{SB\Delta}$ = the covariance of spot price, basis change,

c = the cost of carrying the spot commodity,

μ_S, $\mu_{B\Delta}$ = mean spot price, basis change,

$\sigma_{B\Delta}^2$ = the variance of basis change, σ_s^2 = the variance of spot.

Note that in (5), if the cost of a hedged position (here, c) equals the expected change in the basis, the optimal hedge in Rutledge's model reduces to a hedge ratio quite similar to Johnson's minimum variance ratio, differing by a covariance term and defined in terms of the basis change rather than the futures price change.

Effectiveness of the Rutledge strategy relates to the utility of the hedger. Thus, the effectiveness of the Rutledge strategy is defined as:

$$(6)\quad U(R) = \mu_S - H\mu_{B\Delta} - \lambda(\sigma_S^2 + H^2\sigma_{B\Delta}^2 - 2H\sigma_{SB\Delta})$$

where λ = individual's risk aversion factor.

A summary of the hedge ratio calculations and effectiveness measures of the four models is given in Table One.

C. Data and Empirical Results

1. Data: Spot prices (final index value) and closing prices for the three index futures were obtained from the Wall Street Journal during the period 5/31/82-3/1/83. The cost of carrying the spot commodity was assumed to be the interest cost of the value of the beginning spot price of one contract, expressed in basis points (versus dollar value). The interest cost was calculated using monthly averages of the weekly U.S. T-Bill averages (three month T-Bills) quoted in the St. Louis Federal Reserve's U.S. Financial Data. Two values of the subjective risk aversion parameter, lambda, required for the Rutledge optimization criterion were used, 0.1 and 0.01. Each hedge ratio estimate was calculated using a month of daily price observations; that is, each month's estimate is based on *ex post* data. Thus, the hedge ratio is optimal given perfect knowledge of the hedge month's prices.

For each month on each exchange, three different maturity contracts were used to calculate the ratio estimates: a short maturity (the closest to maturity, or in a delivery month the next closest contract), a long maturity (the farthest from maturity available in that hedge month), and an intermediate maturity. Nine months of hedge ratios were calculated for each exchange (June, 1982 through February, 1983) using three different maturity contracts. Thus, a total of 27 optimal hedge ratios were estimated for each exchange for each hedge model.

2. Working Relationship: To ascertain whether the basis arbitrage hedge strategy developed by Working is applicable to stock index futures, it was necessary to test for the existence of a negative relationship between the size of the basis and its subsequent change (gain or loss on storage with hedging) in stock index futures:

$$\text{(7)} \quad \text{BASIS SIZE} = \beta_0 + \beta_1(\Delta \text{ IN BASIS}) + u$$

A two-month period for each contract trading on the three exchanges was used for the basis relationship: the two-month period beginning one month after the inception of trading in each contract. The basis on a day one month after trading began in that particular contract was used as the initial basis size, and the basis change was calculated using the basis on the day two months from the first observation, or three months from the inception of trading. Thus, the two month period is not over the same calendar time for each contract but depends on when the particular contract started to trade. While it is customary in actual trading to attach greater weight to the basis of contracts closest to maturity, the above procedure was used in order first, to obtain a sufficient number of observations and secondly, to avoid the problem of dependence on one particular time period. A simple OLS regression was performed.

The results indicate a significant negative relationship (β_1 = -0.629, t = -5.25; R^2 = 0.56); it would appear that a hedging decision rule utilizing the relationship might improve the hedger's return over the constant one to one hedge.

Since Working's hedging strategy requires a judgment concerning whether the basis is "sufficiently negative" to hedge, various values of the basis size were used in the decision rule and the results compared. The decision rule was, "if the basis at the beginning of the hedging month is less than or equal to X (the optimal amount of commodity short-hedged), hedge with one futures contract (short). If not, leave the position unhedged." X varied in 1.5 basis point increments from -3.0 to 3.0 points.

3. Empirical Results, Working: It was found that the use of the Working hedging strategy can improve a hedger's gross profit, in some cases quite considerably. Table 2 shows the increase in average profit (in basis points) over the classic one to one hedge strategy according to maturity of contract and basis size used in the decision rule. In all cases, the Working strategy resulted in a gross profit greater than or equal to that from the one to one (profit improvement greater than or equal to 0). For instance, using a short maturity contract on the Kansas City exchange and a basis size of -3.0 in the decision rule, the hedger's average profit improvement would have been 5.98 basis points, or in dollar terms $2,990 ($500 x 5.98). Thus, for the hedger of a

stock portfolio who is interested in profit improvement, the Working strategy represents a viable automatic decision rule.

As can be seen the performance of the Working strategy depends heavily on both the instrument used and the basis size used in the decision rule. With a basis size less than or equal to 0, the short contract performed best of the three maturities, and would appear to be the instrument of choice when using the Working strategy. This result was due primarily to the fact that the longer maturity contracts were less correlated to the index during the period, and tended to rise faster than the spot index or to decrease more slowly, resulting in greater losses on the farther maturities. Further, the Kansas City exchange resulted in general in higher gross profits of the three. Thus, while the choice of contract to use in hedging would be determined primarily by the composition of one's own portfolio, the Kansas City exchange contracts would be preferred if there were a choice based on relative performance alone, as might be the case if the spot portfolio to be hedged were correlated quite similarly to all three index futures. In addition, the results also depended heavily on the size of the basis used in the decision rule. The rule becomes indistinguishable from the constant hedge at a basis size greater than 0.0. With a rule using a basis size of 0.0, -1.5, or -3.0, improved profits were obtained in a majority of cases. At a value of -3.0 or less, the decision rule becomes indistinguishable from a consistent unhedged position. There is thus a very narrow band of values in which the decision rule is applicable. Since the size of the basis can vary, the optimal size of the basis to use in the decision rule can change through time and must be re-examined periodically.

To summarize, the Working hedging strategy led to improved gross profits on average and on a month by month basis over the classic hedge for the period tested. As might be expected, the results are sensitive to the maturity of the hedging contract used as well as to the size of the basis used in the decision rule. Given the automatic nature of the working hedge strategy, it would appear that working strategy is a viable alternative to automatic classic one to one strategy.

4. Empirical Results, Johnson: Empirical estimates of the mean, variance, and range of the Johnson hedge ratios as well as the first order autocorrelation of the hedge ratios for the three exchanges is listed in Table 3. On a month by month basis, the Johnson minimum variance ratio estimates were all less than the classic one, and in a third to a half of the hedges, less than 0.5. In all months examined, then, the classic one to one hedge would have been suboptimal in reducing the variance of the hedged position and in fact would represent a considerable overinvestment in futures, with the attendant overpayment of margin and transaction costs. Values for the ratios ranged from a maximum of 0.7886, or 79% of the value of the spot portfolio position, to a minimum of -.1049, or excluding negative numbers (indicating a long position in the futures), a minimum of 0.1028, or 10% of the spot portfolio position.

The value of the optimal hedge ratio was quite sensitive to the choice of hedging instrument used. The optimal ratios differed systematically between maturity of the hedging contract, and the difference in ratio value between contract maturities was as much as 0.5, or 50% of the value of the spot portfolio position in several hedge months. For the New York and Chicago exchanges (NYSE and S&P500 indexes), the optimal ratio decreased across maturity. This is the same pattern that Ederington (1979) found in the commodity and financial futures he tested. In addition, the hedge ratio estimates for the New York and Chicago exchanges were in general greater than that for the Kansas City exchange (Value Line index), and related to the generally larger variance of the futures price changes on the Kansas City exchange. Thus, all other considerations (especially performance) equal, a potential hedger would prefer to use the long maturity contract and the Kansas City contracts because of the lower margin and transaction costs associated with the smaller hedge ratios. Further, the size of the Johnson optimal hedge ratio found here with stock index futures seems to be lower than most of the ratios estimated for financial or commodity futures (Ederington, 1979; Maness, 1981; Cicchetti, Dale and Vignola, 1981).

However, there are problems for a naive potential hedger with utilizing the Johnson hedging strategy. The value of the optimal ratio differed considerably through time, and as can be seen from the Table, the optimal values in any one month were not closely correlated to the values optimal in the previous or succeeding month. It may thus be extremely difficult for a potential hedger with ordinary skill in forecasting to formulate an estimate of a future optimal hedge using data from a previous period. The need to forecast accurately the various variances and covariances needed to hedge optimally under the models is thus one of the disadvantages inherent in the more complex optimization models.

As to effectiveness of the hedging strategy, the Johnson hedge, as expected, was quite effective at reducing the variance of the hedger's position. The estimates of average effectiveness are listed in Table 4 along with the average effectiveness of the other hedging strategies (classic, Working, and Rutledge) in reducing variance. Effectiveness ranged from a minimum of 0.0203, or 2% of the variance eliminated, to a maximum of 0.902, or 90% of the variance of the long portfolio position eliminated through hedging. The averages centered around 0.65 for the three exchanges, indicating that more than half of the variance of the hedger's portfolio position could be eliminated through hedging.

The effectiveness of the hedges differed between exchanges, with the effectiveness of the Kansas City exchange contracts the lowest in general. Given that the effectiveness measure here is the squared correlation of spot and futures price changes, the futures and spot prices were thus less correlated for the Value Line index than for the NYSE or S&P500 indexes, and the Chicago and New York contracts were more effective in reducing the variance of their own stock index portfolio position than was the Kansas City. This would indicate, then, a trade-off between the lower optimal ratios and lower transaction costs against the lower effectiveness of the Kansas City exchange contracts.

One would expect that the nearest maturity contract would be most highly correlated (most effective) with the underlying commodity or index because traders are more likely to alter their expectations on the nearby contract in response to changes in the underlying commodity price (index value). Surprisingly, this is not the case with index futures: effectiveness did not decline with maturity but instead showed little relation to maturity of contract used. The Chicago exchange was a weak exception; here the effectiveness was highest in the shortest maturity contract in 5 of the 9 months. Thus, given the lack of relation between effectiveness and maturity, there would be a preference for longer maturity/smaller ratio contracts.

Comparing the results using the Johnson ratio with the other three hedging strategy ratios, the Johnson ratio was, as expected, the most effective of the four. The relative performance of the other hedge ratios under the variance reduction criterion is consistent with their own objectives. The Rutledge ratio, for instance, also performed well, and this is probably related to the inclusion in the model of the constraint on variance. Similarly, the Working ratio, a selective strategy concerned with profit maximization, did less well.

Note that in many instances, however, the Working strategy resulted in a larger variance reduction than the classic one to one hedge. This is surprising, since the rationale underlying the constant hedge is risk avoidance. Thus, when risk is defined in terms of the variance of the unhedged position, the simple hedge is not a good choice for the risk averse investor. In fact, comparing the simple hedge to an unhedged, or zero variance reduction position, one notes that the simple hedge in many cases results in variance reduction less than the unhedged. As risk is defined here, a risk averse hedger would do better in reducing his risk in many months by choosing to remain unhedged rather than carry a one to one hedge. This supports Working's (1953) contention that the actual performance of the classic hedge, as a consistent hedging strategy, does not live up to its traditionally good reputation as a risk avoidance strategy.

The average heding effectiveness found here in stock index futures is within the range of that found in financial and commodity futures in previous studies (Ederington, 1979; Maness, 1981; Cicchetti, Dale and Vignola, 1981).

To summarize, the Johnson optimal hedge ratio estimates were all less than the classic one and averaged around 0.50. There was little similarity in hedge ratios across time. On average, the shortest maturity contract had the largest ratio value, and the size of the estimate decreased with increasing maturity on the New York and Chicago exchanges. As to effectiveness, the range in variance reduction was considerable, but averaged 0.65, or 65% of the variance of the unhedged index portfolio. There was little relation between contract maturity and effectiveness. As expected, the Johnson optimal hedge ratios were most effective of the four hedging strategies in reducing the variance of the long portfolio position.

5. Empirical Results, Rutledge: Average values for the optimal ratios using the Rutledge strategy are presented in Table 5. On a month by month basis, the size of the Rutledge hedge ratios were quite similar to the Johnson estimates despite the difference in optimization criteria. They ranged from a minimum of 0.0027, or an essentially unhedged portfolio, to a maximum of 1.2077, or 121% of the portfolio position, averaging between 0.4 and 0.5, depending on the exchange. While the Rutledge ratio estimates were also less than the classic one in most cases, a few of the (monthly) ratio estimates were greater than one, indicating a short position in the futures <u>greater</u> than the amount invested in the underlying portfolio, a more "speculative" position than was optimal in the Johnson strategy. This probably results from the more speculative criterion in the Rutledge model, maximization of mean price change.

Again, the use of the Rutledge strategy required forecasting various components of spot and futures price behavior, and there was little similarity in optimal hedge ratios through time.

Again, the optimal values of the ratios differed according to the maturity of contract used, with the optimal ratio again decreasing with increasing maturity on the New York and Chicago exchanges. Here again, then, the individual hedger would prefer the long maturity contract because of its lower margin and transaction costs.

Since there seems to have been no previous empirical work using the Rutledge strategy, it is difficult to ascertain how "typical" these results are compared to other financial and commodity futures.

The effectiveness of the Rutledge strategy, measured in cardinal utility, is given in Table 6 along with the utility of the other three hedging strategies. Note that the value of relative effectiveness under the Rutledge strategy depends on the specific value of the risk aversion parameter, lambda, chosen for the cardinal utility of the Rutledge optimization criterion.

Surprisingly, while the average values indicate that the Rutledge strategy is "best" under its own criterion, as expected, the Rutledge ratio does not consistently dominate the utility values on a monthly basis under either parameter value.

Using a value of 0.1 for the risk aversion parameter, the Rutledge hedge strategy dominated the others in only half of the 27 monthly hedges on each exchange. In a practical sense, then, the hedger has only an even change of choosing an optimal hedge ratio (under the Rutledge criterion of utility maximization) using the Rutledge ratio estimate in any month. However, no other ratio would be preferred as a specific alternative: the hedger would have a higher probability of using an optimal hedge using the Rutledge ratio consistently. In fact, on an average basis, as can be seen, the hedger would uniformly prefer to follow the Rutledge strategy since this results in the highest cardinal utility on an average basis. As a consistent strategy then the Rutledge hedge does result in the highest utility for the period tested.

In contrast, the classic hedge, the naive standard, does relatively poorly on both on a month by month basis and on average, generally ranking third or fourth in relative performance through the months. None of the other strategies approach the Rutledge in effectiveness, although it appears that the Working hedge strategy is a distant second. Thus, it is not possible to discern a truly "second best" strategy that could perform adequately as a substitute for the Rutledge calculation. It is encouraging, however, that the simple, automatic Working strategy does as well as it does, given the determinate nature of its hedging ratio and hedging decision.

It is difficult to ascertain why the Rutledge ratio is not the best in terms of its own criterion, especially since, as noted previously, the effectiveness measure for the Rutledge optimal ratio is the unconstrained mean return. A possible explanation may lie in the cost factor: the Rutledge hedge "loses" effectiveness during the period with maximum interest cost, July. However, it is not a rigorous explanatory factor, since the relative performance of the Rutledge strategy does not consistently follow interest rate movements through the months.

Considering the relative effectiveness of the various strategies under a risk aversion parameter of 0.01, it is interesting that the two hedge strategies which emphasize risk reduction (the classic and the

Johnson minimum variance) improved their relative performance. This is indeed surprising, since such a loosening of the shadow price of risk could be expected to lessen the relative performance of strategies concerned primarily with risk reduction. Again, the hedger would find the Rutledge strategy works "best" under its own criterion in that it had the highest probability of being optimal in any one month. However, in terms of average utility, the Rutledge strategy performed no better than the Working or Johnson. Thus, the less risk averse the hedger, the less happy he would have been with the Rutledge strategy as a consistent hedge and the more he would have preferred the Johnson or Working strategy as an alternative.

To summarize, the Rutledge ratio estimates were also different from the classic one and on occasion called for more speculative behavior (larger short position) than did the Johnson strategy. The average size of the Rutledge ratio was, however, less than the Johnson. Thus, the hedger using the Rutledge strategy would have, on average, a smaller futures position although in any month the Rutledge strategy may call for a more speculative position. Again, the value of the optimal ratio changed randomly through time, and the optimal hedge ratio within the month increased with decreasing maturity of contract used to hedge.

As to effectiveness, the hedger would have preferred to use the Rutledge hedge both on an average (as a consistent hedging strategy) and individually by month if he believed the optimization criterion particular to the Rutledge strategy and if the hedger were relatively risk averse. However, the Rutledge strategy proved no better on average than the Working or Johnson strategy as the hedger became relatively less risk averse. Finally, the preference for using the Rutledge strategy did not depend consistently on maturity of contract.

D. Conclusions

The three hedge models examined in this paper led to optimal hedging positions in the index futures markedly different from the consistent one to one short hedge, and in some cases called for hedging behavior considered speculative, with long positions in both the futures and the index portfolio, or a short position in the futures greater than the value of the underlying index portfolio. A consistent short position in the stock index futures equal in value to the underlying index, then, would be suboptimal both under criteria emphasizing risk reduction and profit maximization. Further, since in most cases the optimal hedge was less than the classic one, the hedger using the classic strategy would have had a tendency to overhedge under the Johnson and Rutledge framework, overpaying on transaction and margin costs.

In comparing the classic hedge to the selective hedging strategy developed by Working (1953), it was shown that the hedger could improve his profit by using the basis arbitrage hedge, but might sacrifice variance reduction (see Table 4) to do so. Further, it was seen that using the classic hedge strategy could at times result in a larger variance position than leaving the portfolio unhedged. Thus, under a rather wide variety of criteria examined here, the classic hedge strategy proved to be suboptimal if the hedger's motivations matched one of the optimization criteria.

The changing ratio estimates through time clearly showed that successful usage of the strategies depend strongly on skill in forming "correct" expectations on spot, futures, and basis changes. Given the changing magnitude of the optimal ratios, then, it is difficult to assess their usefulness to a practitioner with only average skill in forecasting price levels into future periods. Frequently, the automatic Working strategy gave relatively good results (cf. Rutledge effectiveness measuers), and it may be that the naive forecaster would "satisfice" and use the simpler decision rule at a sacrifice of optimality to avoid the "forecast risk" involved in the more complex hedging strategies.

The estimates obtained here for the Johnson ratios were similar to those found in other financial and commodity futures. One may conclude, then, that hedging with stock index futures is not so different from hedging in the other, older, commodity futures or financial futures.

As expected, the hedge strategies performed best according to their own criterion. As mentioned previously, there seems to be no clear "second best" hedging strategy, although the Working decision rule appeared to be both easy to use and satisfactory in many cases. For instance, use of the Working strategy resulted in the highest cardinal utility, on average, for a risk neutral hedger under the Rutledge model. Thus, the Working strategy may be preferred if the potential hedger desired a quick and easy decision rule.

While the maturity of the hedging contract used affected the size of the optimal hedge ratio, there seemed to be no consistent maturity effect on the performance of the contract as a hedging instrument. Thus, the hedger would be indifferent to selection of maturity of a futures contract, but would need to take maturity into account when calculating the specific size of his position. If transaction costs were related to size of position, then, there would be a preference and a contract maturity with the smallest ratio; in the Johnson and Rutledge framework, this would mean a preference for longer maturity contracts.

No one of the exchanges semeed to be superior over all four strategies in effectiveness, however defined. While using the Kansas City exchange contracts resulted in the largest gross profit on average under the Working strategy, the New York and Chicago exchange contracts resulted in the largest variance reduction under the Johnson strategy. Thus, while a particular exchange may give superior results on average under a particular strategy, the relative performance of an instrument from one of the exchanges depends strongly on the strategy used and also changes from one hedging period to another.

Finally, examining effectiveness in a broader sense, it may be argued that the hedge estimates and their performance are only as good as the model underlying their formulation. Each of the models here focuses on a specific aspect of hedging motivation, and each makes assumptions concerning the particular form that economic relationships take. Some of the ratios are seen to be special cases of others under certain conditions. Thus, the question of a ratio's usage depends heavily on how closely the assumptions underlying the model used to generate it approach a hedger's real situation. Almost all of the models ignore such real constraints as margin costs, taxes, brokerage fees, the indivisibility of futures contracts, and the possibly dynamic nature of the hedge ratio. In addition, there are larger questions concerning the models' assumptions, questions quite similar to those confronting various asset pricing models: the applicability of the two-period formulation, the mean-variance framework, the omission of production uncertainty, the existence of essentially perfect information, etc. (Rausser, 1980). Whether the simplifying assumptions made to formulate the hedge ratios abstract to an essential reality or eliminate a determinate factor in the hedging decision determines the true applicability of the hedge strategy as a solution to the hedger's problem.

TABLE 1

RATIO CALCULATIONS AND EFFECTIVENESS MEASURES

Model	Optimization Criterion/ Decision Rule	Hedge Ratio	Effectiveness Measure
Classical	---	One	---
Working	Basis arbitrage	One or Zero	$\pi = (S_2 - S_1) - H(F_2 - F_1)$
Johnson	Variance minimization	$\sigma_{S\Delta F\Delta}/\sigma^2_{F\Delta}$	$e = \sigma^2_{S\Delta F\Delta}/\sigma^2_{F\Delta}\sigma^2_{S\Delta}$
Rutledge	Utility maximization: mean return with constraint on variance of return	$\frac{\sigma_{S\Delta B\Delta}(\mu_{S\Delta} - c) + \sigma^2_{S\Delta}(\mu_{B\Delta} - c)}{(\mu_{S\Delta} - c)(\sigma^2_{S\Delta} + \sigma_{S\Delta B\Delta}) + (\mu_{B\Delta} - c)(\sigma^2_{B\Delta} + \sigma_{S\Delta B\Delta})}$	$U(R) = \mu_{S\Delta} - H\mu_{B\Delta} - \lambda(\sigma^2_{S\Delta} + H^2\sigma^2_{B\Delta} - 2H\sigma_{S\Delta B\Delta})$

SΔ = Spot price change
FΔ = Futures price change
BΔ = Basis change

H = hedge ratio
c = transaction costs
λ = risk parameter, LaGrangian

TABLE 2

AVERAGE IMPROVEMENT IN GROSS PROFIT
OVER THE CLASSIC STRATEGY
(In points)*

Maturity of Contract	Basis Used in Decision Rule:	-3.0	-1.5	0.0	1.5	3.0
	Kansas City Exchange					
	Short	5.98	5.93	3.00	1.54	0.00
	Intermediate	3.82	2.81	1.48	1.48	0.00
	Long	5.03	1.60	1.44	0.00	0.00
	New York Exchange					
	Short	2.91	2.90	1.12	0.00	0.00
	Intermediate	2.13	0.25	0.14	0.12	0.00
	Long	2.66	0.19	0.00	0.00	0.00
	Chicago Exchange					
	Short	4.72	4.58	1.98	1.75	0.00
	Intermediate	4.64	2.25	1.94	0.00	0.00
	Long	4.76	2.00	1.83	0.23	0.00

*For the dollar value of the profit, multiply by 500.

TABLE 3

ESTIMATES OF OPTIMAL HEDGE RATIOS, JOHNSON MODEL

	Maturity of Contract		
Kansas City Exchange:	Short	Intermediate	Long
Average	.4225	.3013	.4356
Variance	.0195	.0394	.0165
Range	.2558-.6781	-.1049-.5597	.2758-.6794
First-order Autocorr.	-.3798	.1180	-.0308
New York Exchange			
Average	.5423	.4660	.4999
Variance	.0116	.0224	.0085
Range	.3997-.7495	.1306-.7138	.3922-.6881
First-order Autocorr.	-.0550	.0765	-.1820
Chicago Exchange			
Average	.5939	.5819	.5739
Variance	.0166	.0163	.0163
Range	.4403-.7886	.4362-.7866	.4249-.7628
First-order Autocorr.	.0369	.0173	-.0130

OBJECTIVE: Minimization of variance of hedged position, unconstrained.

Hedge Ratio: beta of regression of spot price change on futures price change.

TABLE 4

EFFECTIVENESS, JOHNSON MODEL

(% of Variance of Unhedged Position Eliminated)

Averages

Hedge Strategy	Contract Maturity		
Kansas City Exchange:	Short	Intermediate	Long
Classic	-.9182	-1.5105	-.7810
Working*	-.0879	-.8256	-.1353
Johnson	.5526	.4059	.6009
Rutledge	.4339	.1189	.4408
New York Exchange:			
Classic	.0641	-.2928	-.1448
Working	.0000	-.0564	-.0185
Johnson	.7456	.6686	.7435
Rutledge	.5807	.1708	.5448
Chicago Exchange:			
Classic	.2723	.2318	.2004
Working	.0833	.2809	.2828
Johnson	.7165	.7156	.7140
Rutledge	.5852	.5648	.5352

*Results of using Working strategy with -1.5 as the basis point in the decision rule.

TABLE 5

ESTIMATES OF OPTIMAL HEDGE RATIOS, RUTLEDGE MODEL

	Maturity of Contract		
Kansas City Exchange:	Short	Intermediate	Long
Average	.3703	.4309	.3520
Variance	.0773	.0718	.0738
Range	.0192-.7349	.0044-.9277	.0027-.7969
First-order Autocorr.	-.4030	-.0615	-.3380
New York Exchange			
Average	.4879	.4182	.3331
Variance	.0850	.1158	.0553
Range	.0352-.9637	.0508-1.2077	.0383-.7242
First-order Autocorr.	.2025	-.2440	-.3330
Chicago Exchange			
Average	.5493	.5342	.5183
Variance	.0915	.0982	.1049
Range	.1246-1.0723	.1142-1.0957	.0726-1.0887
First-order Autocorr.	-.2047	-.2073	-.1190

OBJECTIVE: Maximization of Utility of mean return with a constraint on variance of return.

TABLE 6

EFFECTIVENESS, RUTLEDGE MODEL
(Cardinal Utility: $U(R) = X'\mu - \lambda(X'VX-k_1)$)

Averages
(Lambda = .1)

Hedge Strategy	Contract Maturity		
Kansas City Exchange:	Short	Intermediate	Long
Classic	-.7199	-.9674	-.7941
Working*	.0153	-.5372	-.4467
Johnson	-.2071	-.1793	-.2399
Rutledge	.0840	.0503	.0716
New York Exchange:			
Classic	-.2253	-.2406	-.2514
Working	.0342	-.2175	-.1253
Johnson	-.0765	-.0265	-.0717
Rutledge	.0249	.0440	.0385
Chicago Exchange:			
Classic	-.5534	-.5780	-.5773
Working	-.1054	-.2348	-.2841
Johnson	-.2224	-.2251	-.2204
Rutledge	.0876	.0816	.0806

Averages
(Lambda = .01)

Kansas City Exchange:	Short	Intermediate	Long
Classic	-.0348	-.1179	-.1152
Working	.2365	.0391	-.0279
Johnson	.0921	.1014	.0618
Rutledge	.0840	.0503	.0716
New York Exchange:			
Classic	-.0288	.0004	-.0299
Working	.1046	-.0194	.0228
Johnson	.0256	.0614	.0312
Rutledge	.0249	.0440	.0385
Chicago Exchange:			
Classic	.0099	-.0017	.0044
Working	.2102	.1157	.1033
Johnson	.1073	.1036	.1075
Rutledge	.0846	.0816	.0806

*Results of using Working strategy with -1.5 as the basis point in the decision rule.

References

Anderson, R. W. and Danthine, J. P. (1980). "Hedging and Joint Production: Theory and Illustrations," Journal of Finance 35:487-98.

________ (1981). "Cross-Hedging," Journal of Political Economy 89:1182-96.

Black, R. (1976). "The Pricing of Commodity Contracts," Journal of Financial Economics 3:167-79.

Cicchetti, P., Dale, C., and Vignola, A. J. (1981). "Usefulness of Treasury Bill Futures as Hedging Instruments," Journal of Futures Markets 1:379-87.

Dusak, K. (1973). "Futures Trading and Investor Return: An Investigation of Commodity Market Risk Premium," Journal of Political Economy 81:1387-1406.

Ederington, L. H. (1979). "The Hedging Performance of the New Futures Markets," Journal of Finance 34:157-70.

Feder, G., Just, R. E., and Schmitz, A. (1980). "Futures Markets and the Theory of the Firm Under Price Uncertainty," Quarterly Journal of Economics 94:317-23.

Graf, T. F. (1953). "Hedging--How Effective Is It?", Journal of Farm Economics 35:398-413.

Heifner, R. G. (1972). "Optimal Hedging Levels and Hedging Effectiveness in Cattle Feeding," Agricultural Economics Research 24:25-36.

Holthausen, D. M. (1979). "Hedging and the Competitive Firm Under Price Uncertainty," American Economic Review 69:989-95.

Howell, L. D. (1948). "Analysis of Hedging and Other Operations in Grain Futures," USDA Technical Bulletin #971 (August).

________ and Watson, L. J. (1938). "Relation of Spot Cotton Prices to Prices of Futures Contracts and Protection Afforded by Trading in Futures," USDA Technical Bulletin #602.

Johnson, L. L. (1960). "The Theory of Hedging and Speculation in Commodity Futures," Review of Economic Studies 27:139-51.

Maness, T. S. (1981). "Optimal Versus Naive Buy-Hedging with T-Bill Futures," Journal of Futures Markets 1:393-403.

Peck, A. E. (1975). "Hedging and Income Stability: Concepts, Implications, and an Example," American Journal of Agricultural Economics 57:410-19.

Rausser, G. C. (1980). "Discussion: Hedging and Joint Production: Theory and Illustrations," Journal of Finance 35:498-501.

Rolfo, J. (1980). "Optimal Hedging Under Price and Quantity Uncertainty: The Case of a Cocoa Producer," Journal of Political Economy 88:100-16.

Rutledge, D.J.S. (1972). "Hedgers' Demand for Futures Contracts: A Theoretical Framework with Applications to the United States Soybean Complex," Food Research Institute Studies 11:237-56.

Samuelson, P. A. (1973). Economics. New York: McGraw-Hill.

Stoll, H. R. (1979). "Commodity Futures and Spot Price Determination and Hedging in Capital Market Equilibrium," Journal of Financial and Quantitative Analysis 14:873-94.

Ward, R. W. and Schimkat, G. E. (1979). "Risk Ratios and Hedging: Florida Feeder Cattle," Southern Journal of Agricultural Economics 11:71-7.

Working, H. (1953). "Hedging Reconsidered," Journal of Farm Economics 35:544-61.

Yamey, B. S. (1951). "An Investigation of Hedging on an Organized Produce Exchange," The Manchester School 19:305-19.