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Transaction Costs and Equilibrium Pricing of Congested Public Goods with Imperfect Information

Kangoh Lee

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois Urbana-Champaign

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Kangoh Lee, Visiting Lecturer
Department of Economics

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by

Kangoh Lee

Department of Economics
University of Illinois at Urbana-Champaign
1206 South Sixth Street
Champaign, IL 61820

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Abstract

This paper examines the role of transaction costs in an economy with a congested public facility which can be used with variable intensity when there exists information asymmetry between consumers and firms. While a per visit price (toll) is charged in competitive equilibrium, lump-sum pricing may emerge in equilibrium when a transaction cost is required to implement per visit pricing. Since different types of consumers (in terms of demand for visits) cannot be distinguished, and since the number of visits are not counted under lump-sum pricing, high demand consumers have an incentive to patronize firms designed for low demand consumers to exploit a lower lump-sum price gain. Thus, an adverse selection problem may arise, and equilibrium is in general inefficient, depending on the magnitude of the transaction cost.

Transaction Costs and Equilibrium Pricing of Congested Public Goods with Imperfect Information

by

Kangoh Lee*

1. Introduction

In the literature, it is well known that when public goods (facilities) can be used with variable intensity, a per visit price (toll) charged by competitive firms (developers) leads to an optimal allocation with the price equal to the marginal congestion cost or the average cost per visit (see Berglas (1976), Berglas and Pines (1981), Scotchmer and Wooders (1987)). However, per visit pricing requires some type of a transaction cost and it could be prohibitive to implement per visit pricing in case of some public goods. For instance, we observe that all the highways are not financed by toll fees. One of the reasons is that charging a toll fee requires high transaction costs in terms of installing and maintaining toll gates on the part of developers or some public agents and waiting time on the part of consumers. Similarly, Barro and Romer (1987) argue that the reason why Disneyland shifted its pricing scheme from a combination form of per ride tolls and an entry fee to a pure entry fee is the higher transaction cost involved in implementing per visit pricing.[1]

With such a transaction cost, developers may charge a lump-sum price rather than a per visit price in equilibrium since the lump-sum price can be collected with negligible transaction (administration) cost relative to the per visit price.

While the choice between per visit pricing and lump-sum pricing depends simply on the magnitude of the transaction cost, the resulting equilibrium allocations are significantly different under imperfect information. Although consumer types (they differ in terms of demand for visits to a facility as well as the preference for the quality of the facility) are not observable by developers under imperfect information, per visit pricing results in an efficient equilibrium as known in the literature since it possesses a self-selection property. On the other hand, since number of visits are not counted under lump-sum pricing, high demand consumers have an incentive to patronize a facility designed for low demand consumers. By doing so, high demand consumers can exploit a lower lump-sum price gain although they should sacrifice their optimal quality of the facility. Thus, an adverse selection problem may arise and competitive equilibrium may not exist. If exists, it is in general less efficient relative to equilibrium with full information.

This resembles the adverse selection problem in a competitive insurance market analysed by Rothschild and Stiglitz (1976). However, it is not imperfect information alone, but the confounding effect of imperfect information and the transaction cost that prevents competitive markets from achieving an efficient outcome.

Among the works related to this paper, Berglas (1981) analyses a similar issue by an example in a different model. He analyses a water example where metering water consumption is costly. Thus, a local government may charge a poll tax (lump-sum price) when the metering cost is too high. In this case, under imperfect information, the adverse selection problem may arise and an inefficient allocation is achieved. However, this paper is different from Berglas' work in that we consider the issue in a competitive market setting, and that we allow the number of visits to a congested public facility to vary. This specification highlights the difference between per visit pricing and lump-sum pricing.

The plan of the paper is as follows. In the next section, we consider equilibrium with perfect information as the benchmark case. Equilibrium depends on the magnitude of the transaction cost, and is shown to be second-best efficient. In section 3, we introduce information asymmetry between consumers and developers. In contrast with the perfect information case, existence and efficiency of competitive equilibrium is not guaranteed. In Section 4, we present a simple example to help understand the main results. Finally, there is a summary and conclusion.

2. Perfect Information

In this section, we consider equilibrium with perfect information as the benchmark case, which allows us to focus only on the effect of a transaction cost. To simplify the analysis, assume that there are two types of individuals denoted by subscript i =1,2. However, in this section, we omit the type subscript i for notational convenience since individual types are observable with perfect information and hence equilibrium below applies for each type submarket.

Following a club model (Berglas and Pines (1981) and Scotchmer and Wooders (1987)), we assume that a consumer's preferences are given by a well-defined quasi-concave utility function

$$U[x, v, y, g] \tag{1}$$

$$g = nv, \quad (2)$$

where x is a private composite good with the price normalized equal to one, v is number of visits to a given facility (called "club") with characteristics of y and g, y is a physical dimension of the club, and g denotes the degree of congestion formulated as nv with n denoting a number of consumers using a given club. We call a pair (y,g) the quality of a club. The utility function U is assumed to be increasing in the first three arguments and decreasing in the last one.

The cost of providing the club with the quality y and g is

$$C(y,g), \quad (3)$$

where C is increasing in y and g.

To discuss equilibrium, we need to define price mechanisms. A break-even "PVP" (per visit price) to the club with the quality (y,g) is defined as

$$p(y,g) = C(y,g)/g + t, \quad (4)$$

where t is a transaction cost per visit. The transaction cost, t, may be a cost of installing and maintaining toll gates.[2] In the literature, it is assumed that t = 0, and PVP results in an efficient equilibrium.

However, if t > 0 as assumed in this paper, PVP cannot result in the efficient equilibrium. Furthermore, in order to save the transaction cost t, "LSP" (lump-sum pricing) may be used. By LSP we mean that consumers can make as many visits as they want without paying additional PVP once they pay LSP. Then, LSP does not require toll gates and t = 0 under LSP.[3] One might think of LSP as a season ticket price, and PVP as a toll fee. Note that while PVP depends only on (y,g), LSP should depend on v as well as (y,g) since LSP is imposed on a person not on a visit, and since each person makes v vists. Thus, in determining LSP, developers should somehow expect consumers' choice of v under LSP. A developers' reasonable expectation, we believe, is that consumers will choose v so that marginal utility of v equals zero at a given (I,e,y,g) with I and e denoting exogenous income and an LSP respectively. Let the expected v be $\tilde{v}(I,e,y,g)$, which will be written $\tilde{v}(e,y,g)$ whenever clear. Then $\tilde{v}(e,y,g)$ satisfies

$$U^2[I-e,\ \tilde{v}(e,y,g),\ y,\ g] = 0, \tag{5}$$

where the superscript 2 in U denotes the partial derivative with respect to v.[4] It is reasonable to assume that rational consumers actually choose $\tilde{v}(.)$, and that the expectation of $\tilde{v}(.)$ is perfectly realized.

While $\tilde{v}(e,y,g)$ depends on e and g, e and g are also affected by $\tilde{v}(.)$. Thus, in order for any LSP-club to break even, these relationships should be consistent, and hence a break-even LSP in a club with the quality (y,g) is defined to satisfy the following two conditions

$$g = n\tilde{v}(e,y,g) \tag{6}$$

$$e = C(y,g)/n. \tag{7}$$

(6) can be always satisfied since the developer can choose n (or restrict number of consumers to the club) appropriately. (6) and (7) imply that

$$e = [\tilde{v}(e,y,g)/g]C(y,g), \tag{8}$$

which solves for e as a function of (y,g) such that $e = \tilde{e}(y,g)$. That is, the relationship (6) allows us to consider the problem only in terms of (v,y,g). Thus, n does not explicitly appear in the analysis below, but n is implictly chosen to satisfy (6) (the same comment applies to the PVP-club problem by the relationship (2)). Hereafter, $\tilde{e}(y,g)$ denotes a break-even LSP at a given (y,g). Then, $\tilde{v}(e,y,g) = \tilde{v}(\tilde{e}(y,g),y,g)$ denotes v in a break-even LSP-club with the quality (y,g) (to save the space, we will suppress arguments of $\tilde{v}(.)$ whenever clear).

Note that since $\tilde{e}(y,g)$ and $\tilde{v}(e,y,g)$ depend on the utility function, different types of consumers have different functional forms of $\tilde{e}(y,g)$ and $\tilde{v}(e,y,g)$, implying that LSP is type-specific and nonanonymous at a given quality (y,g) while a break-even PVP, $p = C(y,g)/g + t$, is anonymous. This difference between PVP and LSP plays an important role in determining equilibrium with imperfect information in the next section. That is, PVP-club developers need not know consumer types in determining a break-even PVP while LSP-club developers need in determining a break-even LSP, meaning that imperfect information does not affect PVP while it affects LSP.

Before we discuss equilibrium, it proves useful to consider planning problems. From the discussion above, it is clear that a planner's problem in PVP-clubs is to max (1) s.t. $I = x + pv$, where $p = \{C(y,g)/g + t\}$ is a (break-even) per visit tax. Here, we assume that total population of a given type is very large relative to the club size n, so that the integer constraint can be ignored.[5] By assuming an interior solution,[6] let (v^{*a},y^{*a},g^{*a}) be the solution to the PVP-club planning problem, then

$$(v^{*a},y^{*a},g^{*a}) \equiv \arg\max U[I-\{C(y,g)/g+t\}v,\ v,\ y,\ g].$$

Similarly, the planner's problem in LSP-clubs is to max (1) s.t. $I = x + e$, where $e = \tilde{e}(y,g)$ is a (break-even) lump-sum tax, since the planner has the same problem in determining break-even lump-sum tax as developers have. Let (y^{*b}, g^{*b}) be the solution to the LSP-club planning problem, then

$$(y^{*b}, g^{*b}) \equiv \arg\max U[I-\tilde{e}(y,g),\ \tilde{v}(\tilde{e}(y,g),y,g),\ y,\ g],$$

where note that since $\tilde{v}(.)$ and $\tilde{e}(.)$ are common in all break-even LSP-clubs of a given type consumer, (y,g) is the only set of relevant choice variables.

The planning solutions above can be characterized as second-best outcomes as long as the transaction cost t is positive. That is, in the PVP-allocation, extra costs (resources) are required to choose v optimally while these costs are assumed to be zero in the first-best allocation. On the other hand, in the LSP-allocation, v is not optimally chosen because consumers do not consider the (external) effect of their v on g. In either case, the resulting allocation is not first-best efficient. For future reference, call the club with characteristics (v^{*a}, y^{*a}, g^{*a}) $[(y^{*b}, g^{*b})]$ the (second-best) optimal PVP-[LSP-] club, and let u^{*a} and u^{*b} be the resulting utility levels in these optimal clubs.

Given the two types of clubs, a natural question is which type of club is more efficient. As shown in the following lemma, PVP-clubs are superior to LSP-clubs unless the transaction cost t is too large .

Lemma 1. There exists a critical value of t denoted t^* such that $u^{*a} >$ $(<)$ u^{*b} for all $t < (>)$ t^*. t^* could be infinity.

To see the lemma, note first that $u^{*a} > u^{*b}$ when $t = 0$,[7] since u^{*a} then becomes first best efficient. Next, u^{*a} is decreasing in t while u^{*b} is not affected by t. These facts prove the lemma. If $t = t^*$, of course, $u^{*a} = u^{*b}$. However, if the effect of t is not so strong, then it is possible that $u^{*a} > u^{*b}$ for all $t \geq 0$, and that t^* is infinity.[8]

To proceed the analysis, define equilibrium as follows:[9]

Definition 0. Equilibrium is a configuration of clubs such that (i) all the clubs in equilibrium must break even, (ii) no alternative club makes nonnegative profit.[10]

Assuming that either PVP- or LSP-clubs emerge in equilibrium (the possibility that any two-part tariff can emerge in equilibrium will be discussed in Corollary 1 below), to facilitate the analysis we further define equilibrium as follows (based on Definition 0):

Definition 1. Equilibrium is a configuration of PVP-clubs with characteristics (p^*, v^*, y^*, g^*) if

$$p^* = C(y^*, g^*)/g^* + t \tag{9}$$

$$U[I-p^*v^*, v^*, y^*, g^*] \geq \underline{u} \equiv U[I, 0, 0, 0], \tag{10}$$

and there does not exist other PVP-club with $(p', v', y', g') \neq (p^*, v^*, y^*, g^*)$ such that

$$p' \geq C(y', g')/g' + t \tag{11}$$

$$U[I-p'v', v', y', g'] \geq U[I-p^*v^*, v^*, y^*, g^*], \tag{12}$$

and there does not exist an LSP-club with (e'', y'', g'') such that

$$e'' \geq [\tilde{v}(e'', y'', g'')/g'']C(y'', g'') \tag{13}$$

$$U[I-e'', \tilde{v}(e'', y'', g''), y'', g''] \geq U[I-p^*v^*, v^*, y^*, g^*]. \tag{14}$$

An analogous definition applies to the case where equilibrium is a configuration of LSP-clubs. Definition 1 says that equilibrium is a configuration of PVP-clubs if all the PVP-clubs in the configuration break even (Eq. (9)) and satisfy the individual rationality condition (Eq. (10)), and there exists no other profitable PVP-club (Eqs. (11)-(12)), and there exists no profitable LSP-clubs (Eqs. (13)-(14)).

From Definition 1, it is clear that candidate for equilibrium is either a configuration of optimal PVP-clubs with $(p^{*a},v^{*a},y^{*a},g^{*a})$ or a configuration of optimal LSP-clubs with (e^{*b},y^{*b},g^{*b}), where $p^{*a} \equiv C(y^{*a},g^{*a})/g^{*a} + t$ and $e^{*b} \equiv \tilde{e}(y^{*b},g^{*b})$, provided that the individual rationality constraint is satisfied.[11] If the constraint does not hold at either PVP- or LSP-clubs, then equilibrium trivially does not exist. On the other hand, if the constraint holds at only one of PVP- and LSP-clubs, equilibrium is again trivially the one that satisfies the constraint. Thus, to avoid trivial cases, we assume that the constraint holds at least at the optimal (both PVP- and LSP-) clubs. That is,

$$u^{*a} \geq \underline{u}, \text{ and } u^{*b} \geq \underline{u}. \tag{15}$$

From Lemma 1, we have the following proposition.

Proposition 1. If $t < (>)\ t^*$, equilibrium is a configuration of optimal PVP- (LSP-) clubs with characteristics $[p^{*a},v^{*a},y^{*a},g^{*a}]$ $([e^{*b},y^{*b},g^{*b}])$, and is second-best efficient.

To prove the part of the proposition dealing with $t < t^*$, we need to show the two things. First, any potentially (if consumers patronize) break-even club with characteristics other than $[p^{*a},v^{*a},y^{*a},g^{*a}]$ cannot actually break even, and hence cannot be part of an equilibrium. Second, any potentially non break-even (making a positive profit) club cannot actually make a nonnegative profit, and hence cannot upset the club with $[p^{*a},v^{*a},y^{*a},g^{*a}]$. The second requirement is important since a configuration itself is not an equilibrium (because clubs do not break even), but since it may make a nonnegative profit by attracting consumers from the existing clubs with $[p^{*a},v^{*a},y^{*a},g^{*a}]$, which implies that the existing clubs cannot be part of an equilibrium either.

To show the first one, note that any potentially break-even LSP-club cannot be part of an equilibrium since

$$U[I-\tilde{e}(y,g),\ \tilde{v}(\tilde{e}(y,g),y,g),\ y,\ g] \le$$
$$U[I-\tilde{e}(y^{*b},g^{*b}),\ \tilde{v}(\tilde{e}(y^{*b},g^{*b}),y^{*b},\ g^{*b}),\ y^{*b},\ g^{*b}] = u^{*b} <$$
$$u^{*a} = U[I-p^{*a}v^{*a},\ v^{*a},\ y^{*a},\ g^{*a}] \quad \text{for all } (y,g), \qquad (16)$$

where the first inequality follows from definition of (y^{*b},g^{*b}), and the second inequality follows from Lemma 1. Next, from definition of (v^{*a},y^{*a},g^{*a}), it follows that any potentially break-even PVP-club with characteristics $(v,y,g) \neq (v^{*a},y^{*a},g^{*a})$ cannot be part of an equilibrium since

$$U[I-pv,\ v,\ y,\ g] < U[I-p^{*a}v^{*a},\ v^{*a},\ y^{*a},\ g^{*a}]$$
$$\text{for all } (v,y,g) \neq (v^{*a},y^{*a},g^{*a}). \qquad (17)$$

To show the second one, consider an LSP-club with the quality (y,g) charging $e > [\tilde{v}(e,y,g)/g]C(y,g)$ as a lump-sum price. This club can potentially make a positive profit. However, the utility level offered by this club is clearly lower than that offered by LSP-clubs charging $\tilde{e}(y,g)$ as in (16), thus potential profitability cannot materialize. The same argument applies to the potentially profitable PVP-clubs charging $p > C(y,g)/g + t$ as a per visit price.

Therefore, it is clear that any PVP-club with $(p^{*a},v^{*a},y^{*a},g^{*a})$ satisfies definition of equilibrium. The remainder of the proposition (dealing with $t > t^*$) is proved by applying exactly parallel steps to the case with $t < t^*$.

In the discussion above, we rule out a two-part tariff as an equilibrium pricing scheme. However, it is shown that two-part tariff pricing does not affect the equilibrium as follows.

Corollary 1. Two-part tariff pricing does not affect the equilibrium in Proposition 1.

Since the nature of equilibrium in the model is competitve, any form of two-part tariff cannot emerge in equilibrium. To see this, define a two-part tariff as $k + \{t+m(y,g)\}v$ for v visits to the club with the quality (y,g), where k is a fixed part and $\{t+m(y,g)\}$ is a variable part or per visit price. Let $(\hat{v},\hat{y},\hat{g})$ be solution to max $U[I-k-\{t+m(y,g)\}v, v, y, g]$ subject to $k+\{t+m(y,g)\}v = (v/g)C(y,g)+tv$, which is the break-even constraint, and let u^{*c} be the resulting utility level in the club with $(\hat{v},\hat{y},\hat{g})$. Note first that when $k = 0$, two-part tariff pricing reduces to PVP, and hence $(\hat{v},\hat{y},\hat{g}) = (v^{*a},y^{*a},g^{*a})$ and $u^{*c} = u^{*a}$. Next, u^{*c} is decreasing in k while u^{*a} is not affected by k, implying that $u^{*a} > u^{*c}$ for all $k > 0$. Thus, any potentially break-even two-part tariff cannot actually break even, and it cannot be part of an equilibrium. Also, a potentially non break-even (making a positive profit) two-part tariff, say $k + \{t+m(y,g)\}v > (v/g)C(y,g) + tv$, provides a lower utility than u^{*c} (as well as u^{*a}), and hence the existing equilibrium in Proposition 2 cannot be upset by any two-part tariff pricing as claimed.

3. Imperfect Information

In this section, we assume that there exists information asymmetry between consumers and developers in that developers cannot distinguish consumer types. However, the fact that there are two types of consumers and their proportions (θ and $(1-\theta)$ respectively for type-1's and type-2's) in the population are common knowledge, which is a standard assumption in the literature.

Without losing generality, assume (among other differences between the types) that type-2's are higher demanders for v in a break-even LSP-club with the quality (y,g). That is, $\tilde{v}_2(\tilde{e}_2(y,g),y,g) > \tilde{v}_1(\tilde{e}_1(y,g),y,g)$ at a given (y,g). Then, it follows that $\tilde{e}_2(y,g) > \tilde{e}_1(y,g)$ since $\tilde{e}_2(y,g) = [\tilde{v}_2(.)/g]C(y,g) > [\tilde{v}_1(.)/g]C(y,g) = \tilde{e}_1(y,g)$, implying that type-2 break-even LSP-clubs charge a higher LSP than type-1 break-even LSP-clubs if both types of LSP-clubs have the same quality (y,g).

Since equilibrium depends on the magnitude of t, we consider the following three cases.

3-A. The case with $t > t^{*M} \equiv \max \{t_i^*\}$.

In this case, note from Proposition 1 that equilibrium with full information is a configuration of each type optimal LSP-clubs with characteristics $(e_i^{*b}, y_i^{*b}, g_i^{*b})$, i = 1,2. However, with imperfect information, type-2 consumers may patronize type-1 optimal LSP-clubs if e_1^{*b} is sufficiently lower than e_2^{*b}. This is because different types of consumers cannot be distinguished by developers, and because number of visits are not counted in LSP-clubs. In other words, type-2's can enjoy the lower lump-sum price gain (if any) from using type-1 optimal LSP-clubs while satisfying their demand for visits $\tilde{v}_2(.)$.[12] However, since they must sacrifice their optimal quality (y_2^{*b}, g_2^{*b}) of the club, it is in general not clear whether or not type-2 consumers will patronize type-1 optimal LSP-clubs. To make this clear, define a self-selection condition as follows.

Definition 2. The self-selection condition holds in LSP-clubs if

$$u_i^{*b} = U_i[I_i - e_i^{*b}, \tilde{v}_i(e_i^{*b}, y_i^{*b}, g_i^{*b}), y_i^{*b}, g_i^{*b}] \geq U_i[I_i - e_j^{*b}, \tilde{v}_i(e_j^{*b}, y_j^{*b}, g_j^{*b}), y_j^{*b}, g_j^{*b}]$$
$$\text{for } i,j = 1,2 \text{ with } i \neq j. \qquad (18)$$

In other words, if the self-selection condition holds, type-i's must prefer type-i optimal LSP-clubs to type-j optimal LSP-clubs.

While the self-selection condition (18) may or may not hold in LSP-clubs, it holds in PVP-clubs as shown in the following lemma.

Lemma 2. The self-selection condition holds in PVP-clubs (PVP possesses a self-selection property).

To see the lemma, consider the following inequality

$$u_i^{*a} = U_i[I_i - p_i^{*a}v_i^{*a}, v_i^{*a}, y_i^{*a}, g_i^{*a}] >$$
$$U_i[I_i - pv, v, y, g]$$
$$\text{for } i = 1,2, \text{ and all } (v,y,g) \neq (v_i^{*a}, y_i^{*a}, g_i^{*a}), \tag{19}$$

which follows from the definition of $(v_i^{*a}, y_i^{*a}, g_i^{*a})$. Since (19) holds at $(v,y,g) = (v_j^{*a}, y_j^{*a}, g_j^{*a})$, the lemma is proved. Intuitively, if type-i's patronize any type-j break-even PVP-clubs including type-j optimal PVP-clubs, they should pay for each visit (cannot enjoy a possible lower lump-sum price gain) while they should sacrifice their optimal quality (y_i^{*a}, g_i^{*a}) of the club.

From Definition 2 and Proposition 1, we have the following two lemmas:

Lemma 3. Suppose that $t > t^{*M}$. If the self-selection condition holds, equilibrium is a configuration of type-1 and type-2 optimal LSP-clubs with characteristics $(e_i^{*b}, y_i^{*b}, g_i^{*b})$, $i = 1,2$, and is second-best efficient.

To see the lemma, note first that if the self-selection condition holds, equilibrium with imperfect information is the same as one with perfect information. Thus, Proposition 1 applies for each type, and hence the lemma is established.

Lemma 4. If $t > t^{*M}$, type-1's never patronize any break-even type-2 LSP-clubs (the self-selection condition is never binding for any break-even type-2 LSP-clubs).

To see the lemma, note first that the type-1's maximum utility obtainable in a break-even type-2 LSP-club with the quality (y,g) is $U_1[I_1-\tilde{e}_2(y,g), \tilde{v}_1(\tilde{e}_2(y,g),y,g), y, g]$. Next, consider the following inequality

$$\begin{aligned}
&U_1[I_1-\tilde{e}_2(y,g), \tilde{v}_1(\tilde{e}_2(y,g),y,g), y, g] < \\
&U_1[I_1-\tilde{e}_1(y,g), \tilde{v}_1(\tilde{e}_1(y,g),y,g), y, g] \leq \\
&U_1[I_1-\tilde{e}_1(y_1^{*b},g_1^{*b}), \tilde{v}_1(\tilde{e}_1(y_1^{*b},g_1^{*b}),y_1^{*b},g_1^{*b}), y_1^{*b}, g_1^{*b}] = \\
&U_1[I_1-e_1^{*b}, \tilde{v}_1(e_1^{*b},y_1^{*b},g_1^{*b}), y_1^{*b}, g_1^{*b}] = u_1^{*b} \\
&\text{for all } (y,g) \text{ including } (y_2^{*b},g_2^{*b}),
\end{aligned} \tag{20}$$

where the first inequality follows from the fact that $\tilde{e}_2(y,g) > \tilde{e}_1(y,g)$, and the second inequality follows from the definition of (y_1^{*b},g_1^{*b}), which proves the lemma.

Intuitively, if type-1's patronize a break-even type-2 LSP-club, they should sacrifice their optimal quality (y_1^{*b},g_1^{*b}) of the club and pay a relatively higher lump-sum price since type-2 LSP-clubs are designed for type-2's (higher demanders for v).

From Proposition 1 and Lemma 4, we can show the following lemma.

Lemma 5. If $t > t^{*M}$, any type-2 PVP-club cannot be part of an equilibrium.

To see the lemma, note that type-2's are free to choose their optimal type clubs regardless of information available since type-1's never patronize type-2 break-even LSP-clubs in any case as shown in Lemma 4. Thus, it is clear that if $t > t^{*M}$ (implying that $t \geq t_2^{*}$), type-2's will prefer their optimal LSP-clubs to any break-even PVP-clubs from Proposition 1. Also, any non break-even PVP-clubs cannot be part of an equilibrium by definition. This establishes the lemma. Note that the lemma holds regardless of whether the self-selection condition holds or not.

Noting that if the self-selection condition does not hold, type-2 consumers will patronize type-1 optimal LSP-clubs, we have the following lemma:

Lemma 6. Suppose that $t > t^{*M}$. If the self-selection condition does not hold, type-1 optimal LSP-clubs cannot be part of an equilibrium.

To see the lemma, note first that type-2's utility maximizing v in the type-1 optimal LSP-club is $\tilde{v}_2(e_1{}^{*b}, y_1{}^{*b}, g_1{}^{*b})$. Next, from the assumption that type-2's are higher demanders, it follows that $\tilde{v}_2(\tilde{e}_2(y_1{}^{*b}, g_1{}^{*b}), y_1{}^{*b}, g_1{}^{*b}) > \tilde{v}_1(\tilde{e}_1(y_1{}^{*b}, g_1{}^{*b}), y_1{}^{*b}, g_1{}^{*b})$, and that $\tilde{e}_2(y_1{}^{*b}, g_1{}^{*b}) > \tilde{e}_1(y_1{}^{*b}, g_1{}^{*b}) \equiv e_1{}^{*b}$. Assuming that v is a normal good or $d\tilde{v}(I,e,y,g)/dI = - d\tilde{v}(I,e,y,g)/de > 0$, it follows that $\tilde{v}_2(e_1{}^{*b}, y_1{}^{*b}, g_1{}^{*b}) > \tilde{v}_2(\tilde{e}_2(y_1{}^{*b}, g_1{}^{*b}), y_1{}^{*b}, g_1{}^{*b})$, implying that $\tilde{v}_2(e_1{}^{*b}, y_1{}^{*b}, g_1{}^{*b}) > \tilde{v}_1(e_1{}^{*b}, y_1{}^{*b}, g_1{}^{*b})$. Then, it is clear that type-1 optimal LSP-clubs make a negative profit, which proves the lemma. However, as will be discussed below, some type-1 non optimal LSP-club can be part of an equilibrium.

From the discussion above, when the self-selection condition does not hold, any equilibrium must have some device so that type-2's have no incentive to patronize type-1 clubs. There are three possible devices (and hence equilibria); (i) a configuration of type-1 PVP-clubs and type-2 LSP-clubs (called the "separating-PL" configuration), (ii) a configuration of type-1 LSP-clubs with characteristics (e,y,g) and type-2 LSP-clubs (called the "separating-LL" configuration) where (e,y,g) satisfies the self-selection constraint

$$U_2[I_2-e,\ \tilde{v}_2(e,y,g),\ y,\ g] \le u_2{}^{*b}, \tag{21}$$

(iii) a configuration where both types are completely pooled (mixed) in identical LSP-clubs with the proportion of type-1's in each club equal to θ, which is the proportion of type-1's in the economy's population (called the "pooling" configuration).

To see why these three configurations are possible equilibria or why these configurations satisfy the self-selection condition, note first that (i) in the separating-PL configuration, type-2's have no incentive to use type-1 PVP-clubs since PVP possesses a self-selection property as shown in Lemma 2. In other words, per visit pricing plays a role in screening type-2's (higher demanders) from type-1's (lower demanders). (ii) In the separating-LL configuration, certainly type-2's have no incentive to patronize the type-1 LSP-clubs by construction. (iii) In the pooling configuration, neither type-1's nor type-2's have an incentive to move to other clubs since all the clubs are identical. However, developers might have an incentive to construct a club with the proportion of type-1's not equal to θ since by doing so developers may make a positive profit. But, any club cannot choose σ as intended, which follows from the information asymmetry assumption.[13]

To facilitate the analysis, we further define equilibrium (based on Definition 0), similarly to Definition 1, as follows:

> **Definition 3.** Suppose that $t > t^{*M}$, and that the self-selection condition does not hold. Equilibrium is a separating-PL configuration if the clubs in the separating-PL configuration break even and satisfy the individual rationality constraint, and if there exists no other profitable type-1 PVP-club, no other profitable type-2 LSP-club, no profitable type-1 LSP-club (satisfying the self-selection constraint), and no profitable pooled club.

An analogous definition applies to the case when equilibrium is the separating-LL or the pooling configuration. Note from Definition 3 that the only candidate for the equilibrium separating-PL configuration is a configuration of type-1 optimal PVP-clubs with $(p_1^{*a}, v_1^{*a}, y_1^{*a}, g_1^{*a})$ and type-2 optimal LSP-clubs with $(e_2^{*b}, y_2^{*b}, g_2^{*b})$. Similarly, the only candidate for the equilibrium separating-LL configuration is a configuration of type-1 LSP-clubs with (e_1^o, y_1^o, g_1^o) and type-2 optimal LSP-clubs with $(e_2^{*b}, y_2^{*b}, g_2^{*b})$, where (e_1^o, y_1^o, g_1^o) solves the following optimization problem

$$\max U_1[I_1-e,\ \tilde{v}_1(e,y,g),\ y,\ g] \tag{22}$$

$$\text{s.t. } U_2[I_2-e,\ \tilde{v}_2(e,y,g),\ y,\ g] \le u_2^{*b} \tag{21}$$

$$e = [\tilde{v}_1(e,y,g)/g]C(y,g) \tag{23}$$

$$\max U_1[I_1-e,\ \tilde{v}_1(e,y,g),\ y,\ g] \equiv u_1^o \ge \underline{u}_1, \tag{24}$$

where note that $e_1^o = \tilde{e}_1(y_1^o, g_1^o)$ since the solution satisfies the break-even constraint (23). Finally, the only candidate for the equilibrium pooling-configuration is a configuration of pooled LSP-clubs with $[e^*, y^*, g^*]$ which is a solution to max (22)

$$\text{s.t. } U_2[I_2-e,\ \tilde{v}_2(e,y,g),\ y,\ g] = u_2 \ge u_2^{*b} \tag{25}$$

$$e = [\{\theta\tilde{v}_1(.)+(1-\theta)\tilde{v}_2(.)\}/g]C(y,g) \tag{26}$$

$$\max U_1[I_1-e,\ \tilde{v}_1(e,y,g),\ y,\ g] \equiv u_1^* \ge \underline{u}_1. \tag{27}$$

Note that since, in contrast with the other two configurations, the pooled clubs generate a utility frontier, the type-1's utility u_1^* depends on a type-2's utility level u_2 set by developers. While the type-2 utility level in (25) can be arbitrarily determined by developers, it is natural to set at $u_2 \ge u_2^{*b}$, which is available in the other two configurations. Thus, the type-2's utility level does not matter in determining equilibrium, and hence we focus only on the type-1's utility levels. If $u_2 < u_2^{*b}$, then it is clear that the pooling configuration cannot be equilibrium.

From the discussion above, equilibrium is the separating-PL configuration if (28) and one of (29) and (30) hold

$$u_1^{*a} > u_1^o \equiv U_1[I_1-e_1^o, \tilde{v}_1(e_1^o,y_1^o,g_1^o), y_1^o, g_1^o] \tag{28}$$

$$u_1^{*a} > u_1^* \equiv U_1[I_1-e^*, \tilde{v}_1(e^*,y^*,g^*), y^*, g^*] \tag{29}$$

$$u_1^o > u_1^* \tag{30}$$

Similarly, equilibrium is the separating-LL configuration if (28) is reversed and one of (29) and (30) holds while equilibrium is the pooling configuration if both (29) and (30) are reversed.

Before we find equilibrium, it proves useful to discuss existence of equilibrium. Noting that the separating-LL and (or) the pooling configurations may not exist since it is quite possible that no type-1 LSP-club and (or) no pooled club satisfy the relevant constraints, we have the following lemma.

> **Lemma** 7. If there exists a type-1 LSP-club satisfying both the self-selection constraint (21) and the break-even constraint (23), equilibrium exists.

To see the lemma, note first that failing to satisfy the individual rationality constraints ((24) or (27)) does not lead to nonexistence of equilibrium since optimal clubs in the separating-PL configuration are assumed to satisfy the constraints.

Next, suppose that no type-1 LSP-club in the separating-LL configuration satisfies both the self-selection constraint (21) and the break-even constraint (23). For instance, suppose that type-2's are little affected by the quality of a club. Then, type-2's always have an incentive to patronize type-1 LSP-clubs since by doing so type-2's can enjoy the lower sump-sum price gain but lose little from sacrificing their optimal quality of the club. Thus, the only way to satisfy the self-selection constraint (21) is for the type-1 LSP-clubs to charge a very high lump-sum price (greater than a potentially break-even price). Then, the type-1 LSP-clubs make a positive profit (if type-1's patronize), and hence the separating-LL configuration does not exist. Letting u_1^{oo} be the type-1's utility level offered by the type-1 LSP-club in the above example, if $u_1^{oo} > u_1^{*a}$ and $u_1^{oo} > u_1^{*}$, then it is clear that equilibrium does not exist. On the other hand, if there exist type-1 LSP-clubs satisfying both (21) and (23), then $u_1^{oo} < u_1^{o}$. In this case, if the individual rationality constraint (24) is not satisfied, then equilibrium is either the separating-PL or the pooling configuration. Otherwise, equilibrium is one of the three configurations. In any case, equilibrium exists. Note that converse of the lemma may not hold since it is possible that no type-1 LSP-club satisfies (21) and (23), and that $u_1^{oo} < u_1^{*a}$ or (and) $u_1^{oo} < u_1^{*}$.

To find equilibrium, we need to determine the inequalities (28)-(30). First, we show that (30) holds in the following lemma.

Lemma 8. If there exists a type-1 LSP-club satisfying both (21) and (23), the pooling configuration cannot be equilibrium.

This result is proved in the appendix. An intuitive reason is that the separarting-LL configuration problem (21)-(24) is equivalent to the pooling configuration problem (22) and (25)-(27) except that type-1's pay a higher LSP in the pooling configuration, and hence $u_1^{o} > u_1^{*}$. Of course, if there exists no type-1 LSP-club satisfying both (21) and (23), then equilibrium (if exists) is either the separating-PL or the pooling configuration.

In what follows, we show that (28)-(29) depend on the magnitude of the transaction cost. To determine the inequality (28). Let $F(t) \equiv u_1^{*a} - u_1^o$. To sign F(t), note first that from Lemma 1 at $t = t_1^*$, $u_1^{*a} = u_1^{*b}$. Next, it is clear that $(e_1^o, y_1^o, g_1^o) \neq (e_1^{*b}, y_1^{*b}, g_1^{*b})$ (otherwise, the self-selection constraint (21) does not hold), and hence $u_1^o < u_1^{*b}$. These facts imply that $F(t_1^*) > 0$. Recalling that u_1^{*a} is decreasing in t, F(t) is also decreasing in t. Thus, there exists $\hat{t} > t_1^*$ such that $F(t) < (>) 0$ as $t > (<) \hat{t}$.

However, we cannot ignore the possible corner solutions such that $\hat{t}$ equals t^{*M} or infinity. First, since we consider the case with $t > t^{*M}$, if $t^{*M} > \hat{t}$, then $F(t) < 0$ for all $t > t^{*M}$. Thus, in this case, $\hat{t} = t^{*M}$. Second, if the separating-LL configuration does not exist, $F(t) > 0$ for all t, and hence $\hat{t}$ is infinity.

Similarly, to determine the inequality (29), let $G(t) \equiv u_1^{*a} - u_1^*$. To sign G(t), note that $u_1^* < u_1^{*b}$ by the fact that $e^* > \tilde{e}_1(y^*, g^*)$ as shown in Lemma 8 (Appendix) and by the same reasoning as used in (20). Thus, it follows that $G(t_1^*) > 0$. Since u_1^{*a}, and hence G(t) are decreasing in t, there exists $\bar{t} > t_1^*$ such that $G(t) < (>) 0$ as $t > (<) \bar{t}$. Again, it is possible that $\bar{t}$ could be t^{*M} or infinity. An example of the latter case is that $\tilde{v}_2(.)$ is too much larger than $\tilde{v}_1(.)$. Then, the common lump-sum price e^* is too high for type-1's, and hence the individual rationality constraint may not be satisfied in the pooling configuration. We summarize the result as follows.

Proposition 2. Suppose that $t > t^{*M}$, and that the self-selection condition does not hold. (i) If there exists a type-1 LSP-club satisfying both (21) and (23), there exists $\hat{t}$ such that equilibrium is the separating-PL (-LL) configuration for $t < (>) \hat{t}$. (ii) If there exists no type-1 LSP-club satisfying both (21) and (23), there exists $\bar{t}$ such that equilibrium (if exists) is the separating-PL (pooling) configuration for $t < (>) \bar{t}$. $\hat{t}$ and (or) $\bar{t}$ could be t^{*M} or infinity.

To see intuition of the proposition, note that the two equilibrium separating configurations have an advantage and a disadvantage (in terms of efficiency) involved in the choice of self-selecting mechanisms. The separating-PL configuration has an advantage over the LL one in that it can choose the optimal quality for type-1's in PVP-clubs. However, it results in an efficiency loss in the form of the higher transaction cost from implementing PVP. On the other hand, the LL one has a gain from not using PVP, but has a loss since type-1's cannot choose the optimal quality because of the self-selection constraint. Thus, we can expect that the separating-PL configuration is more efficient than the LL one, and hence it is equilibrium provided that the cost of implementing PVP is small such that $t < \hat{t}$. The similar interpretation applies to the relationship between the separating-PL and the pooling configurations.

Regarding efficiency of equilibrium, it is not second-best efficient (which is obtained under the planning problem with perfect information) in any case. This inefficiency is obvious because of imperfect information. One might interpret this efficiency loss from erecting self-selection mechanisms as a screening (or information) cost. However, it is more important to see whether the equilibrium is less efficient than a third-best outcome (a planning solution under imperfect information). As will be shown below, this question is closely related to existence of the type-1 LSP-clubs satisfying (21) and (23).

To see this, consider the planning problem corresponding to the separating-LL configuration. Since the planner can make a lump-sum transfer, and hence solve the following optimization problem

$$\max U_1[I_1-T-e,\ \tilde{v}_1(I_1-T,e,y,g),\ y,\ g] \qquad (31)$$

$$\text{s.t. } U_2[I_2\text{-}T\text{-}e,\ \tilde{v}_2(I_2\text{-}T,e,y,g),\ y,\ g] \leq$$

$$\max U_2[I_2+T\theta/(1-\theta)-e',\ \tilde{v}_2(I_2+T\theta/(1-\theta),e',y',g'),\ y',\ g'] \quad (32)$$

$$e = [\tilde{v}_1(I_1\text{-}T,e,y,g)/g]C(y,g) \quad (33)$$

$$e' = [\tilde{v}_2(I_2+T\theta/(1-\theta),e',y',g')/g']C(y',g'), \quad (34)$$

where T is a lump-sum transfer from type-1's to type-2's (note that T must be nonnegative to satisfy (32) given that the self-selection condition does not hold). If there exist type-1 LSP-clubs satisfying (21) and (23), the maximization problem (21)-(23) in the separating-LL configuration is equvalent to the above planning problem with T = 0, and hence these type-1 LSP-clubs can reach a point (with T = 0) on the utility frontier generated by the above planning problem. Since it is clear that the separating-PL and the pooling configurations also can reach a point on the utility frontier generated by the corresponding planning problems, equilibrium is as efficient as the planning problem.

On the other hand, if there exists no type-1 LSP-club satisfying both (21) and (23), then this type-1 LSP-club cannot reach any point on the utility frontier. Thus, equilibrium is less efficient (or as efficient as) than the planning solution. This is because in this case T must be positive to satisfy both (32) and (33), but because T cannot be positive (lump-sum transfer is not possible) in equilibrium. We summarize this as follows.

Corollary 2. The equilibrium in Proposition 2 is not second-best efficient. If there exists a type-1 LSP-club satisfying both (21) and (23), then the equilibrium in Proposition 2 is third-best efficient.

Note that converse of the corollary may not hold by the same reason showing that converse of Lemma 7 may not hold. Thus, from Lemma 7 and Corollary 2, it follows that if equilibrium exists, it is third-best efficient.

3-B. The Case with $t < t^{*m} = \min \{t_i^*\}$

In this section, we assume that the transaction cost is so low that both types of developers charge PVP with full information. Since the self-selection condition always holds under PVP as shown in Lemma 2, equilibrium with imperfect information is the same as one with full information. Thus, Proposition 1 applies for eaxh type, and we have the following result.

> **Proposition 3.** If $t < t^{*m}$, equilibrium is a configuration of type-i PVP-clubs with $[p_i^{*a}, v_i^{*a}, y_i^{*a}, g_i^{*a}]$, i = 1,2, and is second-best efficient.

One implication of the proposition is that imperfect information alone does not cause nonexistence and (or) inefficiency of equilibrium. It is this confounding effect of the transaction costs and imperfect information that prevents a market from achieving an efficient outcome.

3-C. The Case with $t^{*m} < t < t^{*M}$

Suppose first that $t^{*m} = t_2^*$ (and $t^{*M} = t_1^*$). Then, it is clear that equilibrium is a configuration of type-1 PVP-clubs with $[p_1^{*a}, v_1^{*a}, y_1^{*a}, g_1^{*a}]$ and type-2 LSP-clubs with $[e_2^{*b}, y_2^{*b}, g_2^{*b}]$, and is second-best efficient.

Alternatively, suppose that $t^{*m} = t_1^*$ (and $t^{*M} = t_2^*$). Then, type-2's may or may not like to patronize the type-1 LSP-clubs, depending on the lower lump-sum price gain and the loss from using the different quality of the club. Thus, we have the result very similar to Proposition 2, and need not repeat here.

4. Example

To help understand propositions, we consider a very simple example in this section. Specify the utility function and the cost function as

$$U_1[x, v, y, g] = x + 2yv - 3v^2 g \quad (35)$$

$$U_2[x, v, y, g] = x + 3yv - \beta v^2 g \quad (36)$$

$$C(y,g) = y^2 + g, \quad (37)$$

where $0 < \beta < 4.5$, which guarantees the assumption that $\tilde{v}_2(.) > \tilde{v}_1(.)$. To simplify calculation, assume that y is fixed at $y = 2$, and that $I_1 = I_2$.

4-A. Perfect Information

It is shown that

$$(p_1^{*a}, v_1^{*a}, y_1^{*a}, g_1^{*a}) = [5+2t/3,\ (3-t)^2/108,\ 2,\ 12/(3-t)],$$

$$(p_2^{*a}, v_2^{*a}, y_2^{*a}, g_2^{*a}) = [(8+2t)/3,\ (5-t)^2/36\beta,\ 2,\ 12/(5-t)]$$

$$(e_1^{*b}, y_1^{*b}, g_1^{*b}) = [1/8,\ 2,\ 8]$$

$$(e_2^{*b}, y_2^{*b}, g_2^{*b}) = [3/2\beta,\ 2,\ 16]$$

$$u_1^{*a} = I + (3-t)^3/324,\ u_1^{*b} = I + 1/24$$

$$u_2^{*a} = I + (5-t)^3/108\beta,\ u_2^{*b} = I + 3/4\beta$$

$$t_1^* = 3[1 - (.5)^{1/3}] = .62 < t_2^* = 5 - 3(3)^{1/3} = .67.$$

4-B. Imperfect Information

Let us consider the first case where $t > t^{*M} = t_2^*$. To proceed the analysis, we need to check whether the self-selection condition holds for type-2 consumers since it always holds for type-1's from Lemma 4. It is shown that the self-selection condition holds if $\beta < 3$. That is, if $\beta < (>)\ 3$,

$$u_2^{*b} = I + 3/4\beta > (<)$$

$$U_2[I_2 - e_1^{*b},\ \tilde{v}_2(e_1^{*b}, y_1^{*b}, g_1^{*b}),\ y_1^{*b},\ g_1^{*b}] = I + 9/8\beta - 1/8. \quad (38)$$

Assuming that $\beta > 3$, it is shown that the separating-LL configuration exists, and that

$$(e_1^o, y_1^o, g_1^o) = [6(90-4\beta+A)/(54-4\beta+A)^2,\ 2,\ (54-4\beta+A)/9] \quad (39)$$

$$u_1^o = I + 6(18-4\beta+A)/(54-4\beta+A)^2,$$

where $A \equiv [(54-4\beta)^2 - 288\beta]^{1/2}$. Since the separating-LL configuration (and hence type-1 LSP-club satisfying (21) and (23)) exists, the first part of Proposition 2 applies. Letting $u_1^{*a} = u_1^o$, we have $\hat{t} = 3 - [1944(18-4\beta+A)/(54-4\beta+A)^2]^{1/3}$. If $\beta = 4.3$, $\hat{t} = .85$. However, if $\beta = 4$, $\hat{t} = .66 < t^{*M} = t_2^* = .67$, thus we have a corner solution such that $\hat{t} = t^{*M}$.

5. Conclusion

When per visit pricing of congested public goods requires a transaction cost, a lump-sum price may emerge in a competitive equilibrium if the transaction cost is large. In this case, with information asymmetry between consumers and developers, the higher demanders for visits may have an incentive to patronize lower demanders' clubs since they pay a lower lump-sum price while satisfying their demands. Thus, equilibrium is in general not the same as one with full information. It is shown that equilibrium and its efficiency depend on the magnitude of the transaction cost.

The analysis shows that equilibrium is likely to be inefficient unless transaction cost is low so that PVP is used. Given that many types of local public goods are not financed by per visit price (toll), the adverse selection problem may arise and equilibrium is likely to be inefficient. To improve on the equilibrium allocation, an efficient financing scheme needs to be studied.

Appendix

Proof of **Lemma 8**. Note first that

$$e^* = [\{\theta\tilde{v}_1(e^*,y^*,g^*)+(1-\theta)\tilde{v}_2(e^*,y^*,g^*)\}/g^*]C(y^*,g^*), \tag{A1}$$

since the solution (e^*,y^*,g^*) satisfies the break-even constraint (26). Next, from the assumption that type-2's are higher demanders, and that type-2 LSP-clubs charge a higher LSP than type-1 LSP-clubs, it follows that $\tilde{v}_2(e^*,y^*,g^*) > \tilde{v}_1(e^*,y^*,g^*)$. To see this, suppose the contrary so that $\tilde{v}_2(e^*,y^*,g^*) \leq \tilde{v}_1(e^*,y^*,g^*)$. Then, we have $[\tilde{v}_2(e^*,y^*,g^*)/g^*]C(y^*,g^*) \leq e^* \leq [\tilde{v}_1(e^*,y^*,g^*)/g^*]C(y^*,g^*)$, implying that $\tilde{e}_2(y^*,g^*) \leq e^* \leq \tilde{e}_1(y^*,g^*)$, which contradicts the assumption. Thus, we have $\tilde{v}_2(e^*,y^*,g^*) > \tilde{v}_1(e^*,y^*,g^*)$.

Let $H(\theta) = u_1^o - u_1^*$. Total differentiation of (A1) yields

$$de^*/d\theta = \frac{[\tilde{v}_1(e^*,y^*,g^*)-\tilde{v}_2(e^*,y^*,g^*)]}{g^*/C(y^*,g^*) - [\theta\tilde{v}_1{}^1(e^*,y^*,g^*)+(1-\theta)\tilde{v}_2{}^1(e^*,y^*,g^*)]} \tag{A2}$$

where $\tilde{v}_i{}^1(.)$ denotes partial derivative of $\tilde{v}_i(.)$ with respect to e^*. Given that $\tilde{v}_1(.) < \tilde{v}_2(.)$ as shown above, $de^*/d\theta < 0$ as long as $\tilde{v}(.)$ is a normal good or $\tilde{v}_i{}^1(.) < 0$. On the other hand, from definition of u_1^*, it follows that $du_1^*/d\theta = -(de^*/d\theta)\ \{U_1{}^1[I_1-e^*,\ \tilde{v}_1(.),\ y^*,\ g^*] + \mu U_2{}^1[I_2-e^*,\ \tilde{v}_2(.),\ y^*,\ g^*]\}$, where μ is a positive multiplier associated with the constraint (25). Thus, from (A2), it follows that $du_1^*/d\theta > 0$, implying that $H(\theta)$ is decreasing in θ.

Next, note that the constraint (21) is binding since the self-selection condition does not hold, and that (25) holds as an equality at a solution. Thus, the separating-LL configuration problem (21)-(24) is equivalent to the pooling configuration problem (22) and (25)-(27) when $\theta = 1$. Thus, $H(1) = 0$, implying that $H(\theta) > 0$ for all $0 < \theta < 1$.

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Endnotes

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1. Oi (1971) provides a different explanation (based on monopoly power) why Disneyland shifted its pricing scheme.

2. One might think of t as a time cost in the sense that consumers must wait to pass the toll gate. Assuming that the cost of maintaing the gate is negligible, the analysis may carry over to this case. That is, although consumers do not pay the transaction cost of maintaining the gate, PVP requires the time cost t per visit.

3. Although lump-sum pricing need some administration cost, we ignore it for analytical easiness and for it being negligible relative to t in PVP.

4. We assume that there exists a finite $\tilde{v}(e,y,g)$ since consumption of the public good requires consumption of time.

5. This assumption will be maintained in the rest of the paper.

6. For the necessary conditions and their interpretations, see Berglas and Pines (1981).

7. Since we assume interior solutions, $[v^{*a},y^{*a},g^{*a}] \neq [\tilde{v}(\tilde{e}(y^{*b},g^{*b}),y^{*b},g^{*b}),y^{*b},g^{*b}]$.

8. To see this, let $D(t) \equiv u^{*a} - u^{*b}$. When D(t) is drawn in the two dimensional space with the horizontal axis representing t, if D(t) is convex and approaches the horizontal axis asymptotically, then $D(t) > 0$ for all t.

9. Strictly speaking, this notion of equilibrium is not a competitive equilibrium. Instead, this is a Nash type equilibrium.

10. In (i), "break even" can be replaced by "make a nonnegative profit" without affecting the analysis below.

11. Given that equilibrium is not first-best efficient as will be shown in Proposition 1, the individual rationality constraint may be binding.

12. The lump-sum price e_1^{*b} may be higher than e_2^{*b} if type-1's prefer a higher quality club since e depends on the quality (y,g) as well as the number of visits v.

13. Another reason why all the clubs must have θ is that otherwise there are some leftover consumers (type-1 or type-2) not accommodated to a pooled club, and that they reach, in general, a different utility from that enjoyed by consumers in the pooled clubs. Clearly, this configuration cannot be an equilibrium.