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AN INVESTIGATION OF WEB STRESSES IN REINFORCED CONCRETE BEAMS

BY

FRANK E. RICHART

BULLETIN No. 166

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BY

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I. Introduction

1. *Preliminary.*—This bulletin contains the results of several series of tests of reinforced concrete beams which were made to study the action of web reinforcement (so-called shear reinforcement) under load and to give information on the amount and distribution of stress in such reinforcement. The effectiveness of a variety of types of web reinforcement was investigated. The tests described herein were all of simple beams; it is planned to report similar tests that have been made on restrained beams in a later bulletin.

The action of reinforcement in resisting diagonal tension is not susceptible of exact analysis because of the non-homogeneity of the reinforced concrete member and the high localization of stress in and around the reinforcing steel. The design of web reinforcement, therefore, is usually made by empirical or semi-rational methods. These methods, although based very largely upon observations on existing structures and upon the results of tests of beams of certain types, cannot be expected to apply with any degree of certainty to new and untried types of members or arrangements of reinforcement. It is felt that the measurement of stresses in the web steel of test beams as reported in this bulletin furnishes data from which may be drawn some general conclusions as to the behavior and effectiveness of different types of reinforcement in producing web resistance in beams.

Particular attention is being given at the present time to the limiting value of the shearing unit stress, which is used as a measure of diagonal tension in reinforced concrete, in view of the fact that recent specifications* have raised the allowable value considerably above that permitted by previous standard specifications or building codes. Since high limits of allowable shearing stress are predicated on the use of properly designed web and longitudinal reinforcement, knowledge of what constitutes proper design becomes increasingly important.

In recent years numerous tests have been made on reinforced concrete beams; of these quite a number have included investigations of resistance to diagonal tension failure. The results of tests on rein-

*Report of Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, 1924.

forced concrete beams are given by Talbot in Bulletins 1, 4, 12, 14, 28, and 29 of the Engineering Experiment Station, University of Illinois. Similar tests by Withey are reported in two bulletins, Volume 4, Nos. 1 and 2 Engineering Series, University of Wisconsin. An elaborate series of tests of reinforced concrete beams was reported by Humphrey and Losse in Technologic Paper No. 2, U. S. Bureau of Standards. In France, Considere made tests of concrete beams for the Government, a translated report of which may be found in "Theory and Practice of Reinforced Concrete," by Nathaniel Martin. In Germany, several important series of tests were made by Bach and Graf, and are reported in Heft 39, 45-47, 90-91, and 122-123 of the Forschungsarbeiten aus dem Gebiete des Ingenieurwesens, as well as in Heft 10, 12, 20, 45, and 48 of the Deutscher Ausschuss für Eisenbeton, and in "Der Handbuch für Eisenbeton," 1921. Morsch, in Der Eisenbetonbau, Volume 1, 1920, gives an excellent abstract of the tests of Bach and Graf, as well as a report of several early series of tests made by himself. A recent and very comprehensive series of tests on the web resistance of reinforced concrete beams was made by Slater, Lord, and Zipprodt, Technologic Paper No. 314, U. S. Bureau of Standards, 1926. This last series of tests differs from the other investigations mentioned in that strain gage measurements were taken on the web reinforcement and a great amount of quantitative data on the action of the various kinds of web reinforcement was thereby obtained.

References to these and other investigations on reinforced concrete beams are given in the bibliography in the Appendix.

2. *Acknowledgments.*—The tests described herein were made as a part of the work of the Engineering Experiment Station of the University of Illinois, of which Dean M. S. Ketchum is the director and of the Department of Theoretical and Applied Mechanics, of which Professor M. L. Enger is the head.

The tests form part of an investigation of reinforced concrete beams that has been carried on for many years under the direction of Professor Arthur N. Talbot. The broad scope of the information secured is due to Professor Talbot's adherence to a comprehensive and continued line of investigation, as well as to his careful direction of the planning and performance of the tests.

The testing work was supervised during the twelve-year period covered by D. A. Abrams, W. A. Slater, and H. F. Gonnerman, formerly members of the Engineering Experiment Station Staff, and by F. E. Richart, Research Assistant Professor, and R. L. Brown, Research Associate, all of the Department of Theoretical and Applied Mechanics. Acknowledgment is made to Professor Gonnerman and to G. P. Boomsliter, formerly a member of the Department, for assistance given in the preparation of the bulletin. Assistance in the testing work was given by N. D. Mitchell, R. W. Brooks, J. G. Haeffner, E. A. Schmitz, L. N. Fisher, W. W. Lauer and S. Uchimura, undergraduate and graduate students, to whom credit is due for the care taken and interest shown in the planning and performance of the tests and in the study and correlation of the results.

3. *Scope of Investigation.*—This bulletin embodies the results of tests of 139 reinforced concrete beams made from 1910 to 1922. The various groups of beams may be summarized as follows:

- Series of 1910—25 beams: span 6 ft. 0 in.; width 8 in.; effective depth 10 in.
- Series of 1911—40 beams: same dimensions as Series of 1910.
- Series of 1913—14 beams: span 10 ft. 0 in.; width 8 in.; effective depth 17 in.
- Series of 1917—40 beams: span 9 ft. 6 in.; width 8 in.; effective depth 10 in.
- Series of 1922—12 beams: span 9 ft. 0 in.; width 8 in.; effective depth 21 in. 8 T-beams: span 9 ft. 0 in; width of web 8 in.; effective depth 21 in.; width of flange 20 in.; thickness of flange 6 in.

Figure 1 gives the general dimensions of the test beams used in the various series of tests, and shows the arrangement of reinforcement and loading of a typical beam of each series. The beams were subjected to two equal loads at the one-third points of the span, except in the Series of 1917 where the two loads were applied either 9 or 21 inches on each side of the middle of the beam.

The 139 beams tested were reinforced in a variety of ways, and particular attention was given to types of reinforcement that would offer resistance to diagonal tension and bond. Comparisons were made in two general ways as to the effectiveness of various kinds of web reinforcements: (1) by a study of ultimate shearing unit stresses developed, and (2) by a study of measured deformations in the reinforcement. Disadvantages of the first method are seen in the fact that a considerable number of beams tested failed by tension in the longitudinal steel, and others by bond, or possibly by compression, so that the full resistance of the beams to diagonal tension was not developed. Except when combined with careful observation of the behavior of the beam at

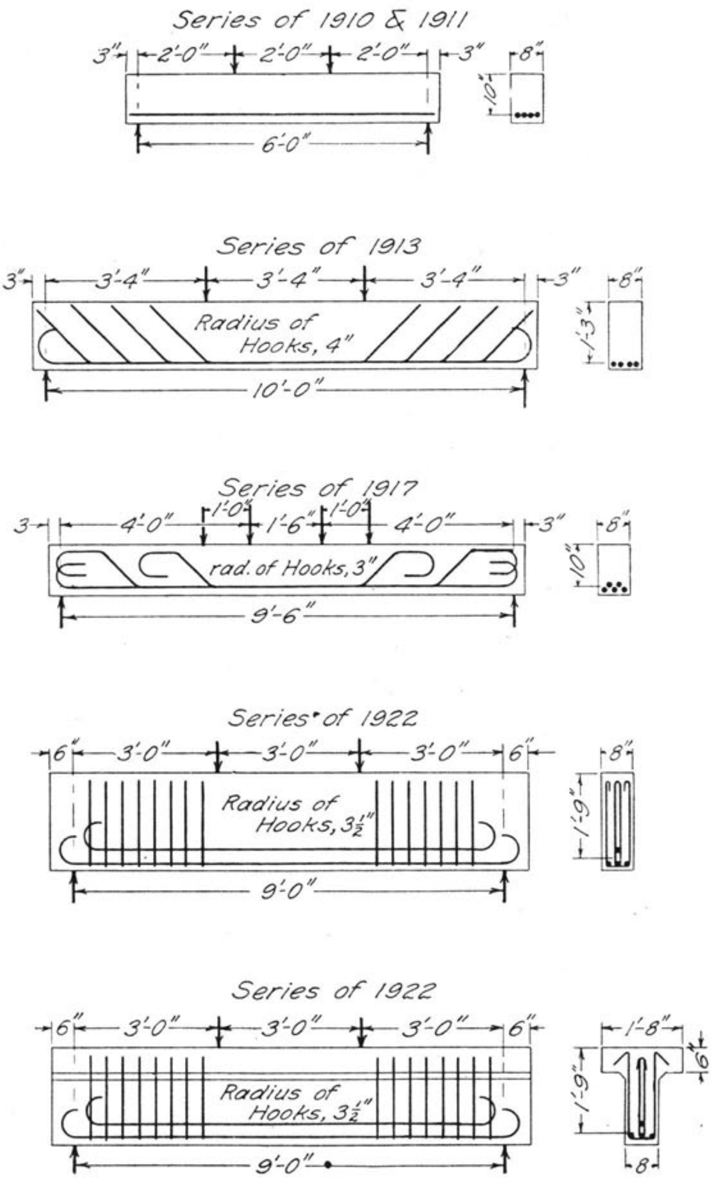

FIG. 1. GENERAL DIMENSIONS OF TEST BEAMS

failure, this method served only to indicate a lower limit of the web resistance that could be developed in certain cases. However, the introduction of the strain gage made possible the measurement of the actual deformations, and hence the stresses, developed in the web reinforcement throughout a test. With known stresses existing in the web reinforcement at failure, it is possible to estimate with a reasonable degree of certainty the ultimate load that might have been carried had failure occurred through diagonal tension. Furthermore, the measurement of stresses furnishes a means of comparing the effectiveness of various types of reinforcement at various stages of the test and of tracing the effect of different factors on the progress of failure in a way that cannot be done by simply considering the ultimate loads carried.

In considering the scope of the investigation, the long period it covered must be remembered. Some of the earlier types of reinforcement are not now in use; others were designed to bring out laws of behavior of the reinforcement rather than to stand as examples of building practice. It is felt, however, that all the tests have a definite bearing on the general subject of web resistance.

4. *Résumé of Analysis of Web Stresses.*—While reinforced concrete beams may fail in a number of ways, the following discussion applies principally to failure by diagonal tension. It is well understood that when both flexural and shearing stresses are present in a beam of homogeneous material, tensile stresses are set up throughout all parts of the beam at various angles of inclination with the horizontal. The angle of inclination and the magnitude of the inclined or diagonal tension stress depend upon the longitudinal and shearing stresses; the diagonal tension will usually be greatest where both moment and shear are large.

In a reinforced concrete beam before the formation of cracks, conditions are somewhat analogous to those in the homogeneous beam. The principal diagonal tension stresses are greatest just above the plane of the longitudinal reinforcement, and make a small angle with the horizontal. At points closer to the neutral surface of the beam the diagonal tension generally decreases and the angle of inclination increases, becoming 45 degrees at the neutral surface, where the longitudinal stress is zero. The angle of inclination also becomes 45 degrees where the bending moment is zero, as at the support of a simple beam or at a point of inflection of a restrained beam. The diagonal tension stress in this 45-degree direction (as well as the diagonal compression that exists at right angles to it) is equal in intensity to the shearing unit stress at the same point.

The diagonal tension stress thus varies from a value equal to the shearing unit stress at the neutral axis to a value considerably greater (perhaps twice as great in beams of the proportions used in the tests) on the tension side of the beam. Cracks may be expected to start at or just above the plane of the longitudinal reinforcement in nearly a vertical direction, and to extend upward with increasing inclination toward the load point until they intersect the neutral axis at about 45 degrees. Such cracks have been found in tests, as may be seen later.

In beams containing web reinforcement, as well as longitudinal reinforcement, the distribution of web stresses differs greatly from that in a homogeneous beam. Little stress is taken by the web reinforcement until the concrete of the web has been overstressed and cracks have formed; when this has taken place the web resistance is furnished principally by the web reinforcement in which the direction of the tensile stress is fixed by the direction of the reinforcement. Since the concrete in the neighborhood of cracks may not transmit either tensile or shearing stresses, the rôle played by the concrete of the web becomes highly indeterminate.

As an aid in furnishing a simpler conception of web stresses, particularly for purposes of design, it has become common practice to use the shearing unit stress as a measure of the diagonal tension and to supplement this approximation with empirical data as to the differences in action observed in test beams of varying proportions of depth to span, varying percentages of reinforcement, varying degrees of fixity of ends, and similar factors. It should be appreciated, however, that the use of the shearing unit stress is an arbitrary rather than a strictly rational procedure. For example, it is obvious that, while the vertical shear diagrams for a simple beam and a restrained beam may be identical, the longitudinal stresses and hence the diagonal tension stresses will differ greatly in the two cases.

As may be inferred from the preceding paragraph the analysis of web stresses in reinforced concrete is admittedly approximate; assumptions and analogies are used that seem reasonable and that provide a simple physical conception of the inter-action of the different structural elements. In the analysis to follow it will be well to consider that the relations derived present merely a reasonable picture of stress phenomena rather than an exact representation of actual conditions.

Shearing and bond stresses are usually calculated on the basis that no longitudinal tension is carried by the concrete and that there is no slipping between concrete and steel; values of these nominal

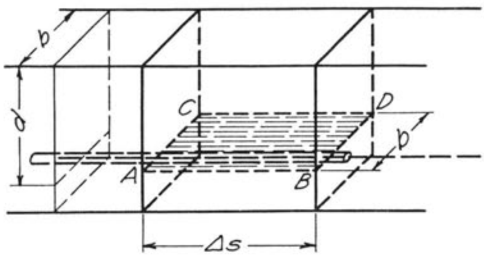

FIG. 2. SECTION OF BEAM SUBJECTED TO SHEARING STRESSES

or conventional shearing and bond stresses may be derived as follows: In the short length Δs of the reinforced concrete beam shown in Fig. 2, a horizontal plane $ABCD$ is passed just above the level of the reinforcement. Denoting the width of the beam by b, the area in horizontal shear over the length Δs is $b\Delta s$, and the total shearing force to be transmitted over this area is equal to the change in total tensile stress in the steel in the distance Δs. Denoting this total stress in the steel at A (equal to the resisting moment or couple divided by the arm of the couple) by $\frac{M_A}{jd}$ and that at B by $\frac{M_B}{jd}$ and the horizontal shearing unit stress by v, the change in stress from A to B is $\frac{M_A - M_B}{jd}$ and is equal to the total shearing force $vb\Delta s$.

The change in moment per unit length is equal to the vertical shear, or $\frac{M_A - M_B}{\Delta s} = V$, so that

$$\frac{M_A - M_B}{jd} = \frac{V\Delta s}{jd} = vb\Delta s, \quad \text{whence} \quad v = \frac{V}{bjd} \tag{1}$$

According to the principles of mechanics, the shearing unit stress v, at any point in a homogeneous beam, is the same in the vertical and horizontal directions. The shearing unit stress transmitted is dependent upon the bond between the concrete and the reinforcement. Thus the total shearing stress $vb\Delta s$, equal to the increment in total tensile stress, is also equal to the total bond stress over the length Δs. Denoting the unit stress in bond by u and the total periphery of the reinforcing bars by mo,

$$vb\Delta s = mou\Delta s$$

whence

$$v = \frac{mou}{b} \tag{2}$$

or
$$u = \frac{V}{m o j d} \tag{2a}$$

The usual assumption of a constant intensity of shearing stress between the longitudinal reinforcement and the neutral surface, which is based on the assumption that the longitudinal tension in the concrete may be neglected, clearly does not apply exactly in actual beams, particularly toward the ends of a beam, where the bending moment is small. Furthermore, it appears probable that the assumption of conservation of plane section during flexure may not hold true in cases where bars are bent up at an angle, or where slipping of bars occurs. However, the relation expressed by equation (2) shows that where an abnormal bond stress is produced, a local variation of the shearing stress will also occur. Thus, if the bond stress is lowered or destroyed by the slipping of a bar, the shearing stress in the immediate locality will vary accordingly, and if a high local bond stress is produced by bending up a reinforcing bar, the distribution of the shearing stress in the adjacent portion of the beam will be affected likewise. To reconcile the variations in horizontal shearing stress produced by variations in bond with the vertical shearing stress indicated by the vertical shear diagram it is seen that local variations in shearing unit stress over the cross-section of the beam must obtain which differ from the conventional distribution of shearing stress based on the straight-line theory of flexure. The intensity of shearing stress may, therefore, vary considerably from the nominal shearing unit stress given by equation (1). A similar statement may be made regarding the actual bond stress and the nominal bond stress given by equation (2). It is probable that many failures attributed to diagonal tension are brought about by initial local bond failure.

By recalling that the horizontal shearing area of Fig. 2 was taken in a plane just above the level of the reinforcing steel, it is evident that equation (1) expressing the value of the nominal shearing unit stress may apply to T-beams as well as to rectangular beams if the width of the beam b be considered as the width of the stem of the T-beam. The flange of the T-beam affects the value of v only as it produces a value of j slightly greater than that for a rectangular beam. Actual differences in the value of v developed in tests will be discussed in a later section.

Types of web reinforcement commonly used may be grouped in three classes; vertical stirrups, inclined stirrups, and bent-up longitudinal bars. The analysis of stresses in web reinforcement has been treated by a number of writers, most of whom have used the same general assumptions. One form of the analysis will be given here, using the following notation:

a_V = cross-sectional area of web reinforcement
f_V = tensile unit stress in web reinforcement
v = nominal shearing unit stress in concrete
b = width of beam (or width of stem of T-beam)
s = spacing of web reinforcing bars, measured along axis of beam
a = spacing of web reinforcing bars, measured at right angles to their direction
r = ratio of web reinforcement $= \frac{a_V}{ab}$
V = total external vertical shear on beam
d = effective depth of beam
jd = distance between centroids of tensile and compressive stresses in beams
$K = \frac{v}{rf_V}$ = efficiency factor for web reinforcement

Beams with Vertical Stirrups

In the various published analyses of stresses in vertical stirrups the assumption has generally been made, either tacitly or expressly, that, regardless of the kind or direction of the web reinforcement, the diagonal compression in the concrete web remains inclined at an angle of 45 deg. to the horizontal. Another assumption is that the action of a beam may be likened to that of a truss, in which the top chord is formed by the compression zone of the concrete, the bottom chord by the longitudinal reinforcement, the tension web members by the stirrups and the compression web members by portions of the concrete web of the beam. The stirrups are considered to be looped around or rigidly attached to the longitudinal steel, while the connection between steel and concrete members is by bond. Figure 3 shows such a truss, in which there is no connection between vertical and diagonal web members except at the plane of the top and bottom chords. The distance jd has been taken as some integral multiple of the spacing a. For such a structure it is easy to show that the number of stirrups cut by a section x-x is $\frac{jd}{a}$ and that the total tensile stress in any vertical member is

$$a_V f_V = \frac{Va}{jd} \tag{3}$$

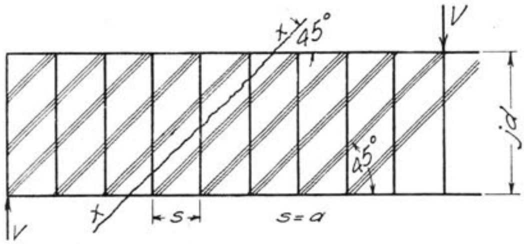

FIG. 3. TRUSS WITH VERTICAL TENSION WEB MEMBERS

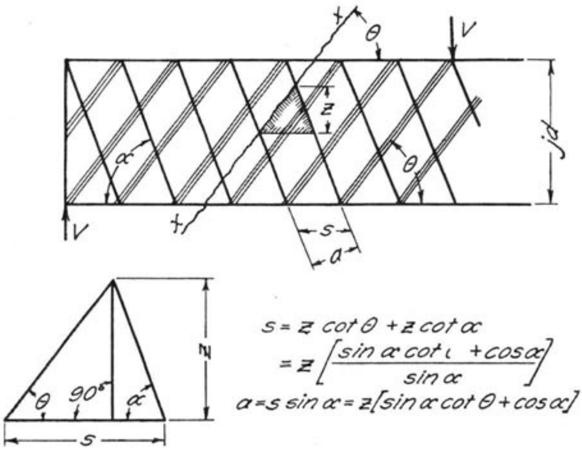

FIG. 4. TRUSS WITH INCLINED TENSION WEB MEMBERS

or since $V = vbjd$ and $r = \dfrac{a_V}{ab}$, $\qquad f_V = \dfrac{v}{r}$ (4)

Thus the tensile unit stress in the stirrup is equal to the shearing unit stress divided by the ratio of web reinforcement.

Beams with Inclined Stirrups

By assuming that the diagonal compression maintains a fixed direction as in the preceding analysis, and again using the analogy between the action of stirrups and that of the tension web members of a truss, a rather general analysis of the stresses in stirrups inclined at an angle α with the horizontal may be obtained. In Fig. 4 consider the inclined stirrups as tension web members of a truss similar to that of Fig. 3, and the diagonal compression zones in the concrete as inclined compression members, here shown making an angle θ with the horizontal. Treating this truss as in the preceding section, by cutting a section x-x parallel to the diagonal compression members, it is seen that the number of stirrups cut by the section is $\frac{jd}{z}$. Hence the vertical component of the external shear V carried by one stirrup is

$$\frac{Vz}{jd} = \frac{Va}{jd}\,\frac{1}{(\sin\alpha\,\cot\theta + \cos\alpha)} = a_V f_V \sin\alpha$$

whence the total stress in a stirrup,

$$a_V f_V = \frac{Va}{jd}\,\frac{1}{(\sin\alpha\cot\theta + \cos\alpha)\sin\alpha} \tag{5}$$

Substituting $r = \frac{a_V}{ab}$ and $bv = \frac{V}{jd}$,

$$f_V = \frac{v}{r}\,\frac{1}{(\sin\alpha\cot\theta + \cos\alpha)\sin\alpha} = \frac{v}{rK} \tag{6}$$

Assuming that regardless of the direction α of the tensile reinforcement the angle θ will remain constant and equal to 45 deg., and substituting this value of θ, equation (6) becomes

$$f_V = \frac{v}{r}\,\frac{1}{(\sin\alpha + \cos\alpha)\sin\alpha} \tag{7}$$

For stirrups inclined at 45 deg. and for vertical stirrups ($\alpha = 45$ deg. and 90 deg.) equation (7) reduces to equation (4)

$$f_V = \frac{v}{r}$$

While the assumption that θ is always equal to 45 deg. has been accepted by many as one that seems a reasonable approximation and is consistent with certain test data, it is well to note the effect of even a small variation in the value of θ. Figure 5 shows values of the quantity

$$K = (\sin\alpha\,\cot\theta + \cos\alpha)\,\sin\alpha = \frac{v}{rf_V}$$

plotted on polar coördinates for values of θ of 40, 45, and 50 deg., and values of α from 0 to 90 deg. For any angle of inclination of stirrups α, the length of the radius vector (or distance from the origin to the point on the curve) represents the value of K. It is seen that when the stirrups are inclined at 45 deg., the values of K are 0.91, 1.00 and 1.10 for values of θ of 50, 45, and 40 deg. respectively, while the respective values for vertical stirrups are 0.84, 1.00 and 1.19.

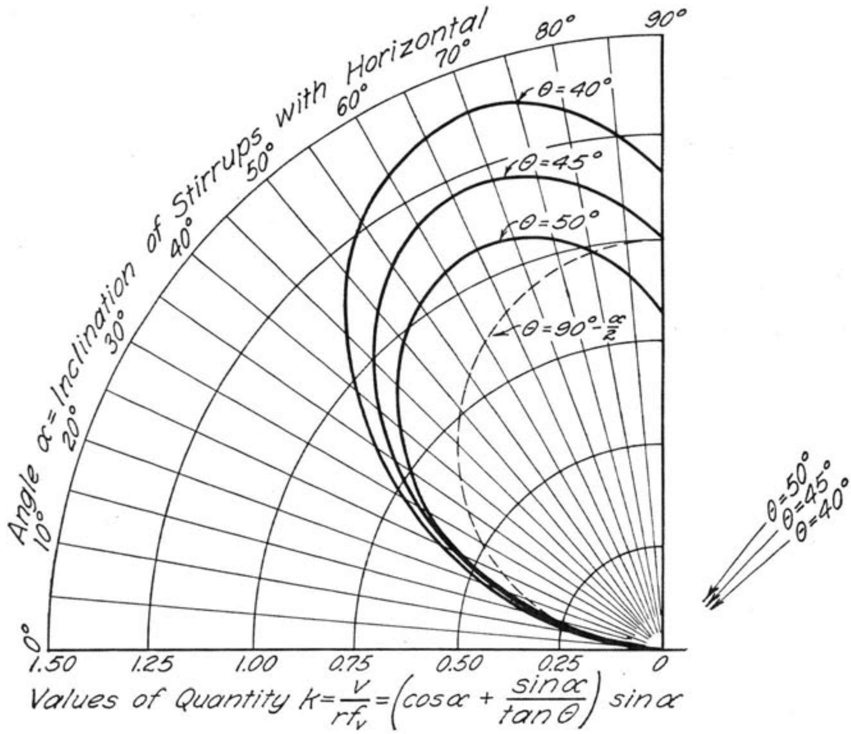

FIG. 5. VARIATION OF QUANTITY $\frac{v}{rf_V}$ WITH ANGLE OF INCLINATION OF STIRRUPS

The effect of a variation from an assumed value of 45 deg. is thus seen to be considerably greater for vertical stirrups than for those inclined at 45 deg. It is seen further that the greatest magnitude of K is found for stirrups inclined at angles α of 65, 67½ and 70 deg. for values of θ of 50, 45 and 40 deg., respectively. This indicates that stirrups inclined 65 to 70 deg. to the horizontal are the most efficient, or will develop the highest value of v per pound of reinforcement (since r may be considered as the ratio of volume of web reinforcement to volume of concrete web). It may be noted that this maximum efficiency is obtained when α is equal to $(90 - \frac{\theta}{2})$ deg., so that the stirrup axis bisects the obtuse angle between the diagonal compression zone and the longitudinal reinforcement.

Other assumptions as to the direction of the diagonal compression have also been investigated.* Figure 5 shows a curve for values of K when θ is taken equal to $(90-\frac{\alpha}{2})$ deg., or the diagonal compression is assumed to act in a line bisecting the obtuse angle between stirrup and longitudinal reinforcement. This assumption is seen to result in relatively low values of K, particularly for the smaller values of α.

As has been noted, the assumption that $\theta = 45$ deg. results in an equal calculated efficiency of stirrups inclined at 45 deg. and at 90 deg., respectively. This condition has been verified by strain gage measurements on test beams by Slater, Lord, and Zipprodt, who found a very slight advantage in the case of the inclined stirrups. The correctness of using a constant value of θ equal to 45 deg. is further substantiated by the fact that cracks commonly occur at about 45 deg. to the horizontal (or along the section x-x, Figs. 3 and 4) in reinforced concrete test beams, so that the diagonal compression members lying between these cracks are necessarily inclined 45 deg. to the horizontal. Considering Figs. 3 and 4 it is seen that in a beam with vertical stirrups the total stress in the stirrup must equal the vertical component of the total diagonal compression, and if the diagonal compression is inclined at 45 deg., its magnitude is equal to $1.41\frac{Vs}{jd}$. In the case of stirrups inclined at 45 deg., the vertical component of stress in the stirrup must equal the vertical component of the total diagonal compression, and if the diagonal compression is inclined at 45 deg. its magnitude is equal to $\frac{Va}{jd}=\frac{0.71Vs}{jd}$. This indicates that the intensity of diagonal compression is twice as great when stirrups are vertical as when they are inclined at 45 deg. Diagonal compression stresses are not usually important unless thin webs or very high amounts of web reinforcement are used.

In the foregoing analysis of inclined stirrups it is seen that the change in longitudinal stress in a distance s must be equal to the sum of the horizontal components of the inclined tension and compression. Presumably the attachment of these diagonal members is largely by bond. It will be seen that in some cases a rigid connection of the inclined web steel to the longitudinal steel is needed to prevent failure

*Equation (6) was derived in an analysis made by Slater, Lord, and Zipprodt, Technologic Paper No. 314, U. S. Bureau of Standards, and was used to investigate the effect of assuming θ equal to 45, $(90-\alpha)$, and $(67\frac{1}{2}-\frac{\alpha}{2})$ deg., respectively. It was concluded that for values of α from 0 to 45 deg., the three assumed values of θ produced results differing but little; for values of α from 45 to 90 deg., the second and third assumptions gave very improbable values, while an assumed value of θ of 45 deg. gave results that seemed reasonable.

through the slipping of the web steel. This will be true in general where the web steel is composed of large bars which do not afford sufficient bond. For this reason, as well as to afford anchorage of the upper ends of the stirrups, the use of small sizes of stirrups is advisable.

Beams with Bent-up Bars

The analysis of the action of bent-up bars as web reinforcement is not so satisfactory as that of stirrups. The variations in longitudinal steel area resulting when bars are bent up at various points bring about conditions of bond and shear that are not consistent with the usual assumptions of conservation of plane section of members in flexure. If a large number of bars are bent up in regular order, the analysis of the action of inclined stirrups may be applied with some degree of justification. The similarity in action of the two methods of reinforcement would be greater if some sort of rigid connection were provided between all reinforcing bars at each point of bending-up. Without such connection, the variations in stress that occur in the longitudinal steel make any quantitative analysis approximate, but generally of sufficient accuracy for design purposes.

The usual basis for determining where to bend up longitudinal bars for web reinforcement is to bend them up only in the portion of the beam where they are not needed to resist bending moment. If a sufficient number of longitudinal bars are used to meet the requirements of the maximum moment, it may be shown that the excess over the moment requirements at other sections will be more than enough for complete web reinforcement. That is, in a length s, the change in longitudinal tension is $\Delta A f_s$ while the total stress in a bent-up web bar may be, from equation (5),

$$a_V f_V = \frac{Vs}{jd}\left(\frac{1}{\sin\alpha \cot\theta + \cos\alpha}\right)$$

Hence, by equating the area available for bending up, ΔA, to the required area of bent-up bar, a_V, the resulting stress in the bent-up bar is

$$f_V = \left(\frac{f_s}{\sin\alpha \cot\theta + \cos\alpha}\right)$$

or, for the angles at which bars are usually bent up, the stress in the bent-up bar will be considerably less than the maximum stress in the horizontal part of the longitudinal bar. In general, however, it would usually be difficult to provide a sufficient number of small bars as tension reinforcement to permit their use as bent-up bars for complete web reinforcement throughout, in beams other than those having low shearing stresses.

The use of bars bent up at a number of sections not only requires great care in the bending and placing of the reinforcement, but also produces conditions of bond and shearing stress that must be considered. Thus, when a number of bars are bent up, there is a concentration of bond stress in the remaining horizontal bars much greater than the nominal bond stress given by equation (2a). The concentration of bond stress will induce a like variation in shearing stress, so that the distribution of shearing stress in beams with bent-up bars may be rather uncertain.

Total Shearing Strength of Beams

For beams having no web reinforcement it has been stated that the diagonal tension stress may be twice as great as the nominal shearing unit stress. Failure by diagonal tension in the concrete may therefore occur at shearing unit stresses of perhaps one-half of the tensile strength of the concrete; calculated ultimate shearing unit stresses as high as 150 to 250 lb. per sq. in. have frequently been observed in beam tests; on the other hand values less than 100 lb. per sq. in. have been observed.

In test beams with web reinforcement the shearing stress found at the ultimate load is somewhat more than that accounted for by the stress in the web reinforcement, and it is frequently stated that a portion of the shear is carried by the concrete. Obviously the strength of the concrete web is largely destroyed when diagonal cracks have formed and the web reinforcement has been brought into action, but undoubtedly some portion of the vertical shear is carried by shearing stress over the uncracked compression area at the top of the beam. This naturally affects the amount of tensile stress developed in the web reinforcement and has led to the assumption, for purposes of design, that a certain portion of the external shear is carried by the concrete and that it may be considered to be uniformly distributed over the area bjd.

The discrepancy between the measured tensile stress in web reinforcement and that calculated by assuming the total shear to be carried by the reinforcement might also be due to the use of too small a factor K in equation (6), resulting from incorrect assumption of the direction of the diagonal compression. However, the tests indicate that the discrepancy is best accounted for as corresponding to the portion of the shear carried by the concrete, as already described.

Two types of design formulas are in common use. In one* it is assumed that one-third of the total vertical shear will be carried by the concrete and two-thirds by the web reinforcement. The other† involves the assumption that the concrete will carry a constant portion of the working shearing unit stress (such as 40 or 50 lb. per sq. in., or a given proportion of the ultimate compressive strength of the concrete) and that the web reinforcement must carry the remainder. The data obtained from the tests discussed herein will throw some light on the correctness of the two types of formula.

II. Materials, Test Pieces, and Methods of Testing

5. *Concrete Materials and Proportions.*—The materials used in making the test beams were similar in character to those used in reinforced concrete beams described in previous bulletins of the Engineering Experiment Station. Data concerning the materials used in making the concrete for all of the test beams, the average proportions of the different mixtures, and the average strengths of the concrete as indicated by tests of control specimens are given in Table 1.

Universal Portland cement, which was furnished to the University for experimental work by the manufacturers, was used in all of the tests. Tests of this cement showed that it complied with standard specifications in regard to fineness, soundness, and, with one slight exception, to strength. The results of briquet tests of the cement are given in Table 1.

The fine aggregate used was in all cases a clean, coarse, well-graded sand from near the Wabash River at Attica, Indiana. The coarse aggregate used in the test beams made previous to 1917 was a broken limestone from Kankakee, Illinois. The coarse aggregate used in 1917 and 1922 was a washed gravel obtained at Attica, Indiana. Both the sand and the gravel consisted of hard, strong, somewhat irregular particles, and were composed largely of calcareous material. The particles of the coarse aggregates ranged from ¼ in. to 1 in. in size. Sieve analyses of the aggregates for the different series of tests are given in Table 2.

With two exceptions, the concrete was mixed in the proportions of 1:2:4 by loose volume. In the series of 1917 an additional mix of pro-

*See Final Report of the Joint Committee on Concrete and Reinforced Concrete, 1917, or Taylor, Thompson and Smulski, "Concrete, Plain and Reinforced," Vol. 1, 1925.

†See Report of Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, 1924; also Specifications of German Committee on Reinforced Concrete, Berlin, 1925; Revised Building Ordinances of the City of Chicago, 1920; Turneaure and Maurer, "Principles of Reinforced Concrete Construction," 1919 and Taylor, Thompson and Smulski, *loc. cit.*

TABLE 1

DATA OF CONCRETE MATERIALS AND OF CONCRETE USED IN TEST BEAMS

Materials: Universal Portland Cement
Torpedo Sand from Attica, Ind.
Coarse Aggregate, Series of 1910, 1911 and 1913, Limestone from Kankakee, Ill.
Coarse Aggregate, Series of 1917 and 1922, Gravel from Attica, Ind.

Item			Series of				
			1910	1911	1913	1917	1922
Beam Numbers, inclusive			280.1 to 285.7	286.1 to 295.5	301.1 to 308.2	16B1.1 to 16B20.2	221.1 to 2210.2
Tensile Strength of Briquets lb. per sq. in.	Neat Cement	Age, 7 days	637	647	595	613	...
		Age, 28 days	675	702	740	769	697
	Ottawa* Sand Mortar	Age, 7 days	209	220	207	232	214
		Age, 28 days	313	290	301	318	311
	Attica* Sand Mortar	Age, 7 days	...	265	...	345	309
		Age, 28 days	...	323	...	465	488
Average Proportions of Mixture	By volume		1:2:4	1:2:4	1:2:4	1:2:4 1:1:2	1:2:4
	By weight		1:2.1:3.6	1:2.1:3.6	1:2.2:3.7	1:2.2:4.2 1:1.1:2.1	1:2.4:4.3
Per cent of Mixing Water, by weight			8.0	9.3	9.4	7.9 10.0	6.8
Water-cement Ratio, by volume			0.80	0.93	0.98	0.88 0.63	0.79
Compressive Strength lb. per sq. in.	6 in. Cubes		3450	....	2580	....	....
	6 by 12 in. Cylinders		....	....	2400	3260 4770	4060
Age, days			93		220	61 61	61
Modulus of Rupture lb. per sq. in.	Control Beams 6 by 8 by 40 in.		372	291	223	...	...
Age, days			71	67	236	...	...

*Mortar of proportions 1:3, by weight.

portions 1:1:2 by loose volume was also used. In the series of 1922 the proportions used were 1:2:4 by tamped volume. The weights of all materials entering into the concrete were recorded in order to obtain a check on the proportioning. All concrete made prior to 1913 was mixed by hand; that made after 1913 was machine-mixed in a batch mixer of one-third cubic yard capacity. The batch was generally mixed four minutes after all ingredients had been put in the mixer. All the work of mixing and placing the concrete was done by men experienced in concrete making, under the supervision of a member of the staff of the Engineering Experiment Station.

TABLE 2
SIEVE ANALYSES OF FINE AND COARSE AGGREGATES
(Made with Tyler Standard Screen Scale Sieves)

Kind of Aggregate	Series	Per cent by Weight Passing Given Sieve								
		100	48	28	14	8	4	⅜ in.	¾ in.	1 in.
Sand...........	1910	3	7	19	42	68	93	99		
	1911	2	7	26	53	76	97	100		
	1913	2	5	27	50	70	92	100		
	1917	3	7	18	42	71	95	100		
	1922	4	10	25	51	78	97	100		
Limestone......	1910				2	3	6	31	93	100
	1911				1	2	5	35	95	100
	1913				2	4	9	46	95	100
Gravel..........	1917						1	14	73	100
	1922						3	13	59	89
Width of Sieve Opening, Inches.....		0.0058	0.0116	0.0232	0.046	0.093	0.185	0.371	0.742	1.05

6. *Steel Reinforcement.*—In the tests made in 1913 and 1917, mild steel was used for reinforcement. In 1922, mild steel was used for stirrups and high-carbon steel for longitudinal reinforcement. Of the fabricated reinforcing units used in 1910 and 1911 part were of mild steel and part were of high-carbon steel. Among the fabricated unit frames which were furnished by their respective manufacturers for the tests were the Corr-bar unit, the General Fireproofing Company's unit, the Gabriel unit, the Monolith unit, and a unit made by the American System of Reinforcing. The first three of these were of high-carbon steel and the remaining two were of mild steel. Reinforcing steel was also furnished for the later series of tests through the courtesy of the Illinois Steel Company, Chicago, Illinois, and the Corrugated Bar Company, Buffalo, New York. Average values of the physical properties of all reinforcing steel are given in Table 3.

For use in computing unit stresses from measured deformations in the steel, the value of the modulus of elasticity of all reinforcing steel was taken as 30 000 000 lb. per sq. in.

7. *Description of Test Beams.*—A somewhat different type of beam was used in each series of tests, as new phases of the subject were being investigated. With the exception of a few beams of the Series of 1922, the test beams were of rectangular cross-section, being 8 in. wide and 10, 15, or 21 in. in effective depth. Eight beams of the Series of 1922 were T-beams. Figure 1 gives representative details of the beams of the different types. A considerable number of beams

TABLE 3
TENSION TESTS OF REINFORCING STEEL

Series	Beam No.	No. of Speci-mens Tested	Description of Bar	Unit Stress at Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Per cent Elongation in 8 inches	Per cent Reduction in Area
1910*	280.3	3	5/8-in. round.........	38 500	55 000	31.4	65.8
	281.1	2	11/16-in. ovoid........	40 000	66 500	22.5	49.6
	281.5	2	11/16-in. ovoid........	37 600	67 100	26.9	53.0
	282.2	1	3/4-in. monolith......	37 700	54 800	18.7	56.0
	283.3	2	5/8-in. round.........	63 300	91 200	18.5	46.5
	283.7	2	5/8-in. round.........	57 000	90 300	20.0	47.7
	284.1	3	5/8-in. corr. round....	63 300	101 100	13.7	28.7
	284.7	3	5/8-in. corr. round....	64 800	102 300	12.3	44.3
	285.3	2	1-in. herringbone.....	54 700	89 400	6.9	7.3
	285.6	2	1-in. herringbone.....	53 000	90 600	11.9	20.8
	281.1	3	3/16-in. round wire...	93 300	99 200	3.3†	39.3
	281.5	3	3/16-in. round wire...	99 400	99 800	3.7†	34.2
	282.2	3	1/4-in. round.........	54 500	67 900	15.8	58.9
	283.3	3	5/16-in. round........	47 100	63 400	28.3‡	57.0
	283.7	2	5/16-in. round........	49 600	64 800	20.6†	54.1
	284.1	3	3/16-in. round........	63 700	74 800	20.0†	68.8
	284.7	3	1/4-in. corr. square...	55 600	74 300	20.6†	48.1
	285.3	3	7/32-in. round........	70 700	84 900	12.5†	57.4
	285.6	3	7/32-in. round........	71 200	84 800	12.9	53.8
1911*	287.6	2	3/4-in. round.........	38 000	54 300	33.4	60.0
	290.3	2	1/2-in. corr. square...	60 000	97 800	14.7	19.8
	290.7	2	1/2-in. corr. square...	56 900	92 600	11.2	13.9
	292.7	2	3/4-in. round.........	44 800	60 600	25.6	51.1
	293.3	2	3/4-in. round.........	34 200	54 000	32.8	59.5
	293.9	2	3/4-in. round.........	38 700	58 300	31.0	43.1
	295.3	2	1-in. monolith........	36 500	67 400	10.6	4.5
	295.6	2	1-in. monolith........	32 700	66 400	9.8	21.6
	287.6	3	1/2-in. round.........	40 700	56 700	31.2†	66.3
	290 3	2	1/2-in. corr. square...	64 400	88 100	20.0	34.9
	290.7	3	0.2-in. round.........	58 500	69 500	12.0†	62.5
	292.7	3	1/2-in. round	39 400	52 800	30.9†	58.5
	293.9	2	1/2-in. round.........	38 800	63 800	24.3†	48.4
	295.3	3	1/4-in. round.........	68 500	74 900	5.0	52.7
	295.6	5	3/8-in. round.........	57 400	72 100	18.6	60.8
1913	.....	20	3/4-in. round.........	36 300	......	....	....
		20	3/8-in. round.........	41 500	......	....	....
1917	.....	2	1-in. round..........	40 500	55 300	34.3	64.5
		21	7/8-in. round.........	45 700	66 300	29.6	51.2
		3	3/4-in. round.........	40 600	58 400	31.2	63.5
		5	5/8-in. round.........	37 500	56 600	30.1	64.6
		6	5/8-in. square........	36 000	56 900	31.7	64.1
1922	.....	4	1-1/8-in. corr. round..	52 400	88 600	21.3	26.7
		4	5/8-in. round.........	39 600	61 300	28.3	63.9
		4	1/2-in. round.........	40 100	57 700	29.2	67.8
		4	3/8-in. round.........	42 900	57 800	30.2	67.3

*Specimens for Series of 1910 and 1911 were taken from beams that had been tested, at points away from the point of failure of the beams. The yield point of stirrups in these two series may be too high, since the stirrup steel in some cases had been bent and straightened before testing.
†Per cent elongation measured on 4-in. gage line.
‡Per cent elongation measured on 2-in. gage line.

were made with no web reinforcement. Others were made with a variety of types of web reinforcement, including both loose and rigidly attached vertical stirrups, rigidly attached inclined stirrups, unit frames, bent-up bars, and various combinations of these types. In many of the beams the longitudinal bars were anchored at their ends by means of hooks or by nuts and washers to prevent slipping. Detailed descriptions of the test beams and data of the tests are given in Chapter III.

8. *Making of Beams.*—All beams were cast in bottomless wooden forms resting on a sheet of building paper placed on the floor of the laboratory. The forms were held in position by wooden clamps spaced at intervals along the length of the beam. In the case of many of the beams on which strain measurements were taken, corks about $1\frac{1}{4}$ in. in diameter and 1 to $1\frac{1}{2}$ in. long were nailed to the sides of the form, the outer reinforcing bars being placed against these corks and secured to the sides of the form with staples. When the forms and the corks were removed the steel was left exposed to facilitate the preparation of strain gage holes.

In pouring the specimens the concrete was carefully puddled around the reinforcing bars. In some of the later beams, trouble arising from the settlement of the concrete away from beneath the reinforcing steel was avoided by suspending the reinforcement from small bars and removing these supports a short time after the concrete was poured and before it had begun to set.

The forms were removed from the beams from two to seven days after they were poured. The beams were generally not moved for several weeks, but were left on the laboratory floor where they had been poured and were sprinkled regularly with water or covered with burlap which was kept wet.

Control specimens were made in all series of tests from the same batch of concrete as used in the beams, and were subjected as nearly as possible to the same conditions of storage as the beams. Three types of control specimens were used. In the tests of 1910, 1911, and 1913, control beams of plain concrete, 6 in. wide, 8 in. deep, and 3 ft. 4 in. long were made. These control beams were tested on a span of 3 ft. to determine the modulus of rupture of the concrete. To determine the compressive strength of the concrete 6-in. cubes were also made. In the 1917 and 1922 tests cylinders 6 in. in diameter and 12 in. long were used for compression specimens instead of the 6-in. cubes.

9. *Methods of Testing.*—In preparing the beams for testing they were given a coat of whitewash, which produced a smooth white surface upon which the formation of fine cracks could be detected more certainly and earlier in their development than on the natural surface of the concrete.

Before placing a specimen in the testing machine the strain gage holes were prepared. Generally, since holes had been left in the concrete to expose the reinforcement at points where gage lines were desired, it merely remained to locate and drill the gage holes. The exact gage length was laid out with a double pointed center punch, care being taken to drill the holes normal to the surface of the bar. It was found desirable to try out the strain gage repeatedly on all gage lines before starting to take any readings, in order to wear down the burr or roughness on the edge of the drilled holes, and to see that the position of the holes allowed a sufficient range of movement in the instrument.

Three testing machines were used in the various series of tests; a Riehle machine of 100 000 lb. capacity, and two Olsen machines of 200 000 and 300 000 lb. capacity, respectively. In the two larger machines the beam rested on bearing plates supported by rocker pedestals or rollers which in turn rested on the wings of the weighing table, while in the 100 000 lb. machine the rollers rested on a steel I-beam placed across the weighing table of the machine.

Load was applied through steel I-beams, rollers, and bearing plates to the load points on the beam. In the earlier tests a layer of rubber belting was used between each bearing plate and the specimen to distribute the load, but in the later tests the practice was to bed the plates in plaster of Paris in position on the beam. In applying load the rate of movement of the testing machine head varied from about 0.03 to 0.06 in. per minute. Load was applied in increments and strain measurements and observations of cracks were taken after each increment of load. The occurrence of special phenomena was also noted as the test progressed. No difficulty was encountered in maintaining a constant load throughout a set of observations except as the ultimate load was approached, when considerable "drop-off" in load occurred through the yielding of the test beam.

10. *Strain Measuring Apparatus.*—In a way, these tests record the progress made in methods of strain measurement during the period of time they cover. Prior to 1910 a Johnson extensometer had been used in tests of beams to measure the deformation of the upper and lower fibers over a single gage length at midspan. In the 1910 tests an effort was made to measure deformations due to the web stresses as well as to the longitudinal stresses. For this purpose graduated dials of the Wissler type were attached to the side of the beam and each one was connected by a fine copper wire to a small plug attached to the side

of the beam approximately 10 in. from the dial. A movement between plug and dial could thus be measured with a precision of 0.0002 in. over the 10-inch gage length. Generally six of these dials were used on a specimen.

In 1911 a strain gage of the Berry type was first employed on the tests together with some of the dial apparatus used in the preceding year. The strain gage made possible the direct measurement of deformations in the steel reinforcement at a large number of points in a reasonable time. After 1911 the strain gage was used almost exclusively. The instrument is now in common use in testing work and need not be described here.* Unit deformations were measured on 4-in. and 8-in. gage lengths with a precision of 0.00001 to 0.00002 in. per in.

In a few cases the slipping of a reinforcing bar due to local bond failure was measured. For this purpose an Ames dial was attached to the beam with its plunger in contact with a piece of stiff wire attached to the reinforcing bar, or with the plunger in direct contact with the exposed end of the bar.

Deflections were measured in the 1910 tests and also in the 1911 tests by means of Ames micrometer dials attached to the middle of the beam. The Ames dials were actuated by contact with small wooden beams which were clamped to the ends of the test beam directly over the supports. No deflections were measured in later tests.

III. Results of Tests

11. *Method of Computing and Tabulating Results.*—The results of the five series of tests are presented in the following sections largely in tabular farm. The methods of calculating quantities are those used in common practice. For example, the percentage of longitudinal reinforcement is calculated from the cross-section of the beam and of the reinforcing steel at midspan. The percentage of web reinforcement given is computed from the ratio r, the ratio of volume of web reinforcement to volume of concrete web reinforced, as defined in Section 4. The stress in the longitudinal steel, f_s, is computed from the equation $f_s = \frac{M}{Ajd}$, wherein M is the maximum bending moment, A is the cross-sectional area of the longitudinal steel, jd is the lever arm of the resisting couple, and d is the effective depth of the beam. Values of j were calculated by use of the methods of Bulletin 4, Engineering Ex-

*For a description of the strain gage and its use, see "Tests of Reinforced Concrete Buildings Under Load," Univ. of Ill., Eng. Exp. Sta. Bul. 64, 1913, or Proc. of A. S. T. M., Vol. 13, p. 1019, 1913.

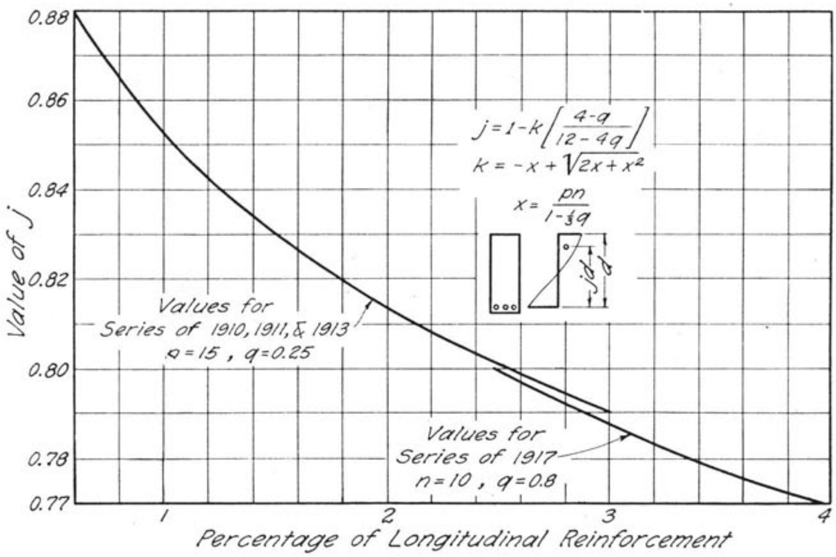

FIG. 6. VALUES OF j FOR VARIOUS PERCENTAGES OF REINFORCEMENT

periment Station, University of Illinois, in which a parabolic variation of the compressive stress in the concrete is considered. The value of j varies with the ratio n of the moduli of elasticity of steel and concrete, with the ratio q of the actual compressive deformation in the concrete at the extreme fiber to the ultimate or crushing deformation of the concrete, and with the percentage of longitudinal reinforcement. From a study of the modulus of elasticity of the concrete, and the magnitude of the compressive strains developed, the values of j given in Fig. 6 were obtained, and were used in all calculations. Values of j for the series of 1922, in which all beams had the same steel area and in which the concrete was of unusual quality, are not shown in Fig. 6; a value of 0.83 was used for rectangular beams, and of 0.89 for T-beams.

The nominal shearing unit stress and the nominal bond stress were calculated by means of equations (1) and (2a), using the values of j given above. It may be noted that in the series of 1910 and 1911 the longitudinal bars in several of the unit frames were of unusual cross-sections. The area of these sections was usually determined quite closely from the weight of a given length of bar; however, the perimeter of the section was not so easily measured, and, therefore, in the calculation of bond stresses the sum of the perimeters of round bars of equal cross-sectional areas has been used. As a result the perimeter used in calculations is probably slightly less than the actual perimeter of the bar, so that in these cases the calculated nominal bond stresses are disproportionately too high. Where part of the bars were bent up for web reinforcement, only the straight bars remaining have been considered in calculating the nominal bond stress. It should be noted that while the tables give values of the maximum applied load, or the load applied by the testing machine, the calculated values of tensile, shearing, and bond stresses have been found by including the weight of the beam, considered as uniformly distributed over the span, and the weight of the loading apparatus, applied at the load points shown in Fig. 1.

Data of individual control specimens are given with the data for each beam, thus furnishing a means of judging whether the concrete used in each case was representative of the general average for the series.

Notes are given regarding the manner of failure of each test beam. In general, mention of failure by "tension in steel" or "bond" refers to the longitudinal reinforcement. The other notes are self-explanatory.

(a) Beams with Unit Frame Reinforcement, Series of 1910

12. *Phenomena and Results of Tests.*—The object of this group of tests was to study the action of beams reinforced with various types of proprietary patented unit frames and to compare them with beams having no web reinforcement. This was done largely by comparing maximum loads carried, and to some extent by measuring and comparing deformations in the webs of the different types of beams. All beams were tested on a span of 6 ft. 0 in., and were subjected to two equal loads at the one-third points of the span. The beams were 8 in. wide and 10 in. in effective depth. The amount of longitudinal reinforcement varied from 1.23 to 1.92 per cent. Data of the test beams and test results are given in Table 4. The various beams are arranged in Table 4 in the order of the maximum shearing stresses developed, the values for each beam being shown graphically in the table. A view of the different types of reinforcing units used in the test beams is shown in Fig. 7, while Fig. 8 gives a sketch of one beam of each kind after failure, showing the position of cracks with reference to the reinforcement.

In analyzing the results and in making comparisons of the different beams it should be noted that those having web reinforcement had more longitudinal reinforcement than those without web reinforcement, that the amount of web reinforcement in the different sets of beams

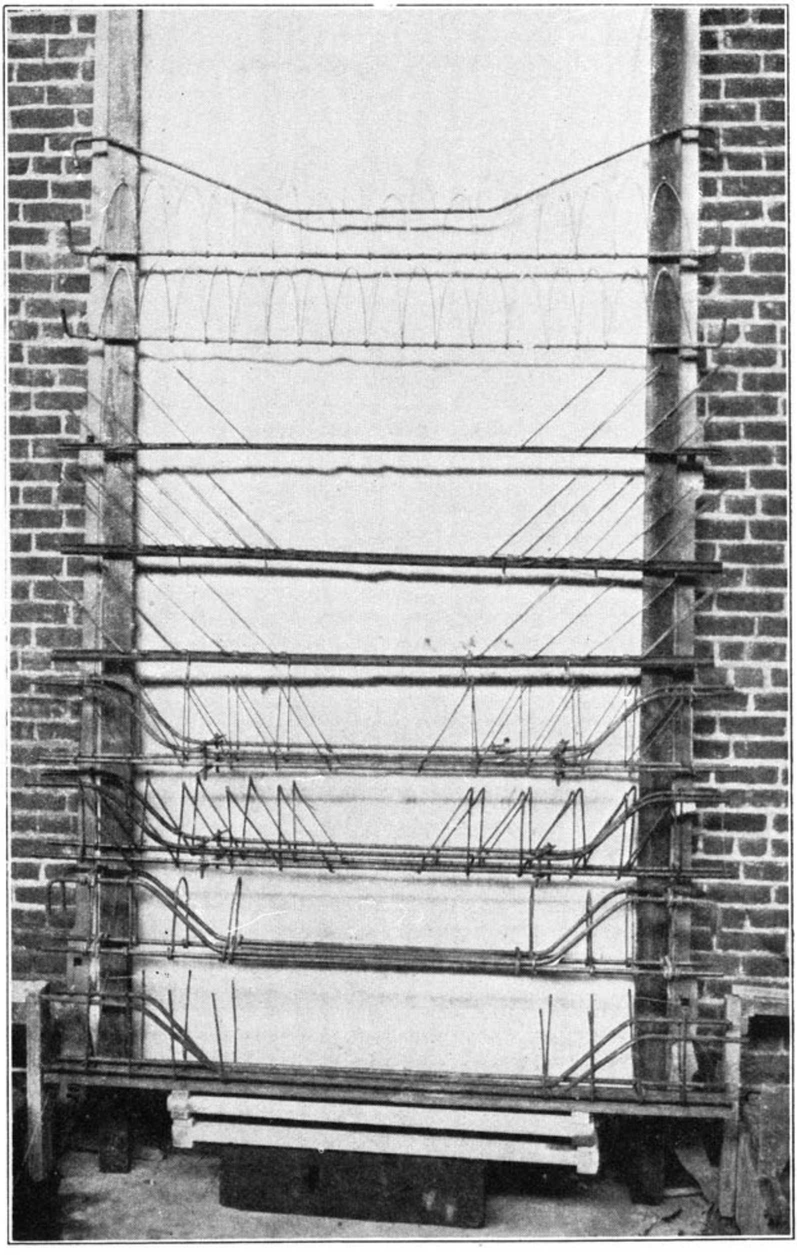

FIG. 7. VIEWS OF UNIT FRAME REINFORCEMENT OF SEVERAL TYPES USED IN SERIES OF 1910

Type 281—Units of Gabriel Steel Co.
Type 282—Unit of Monolith Steel Co.
Type 283—Units of American System of Reinforcing
Type 284—Units of Corrugated Bar Co.
Type 285—Units of General Fireproofing Co.

TABLE 4

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1910

Beam No.	Description of Test Beams: Side Elevation	Section at Center	Max. Shearing Stress lb. per sq. in. (0, 200, 400)	Manner of Failure
280.1 280.2 280.3	3" 2'-0" 2'-0" 2'-0" 3"	8" × 10"	AV	Diagonal Tension. Ditto Ditto
283.5 283.6 283.7	5½ 5½ 5½ 5½ 4½			Diag. Tension & Bond Ditto Ditto
282.1 282.2 282.3	2 5½ 6" 7 4½			Tension in Steel. Ditto Ditto
283.1 283.2 283.3	1½" 6½ 6½ 5 5½			Diag. Tension & Bond Ditto Ditto
285.1 285.2 285.3	6½" 6" 6" 6"			Bond & Slip of Stirrup Ditto Ditto
281.1 281.2 281.3	17 spaces at about 4"			Tension in Steel Ditto Ditto
285.5 285.6 285.7	7 3½" each			Bond & Slip of Stirrups Ditto Ditto
281.5 281.6 281.7	17 spaces at about 4"			Tension in Steel. Diag. Tension & Bond. Tension in Steel
284.1 284.2 284.3	1" 6" 6" 6" 6" 5" Metal Clip			Tension in Steel Ditto Diagonal Tension.
284.5 284.6 284.7	1" 6" 6" 6" 6" 5" Metal Clip			Tension in Steel Ditto Ditto

4-1

TABLE 4 (Concluded)

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1910

Beam No.	Age at Test, Days	Longitudinal Reinforcement: Description	%	Web Reinforcement: Description	%	Maximum Applied Load in Pounds	Computed Stress lb. per sq. in.: Longitudinal Steel, f_s	Shear v	Bond u ¶	Control Beams: Age, Days	Mod. of Rupt. lb. per sq. in.	6-in. Cubes: Age, Days	Compress. Strength lb. per sq. in.
280.1	64	5-1/2″ plain φ, m.s.	1.23	None	0	23800	35200	182	186	77	341	103	3555
280.2	62	Ditto	1.23	None	0	18800	27900	145	148	71	295	97	3097
280.3	61	5-5/8″ plain φ, m.s.	1.92	None	0	21000	20600	166	136	71	378	84	2700
Av.								164	157				
283.5	65	Unit,	1.53	5/16″ φ loose stir-	0.38	32000	38300	246	504	85	393	80	4477
283.6	63	4-5/8″ plain φ,	1.53	rups. 2-5/8″ φ	0.38	27600	33100	213	436	62	314	91	3423
283.7	67	m.s.	1.53	bent-up bars.	0.38 †	31800	38100	244	502	68	379	100	2712
Av.								234	480				
282.1	64	Unit, 2-3/4″	1.40	1/4″ φ loops rig-	0.35	32000	41700	245	371	77	375	103	3223
282.2	69	monolith	1.40	idly attached	0.35	32000	41700	245	371	70	430	95	4758
282.3	65	bars, m.s.*	1.40	to each bar	0.35	33800	44000	259	391	73	340	86	3210
Av.								250	378				
283.1	64	Unit,	1.53	5/16″ φ stirrups	0.32	38900	46500	298	608	85	393	80	4477
283.2	70	4-5/8″ plain	1.53	rig. attached;	0.32	26000	31200	201	412	68	350	94	3723
283.3	61	φ, m.s.	1.53	2-5/8″ φ bent-	0.32	34000	40600	261	534	73	340	86	3210
Av.				up bars.	†			253	518				
285.1	70	Unit, 2 1″	1.75	7/32″ φ loops rig-	0.44	27100	28700	211	286	70	430	95	4758
285.2	66	herringbone	1.75	idly attached	0.44	36600	38600	284	384	62	433	91	3870
285.3	67	bars, h.c.s.*	1.75	to each bar.	0.44	39600	41800	307	415	68	379	100	2712
Av.								267	362				
281.1	61	Unit, 3-11/16″	1.56	3/16″ φ loops	0.52	40000	46800	307	358	77	341	103	3555
281.2	62	ovoid bars,	1.56	wrapped a-	0.52	36400	42700	280	327	71	295	97	3097
281.3	62	h.c.s.*	1.56	round each	0.52	36700	43100	282	330	71	378	84	2700
Av.				bar.				289	338				
285.5	70	Unit, 2-1″	1.75	7/32″ φ loops rig-	0.76	40400	42600	312	422	68	350	94	3723
285.6	66	herringbone	1.75	idly attached	0.76	41900	44100	324	438	67	433	91	3870
285.7	65	bars, h.c.s.*	1.75	to each bar.	0.76	34400	36400	267	361	—	—	—	—
Av.								301	407				
281.5	63	Unit, 2-11/16″	1.48	3/16″ φ loops wrapped a-	0.34	41000	50600	313	549	70	445	97	3423
281.6	63	& 1-5/8″ ovoid	1.48	round each 11/16″	0.34	37800	46700	289	507	62	314	91	3432
281.7	62	bars, h.c.s.*	1.48	bar. 1-5/8″ ovoid	0.34	40000	49300	306	537	71	378	87	2700
Av.				truss bar.	†			303	531				
284.1	60	Unit, 4-5/8″	1.50	3/16″ φ stirrups	0.25	52800	64000	403	822	77	375	103	3223
284.2	65	corrugated	1.50	rig. attached.	0.25	49300	59800	376	768	62	314	91	3423
284.3	68	φ, h.c.s.	1.50	2-5/8″ corrug. φ,	0.25	47500	57800	362	740	73	340	86	3210
Av.				bent-up bars	†			380	776				
284.5	70	Unit, 4-5/8″	1.50	1/4″ φ corr. stirr.	0.56	54500	66100	415	848	70	445	97	3423
284.6	65	corrugated	1.50	rig. attached	0.56	50000	60800	381	780	62	433	91	3870
284.7	66	φ, h.c.s.	1.50	2-5/8″ corrug. φ	0.56	51000	62000	389	796	68	379	100	2712
Av.				bent-up bars.	†			395	808				4-2

* Equivalent round bar used in bond stress computations.
† Bent-up bars not included. ¶ Only straight bars considered.

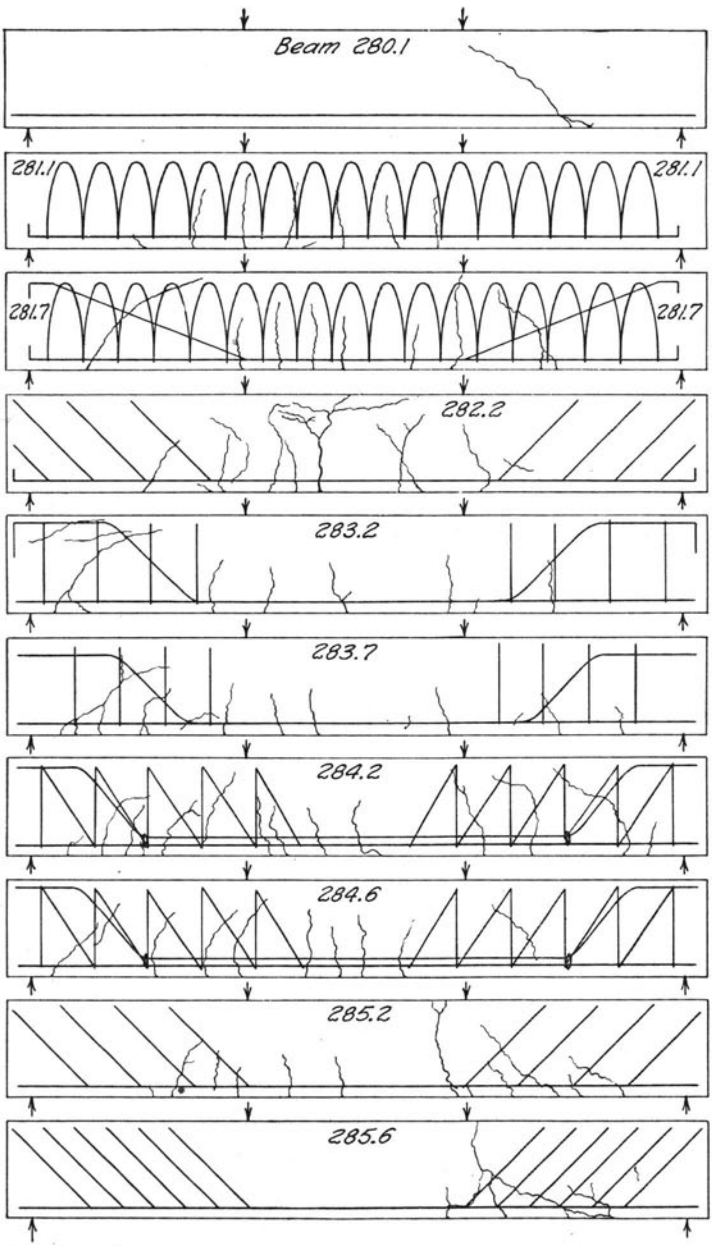

Fig. 8. Sketches of Typical Beams after Failure, Series of 1910

varied, and that different grades of steel were used in the several fabricated systems of reinforcement. Calculated tensile stresses for beams having mild steel reinforcement were in many cases near the yield point and all the failures in one group of beams were by tension.

In the three beams without web reinforcement which may be taken as a standard for purposes of comparison, the characteristic sudden diagonal tension failures occurred. The average shearing unit stress developed by the three beams was 164 lb. per sq. in.

The six beams reinforced with the unit frames of Type 283 (see Fig. 7) failed by diagonal tension and bond, the failure generally occurring quite suddenly. In three of the beams, 283.5, 283.6, and 283.7, the reinforcing steel was loosely placed, and in the other three beams, 283.1, 283.2, and 283.3, the reinforcement had been fabricated into unit frames, in which the stirrups were attached by wrapping the ends two turns around a longitudinal reinforcing bar, and as may be seen in Fig. 7 there was also a rigid connection between the straight and the bent-up bars.

The average shearing strengths developed by the two sets of beams were 234 and 253 lb. per sq. in., respectively. The results indicate that but little advantage was gained by the use of the fabricated frames. These units, although containing both vertical stirrups and bent-up bars, were not as effective in preventing web failure as might have been expected.

The three beams that were reinforced with units of Type 282 developed an average shearing strength of 250 lb. per sq. in. Failure of these beams was by tension in the longitudinal steel. It is evident that the use of mild steel bars for longitudinal reinforcement in these beams did not develop fully the strength of the web reinforcement and that this type of web reinforcement would compare better with some of the other systems if greater longitudinal reinforcement had been provided. The method of anchoring the stirrups to the main bars in these units appeared to give satisfactory results.

There were six beams reinforced with fabricated units of Type 285. These unit frames were of high-carbon steel, but the details and methods of fabrication were not particularly good. In handling the units preparatory to making the six beams several breaks occurred in stirrups and in the clips which attached the stirrups to the horizontal steel. During the tests the clips allowed the diagonal steel to slip long before the elastic limit of the stirrup steel was reached. This caused premature failure of these beams, and did not allow the full strength of either the web reinforcement or the longitudinal reinforcement to be developed. The shearing strengths developed by the two sets of beams were 267 and 301 lb. per sq. in., the average for the six beams being 284 lb. per sq. in.

Beams reinforced with units of Type 281 developed fairly high shearing stresses, the average of six beams being 294 lb. per sq. in. The tests indicated that the inclined bar used in 281.5, 281.6, and 281.7 was more effective in the later stages of the loading than in the earlier stages. The hooks on the ends of the reinforcing bars appeared to be effective in preventing final bond failure. These beams failed generally by tension in the longitudinal steel and consequently the strength of the web reinforcement was not fully developed..

The six beams reinforced with units of Type 284 developed an average shearing strength of 388 lb. per sq. in. The units seemed well proportioned for the loading used. Failure was, in all but one case, by tension in the longitudinal steel. Beam 284.3, which carried the lowest load of any of the beams of this group, failed near the support by diagonal tension.

In comparing the different systems of reinforcement it will be noted that the types of units which gave the highest shearing stresses contained both vertical and inclined web members. In all cases the shearing strength developed by the beams having unit frames for web reinforcement was considerably greater than that developed by the beams with no web reinforcement. Using the average values of shearing stress given in Table 4 and making no allowance for differences in the amount of longitudinal reinforcement in the various sets of beams, the increase in shearing strength produced by the web reinforcement was from 43 to 141 per cent.

(b) Beams with Various Types of Web Reinforcement, Series of 1911

13. *Phenomena and Results of Tests.*—This series of beams was a continuation of the series of 1910. A study was made of the effectiveness of different types of web reinforcement and of the behavior of each type under load. The effect of anchoring the ends of the longitudinal bars on the vertical shearing strength of certain of the beams was also investigated. The test beams were simple beams of 6 ft. span, loaded with two equal loads at the one-third points. The beams were 8 in. wide and 12 in. deep, the effective depth being 10 inches. The amount of longitudinal reinforcement varied from 1.11 to 2.44 per cent. Table 5 gives details of the different types of beams used and also the principal results of each test. The beams are arranged in the

table in the order of maximum shearing stress developed. In making comparisons of the different beams, the amount of reinforcement, the kind of steel used, and the manner of failure should be taken into consideration. Sketches of one beam of each type after failure are shown in Figs. 9 and 10.

The beams having no web reinforcement were of four types, the variation being in the method of anchoring the longitudinal steel. In two sets of beams the longitudinal bars were extended to the end of the beam but were not anchored; in other sets of beams the bars were anchored by means of plates and nuts at the ends, by extending the bars into the overhanging end of the beam, or by bending them into large hooks or loops at their ends. The beams with web reinforcement may be classified as follows: (1) beams with loose vertical stirrups, longitudinal steel unanchored, anchored by means of nuts and plates, or anchored by hooks or loops; and (2) beams reinforced with unit frames, including the American (Type 288), Gabriel (Types 289, 309), Corrugated Bar (Type 290), and Monolith (Type 295) systems. These units were similar to those used in 1910, shown in Fig. 7.

All of the beams without web reinforcement failed by diagonal tension and in most cases the failure was sudden. The results given in Table 5 show that beams 293.1, 293.2, and 293.3, in which anchorage of the longitudinal steel was accomplished by means of plates and nuts not tightened before the test, and beams 294.1, 294.2, and 294.3, in which the longitudinal bars were anchored by embedment in the overhanding ends of the beam, developed shearing stresses but little greater than those developed by the beams having no anchorage of the longitudinal steel. For the first set of beams the average increase in shearing stress was 13 per cent, and for the second set it was 19 per cent. Beams 293.4, 293.5, and 293.6, in which the longitudinal bars were anchored by plates and nuts which were tightened just before the tests, and beams 291.1, 291.2, and 291.3, in which the bars were anchored by hooks at their ends, developed shearing stresses 38 and 29 per cent greater, respectively, than were developed by beams 286.1, 286.2, and 286.3 having unanchored bars.

In the two sets of beams without web reinforcement which had the longitudinal bars anchored by means of nuts and plates, those in which the nuts were tightened just before the tests developed an average shearing stress 22 per cent greater than that developed by those in which the nuts were not tightened. As stated previously, the shearing strength of the latter set of beams was 13 per cent greater than that of the set of beams having no anchorage of the bars. The three beams

TABLE 5

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1911

Beam No.	Description of Test Beams: Side Elevation	Section at Center	Max. Shearing Stress lb. per sq. in. (0, 200, 400)	Manner of Failure
286.1 286.2 286.3	3" — 2'-0" — 2'-0" — 2'-0" — 3" Same dimensions for all beams unless otherwise noted.	8" 10"	Av.	Diagonal Tension Ditto Ditto
286.5 286.6 286.7				Diagonal Tension Ditto Ditto
293.1 293.2 293.3	Nuts not tightened before test.			Diagonal Tension Ditto Ditto
294.1 294.2 294.3	1'-3" — 2'-0" — 2'-0" — 3'-3" to end ℄			Diagonal Tension Ditto Ditto
287.7 287.8 287.9	3" 6" 6" 6"			Diagonal Tension Ditto Ditto
287.4 287.5 287.6	3" 6" 6" 6"			Diagonal Tension Diag. Tension & Bond Diagonal Tension
291.1 291.2 291.3				Diagonal Tension Ditto Ditto
287.1 287.2 287.3	3" 6" 6" 6"			Diagonal Tension Ditto Ditto
293.4 293.5 293.6	Nuts tightened before test.			Diagonal Tension Diag. Tens. & Tension Diagonal Tension
292.1 292.2 292.3				Diagonal Tension Ditto Ditto

5-1-1

TABLE 5 (Continued)

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1911

Beam No.	Age at Test, Days	Longitudinal Reinforcement: Description	Longitudinal Reinforcement: %	Web Reinforcement: Description	Web Reinforcement: %	Maximum Applied Load, Pounds	Computed Stress lb. per sq. in.: Longitudinal Steel, f_s	Computed Stress lb. per sq. in.: Shear v	Computed Stress lb. per sq. in.: Bond u	Control Beams: Age, Days	Control Beams: Mod of Rupt. lb. per sq. in.
286.1	61	3-$\frac{3}{4}$" plain ϕ, m.s.	1.65	None	0	18 000	20400	141	161	61	265
286.2	62	Ditto	1.65	None	0	17 600	19900	138	158	56	232
286.3	64	Ditto	1.65	None	0	22500	25300	176	200		
Av.								152	173		
286.5	69	5-$\frac{5}{8}$" plain ϕ, m.s.	1.94	None	0	17400	17 200	139	114	56	259
286.6	70	Ditto.	1.94	None	0	18500	18 200	147	120	84	304
286.7	64	Ditto.	1.94	None	0	22100	21 600	174	143	86	402
Av.								153	126		
293.1	63	3-$\frac{3}{4}$" plain ϕ,	1.65	None	0	20000	22500	157	178	97	342
293.2	61	m.s., with nuts	1.65	None	0	21400	24000	167	190	60	268
293.3	66	& plates at	1.65	None	0	24800	27800	193	219		
Av		ends						172	196		
294.1	61	3-$\frac{3}{4}$" plain ϕ, m.s.	1.65	None.-Beam	0	25000	28000	194	221	65	228
294.2	70	Ditto.	1.65	ends overhung	0	20 200	22700	156	180	76	218
294.3	67	Ditto	1.65	supports 15"	0	24700	27700	192	219		
Av.								181	207		
287.7	69	5-$\frac{5}{8}$" plain ϕ, m.s.	1.94	$\frac{1}{2}$" ϕ stirrups,	0.82	18 600	18200	147	121	55	282
287.8	64	Ditto	1.94	ends unan-	0.82	14 000	13800	112	92	52	236
287.9	63	Ditto	1.94	chored	0.82	40400	39100	314	257	79	369
Av.								191	157		
287.4	61	3-$\frac{3}{4}$" plain ϕ, m.s.	1.65	$\frac{1}{2}$" ϕ stirrups,	0.82	27 100	30 300	210	238	65	228
287.5	65	Ditto	1.65	ends hooked	0.82	14400	16 300	114	130	52	236
287.6	68	Ditto	1.65		0.82	33 200	37000	257	291	79	369
Av								194	220		
291.1	61	3-$\frac{3}{4}$" plain ϕ, m.s.	1.65	None	0	25 300	28 300	197	224	66	327
291.2	65	with hooks	1.65	None	0	22 500	25 200	176	200	76	218
291.3	59	at ends	1.65	None	0	27700	30 900	215	245	56	212
Av								196	223		
287.1	64	3-$\frac{3}{4}$" plain ϕ, m.s.	1.65	$\frac{1}{2}$" ϕ, stirrups,	0.82	25 300	28 300	197	224	61	266
287.2	69	Ditto	1.65	ends unan-	0.82	23 000	25 800	180	204	84	304
287.3	63	Ditto	1.65	chored	0.82	27 200	30400	211	240	86	402
Av.								196	223		
293.4	66	3-$\frac{3}{4}$" plain ϕ,	1.65	None	0	27400	30 600	213	242	68	248
293.5	63	m.s., with nuts	1.65	None	0	34500	38400	267	303	71	453
293.6	—	& plates at	1.65	None	0	19300	21700	151	172		
Av		ends						210	239		
292.1	59	3-$\frac{3}{4}$" plain ϕ,	1.65	1-$\frac{3}{4}$" bent-up	0	30 700	34 200	238	407	66	327
292.2	65	m.s. with hooks	1.65	bar*	0	28 900	32 300	224	382	68	248
292.3	63	at ends	1.65		0	29800	33300	231	393	64	213
Av.								231	394		

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*Not included in computing per cent web reinforcement or bond stress.

TABLE 5 (Continued)

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1911

Beam No.	Description of Test Beams: Side Elevation	Description of Test Beams: Section at Center	Max. Shearing Stress lb. per sq. in. (0 200 400)	Manner of Failure
293.7 293.8 293.9	3" 2'-0" 2'-0" 2'-0" 3" 3" 6" 6" 6" Nuts tightened before test	8" 10"		Tension in Steel Ditto Ditto
292.5 292.6 292.7	3" 6" 6" 6"			Tension in Steel Ditto Ditto
289.1 289.2 289.3	17 equal spaces			Tension in Steel Ditto Ditto
288.1 288.2 288.3	3" 4" 6" 8"			Tension in Steel Ditto Ditto
290.1 290.2 290.3	5 spaces @ 6"			Tension in Steel Ditto Ditto
309.1	17 equal spaces			Tension & Compression
290.5 290.6 290.7	5 spaces @ 6"			Compression Tension & Compression Tension in Steel
295.4 295.5 295.6	4" 6 spaces @ 5"			Tension & Compression Ditto Ditto
295.1 295.2 295.3	4" 6 spaces @ 5"			Compression Ditto Tension in Steel

5-2-1

TABLE 5 (Concluded)

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1911

Beam No.	Age at Test, Days	Longitudinal Reinforcement: Description	Longitudinal Reinforcement: %	Web Reinforcement: Description	Web Reinforcement: %	Maximum Applied Load, Pounds	Computed Stress lb. per sq. in.: Longitudinal Steel, f_s	Computed Stress lb. per sq. in.: Shear v	Computed Stress lb. per sq. in.: Bond u¶	Control Beams: Age, Days	Control Beams: Mod. of Rupt. lb. per sq. in.
293.7	61	3-3/4" plain φ, m.s.	1.65	1/2" φ stirrups,	0.82	32 500	36 200	251	286	56	212
293.8	61	with nuts &	1.65	ends hooked	0.82	33 000	36 800	255	290	76	218
293.9	105	plates at ends.	1.65		0.82	34 600	38 600	267	303		
Av.								258	293		
292.5	63	3-3/4" plain φ	1.65	1/2" φ stirrups,	1.05	33 000	36 800	255	435	67	304
292.6	63	m.s. with hooks	1.65	ends hooked.	1.05	36 200	40 300	279	475		
292.7	103	at ends.	1.65	1-3/4" bent-up	1.05	35 500	39 500	274	466		
Av.				bar	†			269	459		
289.1	62	3-5/8" ovoid, h.c.s.	1.30	3/16" φ loops	0.50	37 800	53 100	287	370	67	304
289.2	62	3-11/16" ovoid, h.c.s.	1.55	wrapped a-	0.50	31 200	36 600	240	282	52	266
289.3	61	4-1/2" ovoid, h.c.s.	1.11	round each	0.50	42 600	68 600	319	384		
Av.		(All Units) *		bar				282	345		
288.1	64	Unit of 4-5/8"	1.53	5/16" φ stirrups,	0.64	35 100	41 900	271	550	97	342
288.2	69	plain φ, m.s.	1.53	ends unanch.	0.64	31 900	38 100	246	502	52	266
288.3	61		1.53	2-5/8" φ bent-	0.64	46 900	55 800	359	732		
Av.				up bars.	†			292	608		
290.1	70	Unit of	1.25	1/4" φ stirrups,	0.89	34 500	49 800	262	524	62	242
290.2	68	4-1/2" corrugat-	1.25	rig. attached,	0.89	38 500	55 500	292	584	62	318
290.3	106	ed φ, m.s.	1.25	2-1/2" corrug. φ	0.89	42 600	61 300	322	644		
Av.				bent-up bars	†			292	584		
309.1	300	Unit, of 5-1/2" ovoid, h.c.s. *	1.39	1/4" φ loops wrapped around each bar.	1.36	39 000	51 400	300	287	358	260
290.5	68	Unit of	1.25	0.2" φ stirrups,	0.58	38 700	55 800	293	586	60	268
290.6	66	4-1/2" corru-	1.25	rig. attached	0.58	42 000	60 500	317	634	74	366
290.7	117	gated φ, m.s	1.25	2-1/2" corr. φ	0.58	40 300	58 100	305	610		
Av.				bent-up bars.	†			305	610		
295.4	58	Unit of 2-1"	2.44	1/4" φ loops	0.70	41 800	32 400	331	372	62	318
295.5	62	monolith	2.44	rig. attached	0.70	43 800	34 000	347	392		
295.6	109	bars, h.c.s.*	2.44	to each bar	0.70	46 000	35 600	364	412		
Av.								347	392		
295.1	65	Unit of 2-1"	2.44	3/8" φ loops	1.57	46 700	36 200	369	418	64	213
295.2	57	monolith	2.44	rig. attached	1.57	45 600	35 300	361	408	74	366
295.3	119	bars, h.c.s.*	2.44	to each bar.	1.57	55 300	42 800	436	495		
Av.								389	441		

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*Equivalent round bar used in bond stress computations.
†Bent-up bars not included. ¶Only straight bars considered.

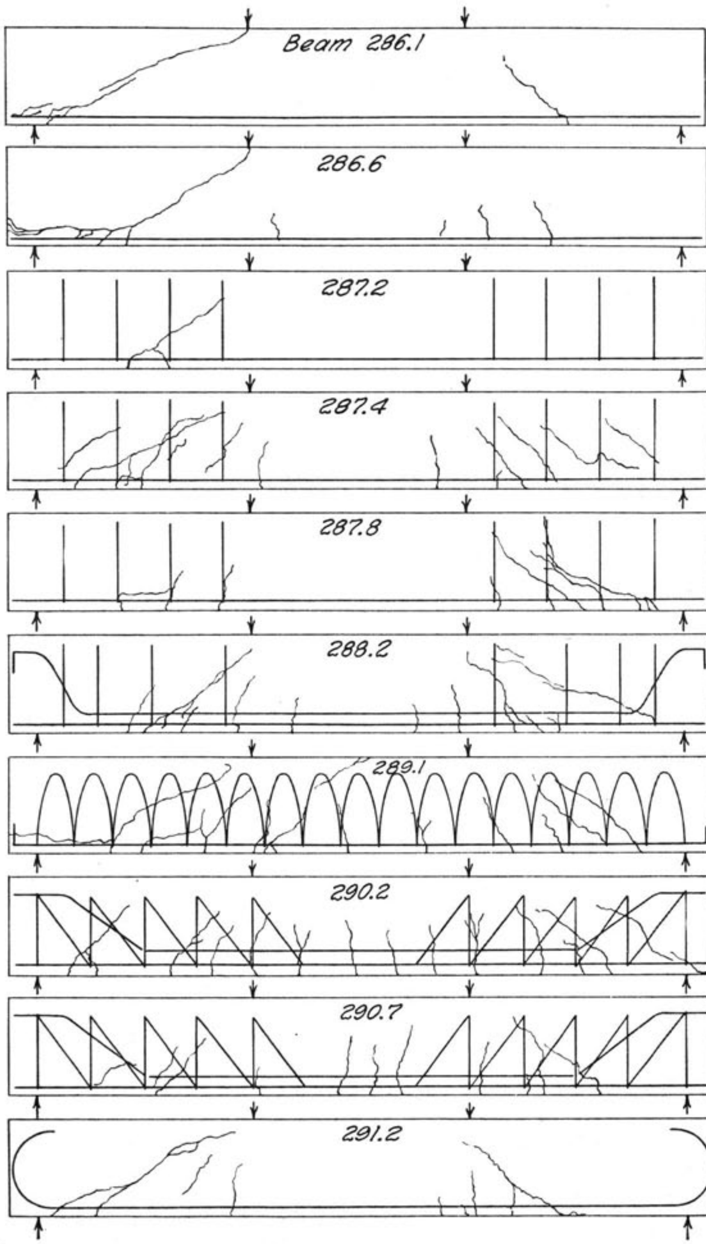


FIG. 9. SKETCHES OF TYPICAL BEAMS AFTER FAILURE, SERIES OF 1911

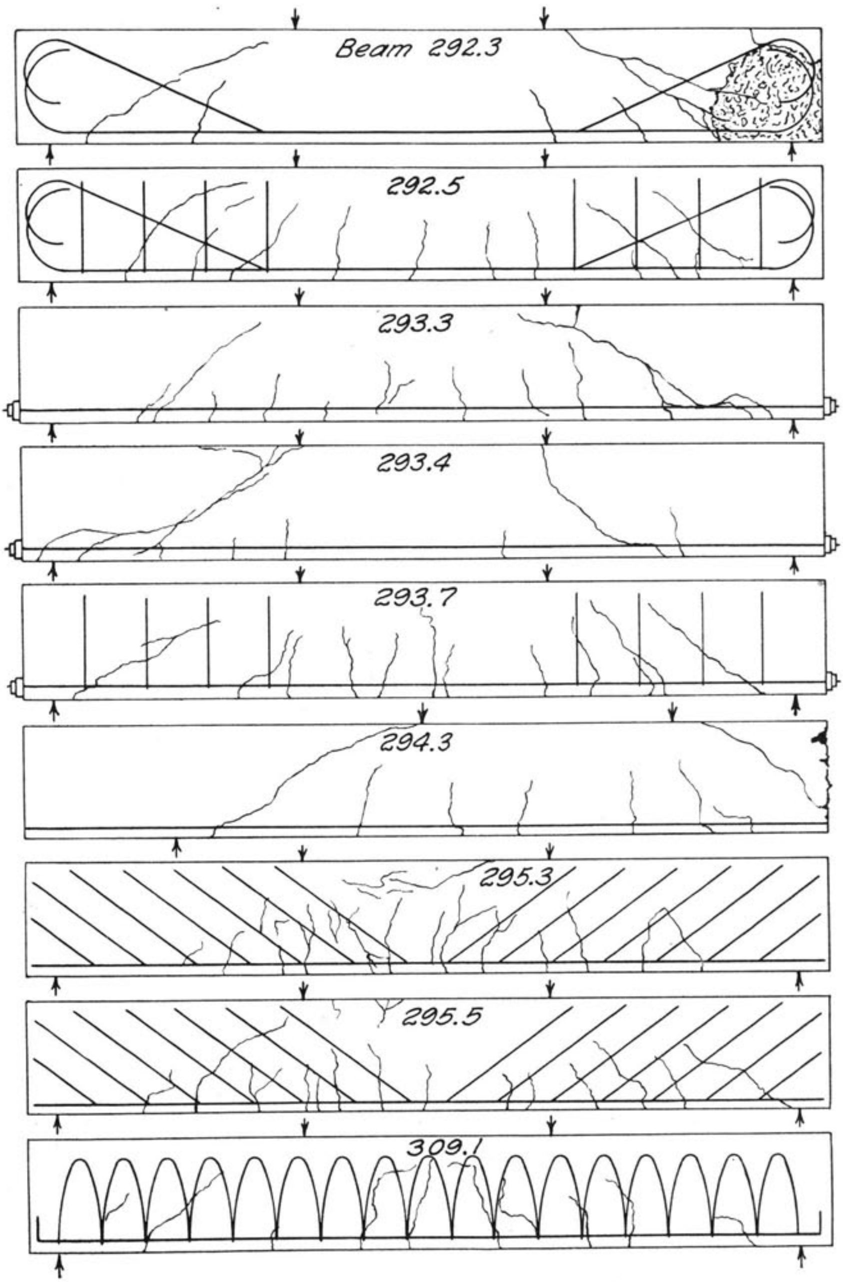


FIG. 10. SKETCHES OF TYPICAL BEAMS AFTER FAILURE, SERIES OF 1911

of this type in which the nuts were tightened and which had vertical stirrups for web reinforcement developed shearing stresses 23 per cent greater than were developed by the corresponding beams without stirrups. The manner of failure of the beams with stirrups was by tension in the longitudinal steel, and consequently the strength of the web reinforcement was not fully developed.

Beams 291.1, 291.2, and 291.3, without web reinforcement, but having longitudinal bars anchored by large hooks, failed rather slowly, the initial failure in diagonal tension being followed by slipping of the bars and straightening of the hooks. This resulted in some splitting and breaking of the concrete at the ends of these beams. As stated before, these beams developed an average shearing stress 29 per cent greater than that developed by the beams not having end anchorage of the bars. Beams 292.1, 292.2, and 292.3, in which the reinforcing bars were anchored with hooks, and one bar was bent up, developed an average shearing stress of 231 lb. per sq. in., a value 18 per cent greater than that developed by the corresponding beams 291.1, 291.2, and 291.3 with all bars straight. Beams 292.5, 292.6, and 292.7, of similar type, but having vertical stirrups in addition to the bent-up bar, developed an average shearing stress of 269 lb. per sq. in. This value of shearing stress is only 16 per cent greater than the average shearing stress developed by the corresponding beams without vertical stirrups, but, as failure occurred by tension in the longitudinal steel, the strength of the stirrup reinforcement was not fully utilized.

All of the beams having loose vertical stirrups for web reinforcement and having no anchorage of longitudinal bars failed by diagonal tension. Beams 287.1, 287.2, and 287.3 and beams 287.7, 287.8, and 287.9 in which the ends of the stirrups were not anchored developed average shearing stresses of 196 and 191 lb. per sq. in., respectively. Although the latter group of beams had a greater percentage of longitudinal steel than the former, the average shearing stress was somewhat less. It will be noted in Table 5, however, that in the latter set of tests there was one beam that gave results abnormally low, and one abnormally high. The average shearing stress for these six beams was about 28 per cent greater than the average shearing stress for the corresponding six beams without web reinforcement.

The average shearing stress developed by beams 287.4, 287.5, and 287.6, which had the stirrup ends hooked, was about the same as that developed by the corresponding beams 287.1, 287.2, and 287.3, which had plain U-stirrups, but it is evident that no very high bond stresses were developed by the stirrups in either set of beams.

The beams reinforced with fabricated unit frames carried higher loads than the other beams of this series. These systems should not be compared directly on the basis of shearing stress developed, since the amount of longitudinal and web reinforcement varied and different grades of steel were used. Failure of these beams generally occurred by tension in the steel or by compression in the concrete, so that the strength of the web reinforcement was not fully utilized. It will be noted in Table 5 that the unit frames of Types 288, 289, and 290 gave about the same shearing strengths, average values being 292, 282, and 298 lb. per sq. in., respectively. It would appear from the appearance of cracks that the bent-up bars in the unit of Type 288 were not fully effective in resisting diagonal tension. The unit frame of Type 295, which had a much higher percentage of both longitudinal and web reinforcement than the other fabricated frames, developed an average shearing stress of 368 lb. per sq. in. in six beams.

All beams having web reinforcement failed gradually, after considerable cracking of the beam, and the maximum load was held for

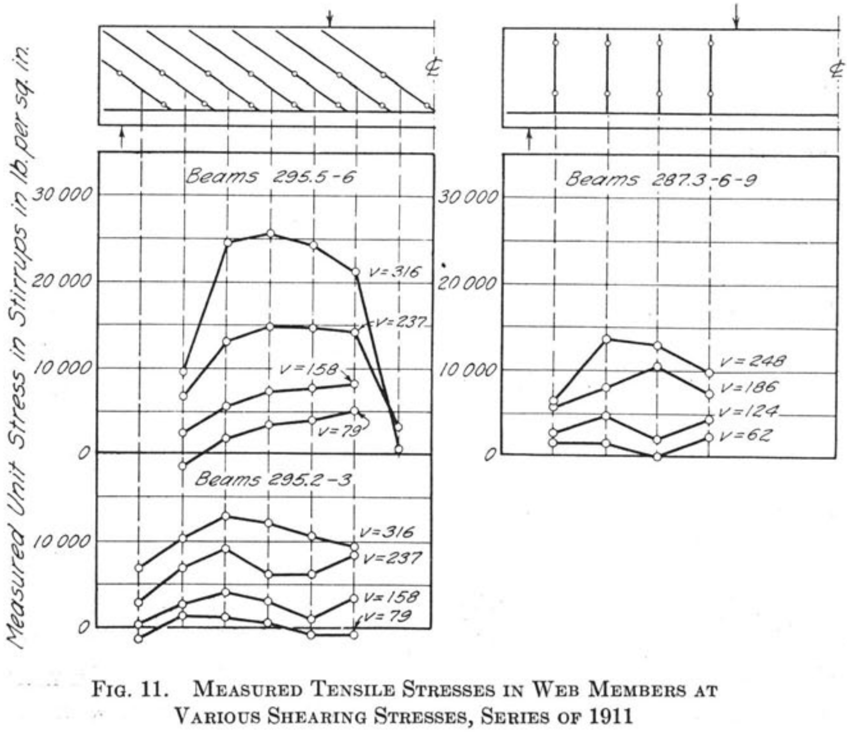


FIG. 11. MEASURED TENSILE STRESSES IN WEB MEMBERS AT VARIOUS SHEARING STRESSES, SERIES OF 1911

an appreciable length of time before giving way. This is in marked contrast to the sudden failure which occurred in the beams having no web reinforcement.

In Fig. 11 are plotted some of the results of strain measurements on inclined and vertical stirrups. These graphs and the strain measurements made on other beams of this series of tests indicate that the highest stress was generally found in the stirrups that were about midway between the support and the load point, the stress in the stirrups near the support and near the load point being low. The maximum stresses developed were about twice as great as the average stresses in all stirrups of a beam. The strain measurements on the stirrups also indicated that considerably more stress was developed in the portion of the stirrup adjacent to the longitudinal reinforcement than in the portion near or above the neutral axis of the beam.

(c) Beams with Rigidly Attached Web Members, Series of 1913

14. *Test Beams and Method of Testing.*—In the beams of this series the effectiveness of various dispositions of rigidly attached web reinforcement was investigated. An important feature of the tests was the careful measurement of the stresses developed in the longitudinal and web reinforcement of the various beams by means of the Berry strain gage.

There were 15 beams in the series, all made of 1-2-4 concrete. The beams were 8 in. in width, 15 in. in effective depth, and 10 ft. 6 in. in length. The longitudinal reinforcement in all cases consisted of four ¾-in. round bars. In two beams no web reinforcement was provided, though the four longitudinal bars were anchored by being hooked at their ends. In two beams two of the longitudinal bars were continued to the ends of the beam and hooked; the other two bars were bent up in the outer thirds of the beam to act as web reinforcement. In two beams the web reinforcement consisted of inverted U-shaped vertical stirrups looped around and welded to the horizontal bars. The remaining nine beams of this series had web reinforcement consisting of straight steel bars securely welded to the longitudinal bars and inclined at an angle of 45 deg. to the horizontal. In these nine beams the size and spacing of the bars forming the web reinforcement was varied.

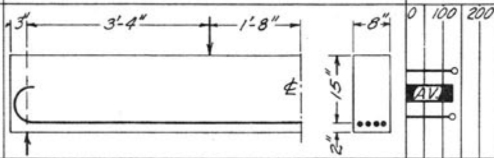
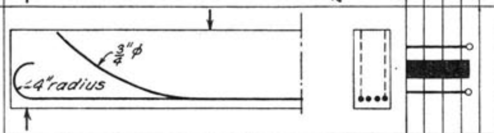
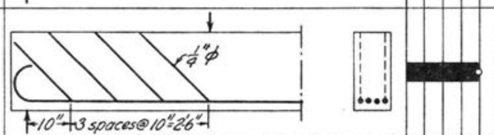
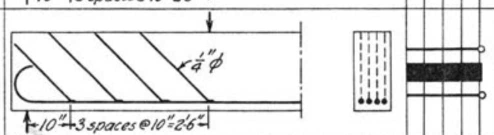
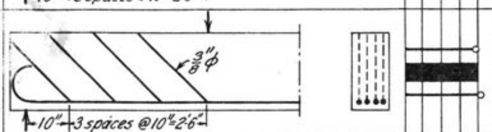
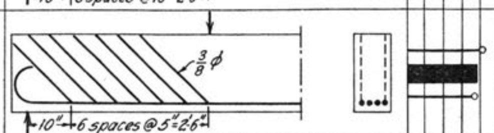
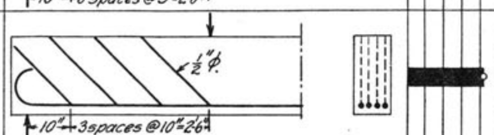
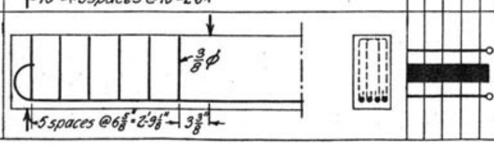
In all of the beams the ends of the longitudinal bars were provided with semi-circular hooks of 4-in. radius to prevent end slip. In none of the beams was the inclined web reinforcement provided with anchorage at the upper end. Table 6 gives details of the test beams together with the principal results of the tests.

The beams were tested on a span of 10 ft., with the loads applied at the one-third points of the span. Strain gage readings were taken along the length of the longitudinal bars and also along the stirrups nearest the sides of the beam. The gage length used was 4 in. Generally there were 4 gage lines on the inclined web members and 3 gage lines on the vertical web members. At each increment of load, strain readings were taken on all gage lines, and generally 5 complete sets of readings were obtained before failure of the beam occurred.

15. *Phenomena and Results of Tests.*—Beams 301.1 and 301.2, which had no web reinforcement, failed by diagonal tension. In beam 301.1 the diagonal tension failure occurred gradually, but in beam 301.2 failure occurred very suddenly. In beams 302.1 and 302.2, which had two ¾-in. bent-up bars for web reinforcement, failure was due to end slip of the unanchored bent-up bars, which permitted the opening and extension of diagonal cracks across the bent-up bars in the outer thirds of the beams. In beam 303.1, which had a small amount of inclined web reinforcement, failure occurred gradually by diagonal tension, after the yield point of the web reinforcement had been reached. There was not enough web reinforcement in this beam to prevent the formation and extension of a large diagonal crack across the region of greatest stress. In all of the remaining beams in the series failure occurred by tension in the longitudinal steel. In beams 304.1 and 304.2 the measured stresses in the web reinforcement were near the yield point of the steel at the time failure occurred, but the diagonal cracks which formed were kept well distributed by the web reinforcement and concentration of web stresses was prevented. Beams 305.1, 305.2, 306.1, and 306.2 failed by tension in the longitudinal steel before the full strength of the web reinforcement was developed. At the time of failure of these beams the maximum measured stresses in the web reinforcement ranged from 20 000 to 26 000 lb. per sq. in. In beam 307.2, which contained a large percentage of web reinforcement, the measured stresses in the web members were small, not exceeding 18 000 lb. per sq. in., and very few cracks opened up. The cracks which did form were small and did not extend to any great length. Beams 308.1 and 308.2, which had vertical stirrups for web reinforcement, also had but few diagonal cracks when failure occurred by tension in the longitudinal steel. The measured stresses in the web reinforcement in these beams at the maximum load did not exceed 18 000 lb. per sq. in.

TABLE 6

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1913

Beam No.	Description of Test Beams — Side Elevation / Section at Center; Max. Shearing Stress lb. per sq. in.	Manner of Failure
301.1 301.2		Diagonal tension. Ditto.
302.1 302.2		Slip of bent-up bars. Ditto.
303.1		Diagonal tension.
304.1 304.2		Tension in steel. Ditto.
305.1 305.2		Tension in steel. Ditto.
306.1 306.2		Tension in steel. Ditto.
307.2		Tension in steel.
308.1 308.2		Tension in steel. Ditto.

6-1

TABLE 6 (Concluded)

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1913

Beam No.	Age at Test, Days	Longitudinal Reinforcement		Web Reinforcement		Maximum Applied Load in Pounds	Computed Stress lb. per sq. in.			Control Beams		6-in. Cubes	
		Description	%	Description*	%		Longitudinal Steel, f_s	Shear v	Bond u	Age, Days	Mod. of Rupt. lb. per sq. in.	Age, Days	Compress. Strength lb. per sq. in.
301.1	225	4-$\frac{3}{4}$" plain ϕ	1.47	None	0	24 900	23 600	130	111	255	258	256	2910
301.2	315		1.47		0	24 200	22 900	126	108	249	253		
Av.								128	110				
302.1	73	4-$\frac{3}{4}$" plain ϕ	1.47	2-$\frac{3}{4}$" plain ϕ, bent-up bars	0.28	33800	31600	174	149			68	2010
302.2	84		1.47		0.28	33000	30900	170	145			106	2620
Av.								172	147				
303.1	238	4-$\frac{3}{4}$" plain ϕ	1.47	8-$\frac{1}{4}$" ϕ at each end of beam	0.17	38 700	36 200	199	170	307	198	307	2910
304.1	225	4-$\frac{3}{4}$" plain ϕ	1.47	16-$\frac{1}{4}$" ϕ at each end of beam	0.35	39 500	36 800	203	172	255	168	256	2550
304.2	225		1.47		0.35	40 200	37400	206	176	262	248	263	2240
Av.								204	174				
305.1	120	4-$\frac{3}{4}$" plain ϕ	1.47	16-$\frac{3}{8}$" ϕ at each end of beam	0.78	38000	35400	195	166			86	2430
305.2	90		1.47		0.78	40 000	37 300	205	175			100	1840
Av.								200	170				
306.1	235	4-$\frac{3}{4}$" plain ϕ	1.47	14-$\frac{3}{8}$" ϕ at each end of beam	0.78	40000	37300	205	175	307	215	307	2450
306.2	234		1.47		0.78	35 650	33300	183	156	298	188	298	2550
Av.								194	165				
307.2	227	4-$\frac{3}{4}$" plain ϕ	1.47	16-$\frac{1}{2}$" ϕ at each end of beam	1.39	40 700	38000	209	178	262	222	262	2340
308.1	80	4-$\frac{3}{4}$" plain ϕ	1.47	12-$\frac{3}{8}$" ϕ U-stirrups at each end of beam.	0.82	44 000	40900	225	192			78	2710
308.2	286		1.47		0.82	44 600	41400	228	194				
Av.								226	193				

6-2

*All stirrups rigidly attached

The average values of shearing unit stress developed by the beams without web reinforcement was 128 lb. per sq. in. This value is somewhat lower than that developed by the beams without web reinforcement in the other series of tests reported in this bulletin. The two beams with bent-up bars for web reinforcement developed an average shearing unit stress of 172 lb. per sq. in. If the bent-up bars had been anchored at their ends, higher shearing stresses would have been developed as end slip of the bent-up bars permitted premature failure of these beams. The results of the tests of these two beams emphasize the importance of proper anchorage when bent-up bars are used as web reinforcement. Beam 303.1, which had a small amount of web reinforcement and which failed slowly by diagonal tension, developed a shearing unit stress of 199 lb. per sq. in., a value 55 per cent greater than that developed by the beams without web reinforcement. Further comparisons on the basis of shearing stresses developed between the other beams of this series of tests can not be made since the beams all failed in tension. It is evident that many of the beams which failed in tension would have developed much higher values of shearing unit stresses than those reported in Table 6 if more longitudinal steel had been provided.

Some typical curves for gage lines where high stresses were measured in the web reinforcement are shown in Fig. 12. At loads below those at which diagonal cracks appeared the measured stresses in the web members were comparatively low, but they increased rapidly after the concrete failed in diagonal tension. For inclined web members a change in slope of the load-stress curves occurred at a load lower than that at which cracks became visible, indicating that even at low loads the concrete near the bottom of the beam had been overstressed in tension, and by cracking permitted a concentration of stress in the web member. It appears also that before failure of the concrete the inclined web members took stress in the same manner as the concrete and, together with the concrete, they resisted the diagonal tension stresses directly. It is seen that at a load of about 10 000 lb., the load-stress curves for the gage lines on inclined web members in many of the beams began to change in slope slightly, and from this load up to failure the increase in stress in the stirrup was roughly proportional to the increase in load. The change in slope occurred when the calculated shearing unit stress was about 50 lb. per sq. in., and at this stress the concrete had begun to crack.

After the formation of well-distributed diagonal cracks, the strain measurements indicate that the web members alone resist the diagonal

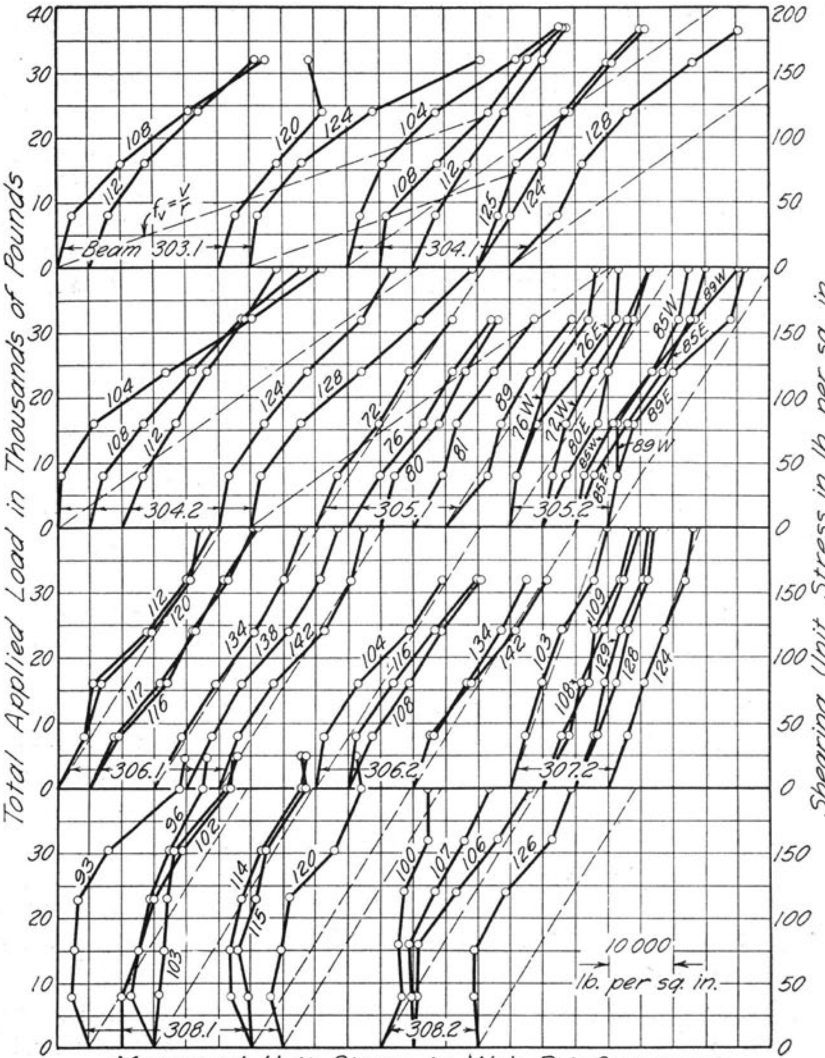


FIG. 12. LOAD-STRESS CURVES FOR GAGE LINES ON WHICH HIGH STRESSES WERE MEASURED, SERIES OF 1913

tension stresses developed, and that the load-stress curves for the various web members are straight parallel lines. It is also seen from the test results that when the web is well reinforced, the diagonal cracks are small and well distributed and rarely extend above the neutral axis; however, when insufficient web reinforcement is used, the stresses become high, and the resultant deformations are likely to be concentrated into a few large diagonal cracks.

Only two beams in this series of tests were made with vertical web members, and consequently the tests are too few in number to give definite results on the action of vertical web members as compared with that of inclined web members. The results obtained indicated that the vertical web members did not take much tensile stress until the load was reached that corresponded to the maximum load for the beams without web reinforcement. In the tests of these beams the diagonal cracks opened and extended much farther than was the case for the beams reinforced with an equivalent amount of inclined web steel.

A comparison of the measured stresses with the stresses calculated by means of the formulas given in Section 4 is made in Fig. 12. It will be noted that for inclined web members at maximum loads the calculated and measured stresses are in fair agreement in some cases, while in others they differ widely. At low loads the calculated stresses are generally greater than the measured stresses.

It should be borne in mind that the web members in the beams of this series were securely welded to the longitudinal bars, and it was due to this fact that they were able to develop their full calculated strength. Unless inclined web members are rigidly attached to the longitudinal reinforcing bars there is danger that at high shearing stresses slipping of the web members will occur at their connection with the longitudinal bars and a diagonal tension failure of the beam will result. In order to prevent slipping of the stirrups at their upper ends it is generally desirable to provide hooks for anchorage. In the beams tested no hooks were provided, but since the stirrups were either of small size or not highly stressed, the bond stresses at the upper end of the stirrups were low.

Figure 13 shows graphically the variation in the measured tensile stress in the longitudinal reinforcement of beam 304.2 at various loads. Calculated stresses found by the method described in Section 11 are also shown in the figure. The variation in the stress measured in the web reinforcement of beam 304.2 is shown in Fig. 14. Graphs similar to those shown in Figs. 13 and 14 were obtained for other beams of this series. It will be noted in Fig. 14 that the intensity of stress varied greatly along the web members. The greatest web stresses were found at the gage lines nearest the longitudinal steel at the bottom of the beam. As the top of the beam was approached the stress in the web member gradually diminished. This was true

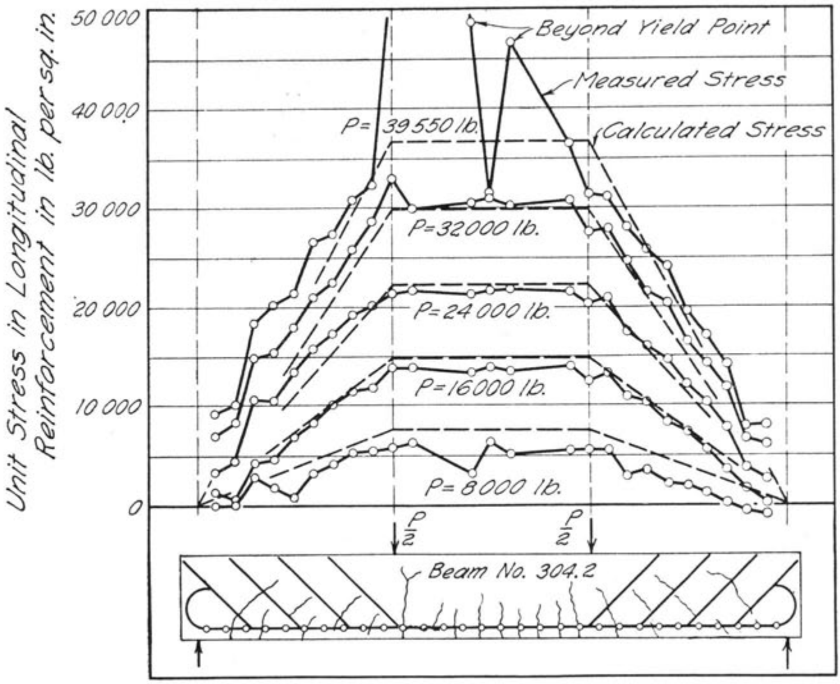


FIG. 13. MEASURED TENSILE STRESSES IN LONGITUDINAL REINFORCEMENT OF BEAM 304.2, SERIES OF 1913

except where the web member was crossed near the upper end by a crack. Furthermore, it will be noted in Fig. 14 and also in Fig. 15 that the highest stresses were found in the web members in the region midway between the load and the support. This might be expected, since the diagonal cracks generally formed first in this region, and there were here no secondary compressive stresses present, as might be the case near the support and near the load points of the beam, due to the applied external forces and reactions.

Figure 16, which is a composite curve for six specimens, shows the variation in measured stress in the web reinforcement in the outer third of the beam length. It will be seen from Fig. 16 that the maximum web stresses were found at a distance from the load point approximately equal to the depth of the beam.

The variation in the measured stress along the web member receiving the highest stress is shown graphically in Fig. 17 for the various beams. It is seen that in general for the higher values of vertical shearing stress the change in stress from gage line to gage line is quite

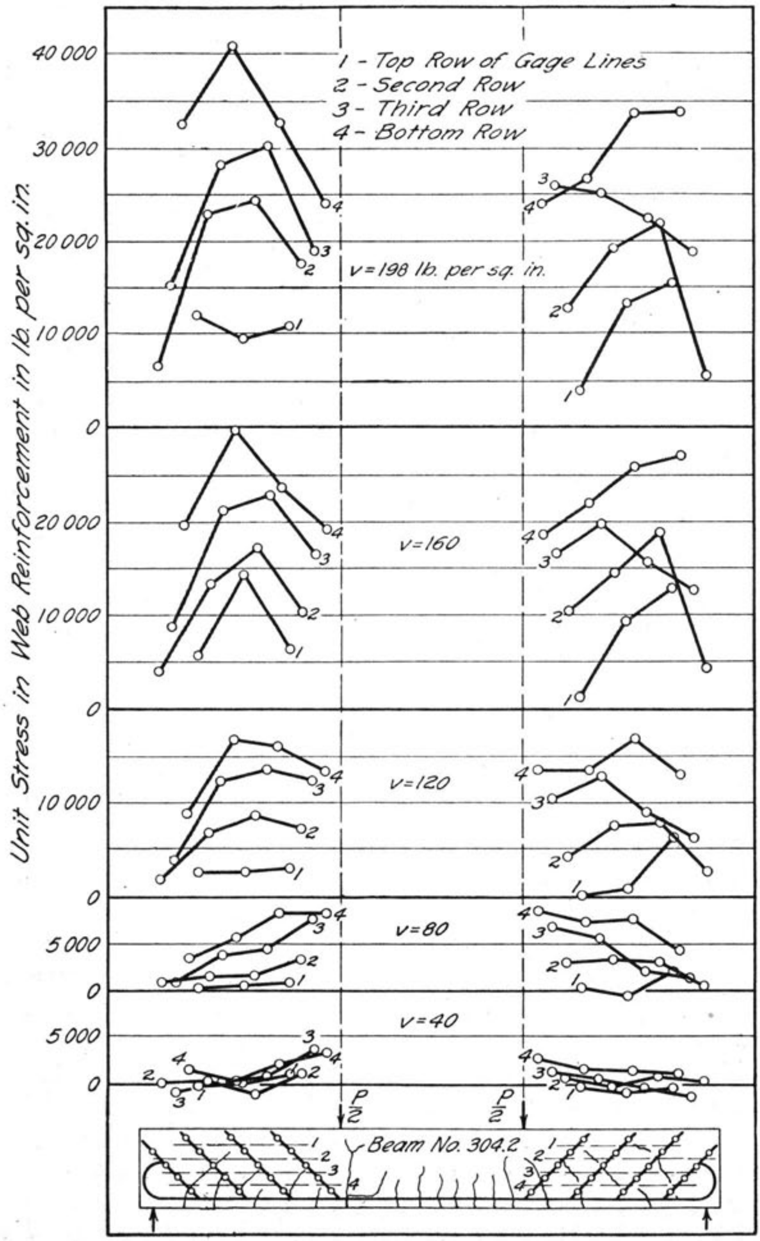


FIG. 14. MEASURED TENSILE STRESSES IN WEB REINFORCEMENT OF BEAM 304.2, SERIES OF 1913

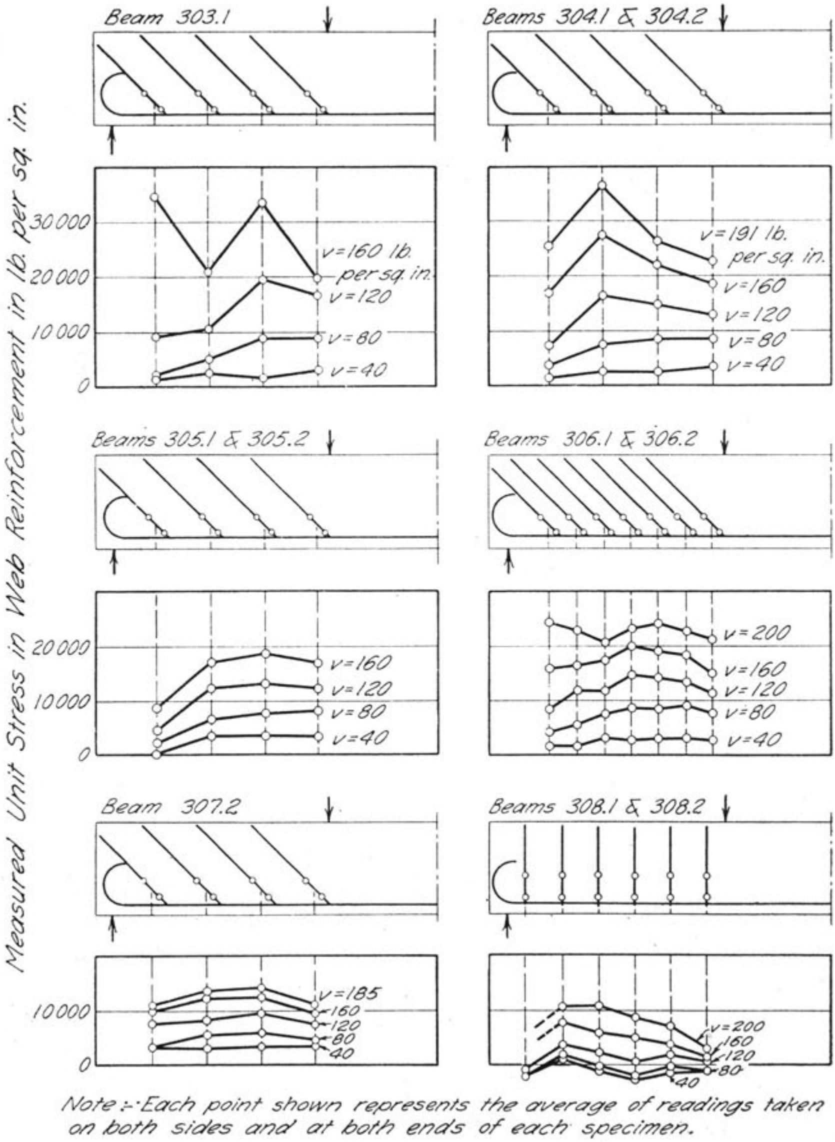


Fig. 15. Variation in Measured Stresses in Web Members between Support and Load Point, Series of 1913

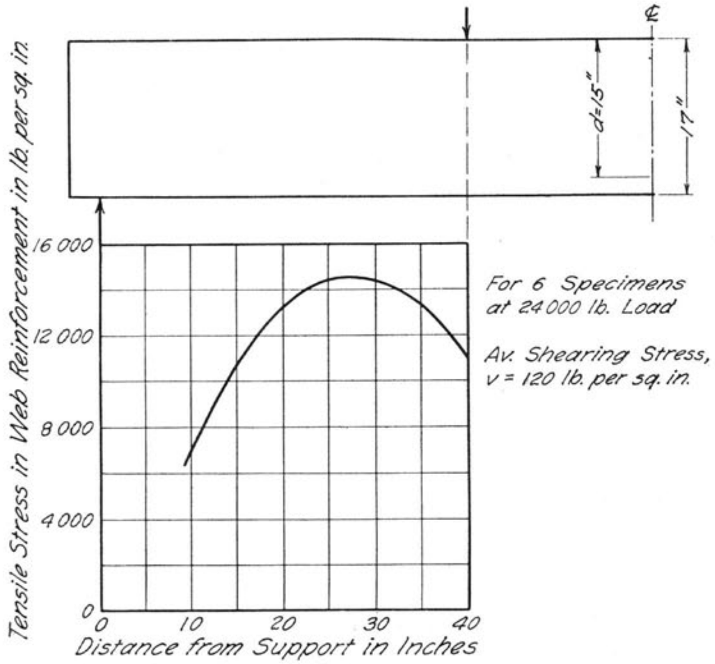


FIG. 16. COMPOSITE CURVE SHOWING VARIATION OF MEASURED STRESSES IN WEB MEMBERS, SERIES OF 1913

rapid. The change in stress can only be accomplished through the bond between the concrete and steel. Assuming that the average stress measured on a 4-inch line is equal to the stress at the mid-points of the gage line considered, and considering the bond area in a length of 4 inches, which is the center to center distance of adjacent gage lines, the average calculated bond stresses for the web members of the various beams at maximum load are as follows:

Beam No.	Diameter of Web Member, in.	Average Calculated Bond Stress, lb. per sq. in.
303.1	¼	230
304.1 and 304.2	¼	214
305.1 and 305.2	⅜	131
306.1 and 306.2	⅜	176
307.1 and 307.2	½	168
308.1 and 308.2	⅜	143

Attention is called to the fact that these stresses are average stresses, and that at individual gage lines the calculated bond stresses were in some instances considerably greater than the values given. It

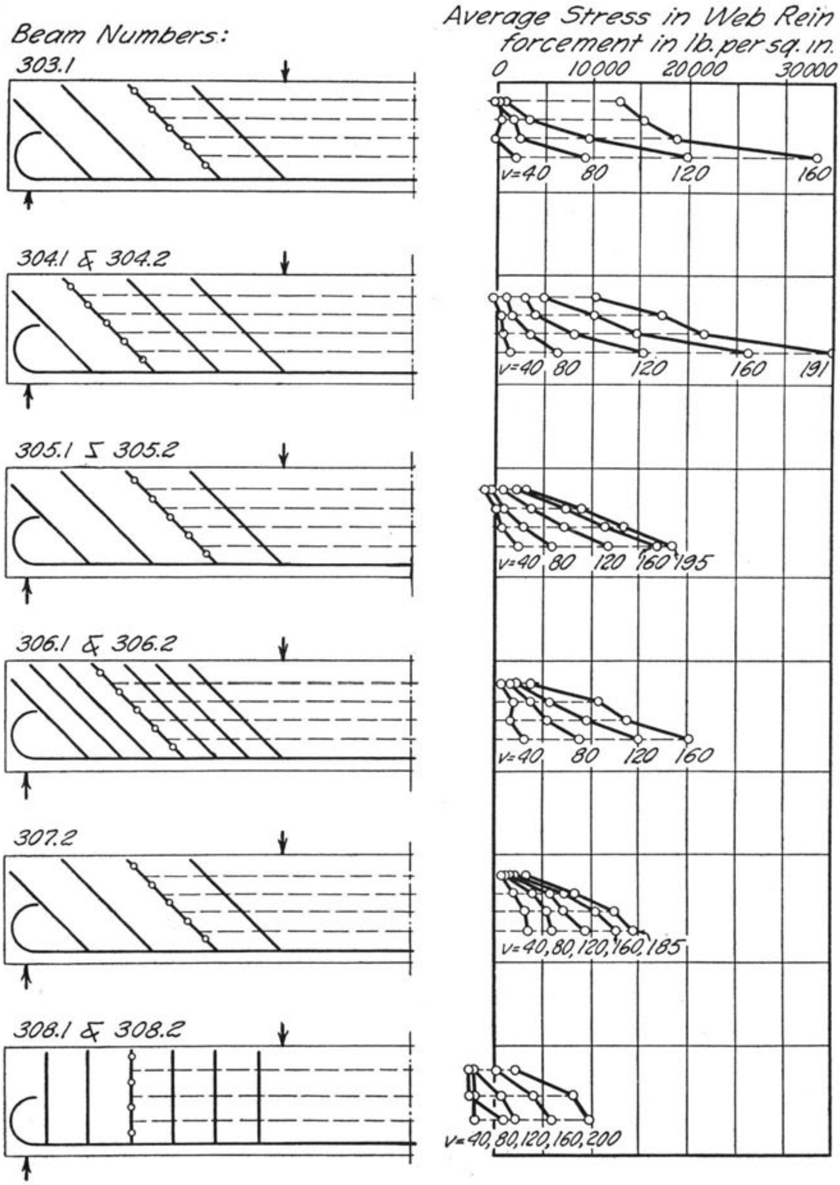


Fig. 17. Variation in Measured Stresses from Upper to Lower End of Web Members, Series of 1913

will be noted that as a result of the small sizes of bar used the values of bond stress developed by the stirrups in the beams of this series were not high. As previously stated, the yield point of the web steel was reached in beams 303.1 while in beams 304.1 and 304.2 the measured stresses were high. In the other beams tension failure occurred before very high stresses were developed in the web reinforcement. It appears that for the stresses developed in the web steel of these beams the depth of beam was great enough to afford ample anchorage for the web reinforcement in the compression side of the beam. If larger bars and a wider spacing had been used the bond on the web reinforcement might have influenced greatly the manner of failure of the beams.

In Figs. 18 and 19 are views of the beams showing the location of the cracks which appeared as the load was applied, as well as the location of gage lines.

(d) Beams with Bent-up Reinforcing Bars, Series of 1917

16. *Test Beams and Method of Testing.*—The beams in this series of tests were made with a view to obtaining information on the effect of the disposition of bent-up bars on web resistance. The effect of varying the amount of steel bent up, of bending up the same amount of steel at different points, and of varying the angle of bend was studied. Strain readings were taken on 4-in. gage lengths at selected points on the beams to determine the stresses developed in the reinforcement at different loads.

Beams of twenty types were tested in duplicate, forty beams in all. The beams were 8 in. wide, 12 in. deep, and 10 ft. long. In order to avoid tension failures as far as possible, and to have available a comparatively large amount of steel for web reinforcement, a high percentage of longitudinal steel (generally 3.7 per cent) was used. This high percentage necessitated the placing of the steel in two layers, giving an effective depth of beam of approximately 10 in. The beams were made of 1-2-4 concrete, except for a zone at the top of each beam 4 in. in depth extending on either side of the center line for a distance of 27 in. The concrete in this zone was of 1-1-2 mix, which was used here as a precaution against premature failure of the beam by crushing of the concrete. The results of tension tests of the reinforcing bars used in the beams (see Table 3) showed that the yield-point stress varied from 36 000 lb. per sq. in. for the ⅝-in. square bars to 45 700 lb. per sq. in. for the ⅞-in. round bars. Details of the various beams

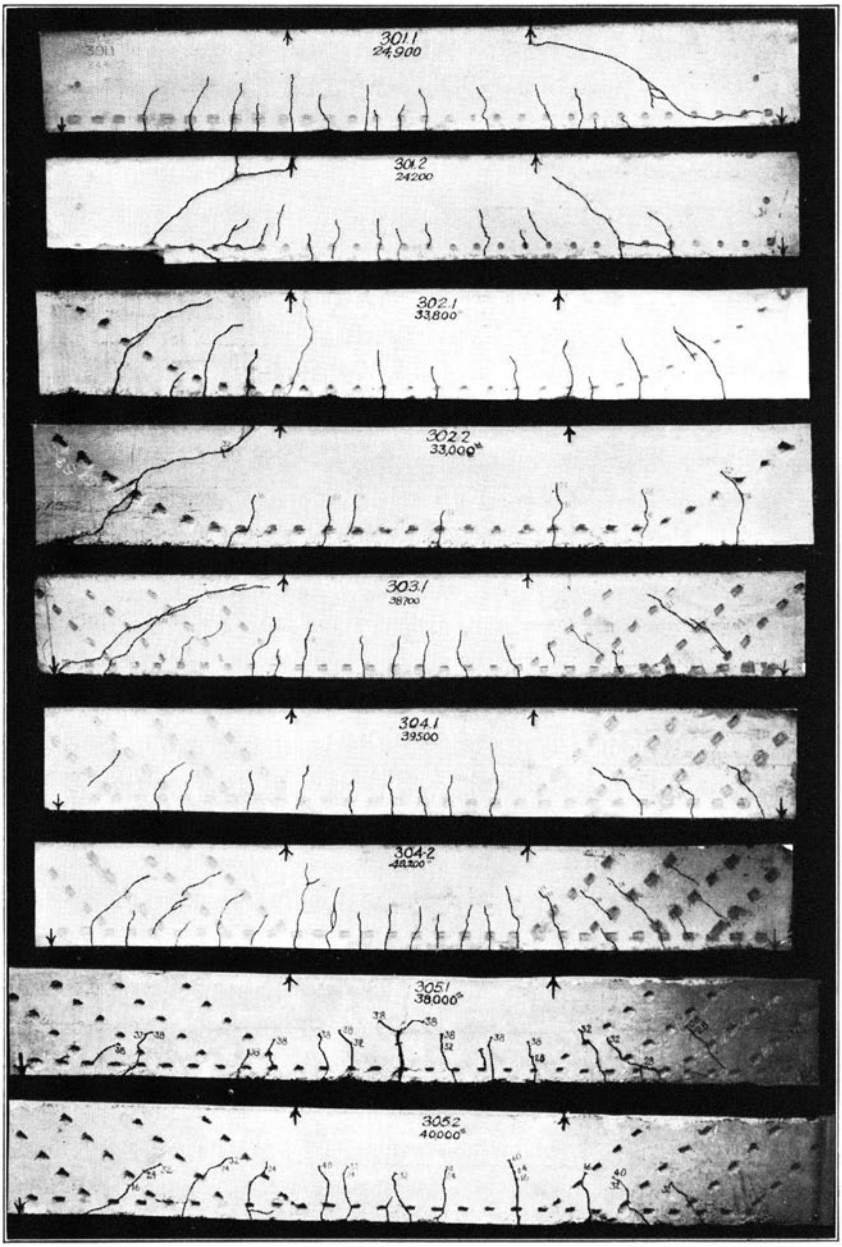


FIG. 18. VIEWS OF BEAMS AFTER FAILURE, SERIES OF 1913

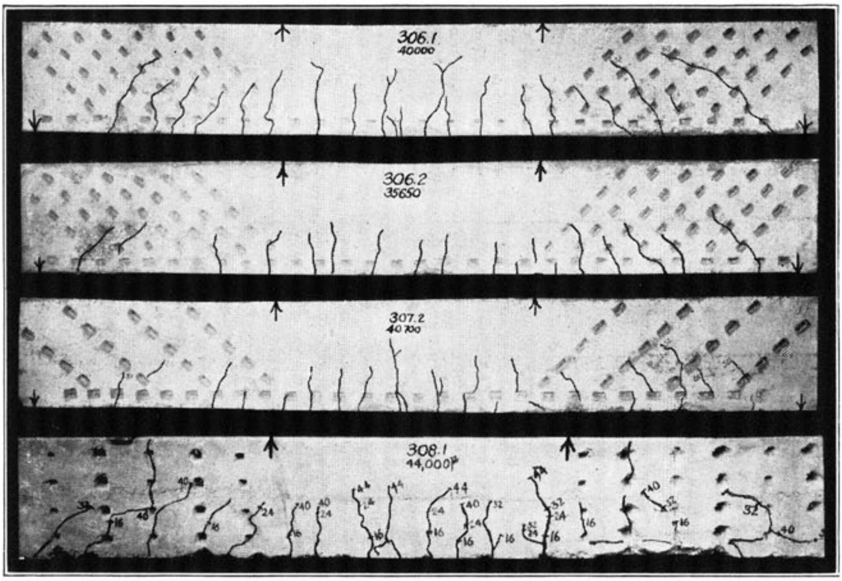


FIG. 19. VIEWS OF BEAMS AFTER FAILURE, SERIES OF 1913

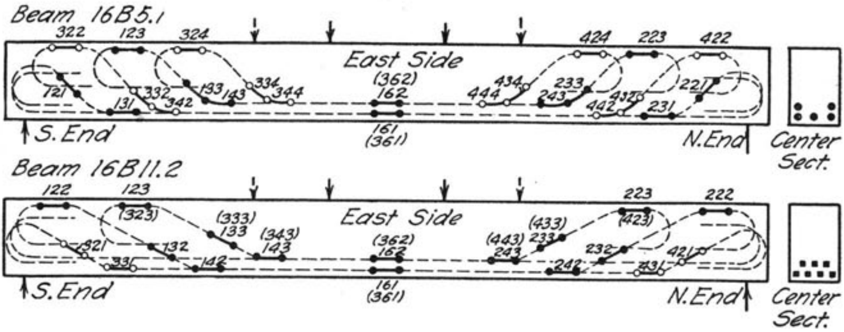


FIG. 20. LOCATION OF GAGE LINES ON TYPICAL BEAMS OF SERIES OF 1917

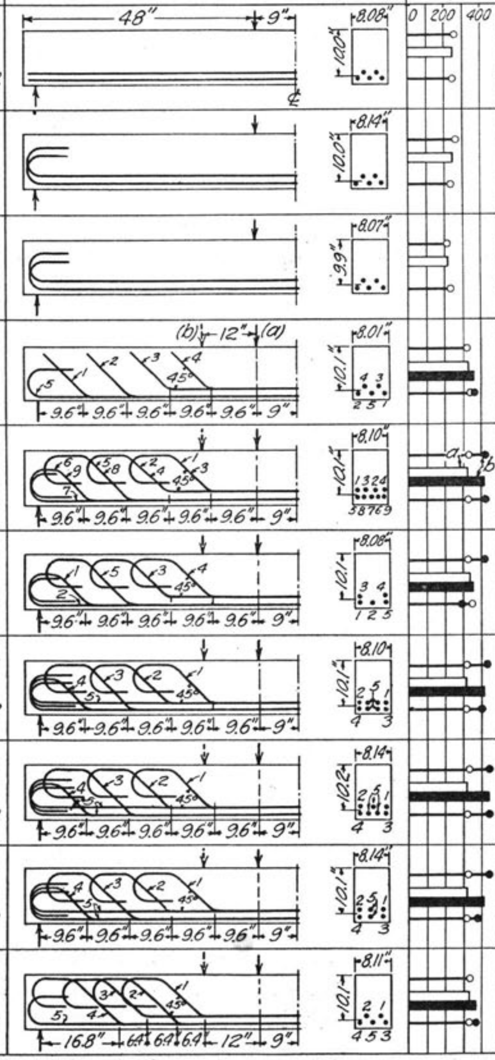
in this series of tests and the results of the tests are given in Table 7. It will be noted in the table that six of the beams had no web reinforcement. In four of the six beams the longitudinal bars extended to the ends of the beams, where they were hooked into the concrete to prevent slipping, and in the remaining two beams the longitudinal bars were not hooked. In two beams bars were bent up to the top of the beam but were not hooked at their ends. In the remaining 32 beams of the series the bent-up bars and the longitudinal bars were provided with hooks of large radius so as to insure adequate anchorage for all reinforcement.

The beams were tested on a span of 9 ft. 6 in. In testing, the load was first applied at two points 18 in. apart placed symmetrically with respect to the center line of the span length. With this arrangement the distance from the support to the load point was 4 ft., thus giving an applied bending moment equal in magnitude to that which would be obtained with a beam of 12-ft. span loaded at the one-third points. With this condition of loading the failure in most cases was by tension in the longitudinal steel, and as soon as the maximum load was reached the test was discontinued, and the beam was later retested with the load points moved 12 in. nearer the supports. In this way the moment for a given load was reduced to three-fourths of its value under the former condition of loading, and the development of greater shear was made possible.

All beams of the series were tested at an age of about 60 days. The load was usually applied in increments of 10 000 lb. and strain gage readings were taken after each increment of load. Figure 20 shows the locations of the gage lines in beams 16B5-1 and 16B11-2, which are typical of the series. In Fig. 20 the gage lines on the near

TABLE 7

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1917



Beam No.	Section at Center	Manner of Failure
16B20.1	8.08" × 10.0"	(a)-D.T. Crack from support to load point.
16B20.2		(a)-D.T. & stripping of bars from bottom of beam.
16B1.1	8.14" × 10.0"	(a)-D.T. Crack from support to load point.
16B1.2		(a)-D.T. & stripping of bars from bottom of beam.
16B2.1	8.07" × 9.9"	(a)-D.T. Near Y.P. at middle.
16B2.2		(a)-D.T. Near Y.P. at middle.
16B3.1	8.01" × 10.1"	(a)-D.T. due to slip of bent-up bar at end. N'r Y.P. middle.
16B3.2		(a)-Tension at middle. (b)-D.T. due to slip of 3rd bent-up bar
16B4.1	8.10" × 10.1"	(a)-T. at middle. (b)-T. at middle & at 2nd & 3rd bends.*
16B4.2		(a)-T. at middle. N'r Y.P. at bends. (b)-T., then crushing at N. load pt.
16B5.1	8.08" × 10.1"	(a)-T. at middle. (b)-T. at middle & near all bends.*
16B5.2		(a)-T. at mid. & load pt. High T. at 3rd bend. (b)-Comp. at middle.
16B6.1	8.10" × 10.1"	(a)-T. at middle. (b)-T. at middle.
16B6.2		(a)-T. at middle & 1st bend. (b)-T. at middle.*
16B7.1	8.14" × 10.2"	(a)-T. at middle, 1st, & 2nd bends. (b)-T. at middle.*
16B7.2		(a)-T. at middle & 1st bend. (b)-T. at middle & 1st bend.*
16B8.1	8.14" × 10.1"	(a)-T. at middle. High T. at 1st bend. (b)-T. at mid. & N. load pt.
16B8.2		(a)-T. at middle. High T. at 1st bend. (b)-T. at middle.*
16B9.1	8.11" × 10.1"	(a)-T. at middle. High T. at 1st & 3rd bends
16B9.2		(a)-T. at middle. High T. at all bends. (b)-T. in through bar near end of beam.

* Final failure by compression.

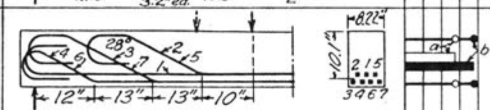
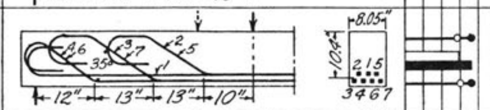
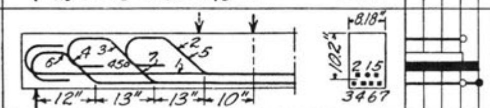
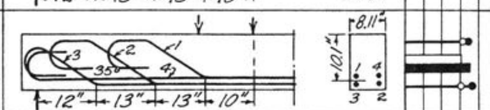
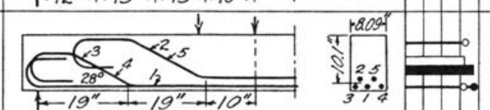
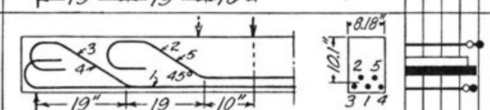
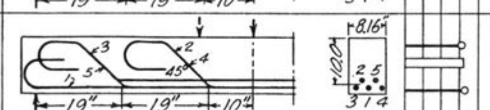
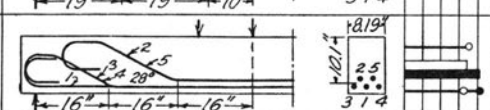
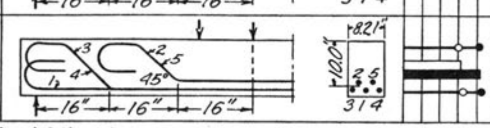
TABLE 7 (Continued)

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1917

Beam No	Age at Test Days	Longitudinal Reinforcement		Web Reinforcement		Max. Applied Load, Pounds		Computed Stresses lb. per sq. in.						1-2-4 Cylinders
								Longitudinal Steel		Shear		Bond		
		Description	%	Description	%	(a)-1'-6" apart	(b)-3'-6" apart	(a)	(b)	(a)	(b)	(a)	(b)	Compress. Strength lb. per sq. in.
16B20.1	62	5-$\frac{7}{8}$" plain ϕ, ends unanchored.	3.71	None	0	31000	—	32600	—	253	—	150	—	3210
16B20.2	59		3.71	None	0	29600	—	31200	—	241	—	143	—	—
Av.										247		147		
16B1.1	61	5-$\frac{7}{8}$" plain ϕ, ends hooked.	3.69	None	0	32000	—	33600	—	259	—	155	—	2450
16B1.2	58		3.69	None	0	28800	—	30200	—	233	—	140	—	2670
Av.										246		148		
16B2.1	60	5-$\frac{3}{4}$" plain ϕ, ends hooked.	2.74	None	0	26600	—	37400	—	213	—	148	—	2450
16B2.2	59		2.74	None	0	29500	—	41800	—	236	—	163	—	—
Av.										224		155		
16B3.1	60	5-$\frac{7}{8}$" plain ϕ, one bar hooked at ends.	3.73	4-$\frac{7}{8}$" plain ϕ, bent up at 45°. Spacing: 9.6".	1.10	39700	—	41500	—	324	—	950	—	3440
16B3.2	61		3.73		1.10	42100	44600	43600	34800	343	364	1005	1065	3170
Av.										333	364	980	1065	
16B4.1	62	8-$\frac{5}{8}$" & 1-$\frac{7}{8}$" plain ϕ, ends hooked.	3.74	8-$\frac{5}{8}$" plain ϕ, bent up at 45°. Spacing: 9.6".	1.12	41600	53000	42400	40600	333	425	995	1269	3210
16B4.2	60		3.74		1.12	41000	52900	41700	40400	329	424	980	1255	—
Av.										331	424	988	1258	
16B5.1	61	5-$\frac{7}{8}$" plain ϕ, ends hooked.	3.65	4-$\frac{7}{8}$" plain ϕ bent up at 45°. Spacing: 9.6".	1.10	41400	52800	42800	41000	331	421	980	1245	3440
16B5.2	61		3.65		1.10	44400	36500	45500	28200	354	293	1050	865	—
Av.										343	357	1015	1055	
16B6.1	62	4-$\frac{5}{8}$" & 4-$\frac{3}{4}$" plain ϕ, ends hooked.	3.65	4-$\frac{5}{8}$" plain ϕ, bent up at 45°. Spacing: 9.6".	0.56	40200	54500	42100	42900	321	434	278	376	3770
16B6.2	61		3.65		0.56	40000	50600	40600	39200	319	404	277	348	—
Av.										320	419	278	362	
16B7.1	62	4-$\frac{5}{8}$" & 3-$\frac{7}{8}$" plain ϕ, ends hooked.	3.64	4-$\frac{5}{8}$" plain ϕ, bent up at 45°. Spacing: 9.6".	0.56	42200	56400	43700	43800	333	444	332	441	3770
16B7.2	60		3.64		0.56	40800	56700	40400	42100	322	447	320	443	—
Av.										328	446	326	442	
16B8.1	62	4-$\frac{3}{4}$" & 2-$\frac{7}{8}$" plain ϕ, ends hooked.	3.60	4-$\frac{3}{4}$" plain ϕ, bent up at 45°. Spacing: 9.6".	0.80	40800	55900	42800	44000	325	444	486	664	3190
16B8.2	61		3.60		0.80	40000	48000	41900	37800	319	382	478	571	—
Av.										322	413	482	618	
16B9.1	60	5-$\frac{7}{8}$" plain ϕ, ends hooked.	3.65	4-$\frac{7}{8}$" plain ϕ, bent up at 45°. Spacing: 6.4".	0.80	41900	—	43300	—	333	—	990	—	3510
16B9.2	61		3.65		0.80	41500	46500	42500	35800	330	370	985	1100	—
Av.										332	370	988	1100	7-1-2

TABLE 7 (Continued)

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1917

Beam No.	Description of Test Beams: Side Elevation; Section at Center — Max. Shearing Stress lb. per sq. in. (0, 200, 400)	Manner of Failure
16B10.1 16B10.2		(a)-T. at N. end & middle. D.T. cracks large. (a)-T. at N. end in through bar.
16B11.1 16B11.2		(a)-T. at middle & 2nd bend. (b)-T. at middle & 2nd bends.* (a)-T. at middle. High T. at bends. (b)-Same as (a).
16B12.1 16B12.2		(a)-T. at middle & 2nd bend. (b)-T. at middle.* (a)-T. at middle. High T. at bends. (b)-T. at outer bends.*
16B13.1 16B13.2		(a)-T. at middle & 2nd bend.* (a)-T. at middle. High T. at 2nd bend.-(b)-T. at 1st & 2nd bends. D.T. followed.
16B14.1 16B14.2		(a)-T. at middle & at bends. (b)-Same, & D.T. cracks & splitting at hook.- (a) T. at middle. High T. at 2nd bend.-(b)-Splitting of concrete in hooks n'r end.
16B15.1 16B15.2		(a)-T. near end & middle. Splitting at hook. (a)-T. at middle. Near Y.P. elsewhere. -(b)-T. in through bar. Splitting at hook at end.
16B16.1 16B16.2		(a)-T. at middle & 1st bend. (b)-T. at middle & 1st bend.* (a)-T. at middle. Near Y.P. elsewhere. (b)-T. at 1st bend.*
16B17.1 16B17.2		(a)-T. at 2nd bend. D.T. followed.* (a)-T. at middle. High T. at outer bends.
16B18.1 16B18.2		(a)-T. & D.T. at bends. (a)-T. at middle, Y.P. at bends. (b)-T. in through bar near end; splitting of concrete at end hooks. Y.P. at middle.
16B19.1 16B19.2		(a)-Y.P. at middle. High T. at 1st bend. Max. load not reached.-(b)-T. at middle & at 2nd bend. -(a)-T. at middle. (b)-T. at 1st bend.*

* Final failure by compression.

TABLE 7 (Concluded)

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1917

Beam No.	Age at Test Days	Longitudinal Reinforcement		Web Reinforcement		Max. Applied Load, Pounds		Computed Stresses lb. per sq. in.						1-2-4 Cylinders
								Longitudinal Steel		Shear		Bond		
		Description	%	Description	%	(a)-1′-6″ apart	(b)-3′-6″ apart	(a)	(b)	(a)	(b)	(a)	(b)	Compress. Strength lb. per sq. in.
16B10.1	60	5-$\frac{7}{8}$″ plain ϕ, ends hooked.	3.55	4-$\frac{7}{8}$″ plain ϕ, bent up at 45°. Spacing: 3.2″.	3.28	36300	—	36700	—	281	—	840	—	3510
16B10.2	59		3.55		3.28	31500	—	31500	—	245	—	730	—	—
Av.										263		785		
16B11.1	63	7-$\frac{5}{8}$″ plain ϕ, ends hooked.	3.28	6-$\frac{5}{8}$″ plain ϕ, bent up at 28°. Spacing: 13″.	1.56	35100	48800	39600	41300	274	380	910	1260	3590
16B11.2	60		3.28		1.56	35000	48600	39100	40700	273	378	903	1246	—
Av.										274	379	907	1253	
16B12.1	62	7-$\frac{5}{8}$″ plain ϕ, ends hooked.	3.26	6-$\frac{5}{8}$″ plain ϕ, bent up at 35°. Spacing, 13″	1.31	36700	46700	40800	39000	285	362	924	1176	3590
16B12.2	61		3.26		1.31	39000	47100	42100	38200	302	365	980	1183	—
Av.										293	364	952	1180	
16B13.1	62	6-$\frac{5}{8}$″ plain ϕ, & 1-$\frac{7}{8}$″ pl. ϕ, ends hooked.	3.55	6-$\frac{5}{8}$″ plain ϕ, bent up at 45°. Spacing: 13″.	1.04	40600	—	42300	—	318	—	955	—	3040
16B13.2	59		3.55		1.04	40100	51600	41800	40300	313	404	942	1205	—
Av.										316	404	948	1205	
16B14.1	60	3-1″ & 1-$\frac{7}{8}$″ plain ϕ, ends hooked.	3.62	3-1″ plain ϕ, bent up at 35°. Spacing: 13″.	1.30	39200	43600	41600	34800	313	349	935	1036	3190
16B14.2	61		3.62		1.30	38000	46000	39400	35800	304	368	908	1090	—
Av.										308	358	922	1063	
16B15.1	61	5-$\frac{7}{8}$″ plain ϕ, ends hooked.	3.65	4-$\frac{7}{8}$″ plain ϕ, bent up at 28°. Spacing: 19″.	1.67	41000	—	42500	—	326	—	970	—	3040
16B15.2	60		3.65		1.67	40000	47400	40900	36400	318	377	945	1120	—
Av.										322	377	958	1120	
16B16.1	62	5-$\frac{7}{8}$″ plain ϕ, ends hooked	3.62	4-$\frac{7}{8}$″ plain ϕ, bent up at 35°. Spacing: 19″.	1.35	44600	51600	46100	40400	352	408	1055	1220	3440
16B16.2	61		3.62		1.35	42200	47100	43100	36200	333	372	1000	1110	—
Av.										342	390	1028	1165	
16B17.1	59	5-$\frac{7}{8}$ plain ϕ, ends hooked.	3.66	4-$\frac{7}{8}$″ plain ϕ, bent up at 45°. Spacing: 19″.	1.10	38200	—	39800	—	306	—	915	—	3440
16B17.2	61		3.66		1.10	40200	—	42000	—	321	—	960	—	—
Av.										313		938		
16B18.1	62	5-$\frac{7}{8}$″ plain ϕ, ends hooked.	3.62	4-$\frac{7}{8}$″ plain ϕ, bent up at 45°. Spacing: 16″	1.96	45500	—	47100	—	359	—	1080	—	3330
16B18.2	60		3.62		1.96	40900	52100	42200	40400	323	412	970	1235	—
Av.										341	412	1025	1235	
16B19.1	62	5-$\frac{7}{8}$″ plain ϕ ends hooked.	3.63	4-$\frac{7}{8}$″ plain ϕ, bent up at 45°. Spacing: 16″.	1.29	37700	52000	39000	40300	299	413	900	1235	3330
16B19.2	61		3.63		1.29	41100	54100	43000	42500	326	429	980	1285	3170
Av.										312	421	940	1260	

7-2-2

side of the beam are shown by solid circles and the gage lines on the far side by open circles. Where gage lines were similarly located on opposite sides of the beams, to avoid confusion in the figure the numbers of the gage lines on the far side of the beam are placed in brackets. Views of the beams after failure showing the location of some of the gage lines are given at the end of this chapter.

17. *Phenomena of Tests.*—All of the six beams of this series which had no web reinforcement failed by diagonal tension. In beams 16B20-1 and 16B20-2, which had unanchored bars, the failures occurred suddenly and were followed by the stripping of the concrete from the reinforcing bars. In the corresponding beams 16B1-1 and 16B1-2, which had the longitudinal bars anchored by hooks, the failure took place rapidly, but not instantaneously, and the stripping of the concrete from the longitudinal bars was not as marked as in the case of beams 16B20-1 and 16B20-2. Beams 16B2-1 and 16B2-2 also failed rapidly by diagonal tension, although at the time of failure the measured stress in the steel at the center of these beams was near the yield point. Beams 16B3-1 and 16B3-2, which had unanchored bent-up bars for web reinforcement, failed in diagonal tension when the slipping of the bent-up bars permitted the opening and extension of large diagonal cracks. In beam 16B3-2 tension failure occurred under the first condition of loading, and it was when the second condition of loading was used that the diagonal tension failure occurred. In beam 16B3-1 the longitudinal steel was stressed nearly to the yield point when the diagonal tension crack opened. The remaining beams of the series failed by tension in the longitudinal steel although in some cases the diagonal cracks which appeared gave indications that diagonal tension failure was imminent.

18. *Results of Tests.*—As has already been noted, all beams of the series without web reinforcement failed by diagonal tension. Of this group beams 16B1-1 and 16B1-2, which had the longitudinal bars anchored at their ends by means of hooks, carried practically the same loads as beams 16B20-1 and 16B20-2, which had the same percentage of reinforcement, but which had no hooks at the ends of the bars. Where hooks were used they were just coming into action at the maximum load. This indicates that the deformation along the bars was sufficient to allow diagonal tension failure without the hooks becoming effective. Beams 16B2-1 and 16B2-2 had 26 per cent less reinforcement than beams 16B1-1 and 16B1-2 and carried 8 per cent less load. Although none of these beams failed by tension in the steel, those with

the smaller amount of reinforcement carried the smaller load, indicating that the load at which diagonal tension failure occurs is a function of the deformation along the longitudinal reinforcement.*

In all beams with web reinforcement failure occurred by tension in the longitudinal steel, except in beams 16B3-1 and 16B3-2 in which slipping of some of the bent-up bars permitted diagonal tension cracks to open. It will be noted in Table 7 that in all the beams with web reinforcement, regardless of the manner of bending up, angle of bend, spacing of bars, etc., the maximum shearing unit stress developed varied only from 263 lb. per sq. in. to 343 lb. per sq. in. when the load points were 18 in. apart, and from 357 to 446 lb. per sq. in. when the load points were 42 inches apart. This result might be expected, since all but two of these beams failed by tension in the steel and the full strength of the web reinforcement was not developed. The distribution of stress in these beams varied somewhat, bars in certain locations taking stress somewhat more rapidly than those in others, and consequently it is from the standpoint of stress in the inclined reinforcement rather than from that of the maximum shearing unit stress developed that the results of these tests will be discussed.

(a) Effect of Angle of Bend upon Location of Maximum Stress in Inclined Members

Beams 16B11, 16B12, and 16B13 were made for the purpose of studying the effect of the angle of inclination of bent-up bars upon the web resistance. In these beams the same amount of reinforcement (two ⅝-in. square bars) was bent up at each point. In types 16B11 and 16B12 a ⅝-in. square bar was used for the through bar in each beam, but in the beams of the 16B13 group a ⅞-in. round bar was used by mistake for the through bar. A comparison of the stresses developed in the reinforcing bars of these beams at the bends is given in Table 8. It will be noted in the table that the highest stresses outside the load points were found on the gage lines located at the bottom of the second bend from the center of the span. For the beams just mentioned, at the given load (30 000 lb.) the percentage excess of stress in the second bend over that in the first bend ranged from 17 for the 28 deg. angle to 45 for the 45 deg. angle. These results may have been affected by the fact that the first bend was always in a bar of the upper layer of reinforcement and the second bend was always in a bar of the lower layer of reinforcement.

*For discussion and test data bearing out this statement see "Tests of Reinforced Concrete Beams, Resistance to Web Stresses," Univ. of Ill., Eng. Exp. Sta., Bul. 29, 1909.

TABLE 8
MEASURED STRESSES IN STEEL FOR BEAMS HAVING BARS BENT UP AT VARIOUS ANGLES, SERIES OF 1917

Beam No.	Angle of Bend, degrees	Spacing of Bars, inches	Measured Stresses at 30 000-lb. load, lb. per sq. in.					
			At First Bend		At Second Bend		At Third Bend	
			Horizontal	Inclined	Horizontal	Inclined	Horizontal	Inclined
16B11-1 16B11-2	28	13	23 800	10 900*	27 900	22 400	18 500	16 100
16B12-1 16B12-2	35	13	21 700	12 200	26 900	18 900	18 000	17 400
16B13-1 16B13-2	45	13	17 800	9 500	25 800	14 200	16 600	11 600
16B15-1 16B15-2	28	19	18 900	16 900	23 200	15 500		
16B16-1 16B16-2	35	19	19 300	13 100	22 500	13 600		
16B17-1 16B17-2	45	19	18 600	10 400	21 900	14 400		
16B18-1 16B18-2	28	16	16 700	10 600	18 500	13 500		
16B19-1 16B19-2	45	16	13 200	7 000	15 300	10 900		

*Stresses measured some distance above bend.

It will be noted in Table 8 that the stress in the inclined portion of the bars was in all cases less than the stress in the horizontal portion of the same bars near the bend. The difference between the stress in the inclined portion and that in the horizontal portion was greater for the larger angles of inclination with the horizontal. The stress in the inclined portion of the bars was greater at the second bend than at the first or third bends. The results of stress measurements on beams 16B15, 16B16, and 16B17 which had 19-in. spacing and on beams 16B18 and 16B19 which had 11-in. spacing, are also given in Table 8. It will be noted that in general the results obtained with these beams were similar in character to those obtained for beams 16B11, 16B12, and 16B13. The results indicate that greater stress reduction occurs at the bend where the angle of bend in the bar is large than where the angle of bend is small.

(b) Effect of the Number of Bars Bent up at a Given Point on Stresses in Bars

In beams 16B4 and 16B5 two-tenths of the total area of the reinforcement was bent up at each of four places. In both groups of

TABLE 9

MEASURED STRESSES IN STEEL FOR BEAMS HAVING SINGLE BENT-UP BAR AND FOR BEAMS HAVING TWO BENT-UP BARS OF SAME TOTAL AREA, SERIES OF 1917

Beam No.	Angle of Bend	Spacing of Bars	Bars Bent Up at Each Point		Measured Stresses at 30 000 lb. load, lb. per sq. in.							
					First Bend		Second Bend		Third Bend		Fourth Bend	
	degrees	inches	No.	Area sq. in.	Horizontal	Inclined	Horizontal	Inclined	Horizontal	Inclined	Horizontal	Inclined
16B4-1 16B4-2	45	9.6	2	0.61	17 300	6 300	18 200	11 700	14 900	12 000	13 800	6 600
16B5-1 16B5-2	45	9.6	1	0.60	17 600	11 000	16 700	9 200	20 600	14 300	16 800	7 900
16B13-1 16B13-2	45	13.0	2	0.78	17 800	9 500	25 800	14 200	16 600	11 600		
16B14-1 16B14-2	35	13.0	1	0.78	17 500	9 100	22 500	15 100	18 400	13 900		

TABLE 10
MEASURED STRESSES IN STEEL FOR BEAMS HAVING VARIOUS RATIOS OF AREA OF BENT-UP BARS TO TOTAL AREA OF LONGITUDINAL BARS, SERIES OF 1917

Beam No.	Ratio of Area Bent up at Each Point to Total area	Measured Stresses at 30 000-lb. load, lb. per sq. in.							
		At First Bend		At Second Bend		At Third Bend		At Fourth Bend	
		Horiz.	Incl.	Horiz.	Incl.	Horiz.	Incl.	Horiz.	Incl.
16B5-1 16B5-2	0.2	17 600	11 000	16 700	9 200	20 600	14 300	16 800	7 900
16B8-1 16B8-2	0.149	19 100	10 800	15 900	9 900	18 100	13 500	14 100	4 900
16B6-1 16B6-2	0.101	21 700	13 600	17 800	11 800	18 400	12 100	12 400	5 200
16B7-1 16B7-2	0.102	19 200	14 700	16 400	9 300	20 900	13 600	12 900	2 100

beams the points of bending were the same, but in beams 16B4 two ⅝-in. round bars were bent up at each place, while in beam 16B5 only one ⅞-in. bar having the same area as the two ⅝-in. rounds was bent up at each place. The test data permit a comparison to be made of the stresses developed when one bar is bent up with those developed when two bars of the same total area are bent up. A similar comparison can be made for beams 16B13 and 16B14 if the effect of difference in angle of bend be neglected.

Table 9 gives the measured stresses in the horizontal and inclined steel at the bends for beams 16B4, 16B5, 16B13, and 16B14. It will be seen from the data given in Table 9 that in general there was but little difference in the unit stresses developed in the steel when one or two bars of the same total cross-sectional area were bent up at a given place. Nevertheless it would seem that the use of the smaller bars would give a better distribution of metal through the web of the beam and consequently would tend to give a better distribution of stress than would the use of a single bar of large size.

(c) Effect of Amount of Steel Bent Up on Stress Developed

In Beams 16B5, 16B6, 16B7, and 16B8 one bar was bent up at each of four points 9.6 in. apart, the first point of bending being 9.6 in. from the load point. The size of the bars in these beams varied and the proportion of the total amount of reinforcement bent up to the total longitudinal steel varied as indicated in Table 10.

It will be noted in Table 10 that within the range covered by the tests there was little difference in the unit stresses developed in the bent-up bars due to varying the proportion of the reinforcement bent up. As in the case of other beams of the series the stress in the inclined portion of the bars was in all cases less than the stress in the horizontal portion of the bar near the bend, and, although the latter stress passed the yield point of the steel in a number of cases, in these four groups of beams there was only one gage line on the inclined portion of a bar in which the stress reached the yield point.

(d) Stresses in Bars Near Top of Beam

The deformations measured in the horizontal portion of the bent-up bars at the top of the beam indicate that little tensile stress was carried through from the bottom of the beam to the horizontal portion of the bar at the top of the beam. For the bars bent up near the load points, considerable compression was developed in the horizontal portion of the bars at the top of the beam. For the bars bent up near the supports, very little stress in either tension or compression was developed in the horizontal portion of the bar at the early stages of the test. At the higher loads the direction of the load deformation curve was frequently reversed, indicating that slip was taking place at one of the gage holes. There was no indication, however, that the amount of load carried was in any case affected by this slip, except in beams 16B3-1 and 16B3-2 in which no hooks were provided, and in which slipping at the ends of some of the bent-up bars apparently hastened diagonal tension failure. In both of these beams the bars near the ends of the beam seemed to have been most affected by end slip.

19. *General Comments.*—Figure 21 shows a composite curve for the 34 specimens having bent-up bars which indicates the measured stress in the inclined bars at various distances from the support. It will be seen in Fig. 21 that for the beams tested the highest stress in the inclined bars was found at a distance of about 18 in. from the support, although stresses nearly as high were found through the region midway between the load point and the support. In beams 16B9 and 16B10, which had the bent-up bars grouped together in the region approximately midway between the load point and the support, the stresses measured in the inclined portion of the outer two bars were considerably higher than those measured in the inclined portion of the inner two bars, and bending up the bars at a considerable distance from the support appeared to permit the formation of diagonal tension cracks between the support and the first bent-up bar. In all cases where the distance from the support to a bent-up bar was less than the depth of the beam the measured stress in the outer inclined bar was

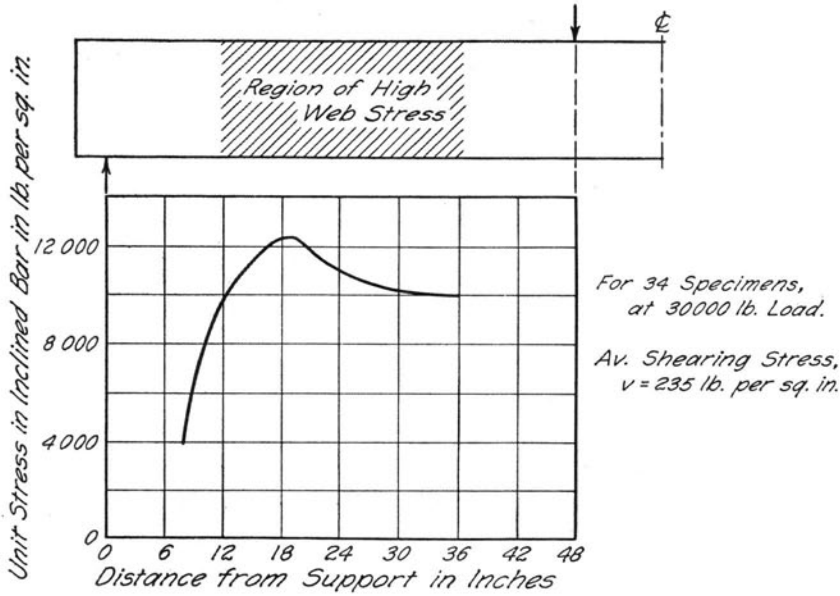


FIG. 21. COMPOSITE CURVE SHOWING VARIATION IN MEASURED STRESSES IN BENT-UP BARS, SERIES OF 1917

comparatively low. The results of the tests also indicate that better stress distribution was obtained when the bent-up bars were well distributed throughout the region of the beam subject to vertical shear than when they were closely spaced in a comparatively small section of the length of the beam.

Table 11 gives values of calculated and measured stresses in bent-up bars of the various beams at a load of 30 000 lb. The calculated stresses were obtained by the use of equation (7). It may be noted that, with only a few exceptions, the measured stresses are about seven-tenths of the calculated stresses, which are based on the assumption that the full vertical shear acts to produce stress in the web members. The exceptions noted were beams with either a very high percentage or a low percentage of web reinforcement. Beams 16B6, 16B7, and 16B8 had a smaller proportion of the total steel area bent up for web reinforcement than did the other beams, and hence a greater amount of longitudinal steel was carried through to the supports. This may have prevented the opening of cracks in the outer portions of the beams and thus reduced the stress in the web members.

In Fig. 22 two typical load-stress curves are shown for a beam of each of the types having bent-up bars. The curves show the stresses

TABLE 11

CALCULATED AND MEASURED STRESSES IN BENT-UP BARS AT LOAD OF 30 000 LB., SERIES OF 1917

Beam No.	Angle of Bend, degrees	Horizontal Spacing of Bars, inches	Web Reinforcement, Area sq. in.	Per cent	Calculated* Stress in Web Reinforcement, lb. per sq. in.	Average Measured Stress‡ in Web Reinforcement, lb. per sq. in.				Max. ratio, [Meas. Stress / Calc. Stress]
						At First Bend From Center	At Second Bend	At Third Bend	At Fourth Bend	
16B3	45	9.6	0.601	1.10	21 800	13 600	8 400	11 100	6 700	0.62
16B4	45	9.6	0.614	1.12	21 200	6 300	11 700	12 000	6 600	0.57
16B5	45	9.6	0.601	1.10	21 500	11 000	9 200	14 300	7 900	0.67
16B6	45	9.6	0.307	0.56	42 100	13 600	11 800	12 100	5 200	0.32
16B7	45	9.6	0.307	0.56	42 000	14 700	9 300	13 600	2 100	0.35
16B8	45	9.6	0.442	0.80	29 400	10 800	9 900	13 500	4 900	0.46
16B9	45	6.4	0.601	1.64	14 300	8 200	9 500	16 200	16 800	1.17
16B10	45	3.2	0.601	3.28	7 000	8 200	7 700	13 100	18 300	2.61
16B11	28	13.0	0.781	1.56	23 300	10 900†	22 400	16 100		0.96
16B12	35	13.0	0.781	1.31	22 000	12 200	18 900	17 400		0.86
16B13	45	13.0	0.781	1.04	22 300	9 500	14 200	11 600		0.64
16B14	35	13.0	0.785	1.30	22 700	9 100	15 100	13 900		0.67
16B15	28	19.0	1.202	1.67	22 300	16 900	15 500			0.76
16B16	35	19.0	1.202	1.35	21 500	13 100	13 600			0.63
16B17	45	19.0	1.202	1.10	21 500	10 400	14 400			0.67
16B18	28	16.0	1.202	1.96	18 800	10 600	13 500			0.72
16B19	45	16.0	1.202	1.29	18 100	7 000	10 900			0.60

*Stresses calculated by use of equation (7).
†Stresses measured some distance above bend.
‡Stresses are the average of the highest four observations on each of two companion beams.

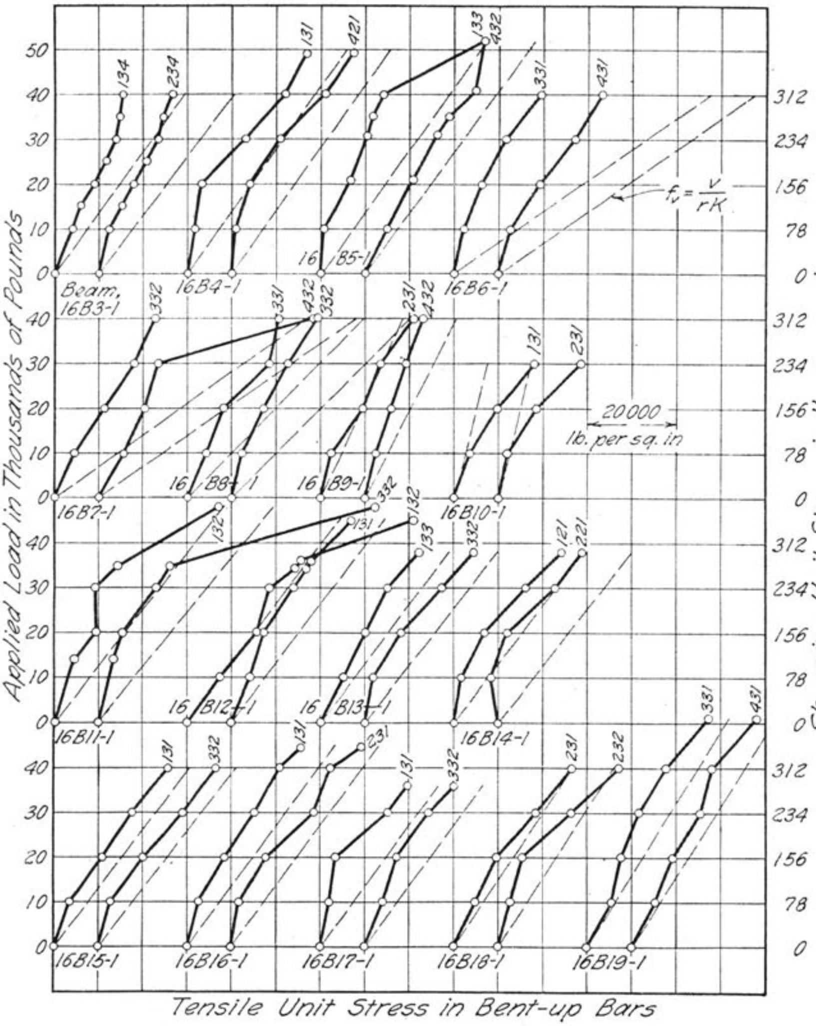


FIG. 22. LOAD-STRESS CURVES FOR GAGE LINES ON WHICH HIGH STRESSES WERE MEASURED, SERIES OF 1917

measured on the more highly stressed gage lines on bent-up bars in the lower part of the web at various loads. Corresponding calculated stresses based upon equation (7) are also shown in the figure by straight broken lines.

Views of the various beams of the series after failure are given in Figs. 23 to 26. These show the location of gage lines and cracks as well as points of application of loads and reactions.

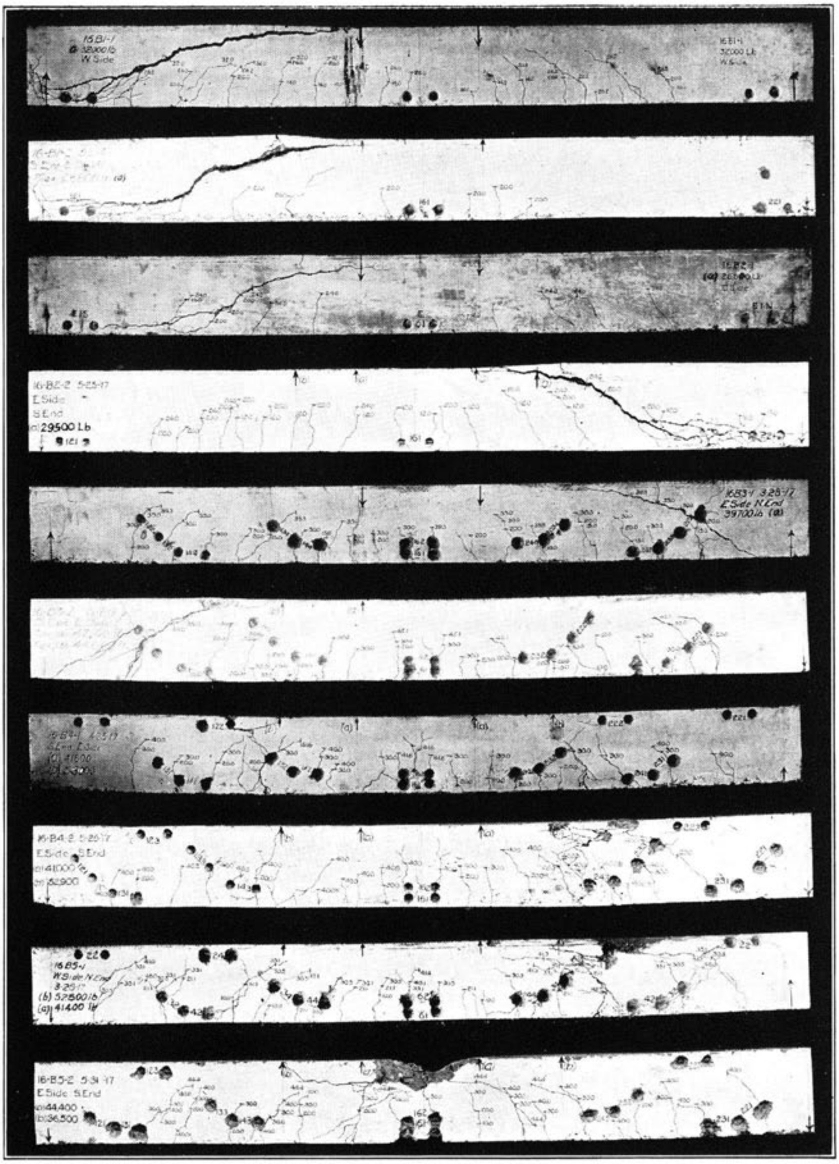


FIG. 23. VIEWS OF BEAMS AFTER FAILURE, SERIES OF 1917

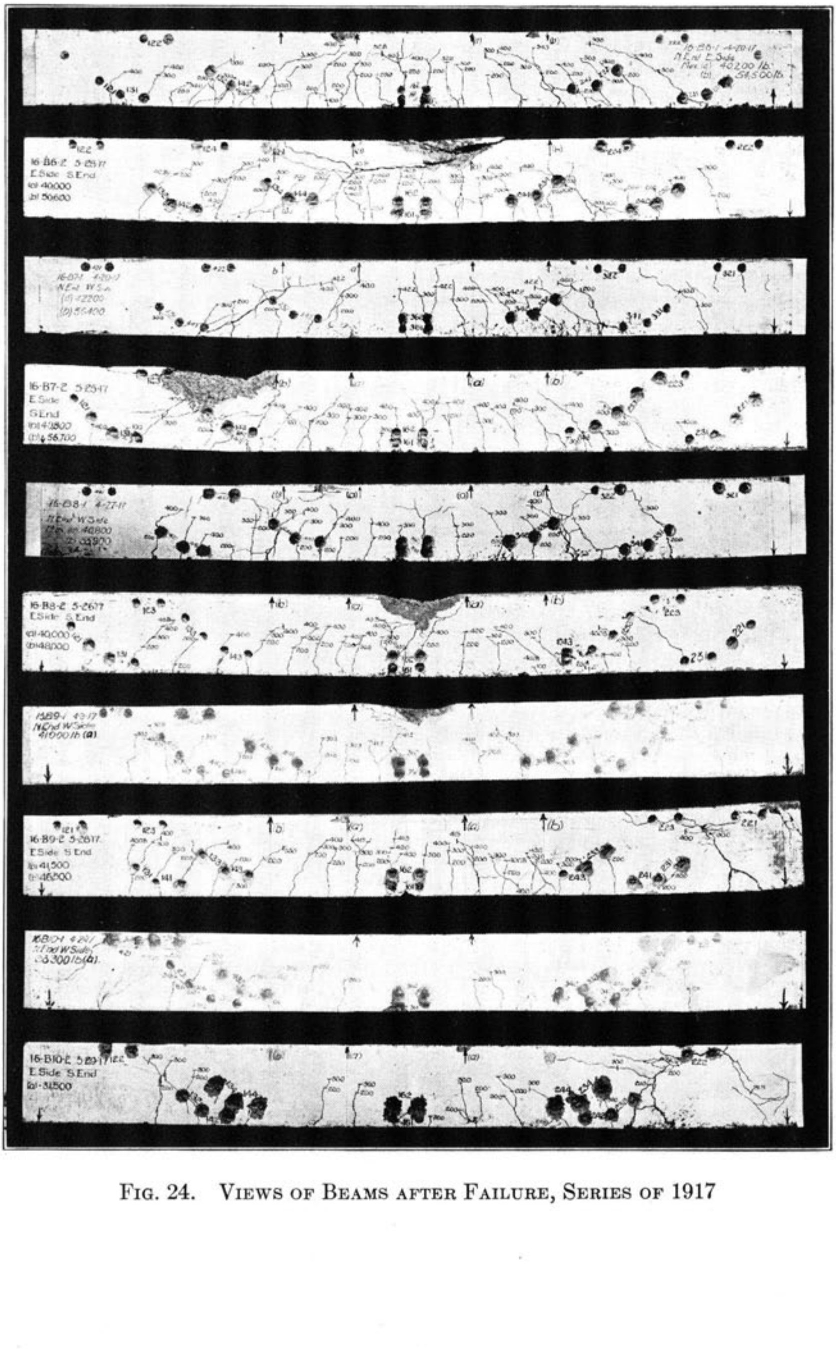


FIG. 24. VIEWS OF BEAMS AFTER FAILURE, SERIES OF 1917

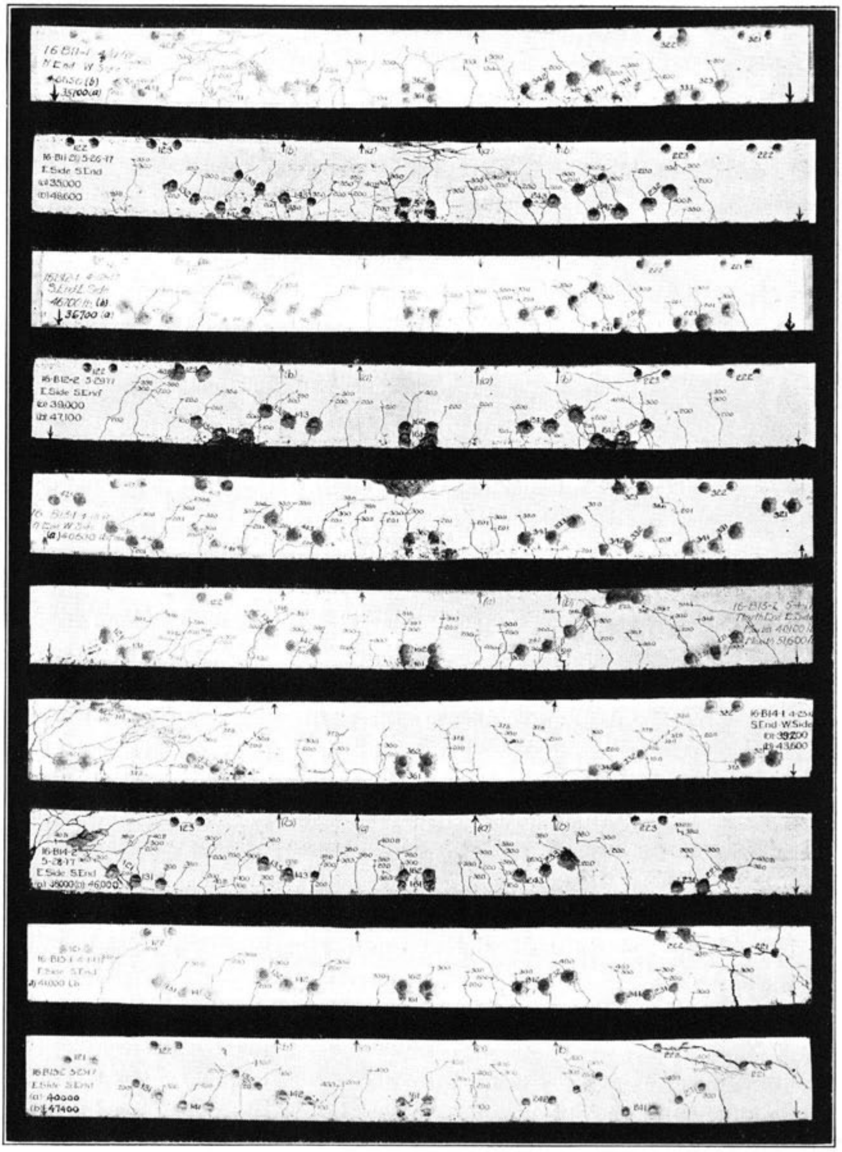


FIG. 25. VIEWS OF BEAMS AFTER FAILURE, SERIES OF 1917

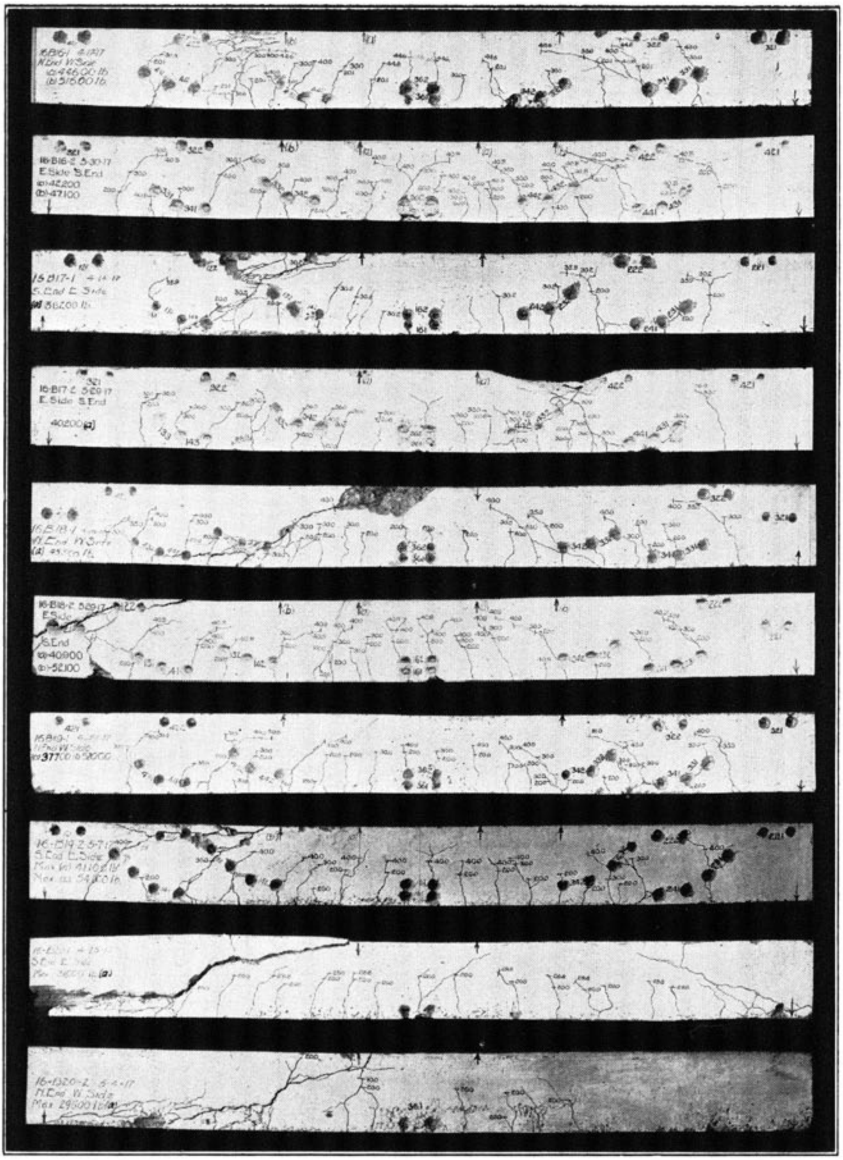


FIG. 26. VIEWS OF BEAMS AFTER FAILURE, SERIES OF 1917

(e) Beams with Loose Vertical Stirrups, Series of 1922

20. *Test Beams and Method of Testing.*—This series of tests was planned to study the effect of stirrup spacing in deep beams, to compare the web resistance of beams of rectangular section and of T-section, respectively, and to determine limiting values of shearing stress that might be developed.

The series includes ten types of beams, with two companion beams of each type. The tests were made on (1) rectangular and T-beams without web reinforcement, (2) rectangular and T-beams with loose vertical stirrups, and (3) rectangular beams with bent-up bars as web reinforcement. All beams were tested on a span of 9 feet, with equal loads applied at the one-third points of the span.

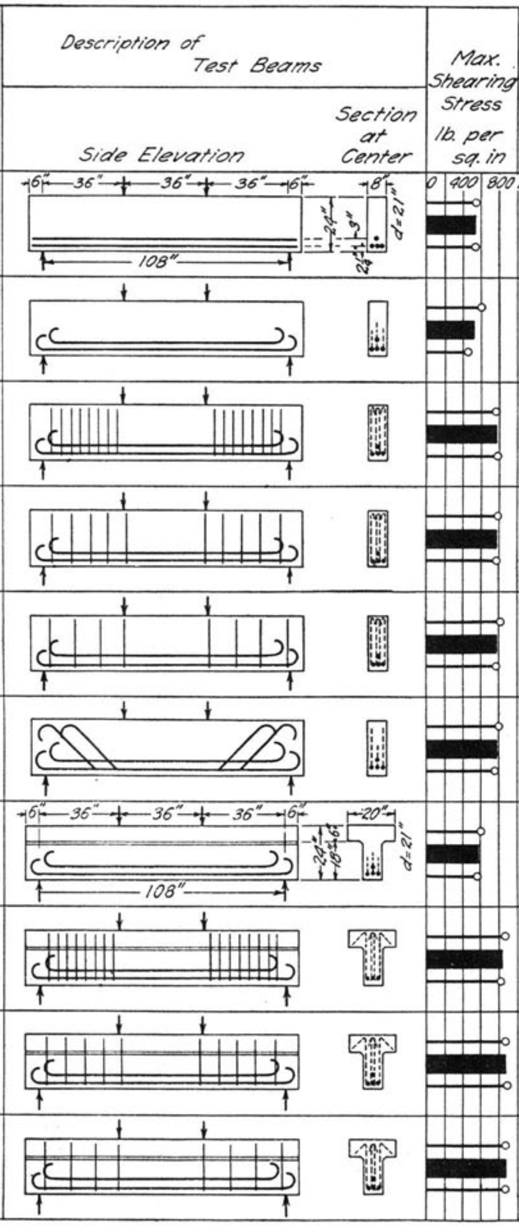
The beams were 24 in. deep, with an effective depth of 21 in. The longitudinal reinforcement in all cases consisted of four $1\frac{1}{8}$-in. deformed round high-carbon steel bars, giving a percentage of reinforcement of 2.33 for the rectangular beams, and 0.93 for the T-beams. The average tensile stress at the yield point of this steel, as given in Table 3, was 52 400 lb. per sq. in. The hooks on the ends of the bars were bent cold to a radius of $3\frac{1}{2}$ in.; the cold bending may explain the breakage of hooks that occurred in one or two of the beams during test. The stirrups were all of mild steel, $\frac{3}{8}$-in., $\frac{1}{2}$-in., and $\frac{5}{8}$-in. round bars, for which the average tensile stress at the yield point varied from 39 600 to 42 900 lb. per sq. in.

The concrete was a 1:2:4 mixture, by tamped volumes, and was of a very stiff consistency, so that the unusually high average compressive strengths of 3230 lb. per sq. in. at 28 days, and 4060 lb. per sq. in. at 61 days, were obtained from the control cylinders made with the beams. The workability is indicated by the average slump of 1 in. in the standard slump test and the flow, or ratio of final to initial base diameter of specimen on the flow table, of 155. It is of interest that concrete of this consistency can be placed around reinforcing steel.

Dimensions of cross-sections and details of reinforcement of all beams of the series are shown in Table 12, together with the principal data of the tests.

The beams were all tested when about 60 days old. Load was applied in increments of 25 000 to 50 000 lb. and strain-gage readings were taken on all gage lines with each increment of load. There were over a hundred gage lines on each beam having stirrups as web reinforcement. Figure 27 shows the location of gage lines and position of cracks at failure for typical beams of the series.

TABLE 12
PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1922



Beam No.	Manner of Failure
221.1	Bond & Diagonal Tension
221.2	Bond & Diagonal Tension
222.1	Bond & Diagonal Tension
222.2	Bond & Diagonal Tension
223.1	Tension in Longitud. Steel
223.2	Tension in Longitud. Steel
224.1	Tension
224.2	Tension
225.1	Tension
225.2	Tension
229.1	Tension
229.2	Tension
2210.1	Diagonal Tension
2210.2	Diagonal Tension
226.1	Tension
226.2	Tension
227.1	Tension
227.2	Tension
228.1	Tension
228.2	Tension

TABLE 12 (Concluded)

PRINCIPAL RESULTS OF TESTS OF SIMPLE BEAMS, SERIES OF 1922

Beam No.	Age at Test, Days	Longitudinal Reinforcement: Description	Longitudinal Reinforcement: %	Web Reinforcement: Description	Web Reinforcement: %	Maximum Applied Load, Pounds	Computed Stress lb. per sq. in.: Longitudinal Steel, f_s	Computed Stress lb. per sq. in.: Shear v	Computed Stress lb. per sq. in.: Bond u	1-2-4 Cylinders: Compressive Strength, lb. per sq. in.
221.1	61	4-1⅛" corrug. ϕ, h.c.s.	2.33	None	0	149 400	39 200	543	308	4076
221.2	61		2.33	None	0	148 000	38 800	537	305	3696
Av								540	307	
222.1	60	4-1⅛" corrug. ϕ, h.c.s.	2.33	None	0	165 500	43 500	599	339	4522
222.2	59		2.33	None	0	126 000	33 100	459	265	4337
Av.								529	302	
223.1	60	4-1⅛" corrug. ϕ, h.c.s.	2.33	⅜" plain ϕ, m.s., Spacing, 4"	1.38	212 500	55 500	769	435	4124
223.2	61		2.33		1.38	216 400	56 600	784	443	3689
Av.								776	439	
224.1	62	4-1⅛" corrug. ϕ, h.c.s	2.33	½" plain ϕ, m.s., Spacing, 7"	1.40	218 500	57 100	790	447	4106
224.2	61		2.33		1.40	216 000	56 700	781	442	3790
Av.								785	445	
225.1	62	4-1⅛" corrug. ϕ, h.c.s	2.33	⅝" plain ϕ, m.s., Spacing, 11"	1.39	227 300	59 200	822	465	3788
225.2	61		2.33		1.39	221 200	55 500	769	435	4041
Av.								795	450	
229.1	62	4-1⅛" corrug ϕ, h.c.s.	2.33	2-1⅛" corrug. ϕ, h.c.s., Bent-up.	0	223 400	58 200	807	457	3931
229.2	60		2.33		0	211 700	55 500	765	434	4203
Av.								786	445	
2210.1	62	4-1⅛" corrug ϕ, h.c.s	0.93	None	0	180 300	44 100	603	346	3610
2210.2	61		0.93	None	0	167 200	41 000	565	322	3570
Av								584	334	
226.1	62	4-1⅛" corrug. ϕ, h.c.s	0.93	⅜" plain ϕ, m.s., Spacing, 4"	1.38	259 500	63 100	872	496	4037
226.2	60		0.93		1.38	245 500	59 500	825	470	4331
Av.								844	483	
227.1	62	4-1⅛" corrug. ϕ, h.c.s.	0.93	½" plain ϕ, m.s., Spacing, 7"	1.40	258 500	62 800	867	495	3799
227.2	60		0.93		1.40	265 800	64 500	892	508	4346
Av.								875	502	
228.1	62	4-1⅛" corrug. ϕ, h.c.s.	0.93	⅝" plain ϕ, m.s., Spacing, 11"	1.39	261 400	63 500	876	500	4058
228.2	60		0.93		1.39	257 200	62 700	865	493	4152
Av.								871	497	

12-2

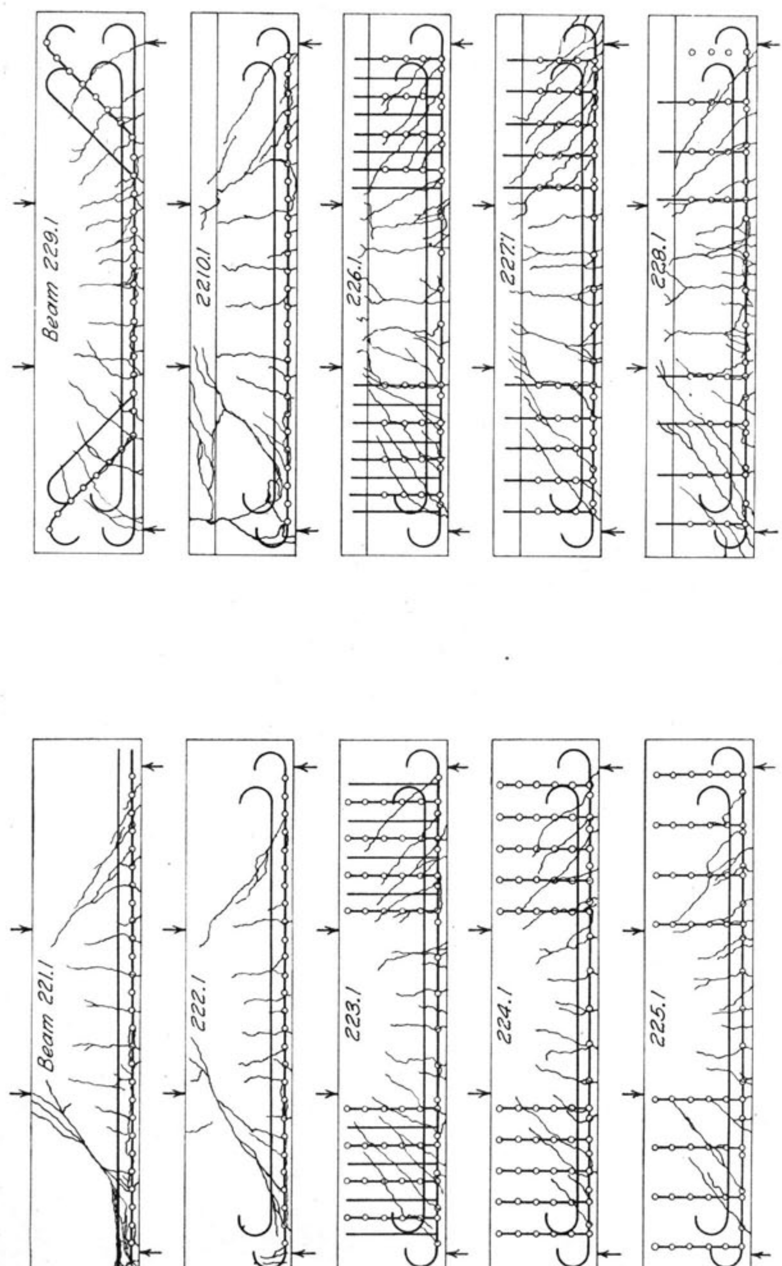


FIG. 27. SKETCHES OF TYPICAL BEAMS AFTER FAILURE, SERIES OF 1922

21. *Phenomena and Results of Tests.*—

(a) Rectangular Beams

There were two types of rectangular beams without web reinforcement, one with straight longitudinal bars, and one with hooks at the ends of all longitudinal bars. Of the first type, beams 221.1 and 221.2, having straight longitudinal bars without hooks, developed shearing unit stresses of 543 and 537 lb. per sq. in. respectively. From strain gage measurements taken on nearly the full length of the longitudinal bars, it was seen that very high bond stresses were developed long before the ultimate load was reached. In beam 221.1, at a unit shearing stress of 269 lb. per sq. in. and a nominal bond stress of about 153 lb. per sq. in., there was apparently some slipping of the bars just outside the load point, accompanied by a very rapid decrease in the tensile stresses at points 10 to 18 in. outside the load points. This slipping progressed toward the support as the load was increased, so that at a load of 83 per cent of the ultimate load, some slipping had extended to within 12 in. of the support. The actual average bond stress on the remaining end portion of the bar as calculated from the measured tensile stress was more than twice the nominal computed bond stress. The failure of the beam had the appearance of a typical sudden diagonal tension failure, but, from the evidence obtained regarding slip of bars, it seems plain that the initial failure was by bond. Beam 221.2 behaved in a manner quite similar to its companion beam, except that at failure the opening of a diagonal crack was accompanied by stripping of the reinforcing bars from the bottom of the beam for some distance. This would seem to give additional evidence of initial bond failure.

Beams 222.1 and 222.2 had the longitudinal bars hooked at the ends. The action of these beams at the lower loads was similar to that of the beams with straight reinforcing bars; a slip of bars progressing from the load point outward toward the support was indicated by the measured tensile stresses. However, while the calculated bond stresses were quite high at failure, the hooks apparently furnished adequate end anchorage for the bars. The reinforcement thus acted as a tie, with nearly constant stress throughout the greater part of its length. Beam 222.1 failed finally by the sudden opening of a large diagonal crack, at a shearing unit stress of 599 lb. per sq. in. Views of this beam after failure are seen in Figs. 28 and 29. The action of beam 222.2 was similar to that of 222.1, but it failed at a shearing unit stress of 459 lb. per sq. in. through the sudden opening of large diagonal cracks and final shearing off of the uncracked portion at the top of

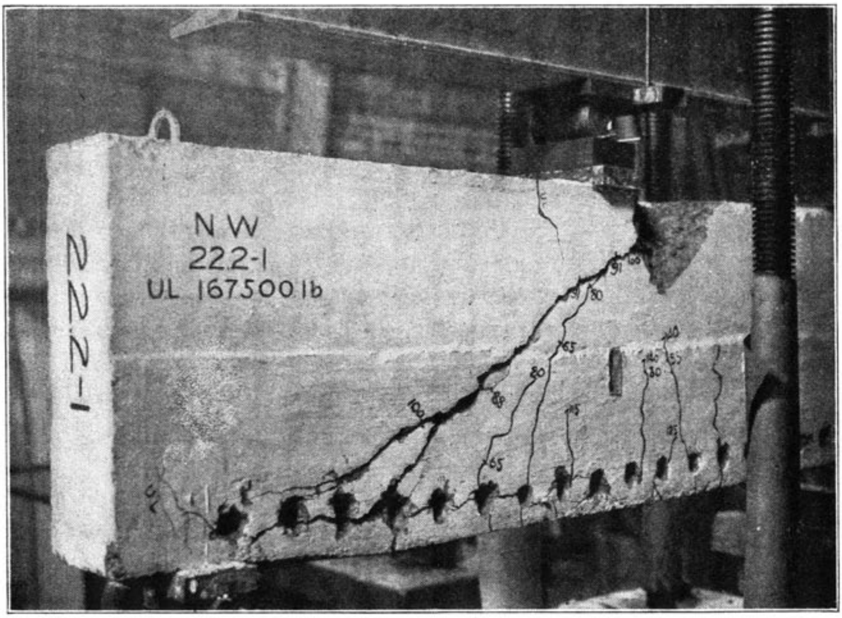


FIG. 28. VIEW OF BEAM 222.1 AFTER FAILURE, SERIES OF 1922

the beam. The suddenness of this failure at a load so much lower than that of its companion beam indicates a possible breakage of the hooked ends of the bars. Such breakage, while not verified in this case, was noted in other beams.

There were three sets of rectangular beams reinforced with vertical stirrups. The stirrups were of varying size and spacing but of the same percentage of web reinforcement. Strain measurements were taken on the outer legs of stirrups, which usually carried five 4-in. gage lines, making a total of 80 to 100 gage lines on the stirrups of each beam.

Beams 223.1 and 223.2 contained ⅜-in. round double U-stirrups, spaced 4 in. apart, beams 224.1 and 224.2 had similar stirrups, ½-in. round, spaced 7 in. apart, and beams 225.1 and 225.2 had stirrups, ⅝-in. round, spaced 11 in. apart. These beams all behaved in a similar manner in the tests. A few small tension cracks were observed at the first increment of load, usually at a shearing stress of 125 lb. per sq. in. These cracks occurred at or between the load points. Diagonal tension cracks became noticeable at shearing stresses of 215 to 285 lb. per sq. in.

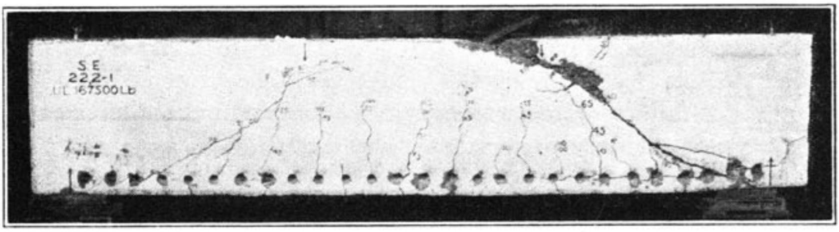


FIG. 29. VIEW OF BEAM 222.1 AFTER FAILURE, SERIES OF 1922

These diagonal cracks increased in length rapidly with increasing load, although their width remained small. In some cases the crack was observed first above mid-height of the web of the beam, developing in the lower part of the web as the load increased. Later the diagonal crack merged with tension cracks that started at the bottom of the beam. It was noted that in the region of highest stirrup stresses the direction of cracks was about the same in the various beams; the angle of inclination of the crack with the horizontal was generally from 45 to 50 deg.

Failure of these beams was by tension in the longitudinal steel. Judging from the stresses observed in the stirrups at the load of 200 000 lb., the stresses in stirrups were still well below the yield point at failure. The highest of 80 or more strain measurements on each beam at the 200 000-lb. load indicated stresses varying from 30 000 to 35 500 lb. per sq. in.

Failure occurred slowly, due to the opening of large vertical cracks as the reinforcement yielded, followed in general by crushing of the concrete at the top of the beam. The shearing stresses for the 6 beams varied from 769 to 822 lb. per sq. in. as noted in Table 12.

Beams 229.1 and 229.2 had two of the longitudinal reinforcing bars bent up for web reinforcement; no vertical stirrups were used. These beams were included in the series to furnish an indication of the comparative behavior of beams with stirrups and with bent-up bars. Both beams failed by tension in the longitudinal reinforcement. While the strain measurements on the bent-up bars indicated the presence of very high bond stresses, the stresses at the hooked ends of the bars were quite low, showing that the ends were anchored effectively. Diagonal cracks were noted at loads about 80 per cent as great as for beams with vertical stirrups, and as loading proceeded the cracks developed were somewhat wider than for beams with vertical stirrups.

The stress in the bent-up portion of the bars was generally less than in the horizontal portion though at and near the point of bending up the stresses were generally high. As might be expected from the fact that the failures were by tension in the longitudinal reinforcement, the average ultimate load for these two beams is identical with that for the six beams with vertical stirrups.

(b) T-beams

Beams 2210.1 and 2210.2 were T-beams without web reinforcement. The longitudinal reinforcement was identical with that of beams 222.1 and 222.2. Failure of both beams was by diagonal tension. The average shearing stresses were 603 and 565 lb. per sq. in., respectively.

There were three types of T-beams made to compare with the three types of rectangular beams having vertical stirrups. Beams 226.1 and 226.2 had 3/8-in. round stirrups at 4-in. spacing, beams 227.1 and 227.2 had 1/2-in. round stirrups at 7-in. spacing, and beams 228.1 and 228.2 had 5/8-in. round stirrups at 11-in. spacing. These beams behaved in a very similar manner under load. In all cases the first tension cracks were observed at a load giving a shearing unit stress of 125 lb. per sq. in. Diagonal cracks were first noted at shearing stresses of from 210 to 280 lb. per sq. in. Failure was in all cases by tension in the longitudinal steel, at shearing stresses of from 825 to 892 lb. per sq. in.

It may be noted that the phenomena of the earlier stages of the tests were much like those for rectangular beams. The loads causing vertical and diagonal cracks were roughly the same in the two sets as also were the direction and distribution of diagonal cracks as loading proceeded. As the maximum loads were somewhat higher than those for rectangular beams, there were more cracks developed, particularly near the supports and load points. Many diagonal cracks started in the middle portion of the web and extended both upward and downward in a diagonal direction.

The reactions at the supports caused rather high bearing stresses and some secondary failures occurred at these points. In beam 227.1, one end was somewhat damaged in placing it in the testing machine; after the yield point of the longitudinal steel had been reached a piece of the end broke away at the place that had been previously cracked. Examination showed that the hooked portion of one of the reinforcing bars had broken off, presumably at the ultimate load. Other beams that showed signs of distress near the supports at high loads were examined after failure for evidence of breakage of hooks, but no other breakage was noted.

22. *General Comments.*—

(a) Rectangular Beams and T-beams

A fair comparison may be made between the loads carried by rectangular beams and by T-beams in the case of the beams without web reinforcement. The rectangular beams of type 221, without hooks, developed an average shearing stress of 540 lb. per sq. in., those of type 222 developed stresses of 599 and 459, or an average of 529 lb. per sq. in., while the T-beams of type 2210 gave an average shearing stress of 584 lb. per sq. in. The second specimen of type 222 carried an exceptionally low load, as noted in Section 21. It seems that the beams of type 222 should have been at least as strong as those of type 221, and presumably stronger. Comparing the shearing strength of types 221 and 2210, the T-beam is seen to be 8 per cent stronger than the rectangular beam. The fact that j is greater for the T-beams has already been taken into account in calculating the shearing stresses. It seems that the 8 per cent advantage in favor of the T-beams may be due to the added resistance of the flange to shearing at final failure.

A comparison may also be made between the six rectangular beams with vertical stirrups and the T-beams with identical reinforcement. The average shearing stress at ultimate load for the six rectangular beams was 785 lb. per sq. in. while that for the corresponding T-beams was 863 lb. per sq. in., an increase of 10 per cent. Since these beams all failed by tension in the longitudinal steel, the excess strength of the T-beams is not easily explained. The difference in values of j, which would affect shearing stresses and longitudinal tensile stresses alike, has already been considered in calculating the shearing stresses noted. An explanation of the greater ultimate strength of the T-beams may lie in the fact that in the rectangular beams the development of the yield point stress in the longitudinal steel was soon followed by a secondary failure of the concrete in compression; in the T-beams a similar action took place, but with a larger margin of strength developed after the yield point stress had been reached in the steel. That is, the flange of the T-beam provided sufficient compression area to develop stresses in the steel considerably beyond the yield point, as shown by both measured and calculated values. The tension cracks in T-beams at failure were very large.

TABLE 13

MEASURED STRESSES IN STIRRUPS AT LOAD OF 200 000 LB., SERIES OF 1922

The stresses given are the highest measured stresses in the four quarters of each beam, in thousands of pounds per square inch.

Beam		West Side		East Side		Average Stress	Ratio of Average Stress to $\frac{v}{r}$
Kind	No.	N. End	S. End	N. End	S. End		
Rectangular beam ⅜-in. stirrups 4-in. spacing	223.1	30.0	34.0	31.0	32.0	31.7	0.61
	223.2	35.5	25.5	30.0	32.0	30.8	0.59
	Av.					31.3	0.60
Rectangular beam ½-in. stirrups 7-in. spacing	224.1	30.0	32.0	26.0	28.0	29.0	0.57
	224.2	30.0	34.0	27.0	29.0	30.0	0.59
	Av.					29.5	0.58
Rectangular beam ⅝-in. stirrups 11-in. spacing	225.1	30.0	23.0	27.0	24.0	26.0	0.51
	225.2	31.0	28.0	28.0	27.0	28.5	0.55
	Av.					27.3	0.53
T-beam ⅜-in. stirrups 4-in. spacing	226.1	32.5	27.0	30.0	30.0	29.9	0.62
	226.2	31.0	28.0	26.0	21.0	26.5	0.55
	Av.					28.2	0.58
T-beam ½-in. stirrups 7-in. spacing	227.1	35.0	25.0	33.0	39.0	33.0	0.69
	227.2	31.0	23.0	32.5	30.5	29.2	0.61
	Av.					31.1	0.65
T-beam ⅝-in. stirrups 11-in. spacing	228.1	24.5	31.0	19.0	29.0	25.9	0.54
	228.2	29.0	29.5	23.0	33.0	28.6	0.60
	Av.					27.3	0.57

(b) Effect of Spacing of Stirrups

The spacings of stirrups used in these tests were 4, 7, and 11 in. or roughly one-fifth, one-third, and one-half of the effective depth, respectively. Apparently this range in the spacing of the stirrups had little effect upon the stresses developed, as long as the percentage of web reinforcement was maintained constant. A study of the measured stresses developed in the stirrups of the various beams under a load of 200 000 lb. may be made from the information given in Table 13. The table gives values of the highest tensile stress measured on individual gage lines on each side and at each end of the several beams, or, in general, the highest four stresses observed on from 48 to 100 gage lines. The average of these four values may properly be considered the maximum stirrup stress in the beam. The shearing stresses at this load are 790 lb. per sq. in. for rectangular beams and 735 lb. per sq. in. for T-beams.

The data of Table 13 show that for a given load there was no very consistent difference in stirrup stresses for the three spacings used, though there was a slight decrease in stress as the spacing increased. Further, there was little apparent difference between the magnitudes of the stresses in rectangular and in T-beams, the grand averages for the six beams of each kind being almost identical.

The last column of Table 13 gives the ratio of the measured stress in the stirrup to the stress calculated by assuming the entire shear to be carried by the steel, according to the equation $f_V = \frac{v}{r}$. It will be noted that at the load of 200 000 lb. the values of the ratio vary from 0.51 to 0.69.

(c) Location of Region of High Stress in Stirrups

As in previous series of tests, it was found that the stirrups in the region midway between support and load point were usually subjected to the highest stresses, and that stirrups near the support and load point usually had very low stresses, and in some cases even showed compressive stresses due to the high local compression produced by the bearing plates. The latter condition is purely local, and has no bearing upon the action of the stirrups in regions of high diagonal tension. Values of the stresses in various stirrups along the beams and at different gage lines on each stirrup are shown in Fig. 30, for rectangular beams and T-beams under a load of 200 000 lb. The high stirrup stresses in the mid-portion of the region subject to shear might be expected from the large number of cracks developed in this region, as shown in Fig. 27.

The variation in stress from bottom to top of stirrup may also be seen in Fig. 30. The highest stress is usually found in the bottom gage line on the stirrup, but fairly high stresses are also found in the second and third gage lines from the bottom, particularly in stirrups near the load point, where diagonal cracks intersect the upper portion of the stirrup. The stresses in the two upper gage lines on rectangular beams were generally unimportant except as an indication of the variation in bond stress developed in the stirrup, and to show the desirability of the hooks that were provided for anchorage at the upper ends of all stirrups.

(d) Load-Stress Curves for Stirrups

A few load-stress curves are shown in Fig. 31 for typical gage lines in the region of highest stirrup stresses for the six types of beam referred to in Fig. 30. The curves are designated by the beam numbers, stirrup numbers, and gage line numbers shown in Fig. 30. These curves show the variation in stirrup stresses as the beams were loaded,

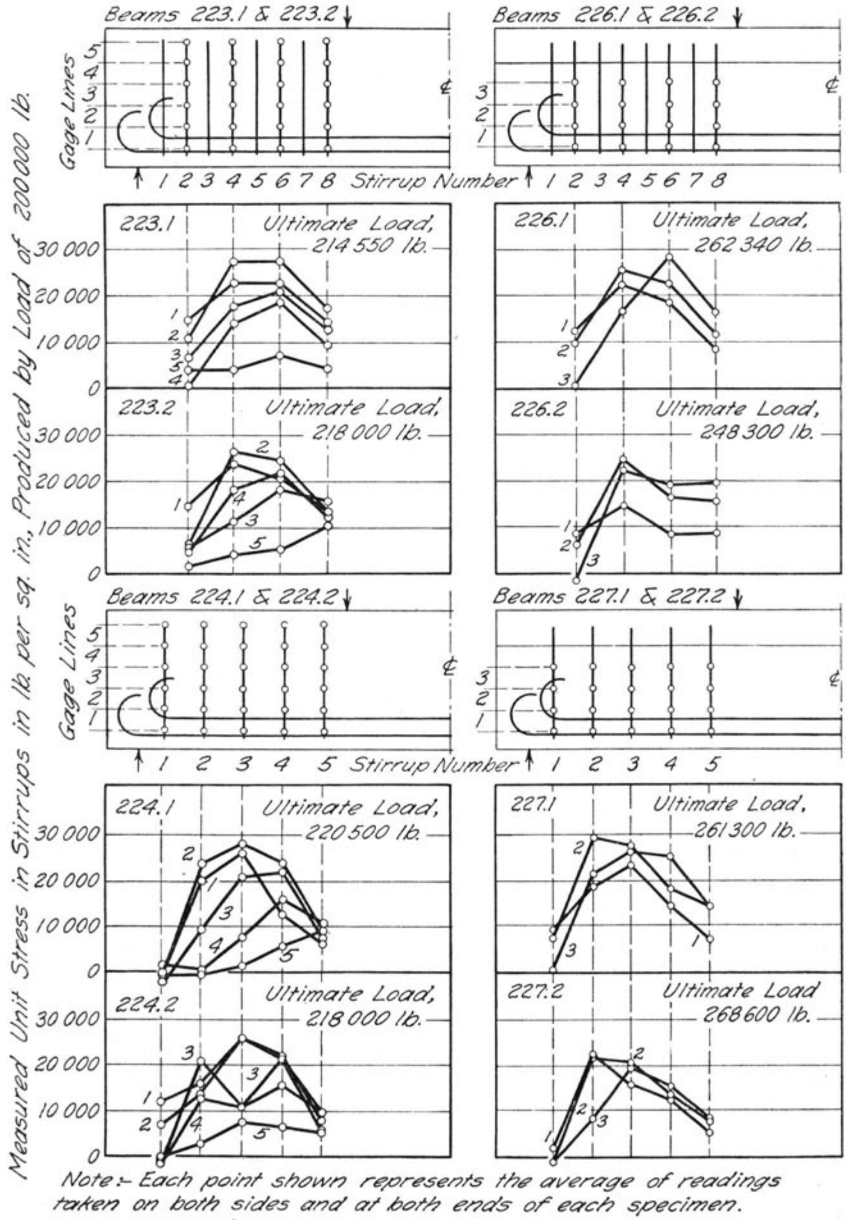


FIG. 30. MEASURED STRESSES IN VERTICAL STIRRUPS OF RECTANGULAR BEAMS AND T-BEAMS, SERIES OF 1922

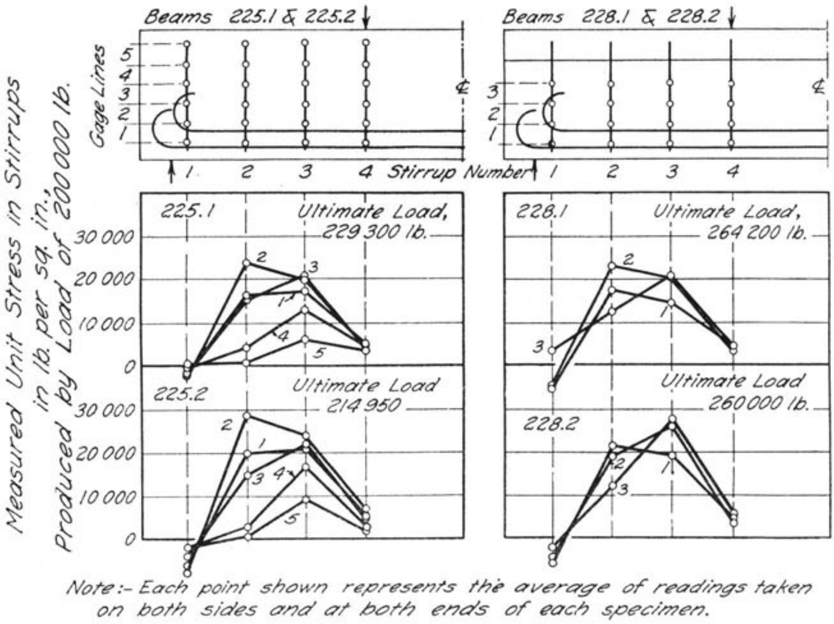


FIG. 30. (Continued) MEASURED STRESSES IN VERTICAL STIRRUPS OF RECTANGULAR BEAMS AND T-BEAMS, SERIES OF 1922

and in particular indicate the loads at which the stirrups began to take appreciable stress. Values of stress calculated by use of equation (4), assuming the stirrups to carry all of the shear, are also shown in the figure for purposes of comparison.

(e) Stresses in Longitudinal Reinforcement

Strain gage measurements were taken on the two outside reinforcing bars for nearly the full length of all beams. The resulting stress-diagrams are similar in form to the diagram of external bending moments, being naturally more irregular in outline. Figure 32 shows typical diagrams of measured stresses for a rectangular beam and a T-beam. A noticeable variation between the measured stresses and the nominal calculated stresses (shown by broken lines) is seen in the curves just outside the load points, where the measured stresses are considerably higher than the calculated ones. A similar variation was found in the 1913 tests, as shown in Fig. 13, but the difference between measured and calculated stress was not so great as that shown in Fig. 32. Such a variation appears to be due to local slipping of the longitudinal bars, beginning at or near the load point and gradually pro-

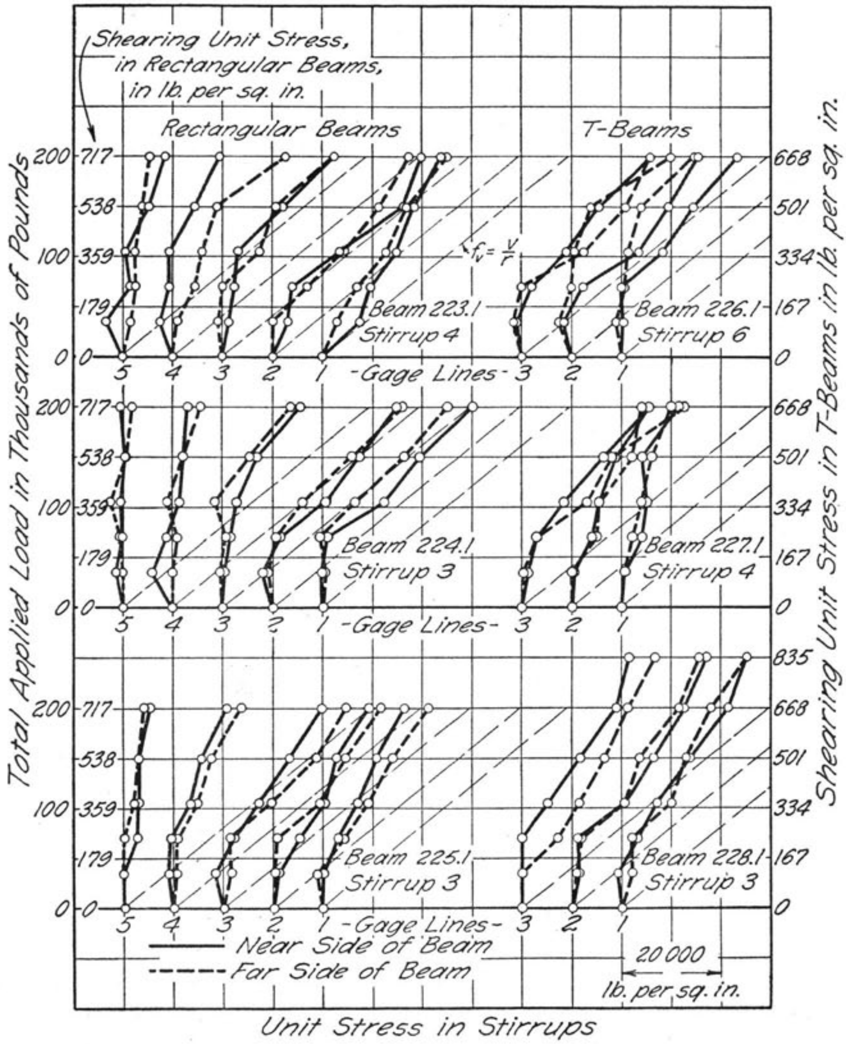


Fig. 31. Load-Stress Curves for Vertical Stirrups, Series of 1922

gressing toward the support. Similar variations were observed by Abrams* in tests made at the University of Illinois in 1912. The effect of the slipping is to increase the bond stress toward the ends of the reinforcing bars considerably above the nominal bond stress, in direct proportion to the slopes of the stress-curves shown. No measurements

*"Tests of Bond between Concrete and Steel," Univ. of Ill., Eng. Exp. Sta. Bul. 71, 1913. Attention is called to the diagrams and discussion on pages 194 to 203.

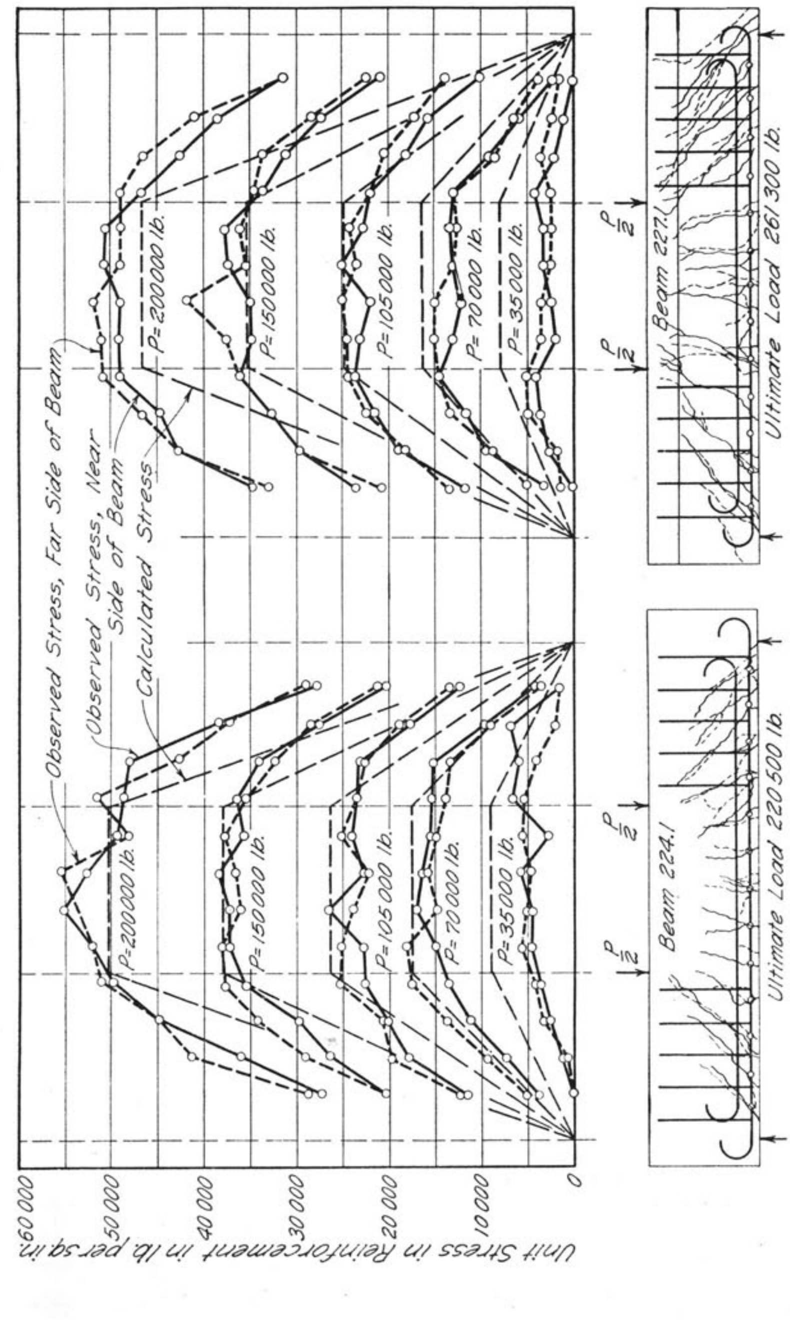


Fig. 32. Measured Stresses in Longitudinal Reinforcement of Rectangular Beam and T-Beam, Series of 1922

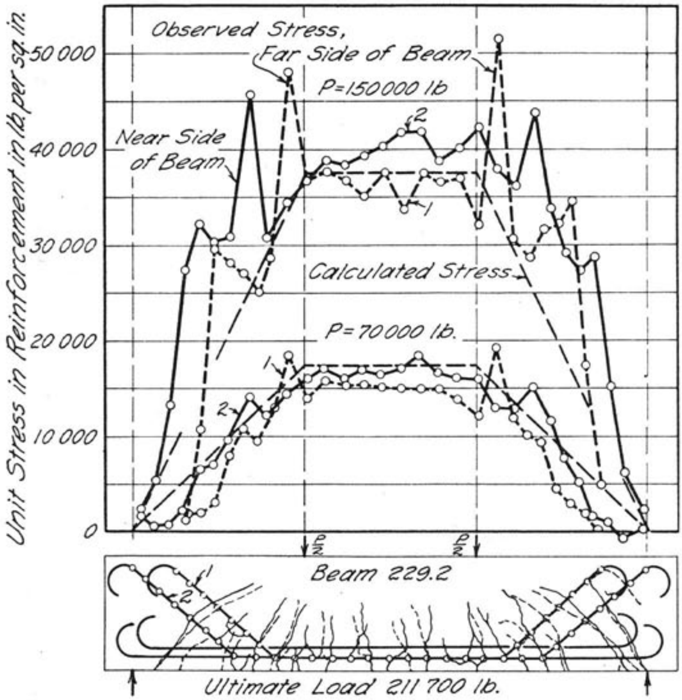


Fig. 33. Measured Stresses in Bent-up Bars, Beam 229.2, Series of 1922

were taken to determine the stresses in the hooked portion of the bars, but since the hook provided a length of bar extending 12 in. beyond the support line, in addition to the anchorage provided by its stiffness and curvature, the average bond stress on this part of the bar apparently was no greater than on portions where strain measurements were taken. The advantages of the hooks in deep beams of this type are obvious.

(f) Stresses in Bent-up Bars

Observations on beams 229.1 and 229.2 furnished information concerning the action of bent-up bars. Figure 33 shows stress diagrams for the bent-up bars of beam 229.2 at loads of 70 000 and 150 000 lb. The computed stresses for the horizontal portion of the bars are also shown in the figure by light, broken lines. A noticeable feature of the diagram is seen in the very high stresses observed on the gage lines at points where bars were bent up. Since these gage lines were located on a curved portion of bar it is probable that a part of the observed strain was due to a slight straightening of the bar, a deflection rather than a direct deformation, so that the stresses shown may be greater than the actual stresses. The figure shows that at the load of 150 000 lb., the bent-up bars had stresses of 25 000 to 30 000 lb. per sq. in. up to the neutral axis of the beam; above this point the stress decreased very rapidly, with accompanying high bond stresses as indicated by the slope of the stress-curves. The hooks at the upper ends of the bars were not highly stressed at the loads shown, but at a load of 200 000 lb. the stress on the gage lines adjacent to the hooks was as much as 20 000 lb. per sq. in. The hooks apparently afforded effective anchorage up to the maximum load on the beam.

IV. General Discussion

23. *Effectiveness of Various Forms of Web Reinforcement.*—The five series of tests described in the preceding chapters were quite naturally directed at different phases of the action of web reinforcement; the data collected may well be reviewed and correlated by the consideration of certain topics in the light of all of the test results.

The effectiveness of the several forms of web reinforcement was generally not fully developed in the tests and it can only be estimated through the consideration of stresses observed in the reinforcement. It may be useful to note to what degree the measured stresses compare with the stresses calculated by use of the equations of Section 4.

(a) Vertical Stirrups

Considering first the case of vertical stirrup reinforcement to which equations (3) and (4) apply, data may be found in the tests of 1913 and 1922. Figures 12 and 31 show load-stress relations for typical gage lines on stirrups in the two series of tests. It is seen that the load-stress curves generally do not show stirrup stresses as great as indicated by equations (3) and (4), and furthermore the curves are not straight lines as the equations would imply. This is to be expected, since at low loads the concrete web of a beam carries a large part of the diagonal tension and there is little stress in stirrups until after cracks have formed. The load-stress curves for beams 308.1 and 308.2 in Fig. 12 (measured on the most highly stressed gage lines on these beams) show negligible stirrup stresses for shearing stresses as great as 75 to 100 lb. per sq. in.; from this point on the curves are approximately parallel to the broken lines representing the equation $f_V = \frac{v}{r}$. In Fig. 31 each group of load-stress curves gives the stress at different points of a single stirrup, the stress in the lowest gage line (No. 1) generally being among the highest stresses measured in the beam. It is seen that even these highest measured stresses are much less than those given by the broken lines representing the value $f_V = \frac{v}{r}$. Here again the load-stress curve shows a low stress up to a shearing stress of perhaps 200 lb. per sq. in., and beyond this the direction of the curve changes rapidly and becomes nearly parallel to the straight line.

From a study of the foregoing and many similar load-stress curves it might be inferred that the maximum stresses observed in vertical stirrups are generally much less than would be indicated by equation (3) or (4); that the load-stress curve is not a straight line through the origin, but could better be expressed by an equation of the form

$$v = C + rf_V \tag{8}$$

This is one of the two forms of design equations in common use, as noted in Section 4. The value of C, which varies from about 90 lb. per sq. in. in the series of 1913 to 200 lb. per sq. in. in the series of 1922, probably depends upon the percentage of web reinforcement used and also on the quality of the concrete. Slater, Lord, and Zipprodt* found from tests of beams with either vertical or inclined stirrups that a similar equation applied except that the term C varied with the web thickness.

The equation is $v = 60 + 25b' + rf_V$ (9)

wherein b' is the web thickness. For a value of b' equal to 8 in., which applies for all beams of this bulletin, equation (9) is identical with equation (8) with a value of C equal to 260 lb. per sq. in.

In the reference mentioned Slater gives a second formula as being simpler and in some ways preferable to equation (9). This equation implies a straight-line variation between v and f_V and hence is of the first type of design equation mentioned in Section 4. The equation is

$$v = (0.005 + r) f_V \tag{10}$$

Instead of a fixed proportion, such as ⅔, of the shearing stress being considered as producing stress in the web reinforcement, this equation implies that the proportion shall vary as $\left[\frac{r}{0.005 + r}\right]$. Only for large values of r does equation (10) approach equation (4).

Summarizing, it may be said that for beams with vertical stirrups,

*"Shear Tests of Reinforced Concrete Beams," by W. A. Slater, A. R. Lord and R. R. Zipprodt. Technologic Paper No. 314, U. S. Bureau of Standards.

equation (8), with values of C of 100 lb. per sq. in. for the series of 1913 and 200 lb. per sq. in. for the series of 1922, agrees fairly well with the shape of the load-stress curves for the stirrups that are most highly stressed. Undoubtedly the value of C depends upon the quality of the concrete, as well as upon the amounts of longitudinal and web reinforcement. In connection with these values of C it will be noted that r was 0.0082 for the 1913 tests and about 0.0140 for the 1922 tests. While equation (10) does not agree with the shape of the observed load-stress curves, it does give values that agree fairly well with observed stresses near the ultimate load, and at lower loads seems to err on the side of safety.

(b) Inclined Stirrups

In considering inclined stirrups the series of 1913 furnishes the principal source of information. Referring again to Fig. 12, the load-stress curves for beams 303.1 to 307.2, which had stirrups inclined 45 deg. to the horizontal, will be examined. Two characteristics of these curves are quite noticeable: (1) the curves approach much nearer to straight lines than did the corresponding curves for vertical stirrups, and (2) the curves for beams with high percentages of web reinforcement agree closely with equation (4), while those for very low percentages of web reinforcement differ widely from this equation. For intermediate values the discrepancy is consistently less. The curves of Fig. 12 may be considered as typical for the most highly stressed gage lines on each of the beams noted. From the characteristics noted, the equation of the load-stress curves might be expected to be similar to equation (10), which involves both the straight-line relation and the variation in effectiveness with the value of r. The value of r for these beams was 0.0017 for beam 303, 0.0035 for beam 304, 0.0078 for beams 305 and 306, and 0.0139 for beam 307. A study of the curves indicates that the average values of the stirrup stresses shown for each beam vary according to a relation similar to that given by equation (10), but that the stresses are about 35 per cent greater than those given by that equation. For a given load the ratio of the maximum measured stirrup stress to that given by equation (4) varies from about one-third for the lowest percentage of web reinforcement to unity for the highest percentage. The latter case is the only one in which the measured stirrup stresses have been found to be as great as indicated by equation (4). It may be well to repeat that only the highest of the measured stresses were considered in this analysis and that the stresses on the majority of the gage lines were very much smaller. There were few beams that failed through overstressing of the web reinforcement, but it seems logical to assume that, since high stirrup stress is almost invariably due to the presence of a large crack across the stirrup, overstressing a single stirrup would allow the crack to spread and produce progressive failure in the adjoining stirrups. Hence the consideration of the highest stirrup stresses is essential in connection with equation (4) or any similar design formula.

(c) Bent-up Bars

With the exception of that furnished by the tests made in 1917, there is not very much information on measured stresses in bent-up bars used as web reinforcement. The 1917 series of tests showed that sufficient longitudinal bars could be bent up in regions where they were not required for providing resisting moment to furnish a complete system of web reinforcement. The only requirements were that the longitudinal reinforcement consist of a number of bars, and that in general these bars could be bent up at different points in the region subjected to shear without interfering with moment requirements. From these tests it appears that the web stresses in bent-up bars may be computed from the same equations as those used for inclined stirrups. As in the case of stirrups, the measured stresses were generally less than the calculated ones. Thus the data of Table 11 show that with one or two exceptions the measured stresses in bent-up bars were considerably less than the stresses calculated by use of equation (7) at a load of 30 000 lb., or roughly three-fourths of the maximum load carried. The most common ratio between the measured and the calculated stresses was about 0.7. Beams of type 16B10 showed a distinct variation from the general rule. The web reinforcement consisted of four ⅞-in. round bars, bent up at 45 deg. and having a 3.2 in. horizontal spacing. In these beams the measured stress at a 30 000-lb. load was 2.6 times as great as the stress calculated from equation (7). It is clear that these bars reinforce a greater length of the beam than is included by four spaces, a distance of only 12.8 in.; that this is logical is seen by imagining the spacing to decrease to zero. It may be concluded that equation (7) should not be applied to irregular spacings of web bars, or to cases where the web reinforcement is concentrated in a small part of the region subjected to shearing stresses. The load-stress curves of Fig. 22 throw further light on the magnitude of the stresses in bent-up bars. Whereas it was noted that the ratio of observed stress to computed stress was approximately 0.7 at a load of 30 000 lb., it is seen from the diagram that at higher loads the observed stresses tend to approach the straight lines representing calculated stresses. At a load of 40 000 lb., the ratio of observed to measured stresses was approximately 0.8. It would be interesting to know what shape the load-stress curves would have taken if failure had occurred in the web steel, but since the observed stresses were well below the yield point of the web steel, the complete form of the load-stress curve was not determined. Some of the curves are of the type represented by equation (8), in which the observed load-stress curve is parallel to the calculated curve, and some are of the form of equation (10) in which the observed stress is seen to be a constant proportion of the stress calculated by use of equation (7). Equation (10) implies that the variation between the observed stresses and the stresses calculated from equation (7) will increase as the percentage of web reinforcement decreases; this is corroborated in a general way by the results of this series of tests. While there is considerable deviation from an average curve, the highest stresses observed in these tests, as shown in Fig. 22, (generally at a load of 40 000 lb.) follow the general trend of the curve represented by equation (10), or are perhaps slightly greater. Equation (10) probably expresses the relations existing at maximum load better than a formula of the type of equation (8), although the latter may be a more accurate index of the actual variation in stresses at various loads.

Beams 229.1 and 229.2 of the 1922 tests furnished valuable information on the variation in stress along bent-up bars. A study of Fig. 33 shows the imperative need for thorough anchorage of the upper ends of bent-up bars, due to the unusually high bond stresses that are developed in the portions below the neutral axis. With proper attention to the spacing of bent-up bars and to the necessary anchorage, such bars will afford efficient web reinforcement.

24. *Effect of Angle of Inclination of Web Members.*—According to equation (7) a stirrup inclined at 67½ deg. to the horizontal is most efficient (considering both the length and cross-section of stirrup), and the effectiveness of stirrups inclined at 45 deg. should be the same as that of vertical stirrups. The data and discussion of Section 23, which apply to stirrups inclined 45 and 90 deg. to the horizontal, indicate that the inclined stirrup takes stress more directly and at lower loads than the vertical one, and that it will develop considerably higher stresses at intermediate loads. Whether the stress in vertical stirrups might have increased at a more rapid rate at higher loads (had failure not occurred in the longitudinal steel) is problematic. Some investigators have found practically equal effectiveness, volume for volume, with stirrups inclined at 45 and 90 deg. to the horizontal in beams that failed by diagonal tension.

Some idea of the relative effectiveness of web members inclined less than 45 deg. to the horizontal is given by the action of bent-up bars in the series of 1917. Referring to Table 11, beams 16B11, 16B12, and 16B13 had the same size and spacing of bars, but angles of inclination of 28, 35, and 45 deg., respectively. The theory of Section 4 would indicate a slightly higher stress in the bars with the smaller inclination, but the measured stresses given are very much greater in the bars with the smaller inclination. Similar conclusions may be drawn from a comparison of stresses in beams 16B15, 16B16, and 16B17, as well as in beams 16B18 and 16B19. For the three groups, it is found that the measured stresses in bars inclined at 28 deg. averaged 81 per cent of the stress computed from equation (7), while the measured stresses in bars inclined at 45 deg. averaged only 61 per cent of the computed values. That is, in addition to the slight reduction in effectiveness indicated by equation (7), the bars bent up at 28 deg. show stresses nearly one-third higher than they should theoretically in comparison with the bars bent up at 45 deg. The inference is that while it may be desirable to bend up bars at small inclinations to avoid abrupt changes in longitudinal stress, such bars do not develop their potential effectiveness as web reinforcement.

25. *Effect of Spacing of Stirrups.*—The data obtained in these tests regarding the effect of varying the spacing of stirrups are rather meager. In the series of 1913, beams 305.1 and 305.2 had four ⅜-in. round inclined web members spaced every 10 in., while beams 306.1 and 306.2 had two similar members spaced every 5 in., giving a value of r of 0.0078 in both cases. Figure 12 indicates that for the same load the stirrup stresses in the beams with the 5-in. spacing were generally considerably greater than those in the beams with the 10-in. spacing.

Another comparison may be found in the data of the series of 1922 from which Fig. 34 has been plotted. The diagram shows measured stresses in vertical stirrups in both rectangular and T-beams at three increments of load. The stirrup spacings were 4, 7, and 11 in. and the ratio of web reinforcement, r, was about 0.0014 in all cases. It is seen that there is a small but consistent decrease in the stress with increase in the spacing in the rectangular beams, and a similar but less definite tendency in the case of the T-beams.

Since the largest spacing shown in Fig. 34 was only about one-half of the effective depth of the beam, it is evident that no great difference in effectiveness between the various spacings could be expected. A difference was noted, however, in the number and size of diagonal cracks developed. While the width of cracks near stirrups was about

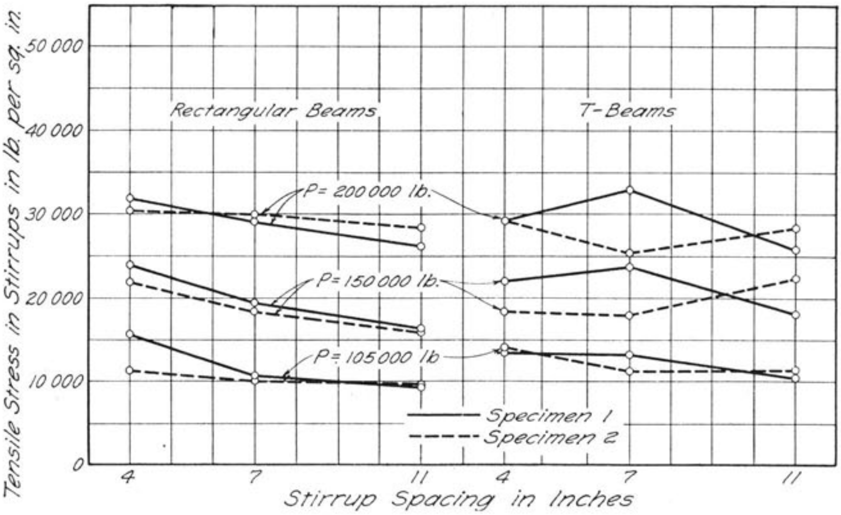


Fig. 34. Effect of Spacing of Stirrups on Tensile Stresses in Stirrups, Series of 1922

the same for all stirrup spacings, a larger number of small cracks were observed in beams with the smaller spacings, while the beams with larger stirrup spacings had wide crack openings midway between stirrups. The greatest widths of the diagonal cracks in the rectangular beams at maximum load were about 0.01 in. with the 4-in. stirrup spacing, and about 0.02 in. with the 11-in. spacing.

26. *Variations in Stirrup Stresses.*—The results of all of the tests agree very well as to the variation in stirrup stresses, from place to place in the beam, as is seen in Figs. 11, 14, 15, 16, 21, and 30. It is quite evident that high stirrup stresses do not exist near the support and load points, where a high concentration of bearing stress exists, and it is not surprising that at such points even a small compression may be developed. In all cases the highest stirrup stresses were found in the portion of the beam covering approximately the middle half of the distance between support and load point, a region in which diagonal cracks were well developed. Reference to the location of cracks and gage lines shows that high concentrations of stirrup stress were found only on gage lines crossed by fairly large cracks. The foregoing observations also apply to the action of bent-up bars.

The data of the 1913, 1917, and 1922 tests also show a fairly regular variation in stress in web members from bottom to top of beam. Figs. 14, 17, 30, and 31 show that the stress was usually a maximum on the bottom gage line, for the most highly stressed stirrups. In some cases the second and even the third gage line from the bottom developed high stresses, especially for stirrups in the vicinity of the load point. This is understood by following the courses of the diagonal cracks, which intersect the stirrups at increasing elevations as the load point is approached. However, few stresses of importance were observed in stirrups above the neutral axis of the beam, although the measured stresses in this region were of value in showing the variation of bond stresses in stirrups.

It should be noted that the variations in stirrup stresses along the beam were all observed on simple beams with two-point loading and should not be assumed to apply directly to beams having other conditions of loading or restraint.

27. *Anchorage of Web Reinforcement.*—Slip of web members was apparently a cause of failure in several beams of the series of 1910, 1911, and 1917. No failures occurred due to slip of stirrups in the 1913 tests, in which the inclined stirrups were welded to the longitudinal reinforcement at the bottom and had no hooks at the top, or in the 1922 tests, in which the vertical stirrups were looped around the longitudinal steel at the bottom and had hooks for anchorage at the top. From a study of the notes of the tests and of the photographs of the beams after failure it seems probable that some of the failures listed as diagonal tension failures were due to slip of longitudinal bars.

In the series of 1910 (see Table 4), beams 283.1-3 and 283.5-7 were made using unit frame reinforcement, one group with loose stirrups and one with stirrups well attached. In all cases the direction of cracks at failure indicate that slipping of the longitudinal reinforcement had occurred. In the case of the beams with loose stirrups without hooks the presence of vertical cracks running along the stirrups indicated that there was some slipping of the stirrups. Such cracks did not occur in the beams having the stirrups anchored at the upper ends.

Beam 281.6 in which a fabricated reinforcing unit was used is also noted as having failed by diagonal tension and bond. In this beam the right-angle bends at the ends of the longitudinal bars were placed too near the surface of the concrete, and the hooks in straightening out split off the outer shell of concrete at both ends of the beam. Large diagonal cracks appeared at both ends, but it is not clear whether initial failure was due to the bond failure at the ends of the longitudinal bars, or to failure of the web reinforcement. It is evident, however, that the anchorage of the web reinforcement was effective in preventing slipping.

Beams 285.1-3 and 285.5-7 were reinforced with fabricated units. In all of these, the slipping or breakage of the metal clips used to attach the stirrups to the longitudinal bars contributed to the failure of the beams. These breakages and slips were noted through the sudden opening of large cracks at the junction of stirrups and longitudinal bars, and were verified after the tests. In four of the six beams, slipping at the ends of the main longitudinal bars was visible at failure, and, from the appearance of the cracks, slipping of longitudinal bars had taken place in all of the beams before failure.

In the series of 1911 (see Table 5) some diagonal tension failures were noted. Beams 287.1-3 and 287.7-9 had ½-in. round stirrups without hooks. No slipping of the stirrups was observed during the tests. Beams 287.4-6, which had stirrups anchored by hooks, failed in the same way as those without hooks. Apparently there was no slipping of the stirrups. No slip of longitudinal steel was observed although slipping was observed in similar beams of the series at bond stresses lower than were developed in these beams.

As noted above, no slipping of stirrups occurred in the 1913 tests, although the upper ends of stirrups were unanchored. Beams 302.1 and 302.2, in which the web reinforcement consisted of bent-up bars without hooks or other anchorage, failed due to end slip of these bars which permitted diagonal cracks to open up.

In the series of 1917 the only beams with web reinforcement that failed by diagonal tension were beams 16B3-1 and 16B3-2, in which the unanchored bent-up bars slipped and permitted the opening of large diagonal cracks. The web reinforcement in other beams of this series was in all cases anchored by large hooks, and no other diagonal tension failures occurred, although in some cases fairly large diagonal cracks formed.

In the series of 1922 all stirrups and bent-up bars were anchored by hooks at the upper ends and there were no signs of slipping of web steel. None of these beams having web reinforcement failed by diagonal tension.

From the foregoing it appears that, while in some cases anchorage of the stirrups was unnecessary due to the fact that the stirrups were of small diameter or were not highly stressed, the use of hooks or other forms of anchorage is highly desirable, if the stirrups are to be fully utilized. The use of hooks seems to be a simple and economical means of insuring against bond failure. Due to the larger diameter of bar, and the limited length available for bond, the use of hooks on bent-up bars seems decidedly advisable in simple beams of the kind that were tested. Hooked ends should in no case be placed too near the surface of the concrete as there is a likelihood of concrete being spalled or split off due to the high crushing stresses inside the hook and to stresses produced by straightening of the hook.

V. Summary of Results

28. *Conclusions.*—The tests described in this bulletin extended over more than 12 years; it may be said, however, that there were certain advantages in the long duration of the investigation as evidenced in the continued thought that was applied to the problem, and in the improvement and development in the planning and carrying out of tests. In view of the large variety of types of test specimens and the extent and diversity of the test information, the results are felt to be of great usefulness.

As noted in the introduction, although the investigation was planned to study the action of web reinforcement, many of the beams with web reinforcement did not fail in the web. The direct measurement of stress in the web reinforcement by means of the strain gage, a method for which these tests represent one of the earliest applications, made it possible to estimate the web resistance, as well as to study the variations in web stress even though direct evidence of the resistance of web members at failure was not obtained.

It is to be remembered that in all of the test beams used an excellent grade of concrete was employed and the workmanship was uniformly good. It is evident from other tests that defects in the concrete and in workmanship will result in material reductions in the values of the shearing unit stress developed, even in beams with web reinforcement. The necessity for good concrete and careful workmanship is obvious.

The tests were all made on simple beams subjected to two-point loading, so that in all cases the web reinforcement was placed in a region of constant shear. The following remarks are intended to apply to beams of this type, although some of them doubtless have a bearing on the action of simple beams under other conditions of loading. The following results of the tests are re-stated here by way of summary:

(1) All beams without web reinforcement failed by diagonal tension, at shearing unit stresses varying from 130 to 250 lb. per sq. in., except in the series of 1922, in which shearing unit stresses

of 530 to 580 lb. per sq. in. were developed; the beams of the 1922 series were short deep beams having a high percentage of longitudinal steel and were made of unusually strong concrete, as shown by Table 12. The beams without web reinforcement all failed quite suddenly, and lacked the desirable characteristics of toughness and ability to yield gradually at failure which were found in those having web steel.

(2) Relatively large diagonal tension stresses were developed when the shearing stress was large in comparison with the flexural stress; this condition is found in short, deep beams and in beams having high amounts of longitudinal reinforcement. Fortunately this type of beam is also well adapted to resist high diagonal stresses; tests have shown that the maximum shearing unit stress obtainable increases with the ratio of depth to span and with the percentage of longitudinal reinforcement.

(3) It appears that higher allowable values of shearing unit stress might be used in reinforced concrete beams than are now common, provided that sufficient web reinforcement is used to satisfy the design formula, and provided that care is taken to guard against other forms of failure. Since initial failures due to bond are often mistaken for diagonal tension failures, it is essential that both web and longitudinal reinforcement be well anchored. The use of unusually high shearing stresses should be accompanied by special care in designing and detailing the reinforcement and in securing a high grade of workmanship in the construction work.

(4) A somewhat systematic variation, evidently dependent upon the manner of loading employed, was found in the measured stresses in the web reinforcement at points along the length of the beams; low stresses were found near the load point and the support, while the maximum stresses were generally found in the middle portion of the region between load point and support. It was in this middle portion that the largest number of diagonal cracks generally formed.

(5) A variation in stress along the length of web members was also observed. Except in a few cases in which the stirrup was near the load point this stress was greatest near the bottom of the web member, decreasing toward the upper end. The position of maximum stress clearly depended upon the intersection of the web member with the main diagonal cracks.

(6) Only on the most highly stressed gage lines did the observed stresses in web members approach the stress calculated by

use of equations (4) or (7), see pp. 16 and 17, and in only a few cases was the calculated stress exceeded. There were many variations in the shape of the load-stress curves for web members, and no one curve can be given as strictly typical. Little stress was generally observed in vertical stirrups at early loads; the load-stress curve frequently ran nearly parallel to the theoretical curve given by equation (7) and might be represented by an equation such as equation (8), p. 90. Inclined stirrups were observed to take stress more nearly in direct proportion to the load, the stress being at a constant proportion (less than unity) of that given by equation (7).

(7) The ratio between measured and calculated tensile stress in web members was noted to increase as the percentage of web reinforcement increased. Such a variation is indicated by equation (10), p. 90, from which it appears that the ratio of measured to calculated stress is equal to $\left[\frac{r}{0.005 + r}\right]$. For maximum stresses in web members, equation (10) gives stresses which are approximately three-fourths as great as were measured in the series of 1913 on rigidly attached inclined web members, which agree fairly well with the stresses measured in the series of 1917 on bent-up bars, and which are slightly higher than those measured in the series of 1922 on vertical stirrups. For expressing maximum stresses the form of equation (10) is probably preferable to that of equation (8), although it will give less accurate results at low loads.

(8) The results of the tests agreed fairly well with the theory in indicating approximately equal effectiveness (pound for pound) of vertical web members and members inclined at 45 deg. to the horizontal. The difference in effectiveness observed seemed to be in favor of the vertical stirrup, but since the stirrup stresses did not extend to failure a definite statement of the difference can hardly be made. This comparison of effectiveness is based on equal percentages, or equal volumes, of web reinforcement. Thus, while less stress is produced in the inclined stirrup than in the vertical stirrup having the same horizontal spacing, the length of the inclined member is correspondingly greater.

(9) It is worth noting that inclined web members have a greater length in which to develop bond than do vertical members; however, in the case of bent-up bars this extra length is usually badly needed since the size of the bars will commonly be much greater than that of stirrups.

(10) In some of the test beams anchorage of web members was not needed, since the stirrups were either of small diameter, or were not highly stressed at failure. In other cases anchorage of the stirrups was of decided advantage. Anchorage was apparently even more necessary for bent-up bars than for stirrups. The provision of properly designed hooks, kept well away from the surface of the concrete, or of other forms of anchorage of bars, appears to be a very desirable means of improving the effectiveness of the action of web members.

(11) For descriptions and discussions bearing on the details of the action of web reinforcement, the text may well be consulted, and the information there contained may be expected to aid in comprehending the behavior of the reinforced concrete beam under a variety of conditions and to give knowledge that may be useful in design and construction—matters that can not be summarized here.

APPENDIX

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