Market Risk Measurement and the Cattle Feeding Margin: An Application of Value-at-Risk

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Value-at-Risk, known as VaR, gives a prediction of potential portfolio losses, with a certain level of confidence, that may be encountered over a specified time period due to adverse price movements in the portfolio's assets. For example, a VaR of 1 million dollars at the 95% level of confidence implies that overall portfolio losses should not exceed 1 million dollars more than 5% of the time over a given holding period. This research examines the effectiveness of VaR measures, developed using alternative estimation techniques, in predicting large losses in the cattle feeding margin. Results show that several estimation techniques, both parametric and non-parametric, provide well-calibrated estimates of VaR such that violations (losses exceeding the VaR estimate) are commensurate with the desired level of confidence. In particular, estimates developed using JP Morgan's *Risk Metrics* methodology appear robust for instruments that have linear payoff structures such as cash commodity prices.

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Introduction

Value-at-Risk (VaR) is considered by many to be the "state-of-the-art" in risk measurement. VaR is receiving considerable attention in the finance literature and more recently, the agricultural economics literature (Boehlje and Lins; Manfredo and Leuthold). Specifically, VaR gives a prediction of potential portfolio losses, with a certain level of confidence, that may be encountered over a specified time period due to adverse price movements in the portfolio's assets. For example, a Value-at-Risk estimate of 1 million dollars at the 95% level of confidence implies that portfolio losses should not exceed 1 million dollars more than 5% of the time over the given holding period (Jorion, 1997).

Currently, Value-at-Risk is being embraced by corporate risk managers as an important tool in the overall risk management process. Initial interest in VaR, however, stemmed from its potential applications as a regulatory tool. In the wake of several financial disasters involving the trading of derivatives products, such as the Barrings Bank collapse (see Jorion, 1997), regulatory agencies such as the Securities and Exchange Commission embraced VaR as a transparent measure of downside market risk that could be useful in reporting risks associated with portfolios of highly market sensitive assets such as derivatives (Linsmeier and Pearson, 1997). Since VaR focuses on downside risk and is usually reported in dollars or returns, it is often considered easier to understand by managers and outside investors that may not be well versed in statistical methods. VaR is commonly used for internal risk management purposes and is further being touted for use in risk management decision making by non-financial firms (Ho, Chen, and Eng; Jorion, 1997; JP Morgan *Risk Metrics*).

VaR is estimated using either parametric or full-valuation procedures. Parametric procedures rely on estimates of volatility and correlations in creating portfolio volatility forecasts which are scaled by a factor corresponding to the desired confidence level. Full-valuation procedures model the entire return distribution with the VaR measure being the quantile associated with the desired confidence level (e.g., 5% quantile for the 95% confidence level). [1] Much of the literature related to Value-at-Risk focuses on the properties of various procedures for estimating the risk measure that fall within these two broad categories. Specifically, parametric and full-valuation procedures are evaluated on their ability to generate VaR estimates that are consistent with the desired pre-determined confidence level. Empirical studies to date (Mahoney; Hendricks; Jackson, Maude, and Perraudin) find that the performance of either parametric or full-valuation procedures is sensitive to the data and portfolio composition examined as well as the predetermined factors of the VaR model itself (e.g., confidence level and time horizon).

Several studies regarding livestock risk management strategies have examined the cattle feeding process in a multiproduct or portfolio framework (Leuthold and Mokler; Peterson and Leuthold). The major market risks to cattle feeding are the variability of fed cattle prices (output price) and the variability of corn and feeder cattle prices (input prices). In fact, studies focussing on the major factors affecting the profitability of cattle feeding operations isolate the variability of these market prices as being particularly influential, especially relative to production risk factors such as feed efficiency (Schroeder et al.; Langemeier, Schroeder, and Mintert; Jones et

[1] Full-valuation procedures are often referred to as non-parametric procedures in the finance literature. However, the term full-valuation is used throughout this paper to avoid confusion with traditional non-parametric statistical methods.

al.). The difference between fed cattle prices and the prices of corn and feeder cattle, under assumed production technology, is referred to as the cattle feeding margin. Thus, the cattle feeding margin serves as a portfolio of assets (fed cattle, feeder cattle, and corn prices) similar to that of a portfolio of financial assets (Peterson and Leuthold).

Considering the recent interest in Value-at-Risk and the variability of the market risk factors of the cattle feeding margin, the overall objective of this paper is to examine VaR measures in the context of the cattle feeding margin. In particular, this paper develops and tests VaR measures estimated using several alternative procedures (both parametric and full-valuation) in predicting large losses in the cattle feeding margin (e.g., the number of times the VaR is exceeded relative to its pre-determined confidence level). This research is important and unique since it provides insight into the performance of procedures, often suggested for use in creating VaR estimates for portfolios of financial assets, in the context of agricultural prices. Given initial evidence of the sensitivity of VaR measures to the procedures and data set used, as well as the increasing interest in Value-at-Risk as a tool in risk management, this research makes advances in understanding VaR estimation techniques and their performance for use in livestock risk management.

Theoretical Constructs of Value-at-Risk

As a downside risk measure, Value-at-Risk concentrates on low probability events that occur in the lower tail of a distribution. In establishing a theoretical construct for VaR, Jorion (1996, 1997) first defines the critical end of period portfolio value as $W^* = W_0(1+R^*)$ where W_0 is initial portfolio value, and R^* is the portfolio return associated with a predetermined level of confidence "c" (e.g., 95%). Hence, W^* is considered the end of period portfolio value when worst possible portfolio returns (R*) occur. Given the pre-determined confidence level "c", these returns should not be encountered more than (1-c) percent of the time. Subsequently, for a general distribution of future portfolio value, f(W), Jorion (1996, 1997) defines Value-at-Risk as:

(1) $$1 - c = \int_{-\infty}^{W^*} f(W)dw$$

such that losses associated with confidence level "c" are isolated in the area of the left tail of the distribution (figure 1).

Full-valuation methods rely on procedures for modeling the entire distribution of portfolio returns and defining the VaR estimate as the quantile associated with 1-c in equation 1. However, parametric estimation of VaR relies on the properties of the normal distribution. Therefore, assuming the general distribution in equation 1 is the standard normal distribution, VaR can be defined as:

(2) $$VaR = W_0 a s$$

where W_0 is initial portfolio value, a is the normal deviate associated with 1-c in equation 1, and s is portfolio standard deviation. Thus, the critical element of parametric VaR (equation 2) is the estimate of portfolio standard deviation (s), also referred to as portfolio volatility.

Several studies have examined the properties as well as the pro's and con's of using both full-valuation and parametric procedures (Linsmeier and Pearson 1996, 1997; Duffie and Pan; Jorion, 1996; Manfredo and Leuthold). For instance, full-valuation procedures are often praised for their flexibility but are often criticized for not being able to capture time-varying volatility often found in financial return series. Parametric procedures, however, are able to capture time-varying volatility through the incorporation of conditional volatility forecasts. In fact, much of the impetus for using parametric VaR stems from the existing and growing literature on volatility forecasting (Bollerslev, Chou, and Kroner; Figlewski) as well as the publicity and popularity of the JP Morgan's *Risk Metrics* method which advocates the use of an exponentially weighted moving average technique for estimating volatility and correlations.

Despite these issues, the central concern with any VaR estimation technique is its ability to adequately capture portfolio values that occur in the lower tail of a distribution commensurate with the pre-determined confidence level. This is especially true since both full-valuation and parametric procedures inherently provide estimates of the total variance of a distribution and do not explicitly model distribution tails. This is often a major criticism of using VaR in general due to leptokurtosis observed with financial as well as agricultural price returns (Yang and Brorsen). Regardless, empirical studies to date (e.g., Mahoney; Hendricks; Jackson, Maude, and Perraudin) have found that both parametric and full-valuation procedures adequately cover large portfolio losses, especially at confidence levels greater than or equal to 95% for the portfolios and data they tested. However, these initial empirical findings also suggest that the performance of any VaR estimation technique is sensitive to the data set used in developing and evaluating the estimates, the predetermined confidence level, forecast horizon, and portfolio composition.

Data

In order to examine various VaR estimation techniques for the cattle feeding margin, price return series are needed. Returns are constructed from Wednesday cash prices of fed cattle, feeder cattle, and corn. Cash prices are used since it is the variability of cash prices that cause the cattle feeding margin to fluctuate over time. Returns are defined as $R_{i,t} = \ln(P_{i,t}) - \ln(P_{i,t-1})$ where $R_{i,t}$ is the weekly return of commodity i, ln is the natural logarithm, $P_{i,t}$ is the current Wednesday price of commodity i and $P_{i,t-1}$ is the previous Wednesday price. Wednesday price data are used since fed cattle and feeder cattle are actively traded only one day per week, with that day typically in mid week (Rob). If a Wednesday price is not available, then a Tuesday price is used. The three data series span from January 1984 through December 1997 providing 14 years (729 observations) of returns for estimation and out-of-sample testing.

Fed cattle prices ($/cwt) are for the Texas-Oklahoma direct market (1100 to 1300 pound steers), feeder cattle ($/cwt) are for the Oklahoma City terminal market (650 to 700 pounds), and corn prices ($/bu) are for Central Illinois (#2 yellow). These data are reported daily in the Wall Street Journal. Furthermore, these prices serve as proxies for local cash market prices since each cattle feeding operation is exposed to specific prices in its particular region that may or may not have different volatility from the prices examined in this study. However, due to the liquidity of these cash markets as well as their frequency and reliability of reporting, these data are assumed robust for examining the performance of alternative VaR estimation methods for the cattle feeding margin.

Methods

In defining the cattle feeding margin, Leuthold and Mokler, Peterson and Leuthold, and Schroeder and Hayenga describe similar cattle feeding scenarios that incorporate fixed feeding technology. It is assumed that cattle are placed on feed at 650 pounds and fed to 1100 pounds, consuming 45 bushels of corn in the process.[2] Based on this technology, the cattle feeding margin is defined as:

[2]Studies examining the hedging of the cattle feeding margin typically assume the consumption of corn to be in the range of 42 to 49 bushels.

(3) margin (\$/head) = (fed cattle price)11 - (feeder cattle price)6.5 - (corn price)45.[3]

On large feedlots, cattle are continually marketed and placed on feed. As well, feedlots with a capacity of 30,000 head or more typically do not maintain corn inventories for more than two weeks (Davies and Widawsky). Therefore, it is assumed cattle feeding is a continuous process with decision makers routinely evaluating the variability of fed cattle, feeder cattle, and corn prices in a portfolio framework. Because of this, VaR measures are estimated and evaluated for weekly horizons consistent with the periodicity of the three price return series.

For parametric VaR estimation, the variance of the cattle feeding margin (portfolio variance) is defined as:

$$(4)\qquad \begin{aligned} s^2_{fm} = {} & w^2_{fc} s^2_{fc} + w^2_{fdr} s^2_{fdr} + w^2_{c} s^2_{c} + 2 w_{fc} w_{fdr} r_{fc,fdr} s_{fc} s_{fdr} + 2 w_{fc} w_{c} r_{fc,c} s_{fc} s_{c} \\ & + 2 w_{fdr} w_{c} r_{fdr,c} s_{fdr} s_{c} \end{aligned}$$

where s^2_{fc}, s^2_{fdr}, and s^2_{c} are the variances of fed cattle, feeder cattle, and corn returns and $r_{fc,fdr}$, $r_{fc,c}$, and $r_{fdr,c}$ are the respective correlation coefficients between returns. The portfolio weights (w_{fc}, w_{fdr}, and w_c) are defined as P_iQ_i where P_i is the price of commodity i and Q_i is quantity of commodity i based on the assumed production technology allowing equation 4 to be expressed in dollar terms (see Jorion, 1997, p. 156). Considering equation 4 in a forecasting framework such that the individual variances (volatility) and correlation coefficients are forecasts, VaR at any given week t is:

$$(5)\qquad VaR_{fm,t} = a\, \hat{s}_{fm,t+1}$$

where $\hat{s}_{fm,t+1}$ is the portfolio volatility forecast of the cattle feeding margin (\$/head) and a is the scaling factor corresponding to the desired confidence level. Several forecasting procedures are

[3] Typically, other variable costs are also subtracted from the margin presented in (3). Since this study focuses on market risk, other variable costs (e.g., vet costs) are held constant.

employed in estimating the individual volatilities and correlations used in equation 4, all of which have been advocated for use in developing VaR measures and/or used in previous empirical studies related to VaR. These methods include a long-run historical average, a 150-week historical moving average, a GARCH (1,1) ~ t, exponentially weighted moving averages advocated by JP Morgan's *Risk Metrics*, and implied volatilities from options on futures contracts.

For both the long-run historical and 150-week moving averages, the volatility forecast is defined as:

(6) $$\hat{s}_{i,t+1} = \sqrt{\frac{1}{T}\sum_{m=0}^{T-1} R^2_{i,t-m}}$$

where $\hat{s}_{i,t+1}$ is the volatility forecast for commodity i, T is the number of past squared returns used in developing the forecast, and $R^2_{i,t}$ is the realized squared return for commodity i in week t where the mean return of the series is constrained to zero.[4] Similarly covariance forecasts, which are needed to calculate correlations between the three commodity price series, take the form:

(7) $$\hat{s}_{ij,t+1} = \frac{1}{T}\sum_{m=0}^{T-1} R_{i,t-m} \cdot R_{j,t-m}$$

where $\hat{s}_{ij,t+1}$ is the forecasted covariance between commodity i and commodity j and $R_{i,t}$ and $R_{j,t}$ are returns for commodity i and j respectively. In the case of the long-run historical average, the sample size is anchored to the first return observation (growing sample size). For the 150-week

[4] In the volatility forecasting literature, it is standard practice that the mean return of a series be constrained to zero when developing volatility forecasts. As well, Figlewski provides empirical evidence that setting the mean to zero provides more accurate volatility forecasts. Therefore, throughout the remainder of this paper, the mean return is set to zero.

historical moving average, T is equal to 150.[5] Moving averages (or moving windows) are very similar to long-run historical averages but are thought to be more sensitive to structural change and observed time variation than models that use a growing sample size.

Due to the popularity of GARCH models in the volatility forecasting literature (Bollerslev, Chou, and Kroner), as well as discussion of their potential use in VaR modeling (Jorion, 1997; Hopper; Duffie and Pan), a GARCH (1,1) ~ t model is also used and defined as:

$$\hat{s}_{i,t+1} = \sqrt{a_0 + a_1 R_{i,t}^2 + b_1 s_{i,t}^2} \tag{8}$$

where a_0, a_1, and b_1 are maximum likelihood GARCH estimates (MLE) generated using the BHHH (Berndt, Hall, Hall, and Housman) algorithm. Unlike the common GARCH (1,1), which assumes the normal distribution, the GARCH (1,1) ~ t uses the Student's-t distribution in the maximum likelihood estimation which better handles leptokurtotic returns and has been found to adequately fit various agricultural price returns (Yang and Brorsen).

The JP Morgan's *Risk Metrics* method of estimating volatilities and covariances (correlations) is also used. The methodology incorporates an exponentially weighted average that relies on a fixed decay factor l. It has been touted for its ease of estimation and its ability to represent time-varying volatility without resorting to GARCH estimation (Mahoney). Because of this, the JP Morgan's *Risk Metrics* methodology for estimating volatilities and covariances (correlations) has been a major impetus for the use of parametric methods in the VaR literature. The *Risk Metrics* volatility forecast is:

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[5] The existing literature provides very little guidance into the number of past observations to use in creating these forecasts. Setting T=150 corresponds with approximately 3 years of past return data, which was deemed adequate in picking up the long-term price variability while being sensitive to time-variation and structural change versus other values of T that were examined.

(9) $$\hat{s}_{i,t+1} = \sqrt{\hat{s}^2_{i,t} + (1 - l)\, R^2_{i,t}}$$

where l is the pre-determined decay factor and $\hat{s}^2_{i,t}$ is the t-period variance forecast. Time varying covariances are forecasted in a similar fashion such that:

(10) $$\hat{s}_{ij,t+1} = l\ \hat{s}_{ij,t} + (1 - l)\, R_{i,t} \cdot R_{j,t}\ .$$

Three fixed decay factors are used, including l =.94 and l =.97, which are recommended by *Risk Metrics* for weekly and monthly data, respectively, as well as a l optimized for weekly data over the respective sample period of the three return series via MLE techniques using the BHHH algorithm in the S-Plus statistical package (l =.96).

Finally, implied volatilities from observed options prices are used for developing parametric VaR estimates. Since exchange traded options contracts written specifically on cash prices do not exist, it is assumed that volatilities implied from options written on live cattle, feeder cattle, and corn futures provide a reasonable proxy for the option market's assessment of future price volatility for these cash prices. Implied volatilities are derived using the Black-1976 model for options on futures contracts using the Financial CAD software. Since the Black-1976 model is a European model, the implied volatilities used are computed as the simple average of the implied volatility from nearby, at-the-money, put and call options. This is done in order to reduce potential bias associated with using a European model for American style options (Mayhew; Jorion, 1995). Since implied volatilities yield annualized estimates, it is necessary to convert these annualized estimates to weekly estimates such that:

(11) $$I\hat{V}_{i,t+1} = \frac{IV_{(annual),i,t}}{\sqrt{52}}$$

where $I\hat{V}_{i,t+1}$ is the volatility forecast for commodity i at week t+1, and $IV_{(annual),i,t}$ is the annualized implied volatility estimate.

Utilizing the above procedures for forecasting volatilities and covariances, correlation forecasts used in equation 4 are computed as:

$$(12) \qquad \hat{r}_{ij,t+1} = \frac{\hat{s}_{ij,t+1}}{\hat{s}_{i,t+1}\hat{s}_{j,t+1}}$$

where $\hat{s}_{i,t+1}$ and $\hat{s}_{j,t+1}$ are the volatility forecasts for commodities i and j and $\hat{s}_{ij,t+1}$ is the forecasted covariance between commodities i and j. It is important to note that both the GARCH (1,1) ~ t and the implied volatilities, equations 8 and 11 respectively, do not have corresponding methods to explicitly develop covariances needed for developing correlation forecasts in equation 12. Because of this, the portfolio variance (volatility) forecast in equation 4 is created using either GARCH (1,1) ~ t or implied volatilities with correlations estimated from the long-run historical average, 150-week moving average, and JP Morgan's *Risk Metrics* method respectively. These are referred to as "mixed" VaR models throughout the remainder of the paper (table 1).

Since the use of *Risk Metrics* volatilities and correlations is advocated as a simple alternative to multivariate GARCH procedures, a constant correlation MGARCH procedure is also used. Specifically the constant correlation MGARCH model presented by Bollerslev (1990) assumes that conditional correlations between commodity price returns are constant over time and that individual commodity price return variances follow a univariate GARCH (1,1) process (Campbell, Lo, and MacKinlay; Bera, Garcia, and Roh). Thus, the model can be shown as:

$$(13) \qquad \begin{aligned} \hat{s}^2_{ii,t+1} &= c_{ii} + a_{ii} R^2_{ii,t} + b_{ii} s^2_{ii,t} \\ \hat{s}_{ij,t+1} &= \hat{r}_{ij,t+1}\hat{s}_{ii,t+1}\hat{s}_{jj,t+1} \end{aligned}$$

where $\hat{s}^2_{ii,t+1}$ is the conditional variance of asset i for t+1, $\hat{s}_{ij,t+1}$ is the conditional covariance, $\hat{r}_{ij,t+1}$ is the constant correlation forecast between assets i and j, and $\hat{s}_{ii,t+1}$ is the conditional standard deviation of asset i at t+1. In all cases, $i \neq j$. This model provides a positive definite covariance matrix if all parameters are positive, $a_{ii} + b_{ii} < 1$, and $\hat{r}_{ij}$ is between –1 and 1. The MGARCH estimates, both assuming a normal distribution as well as a Student's-t distribution in the MLE estimation, are then compared to the more simplistic procedure(s) where volatilities and covariances (correlations) are estimated independent of each other.[6]

Thus using the described parametric methods, weekly VaR measures are estimated from January 1987 through October 1997 providing 564 weekly forecasts of volatility and correlations among the three return series. Value-at-Risk estimates are calculated for the 90% (a =1.28), 95% (a =1.65), and 99% (a =2.33) levels of confidence. Each of the parametric VaR measures developed and tested are described and outlined in table 1.

In addition to the parametric VaR estimates, a simple full-valuation procedure (historical simulation) is also developed for the 90%, 95%, and 99% confidence levels (table 1). The historical simulation method models the entire return distribution with the VaR designated as the quantile associated with the desired level of confidence. The historical simulation procedure used follows the methods of Linsmeier and Pearson (1996). First, at time period t, the cattle feeding margin is calculated as in equation 3. Second, the prices of fed cattle, feeder cattle, and corn observed at time t are exposed to their respective previous 150 weeks of returns such that $P^*_t = P_t(1+R_{t-T})$ for all T = 1...150. Third, the cattle feeding margin is recalculated using these

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[6] Bera, Garcia, and Roh note that assuming a constant correlation structure is a very strong proposition. Several other MGARCH techniques have been suggested that also limit the number of parameters and attempt to ensure a positive definite covariance matrix such as the diagonal vech and BEEK models (see Bera, Garcia, and Roh; Campbell, Lo, and MacKinlay). These specifications, among others, were tried for the cattle feeding margin portfolio. However, model convergence and/or failure to produce a positive definite covariance matrix were consistent problems.

new prices (P^*), creating 150 new values of the cattle feeding margin. Next, each of these new values of the cattle feeding margin is subtracted from the actual feeding margin realized at week t, yielding 150 differences between the cattle feeding margin at week t and the simulated values of the feeding margin previously generated. Finally, from the distribution of these differences, the quantile associated with the desired confidence level (e.g., 5% for the 95% level of confidence) becomes the VaR estimate.

Evaluation

The specific parametric and historical simulation VaR estimates in table 1 are evaluated on their ability to predict large losses (decreases) in the cattle feeding margin resulting from fluctuations in fed cattle, feeder cattle, and corn prices. If actual portfolio losses over the desired horizon (e.g., 1 week) exceed the VaR estimate, a violation occurs. Hence, if violations are in excess to that implied by a particular confidence level, the VaR measure is considered inadequate in measuring large losses of the cattle feeding margin. To determine if violations are commensurate with the designated confidence level of VaR, a likelihood ratio test is developed following the procedures of Lopez. The null hypothesis is $d = d^*$ where d is the desired coverage level (e.g., 5%) corresponding to the given confidence level (e.g., 95%), d^* is X/N where X is the number of realized violations and N is the number of out-of-sample observations. The probability of realizing X violations of VaR for a sample of N is:

(14) $$P_r(X; d, N) = \binom{N}{X} d^X (1-d)^{N-X}$$

and the likelihood ratio test statistic is (Lopez, p. 7):

(15) $$LR(d) = 2\left[\ln\left(d^{*X}\left(1-d^*\right)^{N-X}\right) - \ln\left(d^X (1-d)^{N-X}\right)\right]$$

which has an asymptotic c^2 distribution with 1 degree of freedom.

Similarly, a test of bias in VaR estimates is conducted consistent with the procedures of Mahoney (p. 206 and 207), which is based on a binomial probability distribution. The expectation of the number of violations of a VaR estimate is N(1-c) where N is the number of out-of-sample observations and c is the confidence level expressed in decimal form (e.g., 0.95 for 95% level of confidence). The variance of this estimate is Nc(1-c). Thus, the test for bias is defined as a Z test, which in large samples is distributed normally, such that:

$$(16) \qquad Z_c = \frac{L_{realized} - N(1-c)}{\sqrt{Nc(1-c)}}$$

where $L_{realized}$ = the number of observed violations of VaR at a given confidence level c (Mahoney). Hence, if the Z statistic is significantly positive (negative) then VaR regularly underestimates (overestimates) actual downside risks.[7]

Summary statistics of the VaR violations are also used to evaluate the VaR models including the number of violations realized (X), the percentage of violations that occurred over the sample period (X/N)*100, the average size violation ("sum of violations"/X), the maximum violation, and the minimum violation. Therefore, if several VaR measures are determined to be "well calibrated" (e.g., $d = d^*$) the preferred VaR model among alternatives is the one with the smallest of these summary statistics.

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[7] The likelihood ratio statistic and Z statistic are commonly used in the literature as well as by industry professionals, however, Lopez notes that these tests are designed to determine unconditional coverage, and do not take into consideration potential serial dependence of violations. The development of VaR evaluation measures that examine conditional coverage is a topic of current research (see Lopez; Crnkovic and Drachman).

Empirical Results

Over the sample period from January 1987 to October 1987, the average feeding margin as defined in equation 3 is \$118.37 per head. The largest feeding margin over this time period is \$258.45 per head, realized in the week of 10/2/96, while the smallest feeding margin is -\$44.57 per head in week 7/9/97. The largest single weekly loss (decrease) in the cattle feeding margin (portfolio) over the sample period, –\$53.62, occurred from 6/16/93 to 6/23/93. For all of the VaR measures tested in table 1, this large change resulted in the largest (maximum) violation of VaR for all confidence levels tested; a rare event since losses like these are expected to occur less than 1% of the time. Using this event to illustrate a violation of VaR, the VaR estimate on 6/16/93 is \$39.58 at the 99% confidence level for RM97-VaR. Based on this VaR estimate, it is predicted that the cattle feeding margin would not decrease from its current level of \$124.05 by more than \$39.58 with 99% confidence. However, over the next week the cattle feeding margin decreases by \$53.62 to \$70.43, a violation of the VaR estimate by the size of \$14.05.

The results of the likelihood ratio test, Z test, and summary statistics of the VaR violations are presented in tables 2 through 4. Based on the results of the likelihood ratio and Z tests (equations 15 and 16), the *Risk Metrics* models (RM97-VaR, RM94-VaR, and RMOPT-VaR), the historical moving average model (H150-VaR), and the full-valuation historical simulation (HISTSIM-VaR) provide coverage consistent with all three confidence levels (90%, 95%, and 99%). In other words, in each case, these VaR specifications fail to reject the null hypothesis that the number of violations realized over the sample period equals the number implied by the predetermined confidence level ($d = d^*$). All of the other VaR models, barring IVRM97-VaR at the 90% level, have more violations occurring than accepted by the predetermined confidence level. For those VaR measures that have more violations than allowed, the associated Z statistic is both positive and significant suggesting that those specifications produce estimates that consistently underestimate the true downside risk of the cattle feeding margin.

Overall, among the well-calibrated VaR measures, it is difficult to deem one measure to be the best. However, the *Risk Metrics* specifications, especially RM97-VaR where the decay factor l =.97, appears to provide robust VaR estimates for each of the three confidence levels tested using a wide array of evaluation criteria. However, any improvement provided by the RM97-VaR relative to the other well-calibrated VaR models is fairly minimal and most likely not economically significant. Furthermore, evaluation based on the summary statistics presented, beyond the results of the LR test and Z test, is somewhat subjective. However, RM97-VaR does have the smallest maximum violation among all VaR models for each of the three confidence levels examined. Hence, for the largest violation that occurs from 6/16/93 to 6/23/93, RM97-VaR has the most conservative VaR estimate among all tested.

Examining each of the confidence levels individually, at the 90% confidence level (table 2), several VaR measures fail to reject the null hypothesis of $d = d^*$ including H150-VaR, RMOPT-VaR, RM94-VaR, RM97-VaR, HISTSIM-VaR, and IVRM97-VaR. Of these, RMOPT-VaR has the smallest values of both the likelihood ratio and Z statistics while IVRM97-VaR and HISTSIM-VaR have the largest. However, RM97-VaR, has the smallest average size violation, maximum violation, and minimum violation at $8.081, $31.887, and $0.002 respectively. Of the well-calibrated VaR measures at the 95% confidence level (table 3), again RMOPT-VaR has the smallest values of the test statistics and percent violations (5.5%) that are closest to what would be expected by the pre-determined confidence level of 95%. Both RM97-

VaR and RM94-VaR are next with likelihood ratio statistics and Z statistics at 0.518 and 0.734 respectively. Again in this case, RM97-VaR has the smaller average size violation, maximum violation, and minimum violation with the average size violation being approximately $0.68 per head smaller than that for RMOPT-VaR ($6.838 vs. $7.519) and $1.075 per head smaller RM94-VaR ($6.838 vs. $7.913). It is difficult to judge whether these differences in the size of VaR violations on average over the sample period are economically significant. Interestingly at the 99% level of confidence, where violations of VaR are expected to occur no greater than 1% of the time (table 4), both RM97-VaR and H150-VaR have the same values of likelihood ratio and Z statistics at 0.023 and 0.152 respectively. As with the 90% and 95% confidence levels, RM97-VaR again has the smallest maximum violation.

Since it is well known that composite forecasting techniques often yield superior forecasts (Clemen), a composite VaR estimate is also created and examined versus the individual models. Based on the findings discussed above, the composite measure is constructed as a simple average of RM97-VaR and HISTSIM-VaR (COMP-VaR). These two VaR measures are chosen for combining since they are both constructed using different methods (parametric vs. full-valuation). COMP-VaR is found to provide coverage consistent with all three confidence levels and provides improvement over HISTSIM-VaR individually at all confidence levels based on the percentage of violations realized and subsequently the values of the test statistics. In fact at the 90% level, COMP-VaR provided improvement over both of its component forecasts (RM97-VaR and HISTSIM-VaR) based on the size of the LR and Z statistics.

The results presented in tables 2 through 4 also suggest that performance of any VaR measure is greatly affected by the correlation structure incorporated. Those VaR models that combine univariate volatilities in conjunction with alternative correlation estimates (mixed models) consistently underestimate the true downside risk of the cattle feeding margin. This is evident since the Z statistics for these models are positive and significant and the average VaR estimates are smaller than those of the best performing models (e.g., RM97-VaR). However, performance of the mixed VaR models improve as the correlations go from being long-run historical to conditional *a la Risk Metrics* (e.g., IVHIST-VaR to IVRM97-VaR). The parametric VaR measures found to be well calibrated across confidence levels use volatilities and correlations that are estimated from the same underlying method (i.e., *Risk Metrics*). This observation calls into question the computational validity of using implied volatility or another univariate volatility estimation procedure for VaR that does not have a corresponding way of defining covariances and subsequently correlations. Furthermore, the weak performance of both multivariate GARCH specifications is likely due to the constant correlation assumption incorporated which was necessary to provide a positive definite covariance matrix as well as model convergence.

Interestingly, the majority of violations occurred during the period from April to October. Using RM97-VaR at the 90% level of confidence, 41 out of 62 violations (66% of violations) occur from April to October. During this time, 12 out of the 62 violations (20%) occur in June while 27 violations (43%) occur from June through August. Similarly, at the 95% level of confidence, the April through October time periods saw 22 out of the 32 violations while 8 out of 22 (25%) are realized during the month of June alone. In addition, at the 99% confidence level, 5 out of the 6 violations were realized during the May to October time span. These observations are consistent with known increased price variability of fed cattle and corn prices during these months.

Summary and Conclusions

The methods recommended by JP Morgan's *Risk Metrics*, in particular using l =.97, provide an accurate and robust specification for a covariance matrix in calculating weekly parametric VaR for the examined portfolio (the cattle feeding margin). However, other specifications, such as the other *Risk Metrics* specifications as well as the simple historical simulation (full-valuation procedure), also produce well-calibrated VaR measures for all three confidence levels examined. Therefore, the conclusion as to the superiority of the *Risk Metrics* method using l =.97 is made with caution. The fact that both parametric procedures (e.g. *Risk Metrics*) and full-valuation (e.g., historical simulation) are found to provide well-calibrated VaR measures is most likely due to the fact that the cattle feeding margin, as defined in this study, is a portfolio of linear instruments (cash prices). Overall, it is concluded that at least for this portfolio, correlations are more important to the overall performance of a particular VaR measure than volatilities. Furthermore, the majority of violations of VaR occur during times of observed seasonal increases in the volatility of fed cattle and corn returns. Because of this, risk managers that build VaR models in which the portfolio of interest contains positions in agricultural commodities or other seasonal commodities should be more cautious of their VaR estimates during these known times of increased volatility.

This study is the first known attempt at empirically examining the performance of various VaR measures in the context of an agricultural enterprise portfolio (e.g., cattle feeding). To date, all known empirical studies examining the performance of alternative VaR measures have been conducted in the context of portfolios containing currency, interest rate, or equity data with portfolios often developed randomly (Mahoney; Hendricks). The cattle feeding margin provides a realistic alternative portfolio, as well as new data, for studying existing techniques of VaR estimation. The results of this study also provide an impetus for further research in the area of Value-at-Risk. For instance, research is needed that focuses on the applicability and performance of VaR in the context of other agricultural prices and portfolios (e.g., the soybean crush margin) as well as the performance of alternative parametric and full-valuation procedures when options positions, which have a non-linear payoff structure, are included in a portfolio. Therefore, as interest and use of VaR increases among risk mangers, research should focus on models that are robust for a variety of prices and portfolios.

Figure 1. Illustration of Value-at-Risk

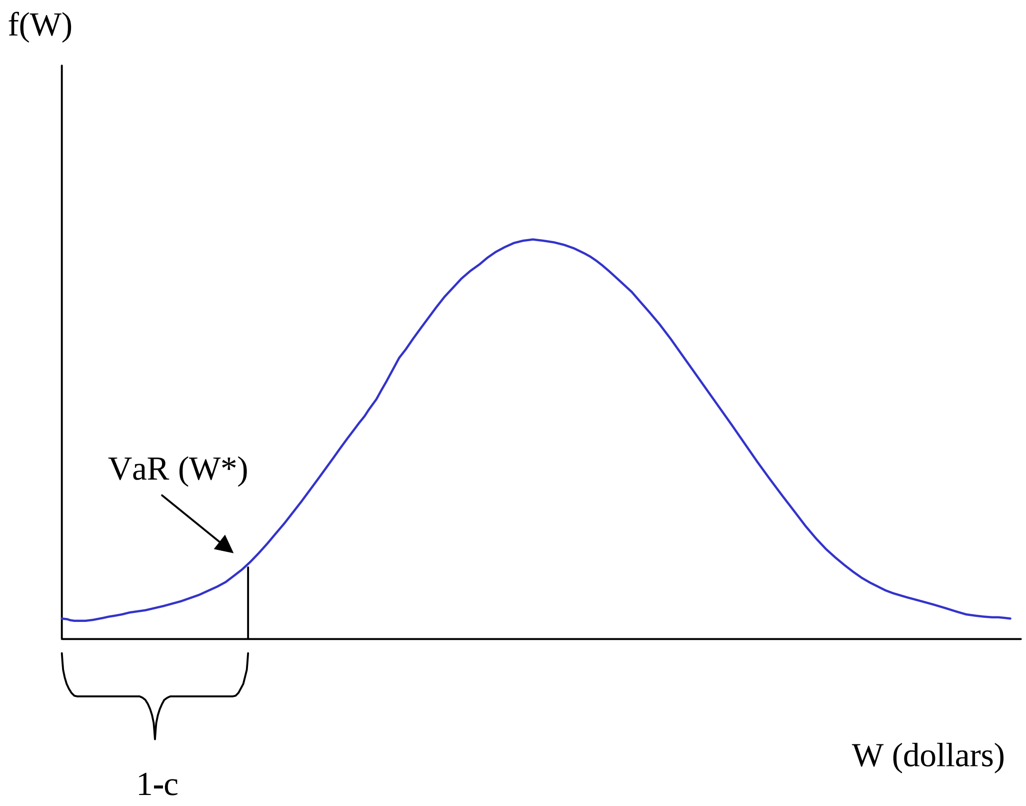

Table 1. Value-at-Risk Measures Key

Abbreviation	Description
MGARCH	Multivariate GARCH (constant correlation)
MGARCH-t	Multivariate GARCH using the Student's t-distribution in the estimation (constant correlation)
H150-VaR	150-day moving average volatilities and correlations
HISTAVG-VaR	Long-run historical average volatilities and correlations
RM97-VaR	*Risk Metrics* volatilities and correlations using λ=.97
RM94-VaR	*Risk Metrics* volatilities and correlations using λ=.94
RM96-VaR	*Risk Metrics* volatilities and correlations using optimized λ=.96
GRM97-VaR	GARCH-t volatilities and RM97 correlations
GH150-VaR	GARCH-t volatilities and H150 correlations
GHIST-VaR	GARCH-t volatilities and HISTAVG correlations
IVRM97-VaR	Implied volatilities (IV) and RM97 correlations
IVH150-VaR	Implied volatilities (IV) and H150 correlations
IVHIST-VaR	Implied volatilities (IV) and HISTAVG correlations
HISTSIM-VAR	Historical simulation
COMP-VaR	Simple average composite of RM97-VaR and HISTSIM-VaR

Table 2. Evaluation of VaR Measures for the 90% Confidence Level

VaR Measure	Number of Violations (N=564)[1]	Percent Violations	Avg. Size Violation[2]	Maximum Violation	Minimum Violation	LR Statistic	Z Statistic	Average VaR Estimate
MGARCH-t	84	14.890	$9.737	$36.105	$0.063	13.251 *	3.874 **	$17.862
MGARCH	73	12.940	$9.013	$34.426	$0.226	5.015 *	2.330 **	$20.107
H150-VaR	61	10.820	$8.206	$32.349	$0.044	0.407	0.646	$21.632
HISTAVG-VaR	72	12.770	$8.425	$32.027	$0.009	4.449 *	2.190 **	$20.280
RMOPT-VaR	59	10.460	$8.621	$32.119	$0.451	0.131	0.365	$21.959
RM94-VaR	60	10.640	$8.726	$32.597	$0.023	0.251	0.505	$21.931
RM97-VaR	62	10.990	$8.081	$31.887	$0.002	0.600	0.786	$21.925
GRM97-VaR	75	13.300	$8.898	$32.280	$0.038	6.243 *	2.611 **	$19.983
GH150-VaR	75	13.300	$9.534	$34.572	$0.279	6.243 *	2.611 **	$19.444
GHIST-VaR	87	15.430	$9.530	$36.649	$0.062	16.101 *	4.295 **	$17.871
IVRM97-VaR	69	12.230	$9.726	$35.815	$0.306	2.941	1.769	$19.351
IVH150-VaR	78	13.830	$9.041	$37.424	$0.070	8.314 *	3.032 **	$18.880
IVHIST-VaR	83	14.720	$9.879	$39.120	$0.261	12.357 *	3.734 **	$17.515
HISTSIM-VaR	69	12.230	$8.367	$33.283	$0.052	2.941	1.769	$20.854
COMP-VaR	60	10.640	$8.738	$33.429	$0.126	0.251	0.505	$21.515

[1]N is the number of weekly VaR estimates and subsequent changes in portfolio value.

[2]Avg. size violation, maximum violation, minimum violation, and average VaR estimate are in dollars per head.

*Significant at the 5% level. The Chi-squared critical value is 3.841.

**Significant at the 5% level. The critical Z value is 1.96.

Table 3. Evaluation of VaR Measures for the 95% Confidence Level

VaR Measure	Number of Violations (N=564)[1]	Percent Violations	Avg. Size Violation[2]	Maximum Violation	Minimum Violation	LR Statistic	Z Statistic	Average VaR Estimate
MGARCH-t	56	9.930	$8.519	$31.040	$0.539	22.073 *	5.371 **	$23.025
MGARCH	44	7.800	$7.771	$28.875	$0.394	8.019 *	3.053 **	$25.920
H150-VaR	34	6.030	$6.580	$26.197	$0.010	1.182	1.121	$27.885
HISTAVG-VaR	39	6.910	$7.583	$25.796	$0.097	3.910 *	2.087 **	$26.143
RMOPT-VaR	31	5.500	$7.518	$25.901	$0.388	0.284	0.541	$28.306
RM94-VaR	32	5.670	$7.913	$26.517	$0.233	0.518	0.734	$28.271
RM97-VaR	32	5.670	$6.838	$25.061	$0.032	0.518	0.734	$28.262
GRM97-VaR	46	8.160	$7.541	$26.108	$0.751	10.015 *	3.439 **	$25.759
GH150-VaR	51	9.040	$7.667	$29.063	$0.033	15.820 *	4.405 **	$25.089
GHIST-VaR	58	10.280	$8.415	$31.741	$0.206	25.739 *	5.757 **	$23.048
IVRM97-VaR	44	7.800	$8.339	$30.666	$0.874	8.019 *	3.053 **	$24.945
IVH150-VaR	45	7.980	$8.712	$32.740	$0.745	8.993 *	3.246 **	$24.338
IVHIST-VaR	54	9.570	$9.057	$34.926	$0.333	19.826 *	4.985 **	$22.578
HISTSIM-VaR	36	6.380	$6.705	$26.624	$0.016	2.096	1.507	$27.929
COMP-VaR	34	6.030	$6.614	$27.200	$0.262	1.182	1.121	$28.258

[1]N is the number of weekly VaR estimates and subsequent changes in portfolio value.
[2]Avg. size violation, maximum violation, minimum violation, and average VaR estimate are in dollars per head.
*Significant at the 5% level. The Chi-squared critical value is 3.841.
**Significant at the 5% level. The critical Z value is 1.96.

Table 4. Evaluation of VaR Measures for the 99% Confidence Level

VaR Measure	Number of Violations (N=564)[1]	Percent Violations	Avg. Size Violation[2]	Maximum Violation	Minimum Violation	LR Statistic	Z Statistic	Average VaR Estimate
MGARCH-t	22	3.900	$6.370	$21.730	$0.400	27.655 *	6.924 **	$32.514
MGARCH	14	2.480	$5.795	$18.674	$1.301	8.863 *	3.538 **	$36.602
H150-VaR	6	1.060	$6.001	$14.891	$0.585	0.023	0.152	$39.377
HISTAVG-VaR	10	1.770	$6.062	$16.036	$0.607	2.768	1.845	$36.917
RMOPT-VaR	10	1.770	$4.478	$14.473	$0.427	2.768	1.845	$39.972
RM94-VaR	11	1.950	$5.021	$15.344	$0.420	4.028 *	2.268 **	$39.922
RM97-VaR	6	1.060	$5.668	$14.050	$0.596	0.023	0.152	$39.910
GRM97-VaR	15	2.660	$5.330	$14.766	$1.217	10.783 *	3.961 **	$36.374
GH150-VaR	15	2.660	$6.388	$18.939	$1.257	10.783 *	3.961 **	$35.394
GHIST-VaR	23	4.080	$6.445	$22.720	$0.147	30.483 *	7.347 **	$32.530
IVRM97-VaR	16	2.840	$4.442	$21.202	$0.533	12.840 *	4.384 **	$35.225
IVH150-VaR	17	3.010	$4.858	$24.131	$0.368	15.026 *	4.808 **	$34.368
IVHIST-VaR	25	4.430	$5.877	$27.218	$0.666	36.409 *	8.193 **	$31.883
HISTSIM-VaR	10	1.770	$5.381	$17.602	$0.273	2.768	1.845	$37.759
COMP-VaR	6	1.060	$6.168	$17.362	$1.648	0.023	0.152	$39.063

[1]N is the number of weekly VaR estimates and subsequent changes in portfolio value.
[2]Avg. size violation, maximum violation, minimum violation, and average VaR estimate are in dollars per head.
*Significant at the 5% level. The Chi-squared critical value is 3.841.
**Significant at 5% level. The critical Z value is 1.96.

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