

UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA-CHAMPAIGN
BOOKSTACKS

H or V	THE HECKMAN BINDERY, INC. North Manchester, Indiana			
	JUST	FONT	SLOT	TITLE
H	CC	1W		BR ACULTY WORKING 19 PAPER
H	CC	1W	B	1988 NO.1468-1485
H	CC	1W		330 B385<"CV"> no.1468-1485 cop.2
H	CC	7W		<IMPRINT> U. of ILL. LIBRARY URBANA

BINDING COPY									
PERIODICAL: <input type="checkbox"/> CUSTOM <input type="checkbox"/> STANDARD <input type="checkbox"/> ECONOMY <input type="checkbox"/> THESIS				NO VOLS THIS TITLE		LEAD ATTACH			
BOOK: <input type="checkbox"/> CUSTOM <input type="checkbox"/> MUSIC <input type="checkbox"/> ECONOMY				AUTH 1ST <input type="checkbox"/>					
ACCOUNT		LIBRARY	NEW	RUB OR SAMPLE	TITLE I.D.	COLOR			
66672 001						6632	WHT	488	
ACCOUNT NAME									
UNIV OF ILLINOIS					ISSN.				
B01912400									
I.D. #2		NOTES	BINDING FREQUENCY	WHEEL	SYS. I.D.				
STX4				1	3	19276			
COLLATING									
35									
ADDITIONAL INSTRUCTIONS									
Dept-STX4 Lot#201 Item#155 HNM#12V#									
1CF2ST3CR MARK BY * B4 81									
SEP. SHEETS		PTS. BD. PAPER	TAPE STUBS	CLOTH EXT	GUM		FILLER		
							STUB		
POCKETS			SPECIAL PREP				LEAF ATTACH		
PAPER	BUCK	CLOTH							
INSERT MAT.		ACCOUNT LOT NO				JOB NO.			
		301				HV365			
PRODUCT TYPE		ACCOUNT PIECE NO				PIECE NO			
HEIGHT		GROUP CARD	VOL. THIS TITLE						
11 2			135						
COVER SIZE									
X									

001247981



STX
1469 10P 2

BEBR

FACULTY WORKING
PAPER NO. 1469

The Present Value of Future Earnings:
Contemporaneous Differentials and the
Performance of Dedicated Portfolios

William R. Bryan
Charles M. Linke

THE UNIVERSITY OF ILLINOIS
AUG 06 1988

BEBR

FACULTY WORKING PAPER NO. 1469

College of Commerce and Business Administration


University of Illinois at Urbana-Champaign

June 1988

The Present Value of Future Earnings:
Contemporaneous Differentials and the
Performance of Dedicated Portfolios

William R. Bryan, Professor
Department of Finance

Charles M. Linke, Professor
Department of Finance



Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

<http://www.archive.org/details/presentvalueoffu1469brya>

Abstract

This paper presents an investigation of the power of the contemporaneous differential, the beginning period differential between interest rates and the rate of growth in wages. The question investigated is whether the contemporaneous differential has a meaningful effect on the realized differential, the over-the-period differential between the ex post yield on a dedicated portfolio and the ex post rate of growth in wages.

Empirical results of the study show that the contemporaneous differential does not have a meaningful impact on the realized differential. Hence, the mean of the realized differential is a robust, minimum error predictor.

I. Introductory Remarks

In an earlier issue of this Journal [1], we offered evidence regarding the estimation of the present value of a stream of future earnings. We addressed the issue of identifying the appropriate differential between the discount rate and the rate of growth in earnings. In that connection, we investigated the use of alternative portfolios dedicated to the replacement of earnings streams of varying duration over the 1953-1984 period. We concluded that dedicated portfolios consisting of U.S. Treasury securities of one-year constant maturity provided the most desirable present value estimates for valuation purposes. Further, we concluded that the appropriate differential was approximately zero.

In recent years atypical differentials have prevailed between interest rates that reflect the time value of money and the rates of growth in wages. Indeed, even a cursory examination of historical data reveals several episodes during which there have been marked divergence between contemporaneous interest rates and wages growth rates. In light of such experiences, it is important to understand the extent to which the contemporaneous differential, the differential existing at the beginning of a loss period, affects the realized, or ex post, differential between the rate of growth in wages and the realized yield on the dedicated portfolio during the loss period.¹

Section II describes a model that permits empirical testing of the contemporaneous/realized differential issue. Regression results are reported in Section III. Concluding remarks are provided in Section IV.

II. The Model

In this section we discuss the adjustment of the economy to discrepancies between the general level of interest rates and the rate of growth in wages. Next, we model the impact of the economy's adjustments on the relation between returns on dedicated portfolios and wages growth rates.

Contemporaneous Interest Rates and Wages Growth

It is common to define the nominal rate of interest (r) as the sum of²

$$(1) \quad r = i + \rho$$

where

i = expected real rate

ρ = inflation premium.

Similarly, it is common to express the rate of growth in wages (g) as the sum of

$$(2) \quad g = \ell + p$$

where

ℓ = rate of growth in the marginal productivity of labor, and

p = rate of change in the general price level.

To the extent that i is related to ℓ and ρ is related to p there is a tendency to drive r and g to a constant relation. To assert a relationship between i and ℓ is merely to assert that the real rate of interest is related to the productivity of labor. The relation between the real rate of interest and the productivity of labor rests on

a relation between the productivity of real capital and the productivity of labor. Relationships among the real rate of interest, the marginal productivity of labor, and the marginal productivity of real capital emerge from steady-state equilibrium conditions of the economy. Suppose we define the marginal productivity of capital as $\pi = mp_K P / P_K$ (where $mp_K P$ = capital's marginal revenue product in constant prices, and P_K = the price of capital) and the productivity of labor as $L = mp_L P / (W/i)$ (where $mp_L P$ = labor's marginal revenue product in constant prices, and W = the nominal wage for labor). Using K to denote the capital stock, N for the number employed and T to denote the state of technology along with other factors affecting the productive process (e.g., natural resources and education), we follow tradition in asserting that

$$(3) \quad \pi = u(K, N, T)$$

where $\frac{\partial \pi}{\partial K} < 0$, $\frac{\partial \pi}{\partial N} > 0$, $\frac{\partial \pi}{\partial T} > 0$, and

$$(4) \quad L = v(K, N, T)$$

where $\frac{\partial L}{\partial K} > 0$, $\frac{\partial L}{\partial N} < 0$, and $\frac{\partial L}{\partial T} > 0$.

In equilibrium, $(mp_K P / i) = P_K$ and $mp_L P = W$; hence,

$$(5) \quad \pi = L = i.$$

Equation (3) says that the productivity of capital declines as additional units of capital are added to an unchanged quantity of labor and an unchanged state of technology. In contrast, capital

becomes more productive as units of labor are added and/or as technology improves. Equation (4) states that the productivity of labor declines as additional units are added to an unchanged capital stock, given a state of technology. But the productivity of labor improves as a consequence of increases in the capital stock and/or improvements in technology. Equation (5) is an assertion that, given the state of technology, adjustments occur in the capital stock and in the quantity of labor until there is equality among the marginal productivity of capital, the marginal productivity of labor and the real rate of interest.

The flow of real investment is given, ceteris paribus, by the real rate of interest, i . Assumptions regarding the pace of steady-state growth (presumably specified in terms of $\frac{\partial \pi}{\partial T}$ and $\frac{\partial L}{\partial T}$) have implications, at length, for the flow of real investment net of depreciation and, hence, for the needed level of i . The adjustments defined in (3) and (4) continue until (5) obtains. These adjustments also bring about the steady-state relation between q and i .

To assert a relationship between ρ and p is merely to assert that expectations are determined, at length, by experience. Thus, for example, we might imagine that ρ is a weighted average of past changes in the price level. If so, the means of ρ and p would be equal, but their variances would differ ($\sigma_{\rho}^2 < \sigma_p^2$).

As a point of departure, suppose that the economic system has equilibrium properties that serve to drive r and g toward equality over long periods of time.³ Given this point of departure, it is plausible to imagine that the economy finds itself with whatever

actual differential exists at the beginning of a loss period, $(r-g)_{t=0}$. We refer to this differential as the contemporaneous differential. We can imagine that the economy acts as though it compares this actual position at $t=0$ with its desired position at the end of period k , $(r-g)_{t=k}^*$. We refer to this desired differential as the equilibrium differential. Finally, we can imagine that adjustments, $\Delta(r-g)_t$, occur to alter the differential during t (from $t=0$ to $t=k$) toward the desired position. The size of that adjustment is given by λ . Thus, we imagine the familiar stock-adjustment model as follows:

$$(6) \quad \Delta(r-g)_t = \lambda[(r-g)_{t=k}^* - (r-g)_{t=0}].$$

Within the framework of the stock-adjustment model, λ is the portion of a discrepancy between the economy's equilibrium and actual $(r-g)$ position to be removed during t . Hence, according to this approach $0 > \lambda \leq 1$. The more rapid the equilibrium adjustment the closer is λ to 1.

Return on Dedicated Portfolios and Wages Growth

To the extent that the adjustments of the economic system can be meaningfully portrayed by equation (6), there can be a similar modeling in terms of realized differentials rather than contemporaneous differentials. But we do not expect the mapping to be one-to-one. Indeed, we regard developments with realized differentials to be only a distant reflection of developments with contemporaneous differentials.

Two things happen to dedicated portfolios as interest rates change. If, for example, interest rates decline while wages growth remains unchanged, there will be capital gains (tending to reduce the size of the initially required portfolio). However, there are reductions in the interest income on the portfolio as reinvestment rates decline (tending to increase the size of the initially required portfolio). The adjustments to be estimated below are affected by how, on balance, shifts such as these impact the realized yield on dedicated portfolios.

Using the stock-adjustment framework specified above, but moving away from first differences of the dependent variable, we rewrite (6) as

$$(7) \quad (r' - g')_{t=k} = \lambda' (r'^* - g')_{t=k} + (1 - \lambda')(r' - g')_{t=0}$$

where

r' = realized yield on dedicated portfolios during period
between $t=0$ and $t=k$

g' = realized rate of wages growth during period between $t=0$
and $t=k$

$(r' - g')$ = realized differential⁴

λ' = coefficient of adjustment for dedicated portfolios

k = length of period.

Recasting (7) into a regression format, we specify that

$$(8) \quad (r' - g')_{t=k} = a_0 + a_1 (r' - g')_{t=0} + \epsilon$$

where

$$a_0 = \lambda'(r'^* - g')_{t=k},$$

$$a_1 = (1 - \lambda'), \text{ and}$$

$$\varepsilon = \text{a stochastic term.}$$

The adjustment coefficient, $\lambda'(\hat{\lambda}' = 1 - a_1)$, measures the extent to which the initial differential (i.e., $(r' - g')_{t=0}$) is eroded during the period of adjustment defined by k . Thus, for example, if \hat{a}_1 were .25 it would mean that λ' would be .75. In turn, this would imply that 75 percent of any discrepancy between $(r'^* - g')_{t=k}$ (i.e., an equilibrium $(r' - g')$ for a period of k years) and the initial position would be removed by the end of k years.

III. Data and Results

A data set of artificial experience with dedicated portfolios was developed for the research reported above. The means by which the dedicated portfolios were developed is discussed fully in [1]. Even so, it is useful to sketch the process briefly. Following the discussion of the data, there is a presentation of results of regression analysis.

Data

The data problem involved identifying the endowment required in a dedicated investment portfolio at $t=0$ to provide exactly for the earnings stream of the average U.S. worker over alternative time periods. Given that initial amount along with the initial annual wages and the ex post growth in wages (g') over the time period, it was possible to solve for the implied ex post return (r') on each dedicated portfolio. With these data it was possible to calculate

$(r'-g')$, the ex post or realized, differential between the discount rate and the growth rate.

The input data for the identification of dedicated portfolios consisted of interest rates on various maturities of U.S. Treasury securities and rates of growth in wages for the 1953-1984 period. Dedicated portfolios were constructed, alternatively, of 1-year, 5-year, 10-year, and 20-year U.S. Treasury securities. That is, each portfolio consisted wholly, at least initially, of securities of a specific maturity. As time passed, any additions to the portfolio consisted of securities of that same maturity, or the longest maturity within the remainder of the loss period. Alternative time periods of investigation included loss periods of 5, 10, 15, and 20 years.

Results

Successive regressions were fitted with dedicated portfolios of alternative composition. Each regression contained all loss periods. Results are shown in Table 1. Our chief interest attaches to estimates of a_1 , for these estimates permit us to determine the size of the adjustment coefficient, λ' . In our model the specification of the equilibrium differential is neglected; hence, its average effect ends up in the constant term. To compensate for the misspecification, various autocorrelation adjustments are used. The order of the adjustments is shown as a superscript on the Durbin-Watson statistic for each regression.

According to the estimates in which $0 > \hat{a}_1 \leq 1$, the economy operated in such a fashion that dedicated portfolios were driven between 42.7 percent and 94.8 percent of the way toward their equilibrium

values. In any one instance (with 20-year securities for a five-year loss period) was there a significant estimate of λ' that was negative. The length of the periods is defined by k , which varies in length in the estimates of a_1 shown in Table 1. To facilitate meaningful comparisons the adjustment periods are restated in terms of what they imply about the length of time required to achieve 95 percent closure of any discrepancy between an initial position, $(r'^*-g')_{t=0}$, and a desired position, $(r'-g')_{t=k}$.⁵ Those data are shown in Table 2.

 Insert Tables 1 and 2 Here

Limiting our attention to coefficients that reach acceptable levels of statistical significance, data in Table 2 indicate that the period of adjustment ranges from 16.3 years to 39.7 years for loss periods of 10 years or more. Hence, by the end of these loss periods the effects of a pre-existing discrepancy would have been eroded substantially.

The estimates of $(1-\lambda')$ for the five-year loss periods are larger than one with dedicated portfolios consisting of securities with maturities equal to or greater than five years. Such estimates are inconsistent with the stock-adjusted framework. For, as indicated, a dominant feature of the stock-adjustment model is that the initial position of the dependent variable is driven toward some central position.⁶ Estimates of $(1-\lambda') > 1$ imply that $\lambda' < 0$. Hence, whatever the initial discrepancy, the adjustment process enlarges upon that discrepancy. No adjustment periods are shown in Table 2 for $\hat{a}_1 > 1$,

although the common sense meaning of such an estimate is that the differential at the beginning of the loss period will persist throughout the period.

Setting aside the stock-adjustment interpretation, it is of interest simply to explore the statistical association between the initial contemporaneous differential and the realized differential for periods of k duration. The question at hand is whether knowledge of the contemporaneous differential permits the formation of a better estimate of the realized differential ($r' - g'$) than is provided by its mean.

For purposes of discussion we limit ourselves to the consideration of realized differentials for twenty-year loss periods, but with regressions fitted to data from 1-year, 5-year, 10-year and 20-year securities. In Table 3 we display realized differentials corresponding to initial contemporaneous differentials ranging from 0 to 5 percent.

Insert Table 3 Here

Dedicated portfolios consisting of 1-year and 5-year securities are least-cost portfolios. That is, these portfolios have the smallest negative ($r' < g'$) or the largest positive ($r' > g'$) realized differentials. Consistently, their realized differentials are larger than realized differentials corresponding to dedicated portfolios consisting of longer-term Treasury securities (see Table 3). Because our interest is in least-cost portfolios, we have limited our consideration to these portfolios.

Dedicated portfolios consisting of 1-year securities show very little differences among realized differentials corresponding to different initial contemporaneous differentials. Dedicated portfolios consisting of 1-year Treasury securities are least-cost portfolios in instances wherein the contemporaneous differential is 2 percent or less. The realized differential corresponding to a zero contemporaneous differential with portfolios consisting of a 5-year securities is -0.47; the realized differential corresponding to a 5 percent contemporaneous differential is .19 percent.

A remaining important issue is whether the predictions provided by the model as fitted provide meaningfully improved estimates of the realized differentials. In our earlier research we concluded that a zero differential is appropriate for valuation purposes. In this paper we are interested in whether the research reported here provides justification for altering the valuation differential--i.e., the realized differential $(r' - g')_{t=k}$ --based on the contemporaneous differential (i.e., $(r - g)_{t=0}$).

The practical answer to that question is no. In addition to the realized differentials, Table 3 reports the standard deviation of residuals around the estimated values. An examination of these residuals indicates that the estimated realized differentials do not differ significantly from zero. All except one of the estimated realized differentials with dedicated portfolios consisting of 1-year or 5-year securities falls within one standard deviation of zero. The single outlier (-0.47) is 1.21 standard deviations below zero. For practical purposes there is no way for a statistically significant

difference to emerge. As contemporaneous differentials depart from their means there are corresponding increases in the standard errors associated with the resulting estimates.⁷ Portfolios consisting of 10-year and 20-year securities are neglected because they are consistently more costly than are portfolios consisting of shorter maturity securities.

IV. Concluding Remarks

This paper is addressed explicitly toward a single issue. Namely, does the contemporaneous differential at the beginning of a loss period provide useful information regarding the realized differential for the loss period considered as a whole? The answer to that question is a resounding no. But, the paper also provides insights into other issues. It reinforces views relating to the equilibrating properties of the economy, and it provides practical advice regarding the appropriate $(r'-g')$ differential for valuation purposes. The appropriate differential is approximately zero.

Footnotes

¹As we pointed out in [1], the focus of the literature relating to the measurement of pecuniary loss in lawsuits has been the contemporaneous relationships between the growth in wages and rates of return on alternative investment media (i.e., $r_t - g_t$). As it turns out, however, contemporaneous relationships are not of primary importance and can be misleading. The covariance of ultimate importance emerges from the relationships between the actual growth in wages (g') and the realized rate of return (r') on what we refer to as dedicated portfolios--that is, portfolios dedicated toward meeting future wage payments as they arise in the actual course of time.

²Equation (1) emerges as follows: $(1+r) = (1+i)(1+\rho)$. Expanding the right-hand side then subtracting one from both sides, we get $r = i + \rho + \rho i$. Following convention, cross-products are ignored. Equation (2) is developed in a similar fashion.

³In point of fact we do not offer this as a theoretical matter. But an empirical matter over the periods we and others have investigated the $(r-g)$ differential has been approximately zero.

⁴In terms of actual measurement, the effective differential at time zero, $(r'-g')_{t=0}$, is the contemporaneous differential at time zero.

⁵By construction, full closure is not attained until $k = \infty$.

⁶The central position need not be a constant. Rather, it may be functionally related to other variables.

⁷For discussion see any standard statistics text. For example, an excellent discussion is given in Maddala, G. S., Econometrics, 1977, pp. 81-82.

Bibliography

1. Bryan, William R. and Charles M. Linke, "Estimating Present Value of Future Earnings: Experience with Dedicated Portfolios," Journal of Risk and Insurance, forthcoming.

Table 1: Regression Results

Dedicated Portfolios Consisting of:	Five Year Loss				Ten Year Loss				Fifteen Year Loss				Twenty Year Loss			
	a_0	a_1	R^2	D-W	a_0	a_1	R^2	D-W	a_0	a_1	R^2	D-W	a_0	a_1	R^2	D-W
1-Year Securities	-1.59 (-5.75)	.235 (5.28) [17.19]	.96	1.99 ⁴	-.43 (-2.46)	.159 (3.04) [16.06]	.79	1.56 ³	-.97 (-10.50)	.062 (3.19) [48.63]	.89	2.27 ⁴	-.27 (-3.04) [29.04]	.052 (1.60) [29.04]	.80	2.19 ²
5-Year Securities	-.50 (-5.75)	1.071 (6.04) [-0.40]	.70	2.04 ⁴	-.46 (-1.95)	.341 (1.53) [2.96]	.41	1.85 ²	-.48 (-2.29)	.313 (1.77) [3.87]	.38	2.03 ³	-.47 (4.15)	.132 (1.27) [8.29]	.23	1.92 ²
10-Year Securities	-1.58 (-2.82)	1.558 (5.07) [-1.82]	.71	1.99 ²	-1.08 (-3.23)	.573 (2.12) [1.56]	.42	2.02 ²	-.92 (-5.11)	.271 (2.10) [5.65]	.70	2.02 ⁴	-1.07 (-3.37)	.204 (0.90) [3.49]	.39	2.04 ²
20-Year Securities	-3.06 (-3.79)	1.840 (4.77) [-2.18]	.66	2.02 ²	-2.02 (-6.30)	.357 (0.98) [1.76]	.46	2.02 ³	-2.32 (-4.73)	.282 (1.30) [3.30]	.52	1.97 ¹	-2.17 (-3.87) [3.44]	.221 (0.98) [3.44]	.57	1.99 ²

NOTE: t statistic for parameters as estimated (i.e., $(1-\lambda')$) are in parenthesis; t statistics in brackets are for λ' . Superscripts of Durbin-Watson ratios indicate the order of the adjustment for autocorrelation of residuals (1 = first order; 2 = second order; etc.).

Table 2: Estimates of Adjustment Periods

Dedicated Portfolios Consisting of:	Number of Years to Equilibrium* for Loss Periods of:			
	5 Years	10 Years	15 Years	20 Years
1-Year Securities	10.3	16.3	16.1	20.3
5-Year Securities	--	27.8	38.7	29.6
10-Year Securities	--	54.2**	34.5	37.7
20-Year Securities	--	29.1**	35.5	39.7

*95 percent closure

**Adjustment coefficient not statistically significant at 1 percent level

Table 3: Contemporaneous and Realized Differentials
for Twenty-Year Loss Periods

Dedicated Portfolios Consisting of:	Realized Differentials in Percents Corresponding to Beginning Contemporaneous (r-g) Differentials of:					
	0%	1%	2%	3%	4%	5%
1-Year Securities	-0.27% (0.36)	-0.21% (0.38)	-0.16% (0.41)	-0.11% (0.46)	-0.05% (0.51)	-0.00% (0.57)
5-Year Securities	-0.47% (0.39)	-0.32% (0.41)	-0.19% (0.45)	-0.05% (0.50)	0.04% (0.56)	0.17% (0.64)
10-Year Securities	-1.07% (0.54)	-0.88% (0.56)	-0.67% (0.61)	-0.47% (0.69)	-0.26% (0.78)	-0.06% (0.89)
20-Year Securities	-2.17 (0.45)	-2.02% (0.46)	-1.79% (0.51)	-1.57% (0.58)	-1.35% (0.66)	-1.13% (0.75)

The standard deviations of the estimated values are shown in parentheses.

HECKMAN
BINDERY INC.



JUN 95

Round - To - Please® N MANCHESTER,
INDIANA 46962

UNIVERSITY OF ILLINOIS-URBANA



3 0112 060296008