BEBR

FACULTY WORKING PAPER NO. 967

A Note on Bias in Capital Budgeting Introduced by Stochastic Life

Chien-Wen Chen
Rene P. Manes

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

FACULTY WORKING PAPER NO. 967

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

July 1983

A Note on Bias in Capital Budgeting
Introduced by Stochastic Life

Chien-Wen Chen, Graduate Student
Department of Accountancy

Rene Manes, Professor
Department of Accountancy

Abstract

In capital budgeting analysis, the use of the expected life of a project instead of the life distribution of the project biases the estimate of expected net present value. In most situations the bias results in an overestimate of the expected net present value of the project. When the exact life distribution is unknown, the bias can be approximated by Taylor series expansion. The sensitivity of the bias to the discount rate, to cash flow patterns, and to income taxes is also investigated.

Introduction

In capital budgeting, there are factors and misapprehensions which bias the calculation of net present value (NPV) towards systematic overestimation. As a result, realized yields on implemented projects tend to be lower than the returns predicted at the time projects are selected. For example, it has long been understood that the tax effects of improper provision for inflation results in an overestimate of a project's NPV (Terborgh, pp. 8-10). More recently, K. Brown [1981] ascribes the overestimation phenomenon to selection bias. Quite reasonably managers select for implementation investment projects from the upper range of a distribution of potential projects. Overestimation of NPV will cause otherwise questionable projects to be accepted, while on the other hand, underestimated projects are more likely to be rejected. Therefore, the schedule of selected projects includes more overestimated ones than does the list of rejected projects. As a result, NPV of implemented projects tends to be overoptimistic.

This note examines another source of bias which in most cases systematically overestimates NPV. In most discounted cash flow models, the calculation of NPV requires the determination of the appropriate discount rate, of cashflows and of project life. Under conditions of uncertainty, attention is focused on problems of estimating cash flows and the cost of capital, project life being treated as given. However, for many projects, asset life is probabilistic. Even if the physical life of assets can be treated as deterministic, technological change, economic conditions or shifts in consumer preference, all can force projects to be terminated at uncertain times before the end of physical life.

The stochastic nature of asset or project life has received some attention in accounting and other research literature. For example, Ijiri and Kaplan [1969] and Jen and Huefner [1970] both consider methods of calculating probablistic depreciation. Greer [1970] addresses the problem in the context of capital budgeting, pointing out that the whole distribution of life, rather than only the expected or the most likely life, should be considered in the calculation of NPV. He also shows how an asset's life distribution can be estimated when beta distribution is assumed. Young and Contreras [Y&C, 1975] and Rosenthal [1978] demonstrate the mathematical processes in the calculation of the expected value and variance of NPV when project life follows certain distributions.

In the above studies, no specific analysis is conducted to determine the potential bias in capital budgeting when stochastic life distribution is ignored. This note shows how capital budgeting decision is biased in this situation. Sensitivity analysis is then performed to see how the bias is influenced by such factors as the discount rate, cash flow pattern, and income taxes. It is shown that the bias is usually in the order of 1 or 2% but can become substantial in some circumstances.

The Bias in NPV when Expected Life is Used Instead of Life Distribution

Y&C directed their attention principally at the expected value of a single payment with stochastic life n. They did demonstrate how the NPV of such a single payment can be adjusted to the NPV of a uniform series of payments for the same period. Because of its emphasis on capital budgeting, this note will focus attention on the overestimation bias in the NPV of a series of payments when expected life is used instead of life distribution. It should be noted, though, that the bias of a series of payments is the opposite of that for a single payment.

We assume that cash flows are uniform in each period. Also assumed is the nonexistence of salvage value.[1] Following Y&C and Rosenthal's notations, the expected PV is:

$$E[P(N)] = \frac{A}{r}\,[1-E(e^{-rN})], \tag{1}$$

where $E[P(N)]$ is the expected value of PV (denoted by P) which depends on the duration (N) of the expected cash flows annuity (A); and r is the nominal continuous-compound interest rate. When $f(N)$, the probability density function of N is known, (1) can be computed by way of the Laplace transformation, i.e.,

$$E(e^{-rN}) = L_N(r), \tag{2}$$

where L_N is the Laplace transformation of $f(N)$.

When the distribution of N is ignored and expected life is used, the expected PV is:

$$E[P(m)] = \frac{A}{r}\,[1-e^{-rm}], \tag{3}$$

where $m = E(N)$.

The bias arises due to the fact that, in general,

$$E[e^{-rN}] \neq e^{-rm}.$$

Subtraction of (3) from (1) gives the measure of bias:

$$E(B) = \frac{A}{r}\,[e^{-rm} - E(e^{-rN})]. \tag{4}$$

When f(N) is unknown, by the Taylor expansion [Rosenthal, p. 164], equation (2) can be stated approximately as:

$$L_N(r) = E(e^{-rN}) \cong (1 + \frac{1}{2}\,v^2r^2 - v^3r^3k/6)e^{-rm}, \tag{5}$$

where v^2 is the standard deviation of N, k is the coefficient of skewness, $k = E[(N-m)^3]/v^3$.

Substituting (5) into (4)

$$E(B) \cong -\,Ae^{-rm}\,(\frac{1}{2}\,v^2r - \frac{1}{6}\,kv^3r^2). \tag{6}$$

A negative value for (6) indicates that the use of expected life, m, overestimates NPV.

It can be easily seen from (5) and (6) that the bias is the function of the first three moments of f(N). It decreases when the mean life (m) increases, and increases when the dispersion (v^2) increases. Moreover, the bias is made larger when f(N) has negative skewness ($k < 0$), while positive skewness serves to alleviate the bias.

Except for the case when the distribution of N is extremely positively skewed such that the parenthesis term in (6) becomes positive, the bias would in general be negative. The bias of symmetric life distributions is always negative. That is, the use of deterministic N results in overestimation of NPV when N is in fact stochastic. The extent of overestimation increases as the dispersion of N increases or as N becomes more skewed to the left.

The intuitive reason for the overestimation is as follows. When a project is terminated before the mean life, a "loss" incurs from the early termination. Conversely, a "gain" is earned when the project lasts longer than the mean life. In the symmetric life distribution, the "loss" is greater than the "gain" since it is discounted at a lower factor. Thus the PV is reduced when the stochastic nature of life is considered. When life distribution is positively skewed, the longer tail on the right hand side of the distribution would offset part of the overestimation. On the other hand, the overestimation would be exacerbated when the project life distribution is negatively skewed.

The bias measured by (6) is only an approximation. When the distribution of N is known, the exact measure of overestimation can be determined. Table 1 presents the notational and numerical illustrations of the bias under some typical distributions of N.[2]

Sensitivity to the Discount Rate and Cash Flow Patterns

The impact of discount rate (r) on the bias is not unequivocal. The first derivative of equation (6) with respect to r is (for simplicitly, assume symmetric distribution of n and omit the skewness term[3]):

$$\frac{\partial B}{\partial r} = -\frac{1}{2} A e^{-rm} v^2 (1-rm). \tag{7}$$

From (7), $\partial B/\partial r$ is negative as long as $rm < 1$, i.e., when either m or r is not very large. This means, for $rm < 1$ the bias becomes more severe as a higher discount rate is used.

The impact of cash flow pattern is more interesting to look at. So far we have assumed a constant cash flow stream which generates the same annuity (A) for each period. If cash flows are not constant, equation (1) has to be rewritten as:

$$E[P(N)] = E[\int_0^N A(n)e^{-rn}dn], \tag{8}$$

where n is a random variable denoting time period, A(n) is the cash flow for each n.

If A(n) increases or decreases exponentially starting from initial cash flow A, i.e.,

$$A(N) = Ae^{an},$$

then (8) becomes

$$E[P(N)] = E[\frac{A}{r-a}(1-e^{-(r-a)N})]. \tag{8a}$$

With the new E[P(N)], (4) becomes

$$E(B) = \frac{A}{r-a}[e^{-(r-a)m} - E(e^{-(r-a)N})]. \tag{9}$$

Equation (9) can be computed by the Laplace transformation as for (2) if f(N) is known. When f(N) is unknown, (9) can be approximated by the Taylor expansion and the bias measured by (6) becomes:

$$E(B) = -Ae^{-(r-a)m}[\frac{1}{2}v^2(r-a) - \frac{1}{6}v^3(r-a)^2k] \tag{10}$$

Comparing (10) and (6), it is seen that the exponential trend of cash flows has the same effect on the bias as do changes in the discount rate. In the symmetric case, when rm < 1, the decreasing trend (a < 0) of cash flows would increase the bias. When the life distribution is negatively skewed, the bias is even larger (see fn. 3), since it is created by two factors which distort the NPV in the same direction.

It can be shown that the above observations can apply to some simple cash flow patterns.[4] In general, the bias is more severe when cash flows are decreasing over time. The decreasing trend, together with the negative skewness of the life distribution, can create substantial bias. Managers need to be especially cautious in making capital budgeting decisions under those circumstances.

Income Taxes

Tax considerations have an impact on the bias discussed above. The impact occurs because the PV of the tax shield might differ for different project lives. In the discussion that follows, it is assumed that the book value of assets is written off (without salvage value) in the year when the project terminates.

Since income taxes are paid periodically, the tax payment and tax shield should be computed on a discrete basis but for mathematical convenience, the after-tax cash flows will be treated as continuous. Assuming constant cash flows, the after-tax version of (1) is:

$$E[P(N)] = (1-t)A\,\frac{1-E(e^{-rN})}{r} + TS(1) \qquad (11)$$

where P(N) is the PV before deducting the cost of asset, t is the constant income tax rate, TS(1) is the tax shield derived from the depreciation charge in the stochastic case.

The analogue to (3) with tax is:

$$E[P(m)] = (1-t)A \frac{1-e^{-rm}}{r} + TS(2). \tag{12}$$

where TS(2) is the depreciation tax shield when deterministic life is used.

The Economic Recovery Tax Act (ERTA) of 1981 stipulates that most newly purchased assets can be depreciated over three or five years. Consider the asset which meets the five-year depreciation requirements. When the mean life and the least estimated life both exceed five years, TS(1) would be equal to TS(2), and the difference between (10) and (11) is the same as (4) except that the bias is proportionally reduced by the tax rate.

When five-year depreciation is used, it can be assumed that the mean life is greater or equal to five years; otherwise, a shorter depreciation period would have been used. When the mean life is less than or equal to five years, it is possible that the life distribution might cover a period of less than five years. In this case, TS(1) would not be the same as TS(2). The difference between (10) and (11) becomes:

$$B' = (1-t)B + [TS(1) - TS(2)], \tag{13}$$

where B is the before-tax bias measured by (6). Recall that B usually takes a negative value. On the other hand [TS(1) - TS(2)] is always positive since additional tax shield is earned when the asset is written off before the five-year depreciation period. Therefore, the tax shield difference offsets part of the bias created by stochastic life.

For the asset whose estimated physical life is less than five years, the estimated life will be the depreciation life. The impact on the bias is the same as above since [TS(1) - TS(2)] will always be positive. There is a "gain" of tax shield for the early write-off of asset, while no "loss" of tax shield is incurred if the asset lasts longer than the depreciation period.

Conclusion

This note shows that the use of expected project life to calculate its NPV usually results in overestimation of NPV. When the exact life distribution is unknown, the bias can be approximated by Taylor series expansion. If F(N) is known, the exact value of the bias can be calculated. The sensitivity of the bias to the discount rate, to cash flow patterns and to income taxes, is discussed. The magnitude of the bias might appear to be small (1% to 2%) but for certain combinations of high discount rates, large variance or negative skewness of life distribution could be larger.

Notes

1. The salvage value can be easily incorporated into the analysis. It can be treated as a lump-sum payment with uncertain timing and magnitude. Y&C provides a detailed discussion for handling this problem.

2. The distributions on Table 1 are those exemplified by Y&C and Rosenthal.

3. When skewness is considered, the derivative in (6) becomes:

$$\frac{\partial B}{\partial r} = -\frac{1}{6} Ae^{-rm}v^2[e(1-rm) - vrk(2-rm)].$$

The sign of $\partial B/\partial r$ would depend on k and the magnitude of rm. When k is negative, the sign determined in the symmetric case still holds, but the impact of r on B is even greater. When k is positive, the sign of $\partial B/\partial r$ becomes a complicated problem. The relative magnitude of each variable has to be compared.

4. For example, consider the case when cash flows follow a linear trend, i.e.,

$$A(n) = A(1+an).$$

By the Taylor expansion (assuming symmetric life distribution), the bias in (9) becomes:

$$B'' \cong B + \frac{a}{r} B(rm-1).$$

Therefore, when $rm < 1$, $|B''| > |B|$ if $a < 0$. The decreasing linear trend increases the bias. This result is consistent with that derived from (10) in which exponential trend is assumed.

References

Brown, K. C., "Uncertainty and the Model Builder," Interfaces (June 1981), pp. 55-58.

Greer, W. R., Jr., "Capital Budgeting Analysis with the Timing of Events Uncertain," Accounting Review (January 1970), pp. 103-14.

Ijiri, Y and R. S. Kaplan, "Probabilistic Depreciation and Its Implication for Group Depreciation," Accounting Review (October 1969), pp. 743-54.

Jen, F. C. and R. J. Huefner, "Depreciation by Probability Life," Accounting Review (April 1970), pp. 290-98.

Rosenthal, R. E., "The Variance of Present Worth of Cash Flows Under Uncertain Timing," Engineering Economist, Vol. 23 (1978), pp. 163-70.

Terborgh, George, Effects of Anticipated Reflection on Investment Analysis, July 1960, MAPI.

Young, D. and L. E. Contreras, "The Expected Present Worths of Cash Flows Under Uncertain Timing," Engineering Economist, Vol. 20 (1975), pp. 257-68.

Table 1 Bias of NPV in Each Life Distribution

Life Distribution (f(N))	Bias (eq. (4))	Numerical Example of Bias[1]	Perc. Δ
1. Exponential	$\frac{A}{r}[e^{-rm} - (1 - \frac{r}{r+1/m})]$	-.60	15.2
2. Uniform[2] ($a \leq N \leq b$)	$\frac{A}{r}[e^{-rm} - \frac{1}{r(b-a)}(e^{-ra} - e^{-rb})]$	-.061	1.6
3. Normal[3]	$\frac{A}{r} e^{-rm}(1 - e^{1/2\, r^2 v^2})$	-.061	1.6
4. Gamma[3]	$\frac{A}{r}[e^{-rm} - \frac{1}{(1+rv^2/m)^{m^2/v^2}}]$	-.059	1.5

Notes:

1. Assume A = 1, r = .10, m = 5. The bias can be compared to the PV in the case of deterministic life, which is 3.935.

2. a = 2.55, b = 7.45 (making m = 5, v^2 = 2).

3. v^2 = 2.