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# Sequential Rationality, Implementation and Communication in Games

*Bhaskar Chakravorti*

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# Sequential Rationality, Implementation and Communication in Games *

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## ABSTRACT

The question motivating this paper is: what conditions define the bounds on a mechanism designer's ability to implement social objectives when the agents indulge in communication prior to participation in the mechanism? Communication may involve mediation, correlation (to expand the equilibrium set) and the transmission of costly signals (to refine the equilibrium set). The objectives of this paper are: to show that a sequential structure is essential to the study of communication; to identify an appropriate equilibrium concept -- sequential mediated equilibrium; to show how standard refinement criteria based on forward induction arguments amy be extended to the problem at hand; and to provide a class of necessary and sufficient conditions for implementation of social performance standards in environments with such communication. Every member of the class is identified with a particular restriction on off-the-equilibrium-path beliefs. A limited characterization of implementability is also given.



JEL classification numbers: 025, 026.

## 1. INTRODUCTION

### *1.1 Summary*

A mechanism designer devises a game to be played by economic agents. If they stand to benefit from it, typically, agents will communicate with each other before playing the game. The question motivating this paper is: what conditions define the bounds on the designer's ability to implement social objectives when the agents have such opportunities for communication? Communication may stem from two seemingly opposing reasons. On the one hand, communication, especially through a mediator, facilitates correlation and enlarges the equilibrium set. This could add attractive outcomes to the set of self-enforcing outcomes. On the other hand, communication may involve the adoption of costly signals to facilitate forward induction. The latter may refine the equilibrium set. This could eliminate unattractive or unreasonable outcomes from the set of self-enforcing outcomes.

The multiple motivations and resulting complexity of communication demands a sufficiently rich model. Within the framework of such a model, the objectives of this paper are: (i) to show that a sequential structure is essential to the study of communication even if there is only a single round of communication and even if the communication is simply cheap talk; (ii) to identify an appropriate equilibrium concept for the induced sequential game; (iii) to show how refinement criteria based on forward induction arguments may be extended to the problem at hand; and (iv) to establish a class of necessary and sufficient conditions for implementability of social performance standards in environments with such communication opportunities. Every member of this class is identified with a particular restriction on off-the-equilibrium-path beliefs. A limited characterization of implementability is also given.

Single-round mediated communication in incomplete information games for enlarging the equilibrium set is generally analyzed within the following model of a communication system (primarily due to Myerson ([18], [20]). Prior to participation in a game, privately informed agents send signals to an unbiased mediator[1] who in turn sends messages (possibly correlated) back to the agents. Finally, the agents choose a move in the underlying game. The signals that the agents send are essentially "cheap talk" and have no effect on payoffs. In addition, by the Revelation Principle (Myerson ([18], [20]), there is no loss of generality in restricting attention to "direct" communication systems where agents signal their types and the mediator recommends moves in the underlying game. The game induced by pre-play communication is solved by applying the concept of *communication equilibrium* (Myerson ([19], [20]) which is a generalization of *correlated equilibrium* (Aumann [1]).

An alternative role of communication has been emphasized in the literature on signaling (Spence [25], Cho and Kreps [5], etc.). Agents indulge in "costly talk" to communicate via forward induction (Kohlberg and Mertens [11]). This serves to refine the equilibrium set.

In general, when agents are confronted with a game there are elements of both the paradigms discussed above that motivates communication. For simplicity, we will assume that there is a single round of communication prior to a game. (The extension to multiple rounds *a la* Forges [8] is discussed later.) It is common knowledge that all agents focus on a particular "language of communication". The selection of a language is

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[1]The mediator does not belong to the set of agents. It is an unbiased outsider and could be a suitably programmed computer. It is easy to show that a mediator can be built by the agents.

similar to the selection of an equilibrium in the sense that the selection process itself is not part of our model.

This paper proceeds in three steps. First, it is argued that there is a fundamental sequential structure to the game induced by communication. The appropriate solution concept for such games is *sequential mediated equilibrium* (*SME*). Conditions for the existence of SME are given.

Especially in light of the sequential nature of the model, a truly general examination of the effects of communication ought to take into consideration the choice of costly talk as well as cheap talk. In a variety of economic applications, costly signals are deliberately chosen by "better types" of agents to be transmitted to other agents. They serve an important purpose of communicating private information and focusing on more "reasonable" equilibria. The implications of costly signals in our model are several. We must consider communication systems that are rich enough to permit the choice of such signals by agents. Models of mediated communication (as in Myerson [18], [20]) and unmediated communication, especially games with pre-play cheap talk (Crawford and Sobel [6], Farrell and Gibbons [7], Mathews and Postlewaite [14], etc.) and signaling games with costly talk (Spence [26], Cho and Kreps [5], Banks and Sobel [3], etc.) would be special cases of the "rich" communication system presented in this paper. The second part of the paper discusses how forward induction based arguments may be applied towards refining the set of SME.

Finally, we employ this structure to address the problem of mechanism design in environments with communication opportunities. Any mechanism that the designer asks the agents to participate in is susceptible to pre-play communication. The designer has no control over the language of communication chosen by the agents or the correlation function chosen by the mediator or the strategies chosen by the agents. In essence, we are confronted by a classic implementation problem (as in Maskin [13], Postlewaite and Schmeidler [25], Palfrey and Srivastava [23]).

The implementation problem considered in this paper is rendered more complex by the fact that agents can communicate prior to participation in the mechanism. Moreover, the communication may be costly which in turn has implications for the equilibrium concept and for social welfare. An appropriate notion of a "performance standard" is defined. For every restriction on off-the-equilibrium path beliefs (and the corresponding refinement of SME), necessary and sufficient conditions and a limited characterization of implementable performance standards are established.

Next, we present an example that motivates the sequential structure. Section 2 presents the model. Section 3 defines the equilibrium concept. Section 4 discusses refinements of the concept and Section 5 discusses implementation. The final section concludes.

### *1.2 An Example*

The following example provides a motivation for explicitly considering the sequentiality in communication and refining the set of communication equilibria. The agents indulge in one round of cheap talk through a mediator before playing the underlying game.

**Example 1:**

There are three agents 1, 2 and 3 playing a game, $\Gamma$. Agent 1's action space is $\{T, M, B\}$, agent 2's action space is $\{L, R\}$ and agent 3 has only one possible action $\{O\}$. Agents 1 and 3 have two possible types $t_1$ and $t_1^*$, where $i$ = 1, 3. Agent 2 has only one possible type $t_2$. Assume that prob$((t_1, t_2, t_3))$ + prob$((t_1^*, t_2, t_3^*))$ = 1. The payoffs in the game $\Gamma$ are given in Figure 1.

*[Insert FIGURE 1 here]*

Consider pre-play communication in this game with a direct communication system as in Myerson [20], i.e. the agents report their types confidentially to a mediator and the mediator privately recommends actions to each agent. A communication equilibrium (a Bayesian-Nash equilibrium of the induced game) is given as follows:

If agent 3 reports $t_3$, then the mediator recommends $(T, L, O)$ with probability one regardless of the other reports.

If agent 3 reports $t_3^*$, then the mediator recommends $(B, R, O)$ with probability one regardless of the other reports.

It may be checked that, assuming common knowledge of the mediator's rule, every player finds it an optimal strategy to report his/her type truthfully and to faithfully follow the mediator's recommendations.

We shall argue that this equilibrium is not sequentially rational. First, observe that there is a particular sequence in the game induced by the communication system. In the first (signaling) stage, every agent reports types and in a later (final decision) stage, actions are taken in the game $\Gamma$. In an intermediate stage, the mediator, whose private information is the agents' reports, sends recommendations. Thus, we have a sequential game with four players.

The agents and the mediator are treated asymmetrically. We shall consider trembles by agents that lead to parts of the game that are assigned zero probability in equilibrium. The mediator, being an unbiased mechanical device, is a dummy player. We presume that it never trembles. The justification generally given to analyses of deviations from equilibrium is as follows. Players typically cannot pre-commit to their components of an equilibrium list of strategies. A deviation from equilibrium may be expected to occur if it was being held in check by a non-credible threat to the deviator. Since the mediator is unbiased (i.e. with a constant utility function), these considerations of "threats" do not arise. Hence, we presume that it always plays according to the equilibrium.

Suppose, for some reason, type $t_3$ of agent 3 were to deviate from truth-telling and were to report $t_3^*$ instead. The mediator's rule given above dictates that *(B, R, O)* be recommended to the agents. Recall that the $t_1$ and $t_3$ types of agents 1 and 3 are perfectly correlated. Agent 1 (of type $t_1$), upon observation of *B*, would conclude that agent 2 has been recommended *R* since the mediator's rule is common knowledge. In addition, it is common knowledge that the mediator does not deviate from equilibrium and that agent 2's strategy in equilibrium is one of faithful obedience. Given these observations, type $t_1$ of agent 1 would deviate from the faithful obedience strategy and choose to play *M* (since he/she gets an increase in payoff of +1). Given that agent 1 will play *M* and agent 2 will play *R* if this deviation from the equilibrium path arises, type $t_3$ of agent 3 will indeed choose to deviate to a report of $t_3^*$ in the first stage (since he/she gets an increase in payoff of +1). Thus, the communication equilibrium described above does not survive.

It must be noted that our concern for sequential rationality is different from that of Myerson [19]. The latter considers a multi-stage game with communication as in Forges [8]. Our emphasis is on the sequence of strategic choices within any single stage of the Myerson/Forges framework.

In this paper, given a set $N$, a set $X_i$ with $x_i \in X_i$ for every $i \in N$, we write $\times_{i\in N} X_i$ as $X$, $(x_i)_{i\in N}$ as $x$ and $(x_j)_{j\in N\setminus\{i\}}$ as $x_{-i}$. Correspondingly, $x_i$ is the $i$-th component of $x$. For any $x \in X$, for all $i \in N$, let $x[x'_i] \equiv (x'_i, x_{-i})$. For any (finite) set $X$, $\Delta(X)$ is the set of probability distributions on $X$. For any sets $X$, $Y$, given a function $g$: $X \to \Delta(Y)$, $g(y|x)$ is the probability assigned to $y \in Y$ by $g$ when $x \in X$ is realized. For every $i \in N$, and any sets $X_i$ and $Y_i$, given a function $g_i$: $X_i \to Y_i$, we write $(g_j(x_j)_{j\in N})$ as $g(x)$. Given a set $X$, co($X$) is the convex hull of $X$.

Given these basic conventions, the model is as follows.

$N$ is a finite *set of agents*. $M_i$ is a finite *set of moves for agent i*. $A$ is a *set of outcomes* and is assumed to be Euclidean[2]. $\xi$: $M \to$ co($A$) is an *outcome function*. A simultaneous-move game (form) $\Gamma$ is a triple $\langle N, M, \xi\rangle$. $\mathfrak{M}$ is a *mediator* and $\mathfrak{M} \notin N$. The mediator receives reports drawn from a finite *report space*, $R_i$ from every agent $i$ and sends messages drawn from a finite *message space*, $Q_i$ to every $i$. The reports and messages are transmitted in a confidential manner. $\mathcal{R}$ and $\mathcal{Q}$ are, respectively, the class of joint reports and the class of joint messages spaces. A *general communication system* is characterized by a language. A *language*, $L$, is a pair $\langle R, Q\rangle$. $\mathcal{L} = \langle \mathcal{R}, \mathcal{Q}\rangle$ is the class of all languages. For every $(r, q) \in \bigcup_{R\in\mathcal{R}} R \times \bigcup_{Q\in\mathcal{Q}} Q$, there is a *trivial language*, $L^{\circ}$ defined by $L^{\circ} = \langle R^{\circ} = \{r\}, Q^{\circ} = \{q\}\rangle$. $\mathcal{L}^{\circ} \subseteq \mathcal{L}$ is the class of all trivial languages. Given $L \in \mathcal{L}$, a *game (form) with communication* is a pair $\langle \Gamma, L\rangle$.

---

[2]For example, $A$ could be interpreted as a set of commodity allocations in an Arrow-Debreu economy.

Each agent $i \in N$ has private information summarized by *i's type*, $t_i$. $T_i$ is the set of possible types for $i$ and is assumed to be finite. Given $L \in \mathcal{L}$, each agent $i \in N$ has preferences on co($A$) that are dependent on the type and report profiles. $i$'s preferences are representable by a VNM *utility function* $u_i^L$: co$(A) \times T \times R \rightarrow \mathbb{R}$. It is assumed that there is a common prior probability distribution on the joint types space $p^o$: $T \rightarrow [0, 1]$ from which $i$'s subjective posterior probability $p_i$: $T_i \rightarrow \Delta(T_{-i})$ can be derived by every $i \in N$ using Bayes' Law.

Remark 1: For purposes of interpretation, consider the following restriction on the class of utility functions: for all $L^o$, $\bar{L}^o \in \mathcal{L}^o$, $u^{L^o} = u^{\bar{L}^o}$. Under this restriction, communication by means of a trivial language corresponds to no communication. Since the restriction is not necessary for our results, we do not impose it.

To summarize, an *environment* $e$ is a list $\langle N, A, \mathcal{L}, T, (u^L)_{L \in \mathcal{L}}, p^o \rangle$. $\mathcal{E}$ is the class of environments. This model is common knowledge, in the sense of Aumann [2].

The definitions that follow are given for $\langle \Gamma, L \rangle$ with $\Gamma = \langle N, M, \xi \rangle$ and $L = \langle R, Q \rangle$.

$\mathfrak{M}(L) \equiv \{\mu^L: R \rightarrow \Delta(Q)\}$. $\mathfrak{M}(L)$ is the *space of mediation plans*. For all $i \in N$, $\tilde{S}_i(R) \equiv \{\sigma_i^L: T_i \rightarrow \Delta(R_i)\}$ and $\tilde{D}_i(L, M) \equiv \{\delta_i^L: T_i \times R_i \times Q_i \rightarrow \Delta(M_i)\}$. $\tilde{S}_i(R) \times \tilde{D}_i(L, M)$ is the *strategy space for agent i*. The strategy space reflects two stages of strategy choice: the first being the *signaling stage* and the second being the *final decision stage*. For all $i \in N$, $B_i(L) \equiv \{\beta_i^L: T_i \times R_i \times Q_i \rightarrow \Delta(T_{-i} \times R_{-i} \times Q_{-i})\}$ is the *space of eventual beliefs for agent i*. To minimize on notation, we shall suppress the superscript "$L$" on $\sigma$, $\mu$, $\delta$ and $\beta$ whenever it is clear that the language being employed is $L$.

The introduction of communication induces a sequence of moves before a game is played. Given a game $\Gamma$, and a language $L$, the sequential game induced by $\langle\Gamma, L\rangle$ is as follows: $N \cup \mathfrak{M}(L)$ is the set of players. The play proceeds in three stages: in the signaling stage every $i \in N$ with information $t_i \in T_i$ and beliefs $p_i$ sends a report $r_i \in R_i$ to $\mathfrak{M}(L)$; in the second stage, $\mathfrak{M}(L)$ with information $r \in R$ sends a messsage $q_i \in Q_i$ to every $i \in N$; in the final decision stage, every $i \in N$ with information $(t_i, r_i, q_i) \in T_i \times R_i \times Q_i$ and beliefs $\beta_i$ selects a move $m_i \in M_i$. The payoffs to the members of $N$ are dependent on the moves chosen, the profile of types and the reports sent in the signaling stage and are given by the utility functions $u^L$. The mediator, $\mathfrak{M}(L)$, is a dummy player in the sense that its utility is independent of all strategies chosen in the game.

Remark 2: The model presented here is a generalization of that of Myerson [20]. The reports spaces that we consider could contain payoff-relevant reports. Moreover, Myerson's focus is on a characterization of the set of all equilibrium outcomes generated by a game paired with all possible languages (with payoff-irrelevant reports). We fix a pair $\langle\Gamma, L\rangle$ and inquire about the set of equilibria generated by this pair. Finally, Myerson studies the induced game in the strategic-form, whereas we consider the extensive-form.

## 3. SEQUENTIAL MEDIATED EQUILIBRIUM

Let $\langle\Gamma, L\rangle$ be given. This section presents the equilibrium concept for the three-stage game induced by $\langle\Gamma, L\rangle$.

For every $t \in T$, $\sigma(\cdot|t)$ is the joint distribution on $R$ induced by $(\sigma_i(\cdot|t_i))_{i\in N}$ and for every $t \in T$, $r \in R$ and $q \in Q$, $\delta(\cdot|t, r, q)$ is the joint distribution on $M$ induced by $(\delta_i(\cdot|t_i, r_i, q_i))_{i\in N}$.

The following notation is used to denote expected utilities in the signaling and in the final decision stages:

for every $i \in N$, $t_1 \in T_1$, and $(\sigma, \mu, \delta) \in \tilde{S}(R) \times \mathfrak{M}(L) \times \tilde{D}(L, M)$,

$$U_1^L(\xi, \sigma, \mu, \delta; t_1) \equiv$$

$$\sum_{t_{-1} \in T_{-1}} p(t_{-1}|t_1) \sum_{r \in R} \sigma(r|t) \sum_{q \in Q} \mu(q|r) \sum_{m \in M} \delta(m|t, r, q) u_1^L(\xi(m), t, r).$$

for every $i \in N$, $t_1 \in T_1$, $(\sigma, \mu, \delta) \in \tilde{S}(R) \times \mathfrak{M}(L) \times \tilde{D}(L, M)$, $\beta_1 \in B_1(L)$, $r_1 \in R_1$, and $q_1 \in Q_1$,

$$V_1^L(\xi, \delta, \beta_1; t_1, r_1, q_1) \equiv$$

$$\sum_{t_{-1} \in T_{-1}} \sum_{r_{-1} \in R_{-1}} \sum_{q_{-1} \in Q_{-1}} \beta_1(t_{-1}, r_{-1}, q_{-1}|t_1, r_1, q_1) \sum_{m \in M} \delta(m|t, r, q) u_1^L(\xi(m), t, r)$$

Given $\mu \in \mathfrak{M}(L)$ and $r_1 \in R_1$, let $\bar{Q}_1(\mu, r_1) \equiv \{q \in Q: \mu(q|r)$ for some $r_{-1} \in R_{-1}\}$.

An equilibrium for the sequential game induced by $\langle\Gamma, L\rangle$ is obtained as follows: fix a strategy for the mediator, say $\mu$, and, presuming that the mediator never deviates from $\mu$[3], define the sequential equilibrium (Kreps and Wilson [12]) of the $N$-player game induced by $\langle\Gamma, L\rangle$ and $\mu$.

**Definition 1:** A quadruple $(\sigma, \mu, \delta, \beta) \in \tilde{S}(R) \times \mathfrak{M}(L) \times \tilde{D}(L, M) \times B(L)$ is a *sequential mediated equilibrium* (*SME*) of $\langle\Gamma, L\rangle$ if

$\forall i \in N$, $\forall t_1 \in T_1$, $\forall \sigma'_1 \in \tilde{S}_1(R)$, $\forall \delta'_1 \in \tilde{D}_1(L, M)$, $\forall r_1 \in R_1$, $\forall q_1 \in \bar{Q}_1(\mu, r_1)$, (i) – (ii)*d* hold.

(i) $U_1^L(\xi, \sigma, \mu, \delta; t_1) \geq U_1^L(\xi, \sigma[\sigma'_1], \mu, \delta; t_1)$ and

(ii) $\exists$ a sequence $\{\sigma^n, \beta^n\}_{n=1}^{\infty}$ such that (ii)a – (ii)d are met:

(ii)a. $\forall n \in \{1, 2, \ldots\}$, $\sigma^n \in \tilde{S}(R)$ is such that $\forall t \in T$, $\forall r \in R$, $\sigma^n(r|t) > 0$,

---

[3] Recall the discussion from the previous section: the mediator is a dummy player. Hence, it does not deviate from equilibrium.

(ii)b. $\forall n \in \{1, 2, \ldots\}$, $\beta^n \in B(L)$ is derived from $(p, \sigma^n, \mu)$ by the application of Bayes' Law.

(ii)c. $(\sigma, \beta) = \lim_{n \to \infty} (\sigma^n, \beta^n)$ and

(ii)d. $$V_1^L(\xi, \delta, \beta_1; t_1, r_1, q_1) \geq V_1^L(\xi, \delta[\delta_1'], \beta_1; t_1, r_1, q_1).$$

In the sequel, whenever we construct $\beta \in B(L)$ from a pair $(\sigma, \mu) \in S(R) \times \mathfrak{M}(L)$ in the manner given in (ii)a – (ii)c above, we shall say that $\beta$ is *justified by* $(\sigma, \mu)$.

$SME(\Gamma, L) \equiv \{(\sigma, \mu, \delta, \beta) \in \tilde{S}(R) \times \mathfrak{M}(L) \times \tilde{D}(L, M) \times B(L)$: $(\sigma, \mu, \delta, \beta)$ is an *SME* for $\langle\Gamma, L\rangle\}$.

The mediator is a dummy player whose utility is constant over all strategies and who does not tremble from an equilibrium. Thus, given any $\mu \in \mathfrak{M}(L)$, the $(N+1)$-player sequential game can be re-written as an $N$-player sequential game. By definition, $SME(\Gamma, L)$ is non-empty if a sequential equilibrium of the $N$-player sequential game induced by $(\Gamma, L)$ and $\mu$ exists. By the general existence theorem for perfect equilibrium (and, therefore, of sequential equilibrium) of finite games (Selten [26]), we have:

Theorem 1: *Let $\langle\Gamma, L\rangle$ be a game with communication.* $SME(\Gamma, L) \neq \varnothing$.

## 4. REFINEMENTS

As is the case with sequential equilibrium and communication equilibrium, the set of SME can be rather large. Much of the communication between agents may be directed not only towards expanding the set of equilibrium outcomes but also towards focusing on more "reasonable" equilibria. It is in the context of such refinement possibilities that the richness of the reports space plays a crucial role. An SME may be rationalized by a set of beliefs that are unreasonable off the equilibrium path. Costly reports often serve to eliminate such equilibria. Consider the following example.

**Example 2:**

Suppose that $N = \{1, 2\}$, $T_1 = \{t_1, \bar{t}_1\}$ and $T_2 = \{t_2\}$. Each type of agent 1 could occur with equal probability. The game $\Gamma = \langle N, M, \xi \rangle$ is such that $M_1 = \{m_1, \bar{m}_1\}$ and $M_2 = \{m_2, \bar{m}_2\}$. Suppose that the agents agree on a language, $L = \langle R, Q \rangle$, where $R_1 = \{r_1, \bar{r}_1\}$, $R_2 = \{r_2\}$, $Q_1 = \{q_1\}$ and $Q_2 = \{q_2, \bar{q}_2\}$. The payoffs for $\langle \Gamma, L \rangle$ are given by Figure 2. The non-genericity of the payoffs is not crucial. The matrices are to be read as follows:

Top left matrix: agent 1 sends report $r_1$ and is of type $t_1$.

Top right matrix: agent 1 sends report $r_1$ and is of type $\bar{t}_1$.

Bottom left matrix: agent 1 sends report $\bar{r}_1$ and is of type $t_1$.

Bottom right matrix: agent 1 sends report $\bar{r}_1$ and is of type $\bar{t}_1$.

*[Insert FIGURE 2 here]*

Consider the following SME:

$\sigma_1(r_1|t_1) = \sigma_1(r_1|\bar{t}_1) = 1;$ $\sigma_2(r_2|t_2) = 1.$

$\mu((q_1, q_2)|(r_1, r_2)) = \mu((q_1, \bar{q}_2)|(\bar{r}_1, r_2)) = 1.$

$\delta_1(m_1|t_1, r_1, q_1) = \delta_1(m_1|\bar{t}_1, r_1, q_1) = 1,$

$\delta_1(\bar{m}_1|t_1, \bar{r}_1, q_1) = \delta_1(\bar{m}_1|\bar{t}_1, \bar{r}_1, q_1) = 1.$

$\delta_2(m_2|t_2, r_2, q_2) = \delta_2(m_2|t_2, r_2, \bar{q}_2) = 1.$

Agent 2's beliefs $\beta_2$ assign greater probability to type $t_1$ in the event that $\bar{q}_2$ is observed.

A forward induction argument would rule out the equilibrium given above. Suppose agent 2 were to observe a disequilibrium message $\bar{q}_2$. Given that it is common knowledge that the mediator does not err and uses the strategy $\mu$, agent 2 would conclude that agent 1 has sent a report $\bar{r}_1$. Type $\bar{t}_1$ is more likely to gain from deviating from the equilibrium report $r_1$ than type $t_1$ regardless of the move that agent 2 chooses. Hence, an observation of $\bar{q}_2$ could be interpreted as an implicit message from agent 1 that she is indeed of type $\bar{t}_1$. If agent 2 assigns a higher probability on agent 1 being of type $\bar{t}_1$, he would switch to $\bar{m}_2$ after observing $\bar{q}_2$. This behavior can be anticipated by type $\bar{t}_1$ who would indeed have the incentive to deviate to $\bar{r}_1$ and obtain a payoff of 2 instead of 1. A more reasonable SME would be:

$$\hat{\sigma}_1(r_1|t_1) = \hat{\sigma}_1(\bar{r}_1|\bar{t}_1) = 1; \qquad \hat{\sigma}_2(r_2|t_2) = 1.$$

$$\hat{\mu}((q_1, q_2)|(r_1, r_2)) = \hat{\mu}((q_1, \bar{q}_2)|(\bar{r}_1, r_2)) = 1.$$

$$\hat{\delta}_1(m_1|t_1, r_1, q_1) = \hat{\delta}_1(m_1|\bar{t}_1, r_1, q_1) = 1,$$

$$\hat{\delta}_1(\bar{m}_1|t_1, \bar{r}_1, q_1) = \hat{\delta}_1(\bar{m}_1|\bar{t}_1, \bar{r}_1, q_1) = 1.$$

$$\hat{\delta}_2(\bar{m}_2|t_2, r_2, q_2) = \hat{\delta}_2(\bar{m}_2|t_2, r_2, \bar{q}_2) = 1.$$

Agent 2's beliefs $\hat{\beta}_2$ assign greater probability to type $\bar{t}_1$ in the event that $\bar{q}_2$ is observed.

The basic intuition behind the elimination of SME discussed above may be formalized by extending the various criteria for restricting off-the-equilibrium path beliefs suggested in the literature, (e.g. McLennan [15], Kohlberg and Mertens [11], Cho and Kreps [5], Banks and Sobel [3], Grossman and Perry [9], Okuno-Fujiwara and Postlewaite [21]).

The bite of any given refinement criterion is considerably dulled in a communication system with a mediator. The receiver of a disequilibrium message would find it more difficult to interpret the message as an implicit statement about an agent's type since there are several senders in addition to a mediator who may introduce noise. Typically, for a disequilibrium message to relay information about types, it must be such that it can be traced back to the deviator.

In the remaining sections of the paper, it will be assumed to be common knowledge that all agents focus on *some* restriction on off-the-equilibrium path beliefs. The restriction will presumably be chosen from the menu of restrictions available in the literature (e.g. Divinity, Universal Divinity, D1, Never Weak Best Response, etc.). We shall refer to the one being employed as restriction $X$. Since the logic underlying a restriction depends on the functions $\sigma$, $\mu$ and $\xi\circ\delta$, given $(\sigma, \mu, \delta, \beta)$, we say that $\beta$ *satisfies criterion* $X(\sigma, \mu, \xi\circ\delta)$ if restriction $X$ is being employed to constrain beliefs. Observe that if the underlying game $\Gamma$ were replaced by some allocation function $z\colon T \times R \times Q \to \text{co}(A)$, the same logic underlying restriction $X$ could be applied and we would say that $\beta$ satisfies criterion $X(\sigma, \mu, z)$.

Given $\Gamma = \langle N, M, \xi\rangle$, let $SME_X(\Gamma, L) \equiv \{(\sigma, \mu, \delta, \beta) \in SME(\Gamma, L)\colon \beta$ satisfies the criterion $X(\sigma, \mu, \xi\circ\delta)\}$.

## 5. IMPLEMENTATION

### *5.1 Definitions*

For the remaining sections in the paper, we relax the requirement that the set of joint moves, $M$, be finite. It is assumed that for all $t \in T$, $p^{o}(t) > 0$. In addition, we shall focus on the set of pure-strategy SME[4].

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[4]The analysis can be extended to include more general prior distributions and the case of mixed strategies. The restrictions are made for

It is assumed that the mediator can randomize over the set of messages. For agent $i$, the pure strategy sets are given by $S_i(R) \equiv \{s_i: T_i \to R_i\}$ and $D_i(L, M) \equiv \{d_i: T_i \times R_i \times Q_i \to M_i\}$. The notation for denoting expected utilities are extended in the obvious manner. The set of pure strategy SME of $(\Gamma, L)$ with beliefs that satisfy restriction $X$ is written as $SME_X^*(\Gamma, L)$.

Let $G \equiv \{g: T \to co(A)\}$. Given $L = \langle R, Q\rangle$, let $\mathcal{F}^L \equiv \{f = (g, s) \in G \times S(R)\}$ and $\mathcal{F} \equiv \times_{L \in \mathcal{L}} \mathcal{F}^L$.

Given $L$, $\mathcal{F}^L$ is a set of pairs of functions. The first component of the pair specifies a type-contingent outcome mixture in co($A$) and the second component specifies a type-contingent report profile.

Let $E(\Gamma, L) \equiv \{f \in \mathcal{F}^L$: $\exists(s, \mu, d, \beta) \in SME_X^*(\Gamma, L)$ such that $\forall t \in T$, $f(t) = ( \sum_{q \in Q} \mu(q|s(t))\xi(d(t, s(t), q)), s(t))\}$.

**Definition 2:** A *performance standard* is a subset of $\mathcal{F}$ and is written as $\varphi = \times_{L \in \mathcal{L}} \varphi^L$. For every $L \in \mathcal{L}$, $\varphi^L$ is assumed to be non-empty. $\varphi_G^L$ and $\varphi_S^L$ are the projections of $\varphi^L$ on $G$ and $S(R)$ respectively. It is assumed that for all $L$, $L' \in \mathcal{L}$, $g \in \varphi_G^L$ if and only if $g \in \varphi_G^{L'}$. In the sequel, given that $\varphi$-optimality of a function $g \in G$ is language-invariant, we shall simply write $g \in \varphi_G$.

A performance standard as defined here is different from the usual notion. Typically, in the social choice/mechanism theory literature, a performance standard associates a set of outcomes (in $A$) with every state of the world (in $T$). The recommendation of the performance standard is based upon some pre-determined optimality considerations. These considerations rely on the presumption that every agent's utility function is defined on the domain $A \times T$. We refer to this as first-best optimality.

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convenience. The more general cases are much more cumbersome.

The society that we are analyzing has opportunities for communication whenever its members are confronted with a game. Eventually, a social planner must invite these agents to participate in a game designed to implement the performance standard. Communication may involve the use of signals that affect the agents' utilities, i.e. every agent's utility function is defined on co($A$) $\times T \times R$ if the language of communication is $L = \langle R, Q \rangle$. Given the distortion caused by the signals, the planner needs to specify some notion of second-best optimality. Hence, for every $t$ in $T$ chosen by Nature, every element of a performance standard must not only specify an outcome in co($A$) but also recommend a profile of reports in $R$. Given ($g$, $s$) in $\varphi^L$, $g$ is based upon first-best considerations and $s$ is chosen so that it minimizes distortion from the first-best. Moreover, the planner has no control over the selection of the language of communication. The agents agree upon a language among themselves. Since the planner's recommendation of $g$ in $\varphi_G$ is based on first-best considerations, it is language-invariant. For every conceivable language $\bar{L} = \langle \bar{R}, \bar{Q} \rangle$ in $\mathcal{L}$, a distortion minimizing signal $s$ in $S(\bar{R})$ must be recommended.

**Definition 3:** A performance standard $\varphi$ is *implementable* if $\exists \Gamma = \langle N, M, \xi \rangle$ such that

(i) $\forall L \in \mathcal{L}$, $E(\Gamma, L) \neq \emptyset$ and

(ii) $\forall L \in \mathcal{L}$, $E(\Gamma, L) \subseteq \varphi^L$.

The definition of implementation has three crucial features. First, it takes account of the fact that a social planner's role is limited to the design of a game or mechanism. Once the agents are confronted with the game, they will choose $L$ as the language of communication. The properties desired of the set of equilibria of the game must hold regardless of which $L$ in $\mathcal{L}$ is chosen. Second, the planner, the mediator and the agents are independent decision-making entities. In any equilibrium of the mechanism, the choice of $(s_i, d_i, \beta_i)$ is made by each $i$ in $N$ and the choice of $\mu$ is made by the mediator. Given that the planner has no control over these choices, Condition (ii) in the definition above ensures that regardless of the choice of $(s, \mu, d, \beta)$, the corresponding equilibrium is $\varphi$-optimal. Third, as Condition (i) implies, we do not insist on "full" implementation. The presumption is that the planner is indifferent between the alternative non-empty subsets of $\varphi$ and, therefore is content with ensuring that $E(\Gamma, L)$ is a non-empty subset of $\varphi^L$ instead of being equal to the set $\varphi^L$ itself. In keeping with the tradition in the literature, the equilibrium concept and the refinement criterion being employed is assumed to be common knowledge among the agents and the planner.

Remark 3: Given the condition in Remark 1 above, communication with a trivial langauge corresponds to no communication. In this situation, it may be checked that our definition of implementation indicates that $\varphi$ is implementable in the absence of communciation whenever it is implementable. Communication opportunities make the implementation problem an ever more difficult one to resolve.

In the sequel, the notation given below is employed:

Given $L = \langle R, Q\rangle \in \mathcal{L}$ let $Z(L) \equiv \{z: T \times R \times Q \to \mathrm{co}(A)\}$.

$$U_j^L(z, s, \mu; t_j) \equiv \sum_{t_{-j}\in T_{-j}} p(t_{-j}|t_j)\sum_{q\in Q}\mu(q|s(t))u_j(z(t, s(t), q), t, s(t)),$$

$$V_j^L(z, \beta_j; t_j, r_j, q_j) \equiv$$

$$\sum_{t_{-j}\in T_{-j}} \sum_{r_{-j}\in R_{-j}} \sum_{q_{-j}\in Q_{-j}} \beta_j(t_{-j}, r_{-j}, q_{-j}|t_j, r_j, q_j)u_j(z(t, r, q), t, r),$$

For every $j \in N$ and $q_j \in Q_j$, given a function $\alpha|_{q_j}: T_j \to T_j$,

$$U_j^L(z, s[s'_j/\alpha_j], \mu; t_j) \equiv$$

$$\sum_{t_{-j}\in T_{-j}} p(t_{-j}|t_j)\sum_{q\in Q}\mu(q|s[s'_j\circ\alpha|_{q_j}](t))u_j(z(t, \quad s[s'_j\circ\alpha|_{q_j}](t), \quad q), \quad t,$$

$s[s'_j \circ \alpha|_{q_j}](t)),$

For all $z \in Z(L)$, $z^{\alpha q i} \in Z(L)$ is defined by $z^{\alpha q i}(t, r, \bar{q}) = z((\alpha|_{q_i}(t_i), t_{-i}), r, \bar{q})$ for all $(t, r, \bar{q}) \in T \times R \times Q$.

Next, we shall define three important properties of performance standards that will be critical for identifying implementable standards.

**Definition 4:** Suppose $g \in \varphi_G$. A performance standard $\varphi$ satisfies *Property 1(X, g)* if the following holds:

$\forall L = \langle R, Q \rangle \in \mathcal{L}$, $\forall j \in N$, $\forall \alpha_j = \{(\alpha|_{q_j} : T_j \to T_j)_{q_j \in Q_j}\}$,

IF

(i) $\exists \{s, \bar{s}\} \subseteq S(R)$, $\mu \in \mathfrak{M}(L)$, $\{z, \bar{z}\} \in Z(L)$ and $\bar{g} \in G$ such that

$\forall t \in T,\ g(t) = \sum_{q \in Q} \mu(q|s(t)) z(t, s(t), q),$

$\bar{g}(t) = \sum_{q \in Q} \mu(q|\bar{s}(t)) z((\alpha|_{q_i}(t_i), \bar{s}_i(t_i), q_i)_{i \in N})$ and

$\forall (t, r, q) \in T \times R \times Q,\ \bar{z}(t, r, q) \equiv z((\alpha|_{q_i}(t_i), r_i, q_i)_{i \in N})$

(ii) $\forall i \in N$, $\forall s'_i \in S_i(R)$, $\forall z' \in Z(L)$, statements (A) and (B) imply statements (C) and (D),

THEN

$(\bar{g}, \bar{s}) \in \varphi^L$,

where (A)-(D) are given by:

(A) $\forall t_i \in T_i$,

$$U_i^L(z, s, \mu; t_i) \geq U_i^L(z, s[s'_i/\alpha_i], \mu; t_i).$$

(B) $\exists \beta_i \in B_i(L)$ such that $\beta_i$ is justified by $(s, \mu)$ and satisfies criterion $X(s, \mu, z)$ and $\forall t_i \in T_i$, $\forall r_i \in R_i$, $\forall q_i \in \bar{Q}_i(\mu, r_i)$,

$$V_i^L(z, \beta_i; t_i, r_i, q_i) \geq V_i^L(z'^{\alpha q i}, \beta_i; t_i, r_i, q_i).$$

(C) $\forall t_i \in T_i$,

$$U_i^L(\bar{z}, \bar{s}, \mu; t_i) \geq U_i^L(\bar{z}, \bar{s}[s'_i], \mu; t_i).$$

(D) $\exists \bar{\beta}_i \in B_i(L)$ such that $\bar{\beta}_i$ is justified by $(\bar{s}, \mu)$ and satisfies criterion $X(\bar{s}, \mu, \bar{z})$ and $\forall t_1 \in T_1$, $\forall r_1 \in R_1$, $\forall q_1 \in \bar{Q}_1(\mu, r_1)$,

$$V_1^L(\bar{z}, \bar{\beta}_1; t_1, r_1, q_1) \geq V_1^L(\bar{z}', \bar{\beta}_1; t_1, r_1, q_1),$$

where $\forall(t, r, q) \in T \times R \times Q$, $\bar{z}'(t, r, q) \equiv z'((\alpha|_{q_j}(t_j), r_j, q_j)_{j\in N})$.

This complicated property has its origins in the Maskin-monotonicity condition (Maskin [13]) which is at the root of the critical properties employed by the implementation literature (Postlewaite and Schmeidler [25], Palfrey and Srivastava [23], Mookherjee and Reichelstein [16], Jackson [10], etc.). Whereas the other properties alluded to can be stated without reference to an allocation or a belief restriction, Property *1(X, g)* is stated by fixing a particular function $g$ in $\varphi_G$ and a restriction $X$ on beliefs.

**Definition 5:** Suppose $g \in \varphi_G$. A performance standard $\varphi$ satisfies *Property 2(X, g)* if the following holds:

$\forall L = \langle R, Q\rangle \in \mathcal{L}$, $\forall j \in N$, $\forall \alpha_j = \{(\alpha|_{q_j}: T_j \to T_j)_{q_j \in Q_j}\}$,

IF

(i) $\exists\{s, \bar{s}\} \subseteq S(R)$, $\mu \in \mathfrak{M}(L)$, $\{z, \bar{z}\} \in Z(L)$ and $\bar{g} \in G$ such that

$\forall t \in T$, $g(t) = \sum_{q\in Q} \mu(q|s(t))z(t, s(t), q)$,

$\bar{g}(t) = \sum_{q\in Q} \mu(q|\bar{s}(t))z((\alpha|_{q_i}(t_i), \bar{s}_i(t_i), q_i)_{i\in N})$ and

$\forall(t, r, q) \in T \times R \times Q$, $\bar{z}(t, r, q) \equiv z((\alpha|_{q_i}(t_i), r_i, q_i)_{i\in N})$

(ii) $\forall i \in N$, $\forall s'_i \in S_i(R)$, $\forall z' \in Z(L)$, statements (B) and (C) are met whenever (A) holds $\forall g' \in G$ satisfying $g'((\alpha|_{q_j}(t_j))_{j\in N}) = z'((\alpha|_{q_j}(t_j))_{j\in N}, \bar{s}[s'_i](t), q)$ for all $t \in T$, for all $q \in Q$ such that $\sum_{q\in Q} \mu(q|\bar{s}[s'_i](t)) > 0$.

THEN

$(\bar{g}, \bar{s}) \in \varphi^L$,

where (A)-(C) are given by:

(A) for all $t_i \in T_i$, for all $r \in \bigcup_{\bar{R} \in \mathcal{R}} \bar{R}$, for all $q_i \in Q_i$, there exists $L' = \langle R', Q' \rangle \in \mathcal{L}$ such that $r \in R'$ and

$$\sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i) u_i^{L'}(g(t), t, r) \geq$$

$$\sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i) u_i^{L'}(g'(\alpha|_{q_i}(t_i), t_{-i}), t, r).$$

(B) for all $t_i \in T_i$,

$$U_i^L(\bar{z}, \bar{s}, \mu; t_i) \geq U_i^L(\bar{z}, \bar{s}[s'_i], \mu; t_i).$$

(C) for all $t_i \in T_i$, $r_i \in R_i$, $q_i \in \bar{Q}_i(\mu, r_i)$,

$$V_i^L(\bar{z}, \bar{\beta}_i; t_i, r_i, q_i) \geq V_i^L(\bar{z}', \bar{\beta}_i; t_i, r_i, q_i),$$

where $\bar{z}' \in Z(L)$ is defined by $\bar{z}'(t, r, q) = z'((\alpha|_{q_i}(t_i), r_i, q_i)_{i \in N})$ for all $(t, r, q) = T \times R \times Q$.

Property $2(X, g)$ also derives from Maskin's [13] monotonicity condition. The relation with Property $1(X, g)$ is discussed later.

**Definition 6:** Suppose that $g \in \varphi_G$. A performance standard $\varphi$ satisfies *multi-lingual incentive compatibility with respect to g (MIC(g))* if $\forall L^o = \langle R^o, Q^o \rangle \in \mathcal{L}^o$, $\forall i \in N$, $\forall t_i, t'_i \in T_i$, given $\{r^o\} = R^o$,

$$\sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i) u_i^{L^o}(g(t), t, r^o) \geq \sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i) u_i^{L^o}(g(t'_i, t_{-i}), t, r^o).$$

The MIC($g$) property is closely related to the standard incentive compatibility or self-selection condition.

Next, we shall state a list of assumptions on the class of environments, $\mathcal{E}$, and on $\varphi$.

Consider the following assumptions on the class of environments $\mathcal{E}$.

*[A1(a)]:* $\forall i \in N$, $\exists \bar{a}^i \in \text{co}(A)$ defined by: $\forall t \in T$, $\forall L = \langle R, Q \rangle \in \mathcal{L}$, $\forall r \in R$, $\forall a \in \text{co}(A) \setminus \{\bar{a}^i\}$, $u_i^L(\bar{a}^i, t, r) > u_i^L(a, t, r)$ and $\forall j \in N \setminus \{i\}$, $\bar{a}^i \neq \bar{a}^j$.

*[A1(b)]* $\exists \underline{a} \in \text{co}(A)$ defined by: $\forall i \in N$, $\forall t \in T$, $\forall L = \langle R, Q \rangle \in \mathcal{L}$, $\forall r \in R$, $\forall a$

$\in co(A)\setminus\{\underline{a}\}$, $u_1^L(\underline{a}, t, r) < u_1^L(a, t, r)$.

*[A2]:* $|N| > 2$.

*[A3]:* $\forall i \in N$, $\forall r \in \bigcup_{R \in \mathcal{R}} R$, $\forall L = \langle R, Q\rangle \in \mathcal{L}$, $\forall L' = \langle R', Q'\rangle \in \mathcal{L}$, $\forall t \in T$, $\forall a, a' \in co(A)$, if $r \in R \cap R'$, then $u_1^L(a, t, r) \geq u_1^L(a', t, r)$ implies $u_1^{L'}(a, t, r) \geq u_1^{L'}(a', t, r)$.

[A1(a)] and [A1(b)] are assumptions that are met in typical "economic" environments with agents' preferences that are strictly monotone in their own consumption vectors. The first assumption states that there is a most preferred outcome for each agent and is distinct from the most preferred outcomes of others. This would correspond to the outcome that allocates the entire resources of the economy to the agent in question. The second assumption requires the existence of a universally least preferred outcome. This corresponds to the outcome that allocates none of the economy's resources to any agent. [A3] is an independence assumption on preferences. *Ceteris paribus,* a given report has the same impact on an agent's preference ordering over the outcomes in co($A$) regardless of the composition of the reports space from which it is drawn. The assumption seems reasonable for most applications. For example, an expensive education (the signal) has the same effect on the payoffs of a job candidate and a potential employer regardless of whether or not there are other alternative signals of the candidate's ability. Furthermore, it does not matter what the alternatives are.

In addition, consider the following assumptions on performance standards.

*[A4]* If $\varphi \subseteq \mathcal{F}$ is a performance standard, then $\forall t \in T$, $\forall g \in \varphi_G$, $g(t) \neq \underline{a}$ and $\forall i \in N$, $g(t) \neq \bar{a}^1$.

*[A5]* If $\varphi \subseteq \mathcal{F}$ is a performance standard, then $\forall L = \langle R, Q\rangle \in \mathcal{L}$, $\forall (g, s) \in$

$\varphi^L$, $\forall i \in N$, $\forall t_i \in T_i$, $s_i(t_i) \in \text{argmax}_{r_i \in R_i} \{ \sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i) u_i^L(g(t), t, r_i, s_{-i}(t_{-i}))\}$.

Given *[A1]*, the assumption *[A4]* is met by various sub-classes of performance standards (defined for "economic" environments) that are generally of interest to economists. For example, *[A4]* is met by the class of all non-dictatorial, Pareto-efficient standards, or the class of all envy-free, Pareto efficient standards or, in environments where every agent has strictly positive initial endowments, the class of all individually rational, Pareto-efficient standards.

Given that efficiency is generally a desideratum, the assumption *[A5]* is justifiable in view of the fact that the role of $s$ in $(g, s) \in \varphi^L$ is to minimize distortion from first-best optimality. The requirement of existence of a function $s$ satisfying the assumption limits the class of environments. However, in the sub-class of environments with *private signaling costs*, where an individual's reports have no externalities, such an $s$ always exists, given finiteness of $R$. This sub-class satisfies assumption *[B]* below. Though *[B]* rules out a number of important applications, it is met in several economic problems of interest.

*[B]* $\forall i \in N$, $\forall L = \langle R, Q\rangle \in \mathcal{L}$, $\forall r, r' \in R$, $u_i^L(\cdot, \cdot, r) = u_i^L(\cdot, \cdot, r')$ if $r_i = r'_i$.

In terms of the education example, [B] would require that education affects the utility of a job candidate and has no benefits to an employer. In other words, from the employer's perspective, its role is purely that of a signal. [B], however, is not necessary for our results.

### *5.2 Results on Implementation*

The following two theorems provide necessary conditions for implementation.

**Theorem 2:** *Let $\varphi$ be a performance standard. If $\varphi$ is implementable, then there exists $g \in \varphi_G$ such that $\varphi$ satisfies Property 1(X, g).*

Proof: Choose $L = \langle R, Q\rangle \in \mathcal{L}$. By definition of implementation, there exists $\Gamma = \langle N, M, \xi\rangle$, $f \in \varphi^L$ and $(s, \mu, d, \beta) \in SME^*_X(\Gamma, L)$ such that for all $t \in T$, $f(t) = (\sum_{q\in Q} \mu(q|s(t))\xi(d(t, s(t), q)), s(t))$. Next, for all $j \in N$, choose $\alpha_j = \{(\alpha|_{q_j}: T_j \to T_j)_{q_j \in Q_j}\}$. Let $g(t) = \sum_{q\in Q} \mu(q|s(t))\xi(d(t, s(t), q))$ and suppose that there exists $\bar{s} \in S(R)$ and $\bar{g} \in G$ such that $\bar{g}(t) = \sum_{q\in Q} \mu(q|\bar{s}(t))\xi((d_i(\alpha|_{q_i}(t_i), \bar{s}_i(t_i), q_i)_{i\in N})$ for all $t \in T$. Define $\bar{d} \in D(L, M)$ by $\bar{d}(t, r, q) = (d_i(\alpha|_{q_i}(t_i), r_i, q_i)_{i\in N}$ for all $(t, r, q) \in T \times R \times Q$. Define $z, \bar{z} \in Z(L)$ by $z = \xi \circ d$ and $\bar{z} = \xi \circ \bar{d}$. Choose $i \in N$ and suppose that part (ii) of the hypothesis of Property 1(X, g) is met, i.e. for all $s'_i \in S_i(R)$ and all $z' \in Z(L)$, (A) and (B) imply (C) and (D) below.

(A) for all $t_i \in T_i$,

$$U^L_i(z, s, \mu; t_i) \geq U^L_i(z, s[s'_i/\alpha_i], \mu; t_i).$$

(B) for all $t_i \in T_i$, $r_i \in R_i$ and $q_i \in \bar{Q}_i(\mu, r_i)$,

$$V^L_i(z, \beta_i; t_i, r_i, q_i) \geq V^L_i(z'^{\alpha q i}, \beta_i; t_i, r_i, q_i).$$

(C) for all $t_i \in T_i$,

$$U^L_i(\bar{z}, \bar{s}, \mu; t_i) \geq U^L_i(\bar{z}, \bar{s}[s'_i], \mu; t_i).$$

(D) there exists $\bar{\beta}_i \in B_i(L)$ such that $\bar{\beta}_i$ is justified by $(\bar{s}, \mu)$ and satisfies criterion $X(\bar{s}, \mu, \bar{z})$ and for all $t_i \in T_i$, $r_i \in R_i$, and $q_i \in \bar{Q}_i(\mu, r_i)$,

$$V^L_i(\bar{z}, \bar{\beta}_i; t_i, r_i, q_i) \geq V^L_i(\bar{z}', \bar{\beta}_i; t_i, r_i, q_i),$$

where for all $(t, r, q) \in T \times R \times Q$, $\bar{z}'(t, r, q) \equiv z'((\alpha|_{q_j}(t_j), r_j, q_j)_{j\in N})$.

Choose $(s'_1, d'_1) \in S_1(R) \times D_1(L, M)$. By definition of SME, we have

(E) for all $t_1 \in T_1$,

$$U^L_1(\xi, s, \mu, d; t_1) \geq U^L_1(\xi, s[s'_1/\alpha_1], \mu, d; t_1).$$

(F) for all $t_1 \in T_1$, for all $r_1 \in R_1$ and all $q_1 \in \bar{Q}_1(\mu, r_1)$,

$$V^L_1(\xi, d, \beta_1; t_1, r_1, q_1) \geq$$
$$V^L_1(\xi, d[d'^{\alpha q 1}_1], \beta_1; t_1, r_1, q_1),$$

where $d'^{\alpha q1}_1 \in D_1(L, M)$ is defined by $d'^{\alpha q1}_1(\cdot, \cdot, \cdot) = d'_1(\alpha|_{q_1}(\cdot), \cdot, \cdot)$.

By setting $z' = \xi \circ d[d'_1]$, the hypothesis of Property $1(X, g)$ yields

(G) for all $t_1 \in T_1$,

$$U^L_1(\xi, \bar{s}, \mu, \bar{d}; t_1) \geq U^L_1(\xi, \bar{s}[s'_1], \mu, \bar{d}; t_1).$$

(H) there exists $\bar{\beta}_1 \in B_1(L)$ such that $\bar{\beta}_1$ is justified by $(\bar{s}, \mu)$ and satisfies criterion $X(\bar{s}, \mu, \xi \circ \bar{d})$ and for all $t_1 \in T_1$, for all $r_1 \in R_1$ and all $q_1 \in \bar{Q}_1(\mu, r_1)$,

$$V^L_1(\xi, \bar{d}, \bar{\beta}_1; t_1, r_1, q_1) \geq$$
$$V^L_1(\xi, \bar{d}[d'^{\alpha q1}_1], \bar{\beta}_1; t_1, r_1, q_1).$$

By definition of an SME, (E) and (F) must hold for all $i \in N$ and all $(s'_1, d'_1) \in S_1(R) \times D_1(L, M)$. Hence (G) and (H) must hold for all $i \in N$ and all $(s'_1, d'_1) \in S_1(R) \times D_1(L, M)$. We conclude that $(\bar{s}, \mu, \bar{d}, \bar{\beta}) \in SME^*_X(\Gamma, L)$.

By construction, $\bar{g}(t) = \sum_{q \in Q} \mu(q|\bar{s}(t))\xi(\bar{d}(t), \bar{s}(t), q))$ for all $t \in T$. By definition of implementation, $(\bar{g}, \bar{s}) \in \varphi^L$. Thus, the conclusion of Property $1(X, g)$ is satisfied. ■

**Theorem 3:** *Let $\varphi$ be a performance standard. If $\varphi$ is implementable, then there exists $g \in \varphi_G$ such that $\varphi$ satisfies MIC(g).*

<u>Proof:</u> The proof follows that of the revelation principle (see Myerson [17]). Let $\Gamma = \langle N, M, \xi \rangle$. Choose $L^o = \langle R^o = \{r^o\}, Q^o = \{q^o\} \rangle \in \mathcal{L}$. By

definition of implementation, there exists $(f^L)_{L \in \mathcal{L}} \in \varphi$ and $(s, \mu, d, \beta) \in SME^*_X(\Gamma, L^o)$ such that for all $t \in T$, $f^{L^o}(t) = (\xi(d(t, s(t), q^o)), s(t))$. Let $g(t) = \xi(d(t, s(t), q^o))$. By definition of a sequential mediated equilibrium, the following must hold:

for all $i \in N$, for all $t_i \in T_i$, for all $\alpha_i^o$: $T_i \times R_i^o \times Q_i^o \to T_i \times R_i^o \times Q_i^o$,

$$V_i^{L^o}(\xi, d, \beta_i; t_i, s_i(t_i), q_i^o) \geq$$
$$V_i^{L^o}(\xi, d[d_i \circ \alpha_i^o], \beta_i; t_i, s_i(t_i), q_i^o) \qquad [3]$$

Since $|R_i^o| = |Q_i^o| = 1$ and for all $i \in N$, $g = \xi \circ d(\cdot, r^o, q^o)$, [3] implies that for all $i \in N$, for all $t_i, t_i' \in T_i$,

$$\sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i) u_i^{L^o}(g(t), t, r^o) \geq \sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i) u_i^{L^o}(g(t_i', t_{-i}), t, r^o). \qquad [4]$$

Thus, $\varphi$ satisfies MIC$(g)$. ■

The following theorem provides conditions under which we can design a game that implements a given performance standard. In conjunction with the two previous theorems, we have an extension of the results on implementation of Maskin [13], Postlewaite and Schmeidler [25], Palfrey and Srivastava [23], Mookherjee and Reichelstein [16] and Jackson [10] to implementation in environments with communication.

**Theorem 4:** *Suppose assumptions [A1] to [A3] on the class of environments* $\mathcal{E}$ *are satisfied. Let* $\varphi$ *be a performance standard. Suppose* $\varphi$ *satisfies assumptions [A4] and [A5]. If* $\varphi$ *satisfies Properties 2*$(X, g)$ *and MIC*$(g)$ *for some* $g \in \varphi_G$, *then* $\varphi$ *is implementable.*

Proof: The proof is by construction of an algorithm, $\mathcal{H}$. When a particular performance standard $\varphi$ and $g \in \varphi_G$ is applied to $\mathcal{H}$, we obtain a game $\mathcal{H}(\varphi,$

g). We claim that if the assumptions of the theorem are met and $\varphi$ satisfies Property $2(X, g)$ and MIC($g$), $\mathcal{H}(\varphi, g)$ implements $\varphi$.

Given $\varphi$ and $g \in \varphi_G$, $\mathcal{H}(\varphi, g) = \langle N, M, \xi \rangle$ is defined by:

(I) $\forall i \in N$, $M_i = \{m_i = (m_i^1, m_i^2, m_i^3) = (t_i(i), g(i), n(i)) \in T_i \times G \times \mathbb{R}_+\}$.

The following definitions shall be used:

**Definition 7:** $\forall i \in N$, $m_{-i}$ satisfies *Condition* $\gamma|i$ if

(i) $\forall j \in N\backslash\{i\}$, $g(j) = g$, and

(ii) $\forall j \in N\backslash\{i\}$, $n(j) = 0$.

$\forall m \in M$, $K(m) \equiv \{i \in N$: $\forall j \in N$, $n(i) \geq n(j)\}$.

(II) $\xi$: $M \rightarrow$ co($A$) is defined by the schematic diagram in Figure 3:

*[Insert FIGURE 3 here]*

The proof of the theorem is given by Lemma 1 and Lemma 4. Lemma 2 and 3 below are required to prove Lemma 4. It is presumed that the conditions of the Theorem are met. The proofs of the lemmata are in the appendix. ■

Lemma I: *For all* $L \in \mathcal{L}$, $\varphi^L \cap E(\mathcal{H}(\varphi, g), L) \neq \emptyset$.

Lemma 2: *For all* $L \in \mathcal{L}$, *for all* $(s, \mu, d, \beta) \in SME^*_X(\mathcal{H}(\varphi, g), L)$, *for all* $t \in T$, *for all* $q \in Q$ *such that* $\mu(q| s(t)) > 0$, $d(t, s(t), q)$ *satisfies Case 1.*

Lemma 3: *For all* $L \in \mathcal{L}$, *for all* $(s, \mu, d, \beta) \in SME^*_X(\mathcal{H}(\varphi, g), L)$, *for all* $i \in N$, *for all* $t \in T$, *for all* $r_i \in R_i$ *for all* $q \in Q$, *if* $\mu(q|r_i, s_{-i}(t_{-i})) > 0$, *then* $d_{-i}(t_{-i}, s_{-i}(t_{-i}), q_{-i})$ *satisfies Condition* $\gamma|i$.

Lemma 4: *For every* $L \in \mathcal{L}$, $E(\mathcal{H}(\varphi, g), L) \subseteq \varphi^L$.

Property $2(X, g)$ and MIC($g$) for some $g \in \varphi_G$ are sufficient conditions for implementation under the asumptions [A1]-[A5]. MIC($g$) is also a necessary condition. A natural question to ask is: under what additional restrictions is Property $2(X, g)$ necessary?

**Definition 8:** A game $\Gamma$ is *mediator neutral with respect to g* if for some $g \in \varphi_G$, $\forall L \in \mathcal{L}$,

(i) $\exists(s, \mu, d, \beta) \in SME_X^*(\Gamma, L)$ such that $\forall t \in T$, $g(t) = \sum_{q \in Q} \mu(q|s(t))\xi(d(t, s(t), q))$,

(ii) $\forall r \in R$, $\forall q \in Q$, $\mu(q|r) > 0$ and

(iii) $\forall t \in T$, $\forall r, r' \in R$, $\forall q, q' \in Q$, $d(t, r, q) = d(t, r', q')$.

Why is mediator neutrality an appealing restriction from a mechanism designer's standpoint? It is apparent that in environments with communication opportunities, the task of a designer is even more difficult than it is in the absence of communication. There are more variables that are beyond the control of the designer, especially the choices of the mediator. The mediator and the designer are distinct entities with different agendas. Mediator neutrality controls some aspects of the problem that are unobservable to the designer. (ii) ensures that off-the-equilibrium beliefs do not have to be constructed. Hence, for each $t$ in $T$, the derivation of the agents' beliefs is known to the designer. (iii) ensures that neither the agent's report nor the mediator's message, both of which are unobservable to the designer, has any effect on the final move chosen by each agent. (ii) and (iii) are required of only one equilibrium that achieves $g$. It appears to be hard to design an implementation game without satisfying mediator neutrality. In light of this fact, the sub-class of mediator neutral games seems to be a natural one in which the designer may search for the right mechanism.

The proofs of Theorem 2 and Lemma 1 yield the following corollary:

**Theorem 5:** *Let $\varphi$ be a performance standard. Suppose that assumptions [A1]-[A5] are satisfied. If $\varphi$ is implementable by a game that is mediator neutral with respect to some $g \in \varphi_G$, then $\varphi$ satisfies Property 2(X, g).*

**Definition 9:** $\varphi$ is *g-implementable* if $g \in \varphi_G$ and $\exists \Gamma\cdot = \langle N, M, \xi \rangle$ such that

(i) $\forall L \in \mathcal{L}$, $g \in E(\Gamma, L)$ and

(ii) $\forall L \in \mathcal{L}$, $E(\Gamma, L) \subseteq \varphi$.

*g*-implementability assures us of implementability of $\varphi$. In addition, it guarantees that $g$ in $\varphi_G$ is always achievable.

The following corollary of the preceding results provides a limited characterization of *g*-implementation.

**Theorem 6:** *Let $\varphi$ be a performance standard. Suppose that assumptions [A1] - [A5] are satisfied. Given $g \in \varphi_G$, $\varphi$ is g-implementable by a mediator neutral game if and only if $\varphi$ satisfies Property 2(X, g) and MIC(g).*

## 6. CONCLUDING REMARKS

This paper establishes a class of necessary and sufficient conditions for implementation in environments with communication possibilities prior to participation in a mechanism. The communication may be rather complex. It may involve correlated messages from a mediator to facilitate expansion of the equilibrium set and costly reports from the agents for a refinement of the equilibrium set. A game with communication induces a fundamental sequential game whose equilibrium is given by SME. SME may be refined by extending a standard restriction on off-the-equilibrium path beliefs. The concept of a "performance standard" must be modified in environments with costly communication possibilities. The definition of implementability also must be strengthened.

Despite the complicated nature of the problem, under a reasonable set of assumptions, it is possible to determine the bounds on implementability. The general structure of the necessary and sufficient conditions is independent of the particular restriction being assumed on the off-the-equilibrium path beliefs.

The Revelation Principle plays no role in our analysis. Since the particular language of communication agreed on by the agents is relevant for computing utilities to agents, we cannot restrict our attention to direct communication systems.

The problem of implementation in environments with communication opportunities has been studied in more specialized contexts by Palfrey and Srivastava [24] and Chakravorti [4]. The former studies unique implementation of incentive efficiency with trading mechanisms that are immune to pre-play communication. The latter considers the implementation problem under alternative communication regimes, i.e. when the channels of communication may be entirely public or entirely private or a mixture of public and private.

A limitation of our analysis is that we have not considered multi-stage communication systems. The analysis given here can be extended to this case. The primary difficulty of such an extension is that the structure of the resulting model is extremely cumbersome. The relevant equilibrium concept is an amalgam of SME and Myerson's [19] *sequential communication equilibria*. Such an extension is a topic for further research.

## REFERENCES


1. AUMANN, R.: Subjectivity and Correlation in Randomized Strategies, *Journal of Mathematical Economics,* 1 (1974), 67-96.

2. AUMANN, R.: Agreeing to Disagree, *Annals of Statistics,* 4 (1976), 1236-1239.

3. BANKS, J. AND J. SOBEL: Equilibrium Selection in Signaling Games, *Econometrica,* 55 (1987), 647-662.

4. CHAKRAVORTI, B.: How Should We Share Information? mimeo, University of Illinois at Urbana-Champaign, 1989.

5. CHO, I AND D. KREPS: Signaling Games and Stable Equilibria, *Quarterly Journal of Economics,* 102 (1987), 179-221.

6. CRAWFORD, V. AND J. SOBEL: Strategic Information Transmission, *Econometrica,* 50 (1982), 1431-1452.

7. FARRELL, J. AND R. GIBBONS: Cheap Talk Can Matter in Bargaining, *Journal of Economic Theory,* 48 (1989), 221-237.

8. FORGES, F.: An Approach to Communication Equilibria, *Econometrica,* 54 (1986), 1375-1386.

9. GROSSMAN, S. AND M. PERRY: Perfect Sequential Equilibria, *Journal of Economic Theory,* 39 (1986), 97-119.

10. JACKSON, M.: Full Bayesian Implementation, mimeo, Northwestern University, 1988.

11. KOHLBERG, E. AND J.-F. MERTENS: On the Strategic Stability of Equilibria, *Econometrica,* 54 (1986), 1003-1037.

12. KREPS, D. AND R. WILSON: Sequential Eqyulibria, *Econometrica,* 50 (1982), 863-894.

13. MASKIN, E.: Nash Equilibrium and Welfare Optimality, mimeo, MIT, 1977.

14. MATTHEWS, S. AND A. POSTLEWAITE: Pre-play Communication in Two-Person

Sealed-Bid Double Auctions, *Journal of Economic Theory,* 48 (1989), 238-263.

15. McLENNAN, A.: Justifiable Beliefs in Sequential Equilibria, *Econometrica,* 53 (1985), 885-904.

16. MOOKHERJEE, D. AND S. REICHELSTEIN: Implementation by Augmented Revelation Mechanisms, mimeo, Stanford University, 1988.

17. MYERSON, R.: Incentive Compatibility and the Bargaining Problem, *Econometrica,* 47 (1979), 61-74.

18. MYERSON, R.: Optimal Coordination Mechanisms in generalized Principal-Agent Problems, *Journal of Mathematical Economics,* 10 (1982), 67-81.

19. MYERSON, R.: Multi-Stage Games with Communication, *Econometrica,* 54 (1986), 323-358.

20. MYERSON, R.: Analysis of Conflict, mimeo, Northwestern University, 1987.

21. OKUNO-FUJIWARA AND A. POSTLEWAITE: Forward Induction and Equilibrium Refinement, mimeo University of Pennsylvania, 1987.

22. PALFREY, T. AND S. SRIVASTAVA: On Bayesian Implementable Allocations, *Review of Economic Studies,* 54 (1987), 193-208.

23. PALFREY, T. AND S. SRIVASTAVA: Implementation with Incomplete Information in Exchange Economies, *Econometrica,* 57 (1989).

24. PALFREY, T. AND S. SRIVASTAVA: Efficient Trading Mechanisms with Pre-Play Communication, mimeo, California Institute of Technology, 1989.

25. POSTLEWAITE, A. AND D. SCHMEIDLER: Implementation in Differential Information Economies, *Journal of Economic Theory,* 39 (1986), 14-33.

26. SELTEN, R.: Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games, *International Journal of Game Theory,* 4 (1975), 25-55.

27. SPENCE, M.: *Market Signaling,* Harvard University Press, Cambridge, MA,

1974.

$(t_1, t_2, t_3)$

| | L | R |
|---|---|---|
| T | 2, 2, 2 | 0, 0, 0 |
| M | 0, 0, 0 | 3, 2, 3 |
| B | 0, 0, 0 | 2, 2, 2 |

$(t_1^*, t_2, t_3^*)$

| | L | R |
|---|---|---|
| T | 2, 2, 2 | 0, 0, 0 |
| M | 0, 0, 0 | 2, 2, 2 |
| B | 0, 0, 0 | 2, 2, 2 |

FIGURE 1

$r_1$

$t_1$

| | $m_2$ | $\bar{m}_2$ |
|---|---|---|
| $m_1$ | 1, 2 | 1, 2 |
| $\bar{m}_1$ | 0, 2 | 1, 2 |

$\bar{t}_1$

| | $m_2$ | $\bar{m}_2$ |
|---|---|---|
| $m_1$ | 1, 2 | 1, 2 |
| $\bar{m}_1$ | 0, 2 | 1, 2 |

$\bar{r}_1$

$t_1$

| | $m_2$ | $\bar{m}_2$ |
|---|---|---|
| $m_1$ | 0, 0 | 1, −1 |
| $\bar{m}_1$ | 1, 3 | 0, 0 |

$\bar{t}_1$

| | $m_2$ | $\bar{m}_2$ |
|---|---|---|
| $m_1$ | 0, −1 | 1, 0 |
| $\bar{m}_1$ | 1, 0 | 2, 3 |

*FIGURE 2*

Proof of Lemma 1: Choose $L = \langle R, Q\rangle \in \mathcal{L}$. By definition of $\varphi$, there exists $s\colon T \to R$ such that $(g, s) \in \varphi^L$. We shall show that there exists $(s, \mu, d, \beta) \in SME_X^*(\mathcal{H}(\varphi, g), L)$ such that for all $t \in T$, $g(t) = \sum_{q\in Q} \mu(q|s(t))\xi(d(t, s(t), q))$. For all $i \in N$, for all $t_i \in T_i$, for all $r_i \in R_i$, for all $q_i \in Q_i$, let $d_i(t_i, r_i, q_i) = (t_i, g, 0)$. In addition, choose $\mu \in \mathfrak{M}(L)$ such that for all $r \in R$, $\mu(r) = \prod_{i\in N}\mu_i(r_i)$, where for all $i \in N$, $\mu_i\colon R_i \to \Delta(Q_i)$ is defined for all $r_i \in R_i$ and all $q_i \in Q_i$ by $\mu_i(q_i|r_i) = \frac{1}{|Q_i|}$. Since Case 1 applies, for all $t \in T$, for all $r \in R$ and all $q \in Q$, $\xi(d(t, r, q)) = g(t) = \sum_{q\in Q} \mu(q|s(t))\xi(d(t, s(t), q))$. We need to check that this outcome is an equilibrium with $\beta$ satisfying criterion $X(s, \mu, \xi\circ d)$.

Suppose agent $i \in N$ contemplates a deviation to $(s'_i, d'_i) \in S_i(R) \times D_i(L, M)$. Choose $t_i \in T_i$, $r_i \in R_i$ and $q_i \in Q_i$ and let $d'_i(t_i, r_i, q_i) = (t'_i, g', n')$. There are two possibilities ((i) and (ii) below) to be considered. Agent $i$ can distinguish between these possibilities since it is common knowledge that for all $j \in N\backslash\{i\}$, for all $t_j \in T_j$, for all $r_j \in R_j$ and all $q_j \in Q_j$, $d_j(t_j, r_j, q_j) = (t_j, g, 0)$.

(i) Suppose $g' = g$. Since for all $(t_{-i}, r_{-i}, q_{-i}) \in T_{-i} \times R_{-i} \times Q_{-i}$, $d_{-i}(t_{-i}, r_{-i}, q_{-i})$ satisfies Condition $\gamma|i$, either Case 1 or 2 applies. Given $(t_{-i}, r_{-i}, q_{-i}) \in T_{-i} \times R_{-i} \times Q_{-i}$, if Case 1 applies, the outcome is unchanged and if Case 2 applies, either $\xi(d'_i(t_i, r_i, q_i), d_{-i}(t_{-i}, r_{-i}, q_{-i})) = \underline{a}$ or $\xi(d'_i(t_i, r_i, q_i), d_{-i}(t_{-i}, r_{-i}, q_{-i})) = g(t'_i, t_{-i})$. Since $\varphi$ satisfies MIC($g$), for all $L^o \in \mathcal{L}^o$, given that $L^o = \langle\{r^o\}, \{q^o\}\rangle$,

$$\sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i)u_i^{L^o}(g(t), t, r^o) \geq \sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i)u_i^{L^o}(g(t'_i, t_{-i}), t, r^o).$$

By assumption [A3], for all $r \in R$,

$$\sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i)u_i^L(g(t), t, r) \geq$$

$$\sum_{t_{-1}\in T_{-1}} p_1(t_{-1}|t_1)u_1^L(g(t_1', t_{-1}), t, r). \qquad [6]$$

(ii) Suppose $g' \neq g$. Since for all $(t_{-1}, r_{-1}, q_{-1}) \in T_{-1} \times R_{-1} \times Q_{-1}$, $d_{-1}(t_{-1}, r_{-1}, q_{-1})$ satisfies Condition $\gamma|i$, Case 3 applies. Given $(t_{-1}, r_{-1}, q_{-1}) \in T_{-1} \times R_{-1} \times Q_{-1}$, either $\xi(d_1'(t_1, r_1, q_1), d_{-1}(t_{-1}, r_{-1}, q_{-1})) = \underline{a}$ or $\xi(d_1'(t_1, r_1, q_1), d_{-1}(t_{-1}, r_{-1}, q_{-1}) = g'(t_1', t_{-1}) \neq \underline{a}$. The latter can occur only if Case 3A applies. By definition of Case 3A, for all $r \in R$, there exists $L' = <R', Q'> \in \mathcal{L}$ such that $r \in R'$ and $\sum_{t_{-1}\in T_{-1}} p_1(t_{-1}|t_i)u_1^{L'}(g(t), t, r) \geq \sum_{t_{-1}\in T_{-1}} p_1(t_{-1}|t_1)u_1^{L'}(g'(t_1', t_{-i}), t, r)$. By assumption *[A3]*, for all $r \in R$,

$$\sum_{t_{-1}\in T_{-1}} p_1(t_{-1}|t_1)u_1^L(g(t), t, r) \geq$$

$$\sum_{t_{-1}\in T_{-1}} p_1(t_{-1}|t_1)u_1^L(g'(t_1', t_{-1}), t, r). \qquad [7]$$

By construction, $\mu$ is a joint distribution induced by $|N|$ independent randomizations $(\mu_j)_{j\in N}$. The observation of $q_1$ yields no information to agent $i$ about agent $j \neq i$. Also, for all $j \in N$, $\mu_j$ assigns positive and equal probability on every element in $Q_j$. We do not need to construct off-the-equilibrium path beliefs. In addition, $d_{-1}$ is independent of $\mu_{-1}(s_{-1}(t_{-1}))$ for all $t_{-1} \in T_{-1}$. Therefore, the only payoff-relevant parameters unknown to $i$ is $t_{-1}$ and $r_{-1}$. Recall that [6] holds for all $r \in R$. Thus, given the assumptions *[A1]* and *[A4]* regarding $\underline{a}$, for all $(t_1, r_1, q_1) \in T_1 \times R_1 \times Q_1$ for which possibility (i) above is true, [6] implies [8] below.

$$V_1^L(\xi, d, \beta_1; t_1, r_1, q_1) \geq$$

$$V_1^L(\xi, d[d_1'], \beta_1; t_1, r_1, q_1). \qquad [8]$$

where $\beta_1$ is justified by $(s, \mu)$.

The argument given in the preceding paragraph also applies to all $(t_1, r_1, q_1) \in T_1 \times R_1 \times Q_1$ for which possibility (ii) above is true. Recall that [7] holds for all $r \in R$. Thus, given the assumptions *[A1]* and *[A4]* regarding $\underline{a}$, for all such $(t_i, r_i, q_i) \in T_i \times R_i \times Q_i$, [7] implies [8].

[8] is true for all $t_i \in T_i$, for all $r_i \in R_i$ and all $q_i \in Q_i$. By construction, for all $j \in N$, $r_j \in R_j$ and $q_j \in Q_j$, $d_j$ is independent of $r_j$ and $q_j$. Thus, a deviation to $s'_i$ does not by itself affect the outcome. Recall that $s$ is such that $(g, s) \in \varphi^L$. By [A5], given that, by construction, $d$ is independent of $r$ and $q$, we have

for all $t_i \in T_i$,

$$U_i^L(\xi, s, \mu, d; t_i) \geq U_i^L(\xi, s[s'_i], \mu, d; t_i). \qquad [9]$$

From [8] and [9] we conclude that $(s, \mu, d, \beta) \in SME(\mathcal{H}(\varphi, g), L)$. By construction, $\mu$ assigns positive probability to every $q \in Q$. Trivially, the criterion $X(s, \mu, \xi \circ d)$ is met. Therefore, $(g, s) \in \dot{E}(\mathcal{H}(\varphi, g), L)$. ∎

<u>Proof of Lemma 2:</u> Choose $L = \langle R, Q \rangle \in \mathcal{L}$, $(s, \mu, d, \beta) \in SME_X^*(\mathcal{H}(\varphi, g), L)$, $i \in N$ and $(t_i, r_i, q_i) \in T_i \times R_i \times Q_i$, where $r_i = s_i(t_i)$ and $\mu(q|s(t)) > 0$ for some $t_{-i} \in T_{-i}$ and some $q_{-i} \in Q_{-i}$. Let $\hat{Q}_{-i}(s, \mu) = \{q'_{-i} \in Q_{-i}: \mu(q_i, q'_{-i}|\ s(t)) > 0$ for some $t_{-i} \in T_{-i}\}$. We shall establish that for all $t'_{-i} \in T_{-i}$, and all $q'_{-i} \in \hat{Q}_{-i}(s, \mu)$, $(d_i(t_i, r_i, q_i), d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i}))$ cannot satisfy any of the cases other than Case 1. We shall write $d(\cdot, \cdot, \cdot) = (d^1(\cdot, \cdot, \cdot), d^2(\cdot, \cdot, \cdot), d^3(\cdot, \cdot, \cdot))$, where the superscripts 1, 2 and 3 on $d(\cdot, \cdot, \cdot)$ denote the restrictions of $d(\cdot, \cdot, \cdot)$ to $T$, $G$ and $\mathbb{R}_+$ respectively.

Choose $t_{-i} \in T_{-i}$ and $q_{-i} \in \hat{Q}_{-i}(s, \mu)$. Let $m = d(t, s(t), q)$. Suppose that $m$ satisfies one of the following: Case 2, Case 3 or Case 4. A contradiction shall be established. Suppose $i$ contemplates a deviation to $d'_i \in D_i(L, M)$. Also, suppose that $d'(t_i, r_i, q_i) = (d^1_i(t_i, r_i, q_i), d^2_i(t_i, r_i, q_i), n')$, where $n'$ is such that for all $t'_{-i} \in T_{-i}$, for all $q'_{-i} \in Q_{-i}$,

$d^3_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i}) < n'$. There are two possibilities to be considered by $i$.

(i) There exists $j \in N$ such that $K(m) = \{j\}$. Therefore, for all $k \in N\setminus\{j\}$, $m_{-k}$ does not satisfy Condition $\gamma|k$. Without loss of generality suppose that $i \in N\setminus\{j\}$. Since $|N| > 2$, and by the assumption [A1(a)], for all $k, k' \in N$ $\bar{a}^k \neq \bar{a}^{k'}$, again without loss of generality we can assume that $\xi(m) \neq \bar{a}^i$. By the construction of $d'_i$, Condition $\gamma|k$ is not met for all $k \in N$ if $i$ deviates to $d'_i$. Since Case 4A applies, $\xi(d'_i(t_i, r_i, q_i), m_{-i}) = \bar{a}^i$.

(ii) There does not exist $j \in N$ such that $K(m) = \{j\}$. Hence, $m$ is such that either Case 2B or 3B or 4B applies, i.e. $\xi(m) = \underline{a}$. If $m$ is such that either Case 2B or 4B applies, then by the outcome rules associated with Cases 2A, and 4A, and given that (by assumption [A4]) for all $g \in \varphi_G$ and all $t \in T$, $g(t) \neq \underline{a}$, we conclude that $u^L_i(\xi(m), t, s(t)) < u^L_i(\xi(d[d'_i](t, s(t), q), t, s(t))$. If $m$ is such that Case 3B applies, then there exists $j \in N$ such that $m_{-j}$ satisfies Condition $\gamma|j$ and for all $k \in N\setminus\{j\}$, $m_{-k}$ does not satisfy Condition $\gamma|k$. Since $|N| > 2$, without loss of generality suppose that $i \in N\setminus\{j\}$. $(d'_i(t_i, r_i, q_i), m_{-i})$ is such that for all $k \in N$, Condition $\gamma|k$ is not met. By Case 4A, $\xi(d'_i(t_i, r_i, q_i), m_{-i}) = \bar{a}^i$.

On the other hand, for all $t'_{-i} \in T_{-i}$, for all $q'_{-i} \in \hat{Q}_{-i}(s, \mu)$, for all $(m_i, d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i})) \in M$ which satisfy Case 1, we have $\xi(m_i, d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i})) = \xi(d'_i(t_i, r_i, q_i), d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i}))$ since $(d'_i(t_i, r_i, q_i), d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i}))$ satisfies Case 2A.

To summarize, for all $t'_{-i} \in T_{-i}$, for all $q'_{-i} \in \hat{Q}_{-i}(s, \mu)$, for all $(m_i, d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i})) \in M$ such that Case 1 is met, we have $\xi(m_i, d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i})) = \xi(d'_i(t_i, r_i, q_i), d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i}))$ and for all $t'_{-i} \in T_{-i}$, for all $q'_{-i} \in \hat{Q}_{-i}(s, \mu)$, for every $(m_i, d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i})) \in M$ that does not satisfy Case 1 we have either $\xi(d'_i(t_i, r_i, q_i), d_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i})) = \bar{a}^i$ or $u^L_i(\xi(m_i, d_{-i}(t'_{-i}, s_{-i}(t'_{-i}),$

$q'_{-i})$, $t$, $s(t)) < u^L_i(\xi(d[d'_i](t, s(t), q), t, s(t))$.. Thus, by [A1], $V^L_i(\xi, d, \beta_i; t_i, r_i, q_i) < V^L_i(\xi, d[d'_i], \beta_i; t_i, r_i, q_i)$.

By definition of sequential mediated equilibrium, the conclusions derived above contradict the hypothesis that $(s, \mu, d, \beta) \in SME^*_X(\mathcal{H}(\varphi, g), L)$. ∎

Proof of Lemma 3: Choose $L = \langle R, Q\rangle \in \mathcal{L}$ and $(s, \mu, d, \beta) \in SME^*_X(\mathcal{H}(\varphi, g), L)$. By Lemma 2, for all $t \in T$ and all $q \in Q$ such that $\mu(q|s(t)) > 0$, $d(t, s(t), q)$ satisfies Case 1. Thus, for all $t \in T$, and all $q \in Q$ for which $\mu(q|s(t)) > 0$, for all $i \in N$, there exists a function $\alpha|_{q_i}: T_i \to T_i$ such that $d_i(t_i, s_i(t_i), q_i) = (\alpha|_{q_i}(t_i), g, 0)$. By the outcome rule associated with Case 1, there exists $\bar{g} \in G$ defined by: for all $t \in T$, $\bar{g}(t) = \sum_{q\in Q} \mu(q|s(t))g((\alpha|_{q_i}(t_i))_{i\in N}) = \sum_{q\in Q} \mu(q|s(t))\xi(d(t, s(t), q))$.

Choose $i \in N$ and suppose that the lemma is not true, i.e. there exists $t \in T$, $s'_i \in S_i(R)$ and $q \in Q$ such that $d_{-i}(t_{-i}, s_{-i}(t_{-i}), q_{-i})$ does not satisfy Condition $\gamma|i$ and $\mu(q|s[s'_i](t)) > 0$. We shall establish that this yields a contradiction.

We shall write $d(\cdot, \cdot, \cdot) = (d^1(\cdot, \cdot, \cdot), d^2(\cdot, \cdot, \cdot), d^3(\cdot, \cdot, \cdot))$, where the superscripts 1, 2 and 3 on $d(\cdot, \cdot, \cdot)$ denote the restrictions of $d(\cdot, \cdot, \cdot)$ to $T$, $G$ and $\mathbb{R}_+$ respectively. Suppose $i$ contemplates a deviation to $d'_i \in D_i(L, M)$. Also, suppose that for all $r_i \in R_i$ and all $q_i \in Q_i$, $d'_i(t_i, r_i, q_i) = (d^1_i(t_i, r_i, q_i), d^2_i(t_i, r_i, q_i), n')$, where $n'$ is such that for all $t'_{-i} \in T_{-i}$, for all $q'_{-i} \in Q_{-i}$, $d^3_{-i}(t'_{-i}, s_{-i}(t'_{-i}), q'_{-i}) < n'$. By definition, $(d'_i(t_i, s'_i(t_i), q_i), d_{-i}(t_{-i}, s_{-i}(t_{-i}), q_{-i}))$ satisfies Case 4A and, therefore, $\xi(d'_i(t_i, s'_i(t_i), q_i), d_{-i}(t_{-i}, s_{-i}(t_{-i}), q_{-i})) = \bar{a}^i$. By assumption [A4], for all $t \in T$, $g(t) \neq \bar{a}^i$. Thus, for all $\hat{t} \in T$, $\bar{g}(\hat{t}) \neq \bar{a}^i$. Moreover, by construction, for all $\hat{t} \in T$ and all $\hat{q} \in Q$ for which

$\mu(\hat{q}|s[s'_1](\hat{t})) > 0$ and $d_{-1}(\hat{t}_{-1}, s_{-1}(\hat{t}_{-1}), \hat{q}_{-1})$ satisfies Condition $\gamma|i$, we have $\xi(d(\hat{t}, s(\hat{t}), \hat{q})) = \xi(d'_1(\hat{t}_1, s'_1(\hat{t}_1), \hat{q}_1), d_{-1}(\hat{t}_{-1}, s_{-1}(\hat{t}_{-1}), \hat{q}_{-1}))$ since $(d'_1(\hat{t}_1, s'_1(\hat{t}_1), \hat{q}_1), d_{-1}(\hat{t}_{-1}, s_{-1}(\hat{t}_{-1}), \hat{q}_{-1}))$ satisfies Case 2A. Thus, by [A1], $V_1^L(\xi, d, \beta_1; t_1, s'_1(t_1), q_1) < V_1^L(\xi, d[d'_1], \beta_1; t_1, s'_1(t_1), q_1)$. This contradicts the hypothesis that $(s, \mu, d, \beta) \in SME^*_X(\mathcal{H}(\varphi, g), L)$. ■

<u>Proof</u> <u>of</u> <u>Lemma</u> <u>4:</u> Choose $L = \langle R, Q\rangle \in \mathcal{L}$ and $(\bar{s}, \mu, \bar{d}, \bar{\beta}) \in SME^*_X(\mathcal{H}(\varphi, g), L)$. By Lemma 2, for all $t \in T$ and all $q \in Q$ such that $\mu(q|\bar{s}(t)) > 0$, $\bar{d}(t, \bar{s}(t), q)$ satisfies Case 1. Thus, for all $t \in T$, and all $q \in Q$ for which $\mu(q|\bar{s}(t)) > 0$, for all $i \in N$, there exists a function $\alpha|_{q_1}: T_1 \to T_1$ such that $\bar{d}_1(t_1, \bar{s}_1(t_1), q_1) = (\alpha|_{q_1}(t_1), g, 0)$. By the outcome rule associated with Case 1, there exists $\bar{g} \in G$ defined by: for all $t \in T$, $\bar{g}(t) = \sum_{q\in Q} \mu(q|\bar{s}(t))g((\alpha|_{q_1}(t_1))_{1\in N}) = \sum_{q\in Q} \mu(q|\bar{s}(t))\xi(\bar{d}(t, \bar{s}(t), q)$. We need to show that $(\bar{g}, \bar{s}) \in \varphi^L$.

To prove that $(\bar{g}, \bar{s}) \in \varphi^L$, we shall appeal to Property $2(X, g)$. Fix $s \in S(R)$. Define $d \in D(L, M)$ by $d_1(t_1, s_1(t_1), q_1) = (t_1, g, 0)$ for all $i \in N$, for all $t_1 \in T_1$ and all $q_1 \in Q_1$. Given the outcome rule associated with Case 1, for all $t \in T$, for all $q \in Q$ such that $\sum_{q\in Q} \mu(q|s(t)) > 0$, $g(t) = \xi(d(t, s(t), q))$. Thus, there exists $z \in Z(L)$ defined by $z = \xi\circ d$ such that for all $t \in T$, $g(t) = \sum_{q\in Q} \mu(q|s(t))z(t, s(t), q)$ and $\bar{g}(t) = \sum_{q\in Q} \mu(q|\bar{s}(t))z((\alpha|_{q_1}(t_1), \bar{s}_1(t_1), q_1)_{1\in N})$. In addition, define $\bar{z} \in Z(L)$ by $\bar{z}(t, r, q) = z((\alpha|_{q_1}(t_1), r_1, q_1)_{1\in N})$ for all $(t, r, q) = T \times R \times Q$. To check that part (ii) of the hypothesis of Property $2(X, g)$ is satisfied, we need to ensure that for all $i \in N$, for all $s'_1 \in S_1(R)$, statements (B) and (C) below are met for all $z' \in Z(L)$ whenever (A) is satisfied for all $g' \in$

$G$ such that $g'((\alpha|_{q_j}(t_j))_{j\in N}) = z'((\alpha|_{q_j}(t_j))_{j\in N}, \bar{s}[s'_i](t), q)$ for all $t \in T$, for all $q \in Q$ such that $\sum_{q\in Q} \mu(q|\bar{s}[s'_i](t)) > 0$.

(A) for all $t_i \in T_i$, for all $r \in \bigcup_{\bar{R}\in\mathcal{R}} \bar{R}$, for all $q_i \in Q_i$, there exists $L' = \langle R', Q'\rangle \in \mathcal{L}$ such that $r \in R'$ and

$$\sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i)u_i^{L'}(g(t), t, r) \geq$$

$$\sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i)u_i^{L'}(g'(\alpha|_{q_i}(t_i), t_{-i}), t, r).$$

(B) for all $t_i \in T_i$,

$$U_i^L(\bar{z}, \bar{s}, \mu; t_i) \geq U_i^L(\bar{z}, \bar{s}[s'_i], \mu; t_i).$$

(C) for all $t_i \in T_i$, $r_i \in R_i$, $q_i \in \bar{Q}_i(\mu, r_i)$,

$$V_i^L(\bar{z}, \bar{\beta}_i; t_i, r_i, q_i) \geq V_i^L(\bar{z}', \bar{\beta}_i; t_i, r_i, q_i),$$

where $\bar{z}' \in Z(L)$ is defined by $\bar{z}'(t, r, q) = z'((\alpha|_{q_j}(t_j), r_j, q_j)_{j\in N})$ for all $(t, r, q) = T \times R \times Q$.

Choose $i \in N$. For all $t_i \in T_i$ and $q_i \in Q_i$, let $d'_i(t_i, s'_i(t_i), q_i) = (t_i, g', n')$ with $g' \neq g$ and $n' > 0$. Consider a deviation from $(\bar{s}, \mu, \bar{d}, \bar{\beta})$ by agent $i \in N$ to $(s'_i, d_i'^{\alpha q i}) \in S_i(R) \times D_i(L, M)$ such that for all $t_i \in T_i$ and $q_i \in Q_i$, $d_i'^{\alpha q i}(t_i, s'_i(t_i), q_i) = (\alpha|_{q_i}(t_i), g', n')$. By Lemma 3, for all $t \in T$, for all $q \in Q$ such that $\mu(q|\bar{s}[s'_i](t)) > 0$, $\bar{d}_{-i}(t_{-i}, \bar{s}_{-i}(t_{-i}), q_{-i})$ satisfies Condition $\gamma|i$. If $g'$ satisfies (D) below, then by Case 3A, $\xi(\bar{d}[d_i'^{\alpha q i}](t, \bar{s}[s'_i](t), q)) = g'((\alpha|_{q_j}(t_j))_{j\in N})$ for all $t \in T$ and $q \in Q$ such that $\sum_{q\in Q} \mu(q|\bar{s}[s'_i](t)) > 0$.

(D) for all $t_i \in T_i$, for all $r \in \bigcup_{\bar{R}\in\mathcal{R}} \bar{R}$, there exists $L' = \langle R', Q'\rangle \in \mathcal{L}$ such that $r \in R'$ and for all $q_i \in Q_i$

$$\sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i)u_i^{L'}(g(t), t, r) \geq$$

$$\sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i)u_i^{L'}(g'(\alpha|_{q_i}(t_i), t_{-i}), t, r).$$

By definition of SME, (E) and (F) below are satisfied whenever (D) holds.

(E) for all $t_1 \in T_1$,

$$U_1^L(\xi, \bar{s}, \mu, \bar{d}; t_1) \geq U_1^L(\xi, \bar{s}[s'_1], \mu, \bar{d}; t_1).$$

(F) for all $t_1 \in T_1$, for all $r_1 \in R_1$ and all $q_1 \in \bar{Q}_1(\mu, r_1)$,

$$V_1^L(\xi, \bar{d}, \bar{\beta}_1; t_1, r_1, q_1) \geq$$

$$V_1^L(\xi, \bar{d}[d_1'^{\alpha q1}], \bar{\beta}_1; t_1, r_1, q_1),$$

where $d_1'^{\alpha q1} \in D_1(L, M)$ is defined by $d_1'^{\alpha q1}(\cdot, \cdot, \cdot) = d_1'(\alpha|_{q_1}(\cdot), \cdot, \cdot)$.

Observe that by setting $z' = \xi \circ d[d'_1]$, given the definitions of $z$ and $\bar{z}$, if (D) implies (E) and (F), then (A) implies (B) and (C).

By Property $2(X, g)$, $(\bar{g}, \bar{s}) \in \varphi^L$. ■

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