BEBR

FACULTY WORKING
PAPER NO. 1045

A Modern Analysis of the Effects
of Site Value Taxation

Jan K. Brueckner

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

A Modern Analysis of the Effects
of Site Value Taxation

Jan K. Brueckner, Associate Professor
Department of Economics

Abstract

    Formal analysis is generally absent from the previous literature

on the effects of site value taxation. This paper analyzes the

impacts of such a system (under which the property tax on improvements

is eliminated, with the tax burden shifted toward land) using standard

modern methods. Specifically, the analysis derives the long-run

impacts on the level of improvements, the value of land, and the price

of housing of a shift to a graded tax system (where the improvements

tax rate is lowered and the land tax rate is raised). The paper also

analyzes the incidence of the short-run windfall gains and losses which

result from gradation of the tax system.

The Effects of Site Value Taxation:
A Modern Analysis

by

Jan K. Brueckner

1. Introduction

Ever since the publication of Henry George's "Progress and
Poverty" in 1879, the possibility of using land value taxation as a
source of government revenue has intrigued economists and other social
commentators. While George's ideas have had little general impact,
land value taxation is practiced in Jamaica and in certain cities in
Australia and New Zealand, and graded property tax systems (where land
is taxed at a higher rate than improvements) are in use in some
Canadian provinces as well as in the city of Pittsburgh.[1]

The literature dealing with land (or site) value taxation is vast
(for an excellent bibliography, see Carmean (1980)). Most writers have
been concerned with predicting the effects of a shift from a typical
property tax system, where land and improvements are taxed at the same
effective rate, to a system of pure site value taxation, where the
improvements tax is eliminated and land is taxed at a higher rate (tax
revenue is held constant). Others deal with the effects of transition
to a graded system (where the improvements tax rate is lowered but
remains positive), recognizing that pure site value taxation is simply
an extreme case of gradation. Consensus has emerged on a number of
points. First, nearly all writers agree that reduction or elimination
of the improvements tax will raise the level of improvements in the
long run, leading to more intensive land-use. Second, there is

agreement that in the short run, windfall gains and losses will result from a movement to a graded system as tax bills rise for certain properties and fall for others. The list of studies which attempt to quantify such impacts includes Schaaf (1970), Smith (1970), Neuner et al. (1974), Lusht (1975), Killoren and Casey (1981), and Stoddard and Fry (undated). Third, the problem of generating accurate land value assessments in the absence of frequent transactions in vacant land is recognized by most writers (a practicing assessor (Back (1970)) claims that accuracy can be achieved). There is considerable interest (but less agreement) regarding the impact of site value taxation on speculation in vacant land, with some commentators arguing that specu- lation will be curbed by higher land holding costs while others predict no effect (see Brown (1927) for an early contribution). The best general discussions of these and other points are provided by Becker (1969), Harriss (1970), and Peterson (1978).

What is remarkable about this large literature is the almost complete absence of modern analysis. Most studies rely on verbal arguments or simple diagrams, and the few analytical efforts (McCalmont (1976) and Cuddington (1978)) are marred by ad hoc assump- tions or misplaced emphasis. While several correct predictions regarding the effect of site value taxation have been derived without the aid of rigorous methods (the predicted increase in land-use inten- sity, for example), the lack of precision of past studies has led to substantial confusion on certain points. A prime example is the question of land value impacts. As is shown below, a lower improve- ments tax raises land value while a higher land tax lowers it. The

net effect is ambiguous but depends in a straightforward way on the
magnitudes of various parameters. While the analysis underlying this
result is quite intuitive, only two authors (Becker (1969) and Harriss
(1970)) correctly grasp the principle involved.[2] Indeed, in his
recent contribution, McCalmont (1976) goes so far as to say that "not
even the direction, let alone the amount, of the change in land rent
can be ascertained from theory alone..." (p. 928). This claim is
false, as the ensuing analysis will demonstrate. Another important
question on which the literature is virtually silent is the impact of
site value taxation on housing prices. Modern analysis gives an
immediate answer, as will be seen below.

The remainder of this paper will elaborate on the above points by
conducting an analysis of the effects of site value taxation using
standard modern methods. Section 2 investigates the long run impacts
of a revenue-preserving shift from a standard property tax system to a
graded system under two different scenarios. In the first case, the
graded tax system is imposed in only a small part of a single city, so
that the price of housing is unaffected. In the second case, imple-
mentation occurs city-wide, so that price effects emerge. In both
cases, the analysis derives the impacts of gradation on the level of
improvements and the value of land. The impact on the price of
housing is also derived for the second case.

While Section 2 assumes that the price of housing is spatially
uniform, Section 3 allows spatial variation. In this setting, improve-
ments and land value vary with location, and short-run windfall gains
and losses result from a switch to a graded tax system. The analysis

is devoted to investigating the spatial pattern of gains and losses under the assumption that the housing price contour is exogenous. Section 4 of the paper briefly presents some empirical evidence relevant to the predictions of the analysis, and Section 5 offers conclusions.

2. Long-run effects

2.1 The case of an exogenous price of housing

In reality, property taxes are levied on a wide variety of types of structures: residential, commercial, and industrial. Typically, the interior space in one type of structure is unsuitable for any other use. In the following analysis, this fact is ignored and the property tax base is assumed to consist of a homogeneous class of structures called "housing." In the model, housing floor space is rented at price p per square foot and is produced using inputs of capital (N) and land (ℓ) under a neoclassical constant returns technology represented by the production function $H(N,\ell)$. Since output is indeterminate under constant returns, the analysis focuses on levels of output and capital input on a per-acre-of-land basis. Housing output per acre is $H(N,\ell)/\ell = H(N/\ell,1) \equiv h(S)$, where S is capital per acre of land (hereafter improvements per acre), a measure of land-use intensity, and $h(S) \equiv H(S,1)$. Note that $h' = H_1 > 0$ and $h" = H_{11} < 0$ by the concavity of H.

The net-of-tax rental prices of capital and land are represented by i and r respectively, and the tax rates on improvements (capital) and land are τ and θ respectively. The gross-of-tax capital and land prices are therefore $(1+\tau)i$ and $(1+\theta)r$ respectively. Note that since

taxes are expressed as a fraction of net rental price instead of
value, conversion to value terms would require multiplication of the
tax rates by the discount rate. Note also that $\tau = \theta$ will hold under
a standard property tax system.

The shift to a graded property tax system is assumed to occur over
a land area of size $\bar{l}$ (referred to subsequently as the "tax zone").
Locational advantages are absent within the tax zone, so that the
housing price p is spatially uniform. Furthermore, in the initial
analysis, the tax zone is viewed as representing a small portion of the
relevant housing market. This means that the change in the tax system
will have a negligible effect on the total supply of housing in the
market, with the result that the price p can be viewed as exogenous.
Finally, given that the analysis deals with the effects of a localized
rather economy-wide tax change, the net return to capital is also taken
to be exogenous (the locality faces a perfectly elastic supply of
capital).

Profit per acre for a housing producer operating in the tax zone
is given by $ph(S) - (1+\tau)iS - (1+\theta)r$. The equilibrium conditions for
the producer require that profit per acre is maximal and that the
maximized value equals zero. The appropriate conditions are

$$ph'(S) = (1+\tau)i \qquad\qquad (1)$$

$$ph(S) - (1+\tau)iS - (1+\theta)r = 0. \qquad\qquad (2)$$

Together, eqs. (1) and (2) determine equilibrium values of improve-
ments per acre S and net land rent r. The impacts on S and r of
changes in the tax rates τ and θ, which are used to derive the effects

of a shift to a graded tax system, are computed by totally differentiating the system (1)-(2). The results are

$$\frac{\partial S}{\partial \tau} = \frac{i}{ph''} < 0 \qquad (3)$$

$$\frac{\partial S}{\partial \theta} = 0 \qquad (4)$$

$$\frac{\partial r}{\partial \tau} = \frac{-iS}{1+\theta} < 0 \qquad (5)$$

$$\frac{\partial r}{\partial \theta} = \frac{-r}{1+\theta} < 0 \qquad (6)$$

By increasing the cost of capital, an increase in the improvements tax rate τ naturally reduces improvements per acre, as seen in (3). In addition, by reducing the profitability of development, a higher improvements tax depresses land rent, as seen in (5) (rent serves to exhaust residual profit). Similarly, (6) shows that a higher land tax rate depresses land rent. In fact, the higher tax is fully capitalized, so that gross-of-tax land cost remains unchanged ((6) yields $\frac{\partial}{\partial \theta} (1+\theta)r = 0$). Finally, (4) indicates that a higher land tax rate has no effect on the level of improvements.

The goal of the analysis is to derive the impacts on S and r of a revenue-preserving shift to a graded tax system. Starting with a standard tax system (where $\tau = \theta$), gradation results from an increase in θ combined with a revenue-preserving change in τ (pure site value taxation emerges when $\tau = 0$). The first step in the derivation is the computation of the derivative $\partial \tau / \partial \theta$, which gives the revenue-preserving change in τ accompanying an increase in θ. Noting that tax

revenue originating from the tax zone equals $\bar{\ell}(\tau iS+\theta r)$, $\partial\tau/\partial\theta$ must satisfy $d(\tau iS+\theta r)/d\theta = 0$, or

$$\frac{\partial\tau}{\partial\theta}\, iS + \tau i\left(\frac{\partial S}{\partial\theta} + \frac{\partial S}{\partial\tau}\frac{\partial\tau}{\partial\theta}\right) + r + \theta\left(\frac{\partial r}{\partial\theta} + \frac{\partial r}{\partial\tau}\frac{\partial\tau}{\partial\theta}\right) = 0 \tag{7}$$

Substituting from (3)-(6) and rearranging, (7) yields

$$\frac{\partial\tau}{\partial\theta} = \frac{-r}{iS}\left\{1 + \frac{(1+\theta)\tau i}{ph''S}\right\}^{-1}. \tag{8}$$

Eliminating h" using the relationship

$$\sigma = \frac{-h'(h-Sh')}{Shh''}, \tag{9}$$

where σ is the elasticity of substitution between capital and land in housing production,[3] and noting that land's factor share μ_ℓ (which equals $(1+\theta)r\ell/pH$) may be written $(h-Sh')/h$, (8) reduces to

$$\frac{\partial\tau}{\partial\theta} = \frac{-r}{iS}\left\{1 - \frac{(1+\theta)\tau\sigma}{(1+\tau)\mu_\ell}\right\}^{-1}. \tag{10}$$

Inspection of (11) shows that the sign of $\partial\tau/\partial\theta$ is ambiguous, so that a revenue-preserving change in τ may involve either a decrease or an increase. The outcome depends crucially on the magnitude of the elasticity of substitution, which, for given values of τ, θ, and μ_ℓ, determines the sign of the expression in braces in (10). Inspection of (10) indicates that for σ sufficiently close to zero, $\partial\tau/\partial\theta$ will be negative, while for σ sufficiently large, $\partial\tau/\partial\theta$ will be positive. To gain an intuitive understanding of this result, the first step is to note that (10) equals minus the ratio of the derivative of tax revenue with respect to θ ($r+\theta\partial r/\partial\theta$) and the derivative of revenue with respect to τ ($iS + \tau i\partial S/\partial\tau + \theta\partial r/\partial\tau$). Since the first derivative is always positive (revenue is always increasing in θ),[4] the sign of $\partial\tau/\partial\theta$

depends on the sign of the latter derivative, which depends in turn on
two separate effects. First, since $\partial r/\partial \tau < 0$ by (5), an increase in τ
indirectly depresses revenue from the land tax, making the last term in
the derivative negative. The effect of a higher τ on improvements tax
revenue (captured by $iS + \tau i\partial S/\partial \tau$) is ambiguous, however, and depends
on the magnitude of σ. A low (high) value of σ means that improvements
tax revenue is increasing (decreasing) with τ due to weak (strong)
substitution away from capital as τ rises.[5] When σ is large, a higher
τ therefore depresses revenue from both the improvements and land
taxes, cancelling the revenue gain from a higher θ. In this case,
$\partial \tau/\partial \theta > 0$ must hold for revenue to remain constant. When σ is suf-
ficiently small, however, the increase in improvements tax revenue
resulting from a higher τ dominates the decline in land tax revenue,
and total revenue rises with τ. In this case, τ must fall as θ rises
to keep total revenue constant.[6]

Whether $\partial \tau/\partial \theta$ is negative or positive for plausible values of σ
depends on the magnitudes of the other parameters in (10). To make
matters simple, suppose first that $\partial \tau/\partial \theta$ is evaluated under a standard
property tax system, so that $\tau = \theta$ holds. In this case, the sign of
(10) is the same as the sign of $\sigma - (\mu_\ell/\tau)$. Focusing first on land's
share, published (or implied) estimates of μ_ℓ range from 20% to 50%,
with most values lying in the middle or lower end of this range.[7] In
addition, data compiled by the Advisory Council on Intergovernmental
Relations (1983, Table 37) show that the average effective property
tax rate in the U.S. in 1981 for single family homes with FHA insured
mortgages was 1.26%. With a value-to-rent ratio between 10 and 20 (a

discount rate between 5% and 10%), this yields a τ between 12% and 26% (recall that τ is the tax rate on net rent, not value). Together, these μ_ℓ and τ values imply that μ_ℓ/τ lies between .8 and 4.2, with a plausible value falling near the middle of this range. Since published estimates of the elasticity of substitution in housing production are almost always smaller than unity (see McDonald (1981) for a survey), it follows that $\sigma - (\mu_\ell/\tau)$ is almost certainly negative, implying $\partial\tau/\partial\theta < 0$. This implication is welcome given that a positive value for $\partial\tau/\partial\theta$ would be quite unnatural.

While the initial shift away from a standard property tax will typically require a decline in τ, the behavior of τ as the rate falls below θ is harder to determine. If it could be shown that $d[(1+\theta)\tau/(1+\tau)\mu_\ell]/d\theta < 0$ holds, then (assuming a constant σ) the expression in braces in (10) would remain positive as θ rises away from τ, and $\partial\tau/\partial\theta < 0$ would hold globally. Unfortunately, the above derivative is ambiguous in sign, so that the initial negativity of $\partial\tau/\partial\theta$ does not guarantee global negativity. Note, however, that for τ sufficiently close to zero, $\partial\tau/\partial\theta$ must be negative (see (10)). As a result, it is safe to say that $\partial\tau/\partial\theta$ will be negative at the beginning (τ near θ) and at the end (τ near zero) of a transition to pure site value taxation. Intermediate cases may show $\partial\tau/\partial\theta > 0$, although, as mentioned earlier, this outcome would be most unnatural.

Having computed $\partial\tau/\partial\theta$, it is now possible to derive the impacts on S and r of a shift to a graded property tax system. Since $\partial S/\partial\theta = 0$ by (4), the impact on S is simply

$$\frac{dS}{d\theta} = \frac{\partial S}{\partial \tau} \frac{\partial \tau}{\partial \theta} \, . \tag{11}$$

Given that $\partial S/\partial \tau < 0$ by (3), $dS/d\theta$ will be positive in the normal case

where $\partial \tau/\partial \theta$ is negative. This is the outcome recognized in the

earlier literature: a shift in the property tax burden toward land

and away from improvements will raise the level of improvements.[8]

While earlier writers were correct on this point, they typically

failed (as mentioned above) to appraise correctly a graded tax

system's effect on land value. The present analysis gives an imme-

diate answer since it follows (using (5), (6), and (10)) that

$$\frac{dr}{d\theta} = \frac{\partial r}{\partial \theta} + \frac{\partial r}{\partial \tau} \frac{\partial \tau}{\partial \theta}$$

$$= \frac{\Lambda r}{(1+\theta)(1-\Lambda)} \, , \tag{12}$$

where $\Lambda \equiv (1+\theta)\tau\sigma/(1+\tau)\mu_\ell$ is the expression inside the braces in (10).

Since $\partial \tau/\partial \theta \lessgtr 0$ as $1 - \Lambda \gtrless 0$, (12) implies that

$$\frac{dr}{d\theta} \gtrless 0 \quad \text{as} \quad \frac{\partial \tau}{\partial \theta} \lessgtr 0. \tag{13}$$

In other words, land value (which is proportional to r) rises (falls)

with θ when the revenue-preserving change in τ is negative (positive).

In the intuitively sensible case where $\partial \tau/\partial \theta$ is everywhere negative,

this result implies that land value rises continuously as θ increases,

reaching a maximum in the case of pure site value taxation.

Some understanding of these results can be gained by inspection of

the first line of (12). Since both $\partial r/\partial \theta$ and $\partial r/\partial \tau$ are negative, it

is clear that if a higher θ requires a higher τ to maintain tax reve-

nue ($\partial \tau/\partial \theta > 0$), then land value declines unambiguously, as indicated

by the lower inequalities in (13). When $\partial\tau/\partial\theta < 0$ holds, however, a lower improvements tax raises land value at the same time that the higher land tax depresses it. While the net effect might appear to be ambiguous (and was identified as such by those earlier writers astute enough to recognize the trade-off involved), the mathematics unambiguously indicate that the impact of the lower improvements tax wins out and land value rises. Consideration of the case where $\sigma = 0$ and capital-land substitution does not occur makes this outcome somewhat clearer. First, note from (10) that $\partial\tau/\partial\theta$ is closer to zero when $\sigma = 0$ than when $\sigma > 0$ ($\partial\tau/\partial\theta < 0$ is assumed). The reason is that when $\sigma > 0$, S increases in response to a lower τ, which means that the revenue-preserving decline in τ must be greater than in the case where $\sigma = 0$ and S is fixed. Next, note from (12) that when $\sigma = 0$, $dr/d\theta = 0$ holds. Since a revenue-preserving tax change thus leaves r unchanged when $\sigma = 0$, and since constant revenue requires a faster decline in τ when $\sigma > 0$, the value-increasing effect of a lower τ must dominate in the latter case, leading to an increase in r. This surprising conclusion, which establishes that transfer of the tax burden from improvements to land actually <u>raises</u> land value, shows the usefulness of modern analysis applied to the present problem.

Summing up, the above analysis has shown that for representative parameter values, an initial shift away from a standard property tax system toward a graded system will lead to an increase in both improvements per acre and land value. Provided that $\partial\tau/\partial\theta < 0$ continues to hold as θ increases, the initial impacts are simply magnified as the tax burden on land increases, with improvements per acre and land

value eventually reaching a maximum under a system of pure site value taxation.

2.2 The case of an endogenous price of housing

In this section, the tax zone is assumed to encompass the entire housing market. The assumption that locational advantages are absent is maintained, however.[9] In this case, a change in the property tax system will have an impact on the price of housing, and a market-clearing equation must be added to the previous equilibrium system (1)-(2). Letting $D(p)$ denote the aggregate demand function for housing,[10] which satisfies $D' \leq 0$, the expanded equilibrium system consists of the earlier equations together with

$$\overline{\ell}h(S) = D(p). \tag{14}$$

The separate impacts of τ and θ on p, S, and r are derived by totally differentiating (1), (2), and (14). The results are given by (4), (6), and

$$\frac{\partial p}{\partial \tau} = \frac{(\overline{\ell}h'/D')i}{ph'' + (\overline{\ell}h'^2/D')} > 0 \tag{15}$$

$$\frac{\partial p}{\partial \theta} = 0 \tag{16}$$

$$\frac{\partial S}{\partial \tau} = \frac{i}{ph'' + (\overline{\ell}h'^2/D')} < 0 \tag{17}$$

$$\frac{\partial r}{\partial \tau} = \frac{h\partial p/\partial \tau - iS}{1+\theta} \gtrless 0 \tag{18}$$

As before, a higher land tax is fully capitalized (leaving $(1+\theta)r$ unchanged) and has no effect on S. As a result, there is no impact on

p, as shown in (16). A higher improvements tax once again lowers the
level of improvements (see (17)), but its effect on land rent is ambig-
uous, as shown in (18). The latter result is due to the fact that the
improvements tax is shifted forward, raising the price of housing (see
(15)). Since the higher p tends to increase the profitability of
development at the same time that the higher τ reduces it, the net
impact on r is indeterminate. This outcome makes $\partial r/\partial \tau$ numerically
larger than before (compare (18) and (5)), and comparison of (17) and (3)
shows that $\partial S/\partial \tau$ is also larger (less negative) in the present case.
This follows from the stimulating effect of a higher p, which moder-
ates the decline in S resulting from the higher improvements tax.

Substituting the above results in (7) to compute $\partial \tau/\partial \theta$ yields

$$\frac{\partial \tau}{\partial \theta} = \frac{-r}{iS} \left\{ 1 + \frac{(1+\theta)\tau i + (\theta \overline{\ell} h h'/D')}{S[ph'' + (\overline{\ell} h'^2/D')]} \right\}^{-1} . \tag{19}$$

To simplify (19), let $\eta \equiv (p/h)(\partial h/\partial p) = -h'^2/hh'' > 0$ represent the
elasticity of housing supply per acre of land.[11] Also, let
$\varepsilon \equiv pD'/D = pD'/\overline{\ell} h \leq 0$ represent the elasticity of housing demand.
Substituting η and ε together with σ and μ_ℓ into (19), the equation
reduces to

$$\frac{\partial \tau}{\partial \theta} = \frac{-r}{iS} \left\{ 1 - \frac{\sigma[(1+\theta)\tau \varepsilon + \theta(1+\tau)]}{(1+\tau)\mu_\ell(\varepsilon - \eta)} \right\}^{-1} . \tag{20}$$

To see that the earlier solution for $\partial \tau/\partial \theta$ is just a special case of
(20), note that (20) reduces to (10) when $\varepsilon \rightarrow -\infty$ (when p is
exogenous). While the sign of $\partial \tau/\partial \theta$ is again ambiguous, it is easy to
see that (20) is negative whenever the corresponding derivative in

(10) is negative. To establish this fact, note first that the expression in braces in (20) is greater than the corresponding expression in (10).[12] When the latter expression is positive (implying $\partial\tau/\partial\theta < 0$ in the p-exogenous case), it follows that the expression in (20) is a greater positive quantity, implying that $\partial\tau/\partial\theta$ from (20) is negative but less in absolute value than (10). Therefore, when $\partial\tau/\partial\theta$ is negative in the p-exogenous case, the p-endogenous derivative is also negative but closer to zero. The reason for this result is that the expressions in (10) and (20) are proportional to the derivative of tax revenue with respect to τ. Since $\partial S/\partial\tau$ and $\partial r/\partial\tau$ are both larger in the p-endogenous case, tax revenue responds more vigorously to a change in τ. In the case where revenue increases with τ in the p-exogenous case, it increases at a faster rate when p is endogenous, meaning that a smaller decrease in τ is needed to keep revenue constant as θ rises.[13]

Computation of the impacts of a switch to a graded tax system proceeds as before. First, since $\partial p/\partial\theta = 0$, it follows that

$$\frac{dp}{d\theta} = \frac{\partial p}{\partial\tau} \frac{\partial\tau}{\partial\theta} , \tag{21}$$

which (noting (15)) is negative as long as $\partial\tau/\partial\theta < 0$. Since a higher θ has no impact on p while a lower τ reduces it, the total effect of gradation is to reduce the price of housing. In addition, since $\partial S/\partial\theta$ again equals zero, $dS/d\theta$ is again given by (11). Therefore, the level of improvements again increases with θ provided that $\partial\tau/\partial\theta$ is negative. Upon substituting (6), (18), and (20) into the first line of (12) to compute $dr/d\theta$, simplification yields the result

$$\frac{dr}{d\theta} = \frac{\sigma[1+\tau(1+\epsilon)]}{(1+\tau)\mu_{\ell}(\epsilon-\eta)} \frac{r}{1 - \Lambda'} , \tag{22}$$

where $1 - \Lambda'$ is the expression in braces in (20). When $1 - \Lambda' > 0$, so that $\partial\tau/\partial\theta < 0$, the sign of (22) is the same as the sign of $-[1+\tau(1+\epsilon)]$, which depends in turn on the magnitude of ϵ. If housing demand is inelastic, so that $-1 \leq \epsilon \leq 0$, then $1 + \tau(1+\epsilon) > 0$ and $dr/d\theta < 0$. When demand is elastic, $1 + \tau(1+\epsilon)$ can have either sign, with $1 + \tau(1+\epsilon) \gtrless 0$ holding as $\epsilon \gtrless -(1+(1/\tau))$. Substituting a representative τ value of .20 to appraise the effect of an initial shift toward a graded tax system, it follows that $dr/d\theta \gtrless 0$ as $\epsilon \lessgtr -6$. Since most estimates indicate that housing demand is price inelastic (see Mayo (1981) for a survey), $dr/d\theta < 0$ seems the likely outcome. Note that as τ falls toward zero, the case against $dr/d\theta > 0$ is even stronger (demand would have to be even more elastic in order for the inequality to hold). As a result, it appears that $dr/d\theta$ is globally negative.[14]

This result is clearly the opposite of the outcome in the p-exogenous case (where land value rises), and the explanation lies in the behavior of the housing price. Since p falls with θ in the present situation, a new force which reduces the profitability of development (and hence the value of land) enters the analysis. Since it may be shown that the price of housing falls faster with θ the less elastic is the demand for housing (the absolute value of $dp/d\theta$ is larger when ϵ is closer to zero), the specific results above become clearer. Inelastic or moderately elastic demand leads to a sharp decline in p and a correspondingly large depressing effect on r. This effect, which was not present before, is sufficient to reverse the previous outcome and lead to a decline in land value. When demand is

highly elastic, the decline in p is moderate and the depressing effect on r is not sufficiently strong to reverse the earlier positive impact, so that land value rises.

Summing up, the preceding analysis has shown that when $\partial\tau/\partial\theta$ is negative, an increase in θ depresses the price of housing, raises the level of improvements, and, as long as housing demand is not highly elastic, depresses the value of land. The maximum impacts will be observed when a system of pure site value taxation is reached. While the sign of $\partial\tau/\partial\theta$ is generally ambiguous, so that the above conclusions cannot be viewed as automatic, $\partial\tau/\partial\theta < 0$ seems natural. In any case, since it was shown that $\partial\tau/\partial\theta$ evaluated at $\tau = \theta$ is almost certainly negative in the p-exogenous case, and since negativity implies that $\partial\tau/\partial\theta < 0$ also holds for $\tau = \theta$ when p is endogenous (see above), we can conclude with a high degree of confidence that starting with a standard property tax system, the initial shift toward a graded system will have the impacts listed above.

3. An analysis of temporary gains and losses

In long run equilibrium, housing producers are indifferent to the features of the property tax system since profit is identically zero. Before full market adjustment occurs, however, producers can experience windfall gains or losses from a change in the tax system. While the uniformity of land-use within the tax zone means that such gains and losses will never emerge in the model of Section 2, land-use heterogeneity within the zone (differences in improvements per acre and land values) will generally lead to windfalls. The purpose of this section is to present a stylized analysis of such windfalls using

a model where the price of housing (and thus the levels of S and r)
varies within the tax zone.

Without loss of generality, the tax zone is taken to be a rec-
tangle of unit width, and the price of housing p is assumed to be a
decreasing function of distance x from one end of the rectangle
($\partial p/\partial x < 0$). This could reflect the impact of differential commuting
costs within the tax zone or the influence of an amenity or disamenity
near the zone. The housing price function is taken to be exogenous.
The first step in the analysis is to deduce the spatial behavior of S
and r implied by spatial variation in p. This is done by totally dif-
ferentiating (1) and (2) with respect to x, which yields

$$\frac{\partial S}{\partial x} = \frac{-h'}{ph''} \frac{\partial p}{\partial x} < 0 \tag{23}$$

$$\frac{\partial r}{\partial x} = \frac{h}{1+\theta} \frac{\partial p}{\partial x} < 0, \tag{24}$$

indicating that both improvements per acre and land value decline
with x (these results are familiar from urban spatial analysis). The
levels of S and r prevailing prior to a change in the tax system will
be denoted $\bar{S}$ and $\bar{r}$. While $\bar{S}$ and $\bar{r}$ are implicitly functions of x, the x
argument will be suppressed for simplicity in the analysis.

The analysis investigates the effect of a shift to a graded prop-
erty tax system over both short- and medium-run horizons. The feature
which is common to both the short and medium run is that improvements
per acre at any given location are fixed at the level prevailing prior
to the change in the tax system (S is fixed at $\bar{S}$). This is a reflec-
tion of the longevity of housing capital, which makes an immediate

response to a tax change uneconomical. What distinguishes the short-
and medium-run cases is that in the short run, the assessed value of
land will remain at its previous level, whereas in the medium run, the
effects of reassessment will emerge. In both cases, however, land
cost net of the tax liability will remain fixed at $\bar{r}$, reflecting the
fact that payments on existing mortgages will be unaffected by a
change in the tax system.

Since revenue per acre $(ph(\bar{S}))$ and net-of-tax improvements and
land costs $(i\bar{S}$ and $\bar{r})$ will remain fixed in the short and medium run,
gains and losses will be due entirely to changes in tax liabilities.
To analyze the short-run case, the first step is to note that the tax
payment at a given location equals $\tau i \bar{S} + \theta \bar{r}$ and that total revenue
from the tax zone is given by $\tau i \int_0^b \bar{S} dx + \theta \int_0^b \bar{r} dx$, where b is the length
of the zone (recall that its width is unity). Letting $\bar{S}_A \equiv \int_0^b \bar{S} dx / b$
denote the average level of improvements per acre in the zone, with
$\bar{r}_A$ defined analogously, tax revenue is given by $\bar{\ell}(\tau i \bar{S}_A + \theta \bar{r}_A)$ (note
$b = \bar{\ell}$). Constancy of total tax revenue then requires

$$\frac{\partial \tau}{\partial \theta} = \frac{-\bar{r}_A}{i \bar{S}_A} < 0. \tag{25}$$

To find the short-run change in the tax liability at a particular
location, (25) is substituted into $i\bar{S} \partial \tau / \partial \theta + \bar{r}$, yielding

$$\bar{S}[(\bar{r}/\bar{S}) - (\bar{r}_A/\bar{S}_A)]. \tag{26}$$

Eq. (26) indicates that parcels with high ratios of land value to
improvements face higher taxes as θ rises and τ falls, with taxes
declining for parcels with low $\bar{r}/\bar{S}$ ratios. Note that if land-use is

uniform within the tax zone, so that $\bar{r} \equiv \bar{r}_A$ and $\bar{S} \equiv \bar{S}_A$, then (26) is zero and the tax liability is unchanged at each location.

The spatial pattern of gains and losses is deduced by taking the derivative of $\bar{r}/\bar{S}$ with respect to x. Calculations based on (23) and (24) yield

$$\frac{\partial \bar{r}/\bar{S}}{\partial x} = \frac{(1-\sigma)h}{1+\theta} \frac{\partial p}{\partial x},$$ (27)

indicating that the expression in parentheses in (26) is an increasing (decreasing) function of x as $\sigma \gtrless 1$. In the realistic case where $\sigma < 1$, this implies that (26) is positive (negative) for low (high) values of x ((26) must change sign in order for tax revenue to remain constant). Therefore, windfall losses (higher taxes) accrue to parcels with low x's while windfall gains (lower taxes) accrue to parcels with high x's. A shift to a graded property tax system thus imposes short-run losses on the most intensively developed parcels while bestowing short-run gains on the least intensively developed sites. This result might at first appear counterintuitive since parcels with high improvements per acre stand the most to gain from a lower improvements tax. This observation, however, ignores the fact that such parcels also have high land value, which makes an increase in θ especially burdensome. When $\sigma < 1$, the latter effect dominates and the total tax liability rises.

Although the above analysis applies to a tax zone with a single type of real estate, the conclusions based on (26) apply even in the case of mixed land uses. That is, regardless of what types of property are located in the tax zone, comparison of the $\bar{r}/\bar{S}$ ratio for a

given parcel to the ratio of average values for the zone tells whether taxes rise or fall for that parcel in the short run. Using this principle, the impact analyses cited in the introduction attempt to predict the short-run incidence of a shift to pure site value taxation for various municipalities. Typical findings show that many commercial and industrial properties would face higher taxes, while single family homes would generally benefit from lower tax bills.

As mentioned above, the difference between the short and medium run is that land will be reassessed for tax purposes in the latter case. Reassessment will reflect land's market value, which is based on the profitability of new development. Specifically, value will be given by the r solution to (1)-(2) with S freely variable, even though improvements are currently frozen at $\overline{S}$. With reassessment, the tax liability becomes $\tau i \overline{S} + \theta r$, and total revenue from the tax zone is given by $\tau i \overline{S}_A \overline{\ell} + \theta \int_0^b r dx$. Using (5) and (6) to compute $\partial \tau / \partial \theta$ yields a solution identical to (25), so that $\partial \tau / \partial \theta$ is the same regardless of whether or not land is reassessed. The tax change for a given parcel is now $d(\tau i \overline{S} + \theta r)/d\theta$, which, using (5), (6), and (25), reduces to (26) divided by $(1+\theta)$, implying that the change in the post-reassessment tax liability is proportional to the change in the pre-reassessment liability (absolute post-reassessment changes are somewhat smaller). As a result, the previous conclusions hold for the medium-run case: parcels with high (low) $\overline{r/S}$ ratios face higher (lower) post-reassessment taxes, with the losers (gainers) being located at low (high) x values. It is interesting to note that since

$dr/d\theta$ may be shown to be exactly the negative of the tax change, post-reassessment land values fall where values are high (at low x's) and rise where they are low (at high x's).

Summing up, the analysis in this section has shown that a shift to a graded property tax system will generate temporary gains and losses in a distinct spatial pattern. Provided that $\sigma < 1$, the most intensively developed parcels will suffer windfall losses as their tax liabilities rise while the least intensively developed sites reap windfall gains in the form of lower tax bills. This pattern holds both before and after land is reassessed.[15]

4. Empirical evidence

The ideal empirical test of the predictions of the long-run analysis of Section 2 would rely on a cross-section regression relating the level of one of the endogenous variables (p, S, or r) in various cities to a measure of the extent of gradation of the city's property tax system as well as other explanatory variables. The rarity of graded tax systems, however, means that the data for such an exercise is simply unavailable. A less reliable but nevertheless suggestive test would involve a comparison of the features of two cities whose underlying characteristics are identical except for the presence of a graded property tax system in one and a standard system in the other. Archer (1972) presents what seems to be the only such comparison in the existing literature. The data he offers allow comparison of the level of improvements per acre in the downtown parts of the Australian cities of Sydney (where pure site value taxation has been in use in 1916) and Melbourne (which has a standard property tax system). Archer is careful to

recognize the importance of holding other things equal in making such a comparison, and he notes that the two cities have nearly identical size populations and downtown work forces.[16]   Table 1 reproduces Archer's table (1972, p. 32) listing the distribution of downtown building heights in the two cities.

Table 1

Building Height	Sydney	Melbourne
vacant site	1	1
building in construction	3	0
ground floor only	0	0
2 floors	3	3
3 floors	2	8
4 floors	2	1
5 floors	1	9
6 floors	0	7
7 floors	6	5
8 floors plus	64	42
Total	4 sites and 78 buildings	1 site and 75 buildings

Inspection of the table shows that a greater proportion of Sydney buildings fall in the 8-floors-plus category, confirming the prediction that site value taxation leads to a higher level of improvements per acre. Unfortunately, Archer does not present data which would allow testing of the theory's predictions regarding land values and housing prices.

5. Conclusion

This paper has presented rigorous analysis of two of the principal issues treated by the previous literature on site value taxation: long run effects and the incidence of short term gains and losses. The long-run analysis established that when a graded property tax

system is instituted over an area small enough so that the price of
housing is unaffected, two distinct effects emerge in the normal case
where $\partial\tau/\partial\theta < 0$: the level of improvements per acre rises, as does
the value of land. The increase in land value is a surprising result
given that gradation of the tax system increases the tax burden on
land. When the tax zone is large enough so that gradation affects the
price of housing, the outcome is somewhat different. Improvements per
acre still rise when $\partial\tau/\partial\theta < 0$, but this is accompanied by a decline in
the price of housing coupled with a decline in the value of land, with
the latter effect emerging as long as housing demand is not highly
elastic. While the impact of gradation on the level of improvements
was well understood in the earlier literature, the predictions relating
to land values and the price of housing are entirely new.

The contribution of the short-run analysis is to show that the
windfall gains and losses which result from gradation of the tax
system have a rather surprising spatial incidence. Contrary to a com-
mon impression, the most intensively developed parcels suffer windfall
losses in the form of higher taxes, while the least intensively devel-
oped parcels benefit from windfall gains.

It must be pointed out that a principal limitation of the analysis
in this paper is its partial equilibrium nature. For example, as a
result of the implicit assumption that landlords are absentee (see
footnote 9), the analysis in Section 2.2 fails to capture the impact
of changes in rental income on the demand for housing. Although a
general equilibrium model would capture such feedbacks, it is an open
question whether the type of analysis done in this paper would be
tractable in such a framework.

Finally, it should be noted that the paper has been silent on the normative question of whether a shift to a graded property tax system (or adoption of pure site value taxation) would be desirable. The answer to this question, however, is implicit in the optimal tax literature (see Sandmo (1976)) and affirms Henry George's original claim. Specifically, a principal lesson of optimal tax theory is that efficiency requires tax rates to be inversely related to demand (or in the case of inputs, supply) elasticities. When the relevant elasticity for a given commodity is zero, as in the case of land supply, it is optimal for society's tax burden to be carried by that commodity alone. As a result, modern theory vindicates George's belief in the efficiency of site value taxation.

Footnotes

[1] Lent (1967) provides a complete list of countries using variants of land value taxation. Holland (1969) gives a lengthy description of the institutional aspects of Jamaica's tax system, while Breckenfield (1983) discusses Pittsburgh's system.

[2] Turvey (1955) also claims that the impact of site value taxation on land values is ambiguous, but his reasoning is unclear.

[3] This expression results from rewriting $\sigma \equiv H_1 H_2 / H_{12} H$ in terms of the function h.

[4] Using (6), $\partial \theta r / \partial \theta = r/(1+\theta) > 0$.

[5] Using (3), $\partial \tau i S / \partial \tau = iS(1-\tau\sigma/(1+\tau)\mu_\ell)$.

[6] Another way of expressing this result is that a necessary (but not a sufficient) condition for $\partial \tau / \partial \theta < 0$ to hold is that the relevant range of the improvements tax "Laffer curve" is upward sloping.

[7] Direct μ_ℓ estimates can be found in Richman (1965), Gottlieb (1969), and Harriss (1970). Implied μ_ℓ estimates can be computed from data contained in the impact studies cited in the introduction.

[8] Pollock and Shoup (1977) provide an empirical estimate of the magnitude of the impact on improvements using a model which posits a value for $\partial \tau / \partial \theta$ and makes use of a particular parameterization of eq. (1).

[9] Since $\overline{\ell}$ is exogenous, the analysis ignores the possibility that a change in the tax system could affect the spatial size of the city. Analysis of such an effect, which requires use of a monocentric city model, unfortunately proved to be intractable.

[10] The fact that housing demand does not depend on net land rent r (which determines the income of land owners) reflects the implicit assumption that land owners are absentee, living outside the tax zone.

[11] Note that $\partial h / \partial p = h' \partial S / \partial p$, with $\partial S / \partial p = -h'/ph''$ from (1).

[12] The difference between the two expressions equals $-(1+\theta)(\eta+1)\sigma\tau/(1+\tau)\mu_\ell(\varepsilon-\eta) > 0$.

[13] When revenue decreases with τ in the p-exogenous case, revenue can either increase or decrease with τ when p is endogenous, yielding an ambiguous sign for $\partial \tau / \partial \theta$.

[14] When $\Lambda' < 0$, so that $\partial \tau / \partial \theta > 0$, this conclusion is reversed.

[15] In the long run, the outcome in the present framework is similar to previous results. If the expression in the denominator of (10) is positive at each x, $\partial\tau/\partial\theta < 0$ holds and $dS/d\theta > 0$ is satisfied for all x. Although $dr/d\theta$ can have either sign at a given location, total land value in the tax zone rises with θ provided $\partial\tau/\partial\theta < 0$.

[16] Since the analysis requires tax revenue to be held fixed, a substantial public spending difference between the cities (which is unlikely in any case) would invalidate its predictions.

References

Advisory Commission on Intergovernmental Relations, 1983, Significant
    features of fiscal federalism, 1981-82 edition (ACIR: Washington,
    D.C.).

Archer, R. W., 1972, Site value taxation in central business district
    development (Sydney, Australia) (Urban Land Institute, Washington,
    D.C.).

Back, Kenneth, 1970, Land value taxation in light of current theory
    and practice, in: Daniel M. Holland, ed., The assessment of land
    value (University of Wisconsin Press, Madison) 37-54.

Becker, Arthur P., 1969, Principles of taxing land and buildings for
    economic development, in: Arthur P. Becker, ed., Land and
    building taxes: Their effect on economic development (University
    of Wisconsin Press, Madison) 11-47.

Breckenfield, Gurney, 1983, Higher taxes that promote development,
    Fortune, August.

Brown, Harry G., 1927, Land speculation and land-value taxation,
    Journal of Political Economy 35, 390-402.

Carmean, Patricia, A., 1980, Site value taxation (International
    Association of Assessing Officers, Chicago).

Cuddington, John T., 1978, Estimating impacts of a property tax
    reform, Land Economics 54, 362-372.

George, Henry, 1879, Progress and poverty (Modern Library, New York).

Gottlieb, Manuel, 1969, Site value taxation and urban renewal, part I,
    Assessors Journal 4 (no. 2), 3-27.

Harriss, C. Lowell, 1970, Transition to land value taxation: Some
    major problems, in: Daniel M. Holland, ed., The assessment of
    land value (University of Wisconsin Press, Madison) 213-251.

Holland, Daniel M., 1969, A study of land taxation in Jamaica, in:
    Arthur P. Becker, ed., Land and building taxes: Their effect on
    economic development (University of Wisconsin Press, Madison)
    239-286.

Killoren, D. J. and Casey, J. M., 1981, Comparison study of St. Louis
    city real estate taxes with site value taxes producing equal reve-
    nues using 1980 assessed valuations, unpublished paper, Public
    Revenue Education Council, St. Louis.