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RESISTANCE OF TUBES TO COLLAPSE

BY

A. P. CARMAN



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BULLETIN No. 5

JUNE 1906

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RESISTANCE OF TUBES TO COLLAPSE

BY ALBERT P. CARMAN, PROFESSOR OF PHYSICS  
AND  
MAURICE L. CARR, ASSISTANT IN PHYSICS

This paper describes a series of experiments made on the resistance of metal tubes to collapse when subjected to external hydraulic pressure. The principal use of the results of such experiments is probably their application to the design and inspection of the fire flues of steam boilers, but the results are also of great interest and importance in the theory of elasticity and the strength of materials. Engineers have used various empirical rules and formulæ for the collapse of tubes, but these formulæ have been derived from a few unsatisfactory experiments and have within recent years been generally distrusted. This distrust has not been lessened by the mathematical discussions of the last few years, in which students of elasticity have attempted to derive a rational formula for tube collapses. These proposed rational formulæ differ, as we shall see, from the empirical rules in use by engineers.

The present work was first planned by L. P. Breckenridge and A. P. Carman in the spring of 1904, but University duties prevented its prosecution at that time. In the winter of 1904 to 1905, a series of experiments on the collapse of small seamless brass tubes was carried out by A. P. Carman. These experiments were described and the results discussed in a paper read before the American Physical Society at Chicago, April 23, 1905, and published later in the *Physical Review*.<sup>1</sup> It was there shown that the form-

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<sup>1</sup>A. P. Carman, Collapse of Tubes, *Physical Review*, Vol. XXI, pp. 381-387.

ulæ in general use, all of which are based on that of Sir William Fairbairn, are inadequate; that there is a certain "critical minimum length" beyond which the resistance to collapse of a tube is independent of the length; and that a formula of the rational type proposed by Professor G. H. Bryan was probably nearly true for these small brass tubes.

The experiments discussed in this bulletin have been made by the department of Physics of the Engineering Experiment Station of the University of Illinois. They were begun in the fall of 1905, but owing to delays in getting apparatus and materials were not completed until May 1, 1906. M. L. Carr, B. S., has been the assistant in carrying on this series of experiments. He has made the observations, and to him are due several of the special devices used as well as much of the completeness and accuracy of the calculations. Before describing our own experiments and discussing the results, we shall give a brief account of previous experiments and of the empirical and theoretical formulæ which have been proposed.

#### HISTORICAL

The first and until very recently the only systematic experiments on the collapse of tubes were those of Sir William Fairbairn made nearly fifty years ago<sup>1</sup>. Fairbairn's work was done at the suggestion and with the aid of the Royal Society and of the British Association for the Advancement of Science. The common steam pressure in that day was 50 pounds per square inch or less, and so the highest pressure thought necessary by Fairbairn was less than 500 pounds. The tubes were "composed of a single thin iron plate bent to the required form upon a mandril and riveted and also brazed to prevent leakage into the interior". The ends were closed by cast-iron disks or plugs, and the tube was placed in a large cast-iron cylinder and there subjected to hydraulic pressure. Fig. 1 is reproduced from Fairbairn's original paper and shows his arrangement. The cast-iron cylinder was 8 feet long, 28 inches in diameter and the walls were 2 inches thick. The cylinder was placed in a vertical position and the tube was

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<sup>1</sup> William Fairbairn, On the Resistance of Tubes to Collapse, Philosophical Transactions of the Royal Society of London for 1858, pp. 389-413; also in Fairbairn's Useful Information for Engineers, Second Series, London, 1867.

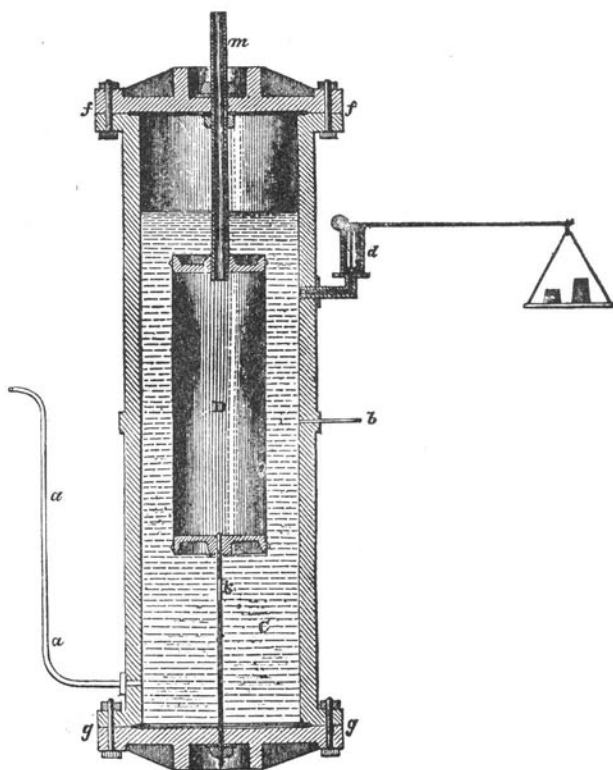


FIG. 1 SIR WM. FAIRBAIRN'S APPARATUS  
FOR TESTING TUBES, REPRODUCED  
FROM PHIL. TRANS. FOR 1858

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supported from both ends, as shown in the figure. The interior of the tested tube was connected with the atmosphere at the upper end. The pressure was produced by a hydraulic pump and was read by steam gauges. A safety valve set at 500 pounds limited the pressure. The diameters of the tubes collapsed were 4, 6, 8, 9, 10 and 12 inches with the exception of one which was  $18\frac{3}{4}$  inches in diameter. The length of these tubes ranged from 19 to 60 inches, and nearly all had the same wall thickness of .043 of an inch. In all, about twenty-five satisfactory collapses were made. Fairbairn sums up his results in this well-known formula:

$$P = 9,675,600 \frac{t^{2.19}}{ld}$$

“Which is”, he says, “the general formula for calculating the strength of wrought-iron tubes subjected to the external pressure within the limits indicated by the experiments; i. e., provided their length is not less than 1.5 feet, and not greater than 10 feet”. In the above formula,  $P$  = pressure in pounds per square inch;  $t$  = thickness of wall in inches;  $l$  = length of tube in inches; and  $d$  = diameter in inches. Fairbairn adds as a foot-note the following: “By taking 2 instead of 2.19 for the index of  $t$ , this formula becomes

$$P = 9,675,600 \frac{t^2}{ld}$$

whence the value of  $P$  may be calculated by ordinary arithmetic. For thick tubes of considerable diameter and length, this formula may be regarded as sufficiently exact for practical purposes”. This last or approximate form is sometimes known as Fairbairn’s second form. Fairbairn’s formula has been the text for practically all the discussions on the collapse of tubes for the last fifty years. Aside from Fairbairn’s experiments, there have been no systematic experiments described until within the last year. It is noteworthy that a problem so widely discussed, the solution of which has not only great scientific interest but also valuable technical applications, should remain for so many years with so little experimental work. The only reasons that can be assigned are the considerable expense involved in the experiments and the fact that the appliances needed are not commonly available in testing laboratories.

TABLE I

FAIRBAIRN'S DATA ON THE RESISTANCE OF TUBES TO COLLAPSE

Diameter, inches	Length, inches	Thickness of Wall, inches	Pressure of Collapse, lb. per sq. in.	Remarks
4	19	.043	170	Tubes formed of plates of uniform thickness.
4	19	.043	137	Tubes formed of plates of uniform thickness.
4	40	.043	65	Tubes formed of plates of uniform thickness.
4	38	.043	65	Tubes formed of plates of uniform thickness.
4	60	.043	43	Tubes formed of plates of uniform thickness.
4	60	.043	140	Made in 3 sections, 1 ft. 7 in. each.
4	60	.043	47	
4	30	.043	(195)	Ends of tube fractured, allowing water to enter and cause reacting pressure.
4	30	.043	93	
4	15	.043	147	
6	30	.043	48	Cast-iron ends fractured, causing collapse before outer shell attained maximum resistance.
6	29	.043	47	
6	59	.043	32	
6	30	.043	52	
6	30	.043	65	
6	30	.043	85	Rod placed down axis to prevent end from approaching. Tin ring left in caused increased pressure.
8	30	.043	39	
8	39	.043	32	
8	40	.043	31	
8	60	.043	22	
8	30	.043	36	
10	50	.043	19	
10	30	.043	33	
12	30	.043	22	
12	60	.043	12.5	
12.2	58.5	.043	11	
9	37	.25	(450)	Uncollapsed.
18.75	61	.25	420	
9	37	.14	262	Lap joint.
9	37	.14	378	Butt joint.
14x10.25	60	.043	6.5	Elliptical tubes.
20.75x15.5	61	.25	127.5	Elliptical tubes.

The most remarkable feature of Fairbairn's formula is the dependence of the collapsing pressure upon the length. This will be discussed later. Immediately after the publication of Fairbairn's paper, one writer after another began to discuss the data for the purpose of deducing a more general and convenient empirical formula. The following is a partial list of the best known of such formulæ. The dimensions are given in inches just as in the form of Fairbairn's formula:

(1) By F. Grashof: in *Zeitschrift des Vereines deutscher Ingenieure*, p. 234, 1859.

(a) For thick tubes:

$$P = 1,033,620 \frac{t^{2.081}}{l^{0.564} d^{0.889}}$$

(b) For thin tubes:

$$P = 24,481,000 \frac{t^{2.315}}{l d^{1.278}}$$

(2) By G. H. Love: in *Todhunter and Pearson's History of Elasticity*, Vol. II, p. 667. Appeared in *Civilingenieur*, 1861.

$$P = 5,358,150 \frac{t^2}{ld} + 41906 \frac{t^2}{d} + 1323 \frac{t}{d}$$

(3) By J. W. Nystrom: Quoted here from *Van Nostrand's Engineering Magazine*, Vol. XXIV, p. 213. Original reference, *Treatise on Steam Engineering*, by J. W. Nystrom, p. 106.

$$P = 692,800 \frac{t^2}{l^{0.5} d}$$

(4) By W. C. Unwin: in *Minutes of Proceedings of Institution of Civil Engineers*, Session 1875-1876, Part IV, p. 225.

$$P = 15,547,000 \frac{t^{2.35}}{l^{0.9} d^{1.16}}$$

for flues with longitudinal and circumferential joints.

(5) By F. Wehage: in *Dingler's Polytechnisches Journal*, pp. 236-243, 1881.

$$P = \begin{cases} 368,000 & 8t \\ 490,000 & d \end{cases} \sqrt[3]{\frac{t}{ld}}$$

The coefficient 368,000 to be used for flues with lap joints riveted; the 490,000 to be used for flues with flap joints riveted on.

(6) By Theodore Belpaire: *Note sur la résistance des tubes pressées de l'extérieur*, par Theodore Belpaire in *Annales du Génie civil*, March, 1879. Quoted from *Van Nostrand's Magazine*, Vol. XXIV, 1881.

$$P = 3,427,152 \left[ \frac{t^2}{ld} \right] - 56,892,400 \left[ \frac{t^3}{ld^2} \right]$$

All of the above formulæ have the same fundamental form as Fairbairn's. Indeed, we might expect this, for they are simply

empirical expressions made to fit Fairbairn's observations. Most writers had seen that there must be a limit to the length of tubes for which these formulæ should be applied, but no results of experiments had been published previous to 1905 regarding this.

The previous experiments by the writer were made to test this principal characteristic of the Fairbairn formula, viz., that the collapsing pressure varies inversely as the length. Twenty-five small seamless brass tubes were collapsed by hydraulic pressure. The diameter, thickness of wall, length and collapsing pressure are shown in the following table:

TABLE II

TABLE OF COLLAPSING PRESSURES OF SMALL SEAMLESS BRASS TUBES<sup>1</sup>

Mean Diameter, inches	Thickness of Wall, inches	Length, inches	Collapsing Pressure, lb. per sq. in.
.350	.0163	.315	4125
.350	.0163	.472	3415
.350	.0163	.709	3200
.350	.0163	1.063	2248
.350	.0163	1.570	1778
.350	.0163	3.150	1850
.350	.0163	3.540	1850
.350	.0163	1.570	9525
.441	.0315	2.280	6975
.441	.0315	2.715	6690
.441	.0315	3.345	6690
.441	.0315	3.820	6400
.441	.0315	7.480	6690
.721	.0315	1.220	7750
.721	.0315	1.733	6620
.721	.0315	2.280	6260
.721	.0315	3.030	5120
.721	.0315	8.200	4980
.721	.0315	3.420	11940
.701	.0531	3.500	12200
.701	.0531	5.120	12090
.701	.0531	5.120	12020
.701	.0531	5.190	11940
.701	.0531	5.270	12090
.701	.0531	8.270	12090

The curve, Fig. 2, shows the relation between length and collapsing pressure for tubes 0.35 of an inch in diameter. The curves for the other diameters are of the same shape. The following conclusions were drawn. "An inspection of these data and curves shows immediately that there is a minimum length for

<sup>1</sup>Data taken by Professor A. P. Carman, from Physical Review, Vol. XXI, No. 6, Dec., 1905.

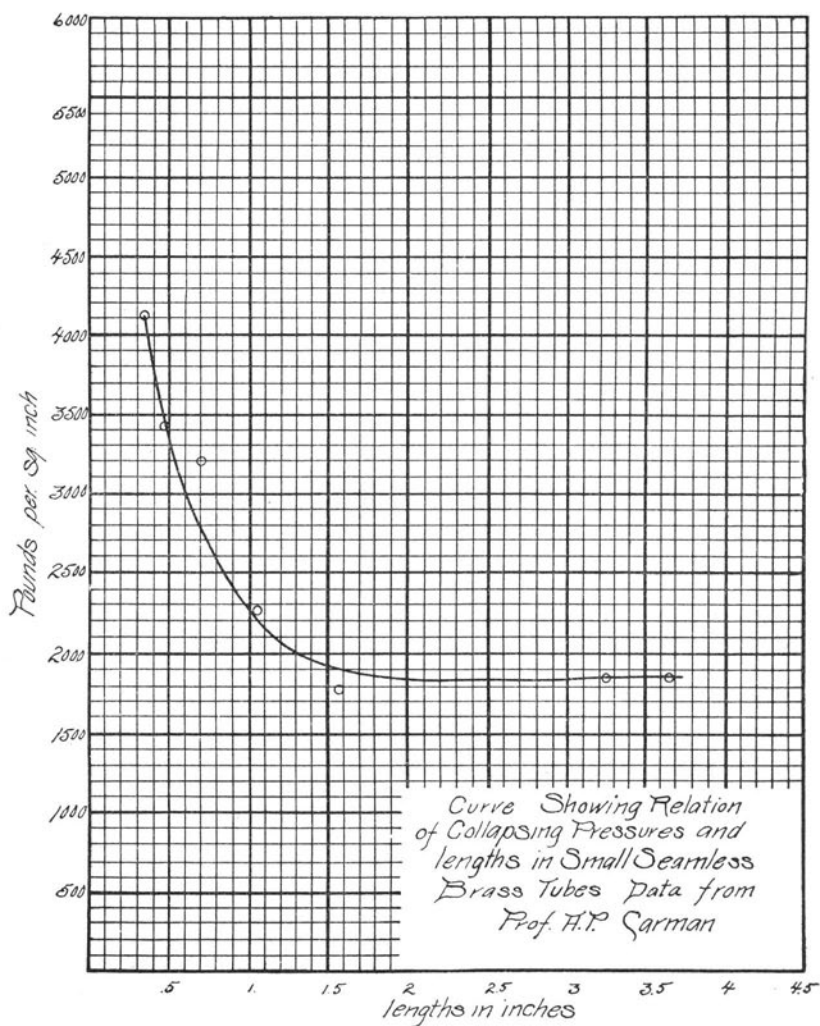


FIG. 2 CURVE SHOWING RELATION OF COLLAPSING PRESSURE  
TO LENGTH IN SMALL SEAMLESS BRASS TUBES

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each tube beyond which the collapsing pressure is constant; and further, that this minimum length is quite definite. Again, we see that for lengths less than this critical minimum length, the collapsing pressures rise rapidly. As definitely as can be determined from these small tubes, the collapsing pressure varies inversely as the length, for lengths less than the critical length. In this they follow Fairbairn's formula, and suggest that Fairbairn's tubes were all shorter than their critical lengths. An inspection of the woodcuts which Fairbairn gives for each of his experiments, and a comparison of these with the shapes of the brass tubes which have been collapsed in our experiments confirm this. Figs. 3 and 4 show shapes and sections of the collapsed tubes of the curve, Fig. 2. Fairbairn found exactly these shapes which we have obtained for lengths less than the critical length".<sup>1</sup>

These previous experiments had thus shown the inadequacy of Fairbairn's formula, and they had particularly shown the very narrow range of the law of inverse lengths. When the present series of experiments on standard steel boiler flues was begun, little attention was therefore paid to the law of lengths, except to see that the tubes were longer than the critical minimum length.

## METHODS AND DATA OF EXPERIMENTS

The method followed in the present tests was similar to that used by Fairbairn. The tube to be tested was closed at both ends, placed inside a stout steel cylinder, and there subjected to increasing external water pressure until the tube collapsed. The pressures were read by hydraulic gauges. The cylinder used in all these experiments was a section of a nickel steel naval gun tube, kindly furnished at a nominal price by the Bethlehem Steel Company, Bethlehem, Pennsylvania. The dimensions of this gun tube were: length, 12 feet; external diameter, 7 inches; internal diameter, 5 inches. For a distance of six inches at one end, the external diameter was left at 9 inches, thus making a shoulder against which the end plug could be clamped. This plug was a steel disk with a projection, and was held in place by heavy cast-iron rings and eight 1½-inch steel bolts. A lead gasket with circular grooves in the end face of the tube prevented leakage even at the highest pressures. The other end of the nickel steel cylinder was closed by a cast-iron plug, six inches long, shrunk into place.

<sup>1</sup> A. P. Carman. Am. Physical Society, April, 1905.

A  $\frac{3}{4}$ -inch hole for stay rods was drilled through the center of each end plug. Leakage about the rods was prevented by packing held in place by bushing nuts. These stay rods were made of  $\frac{3}{4}$ -inch steel shafting, and were screwed, one into each end plug of the tube to be tested. The tube could thus be put under tension so as to take up the end pressures. One of these stay rods had a small hole through it connecting the interior of the tube with the atmosphere. By rubber tubing, the interior could be connected with a U manometer. This manometer was very useful in indicating any leakage.

In the first experiments, the tube to be tested was closed by steel plugs, fitted and soldered in the ends. Experience soon showed that soldering the plugs was both a tedious and an uncertain method of closing the ends. Several other plans were tried before a satisfactory and convenient method was found. The final method is shown in Fig. 5. Tool-steel clamps were made in the shape of split rings, hinged on one side and held together on the other side by bolts. These ring clamps were placed on the tube near the end as a grip, and slipping was prevented by burring the tube with a cold chisel. A steel disk faced with a sheet of lead was then drawn tight against the plane ends of the tube by bolts screwing into the clamps. After the ends were clamped on the tube it could be tested for leakage by placing the whole tube in a trough of water, and pumping air into it with a foot bicycle pump through the bored stay rod. It was seldom that a tube tested in this way leaked when subjected even to high external water pressure. The connections with the gauges, the pump and the hydrant were made through small holes bored in the nickel steel cylinder. These holes were made tight by special screw plugs and leather gaskets. The cylinder was mounted on two heavy trestles and inclined so as to allow the easy escape of the air when filling it with water. The pressures were produced by a Cailletet pump made by the Société Genevoise, of Geneva, Switzerland. The pump was capable of producing pressures of 1000 kilograms per square centimeter, or approximately 14,000 pounds per square inch. Copper pressure tubing was used to connect the pump and the gauges with the cylinder. Four gauges made by Shæffer and Budenberg were used, viz.:

No. 1, reading to 8000 pounds per square inch.

No. 2, reading to 1000 kilograms per square centimeter.

No. 3, reading to 3000 pounds per square inch.

No. 4, reading to 300 kilograms per square centimeter.



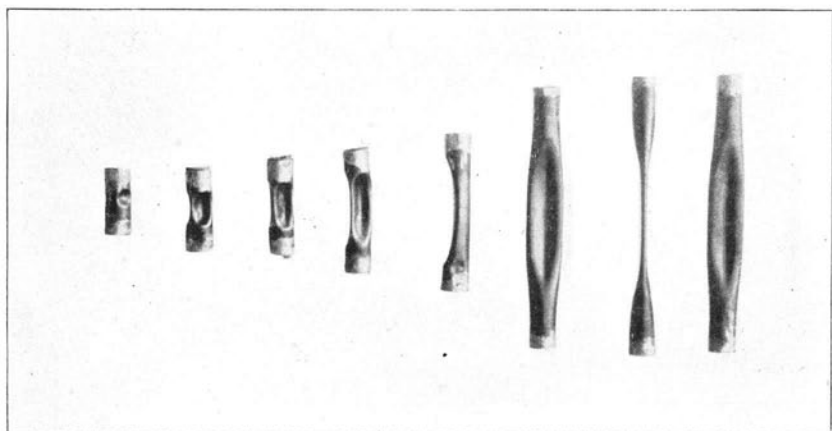


FIG. 3 APPEARANCE OF COLLAPSED SMALL SEAMLESS BRASS TUBES

Referred to in Figs. 2 and 4



FIG. 4 SECTIONS OF COLLAPSED SMALL SEAMLESS BRASS TUBES

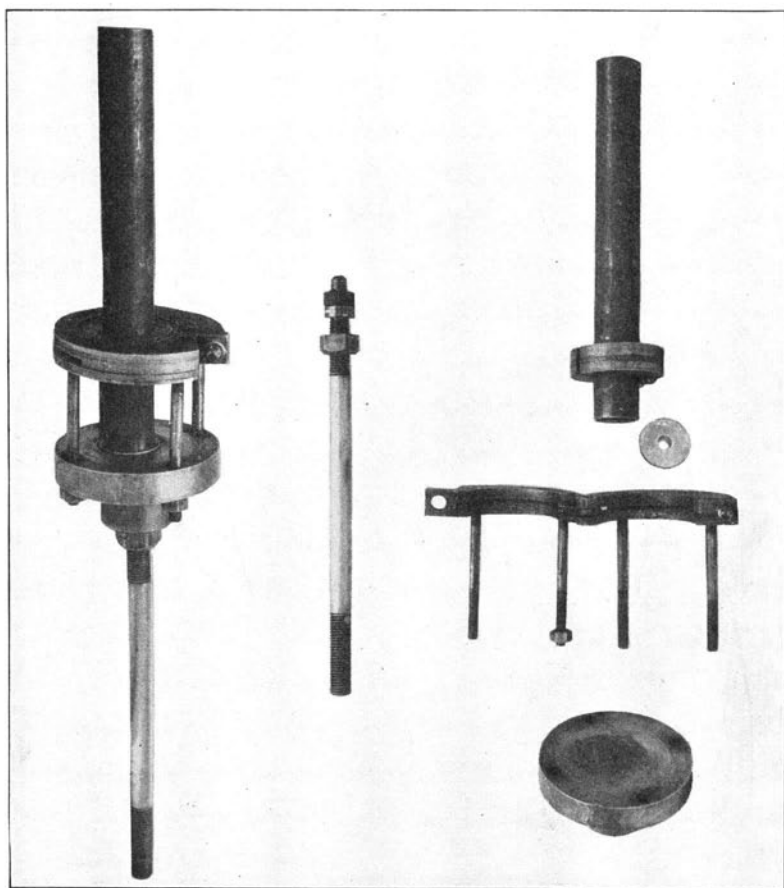


FIG. 5 PARTS OF CLAMP FOR STOPPING END OF TUBE  
(on the right)

CLAMP IN PLACE ON TUBE  
(on the left)

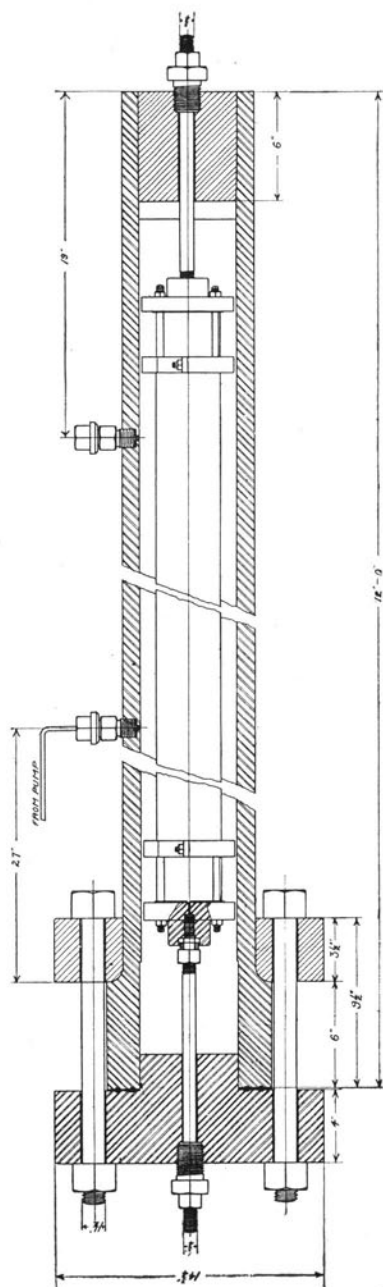


FIG. 6 SECTION OF NICKEL STEEL CYLINDER, WITH TUBE TO BE TESTED IN PLACE, SHOWING  
END PLUGS AND CONNECTION TO PUMP

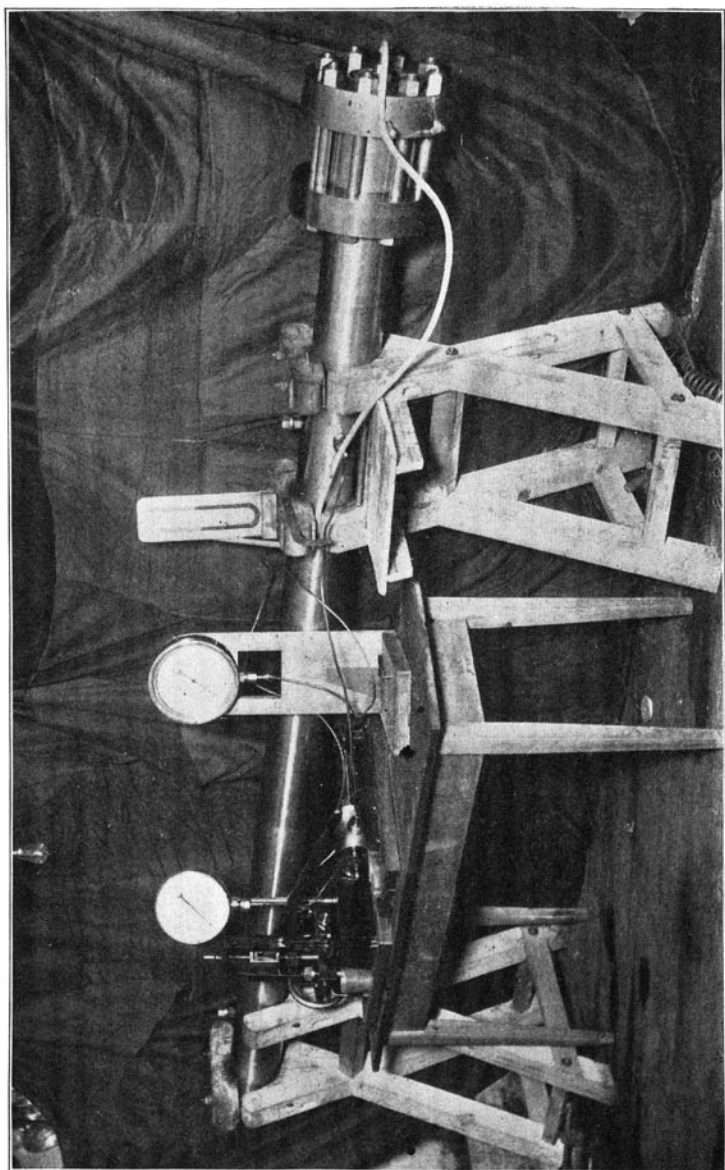


FIG. 7 APPARATUS FOR COLLAPSING TUBES

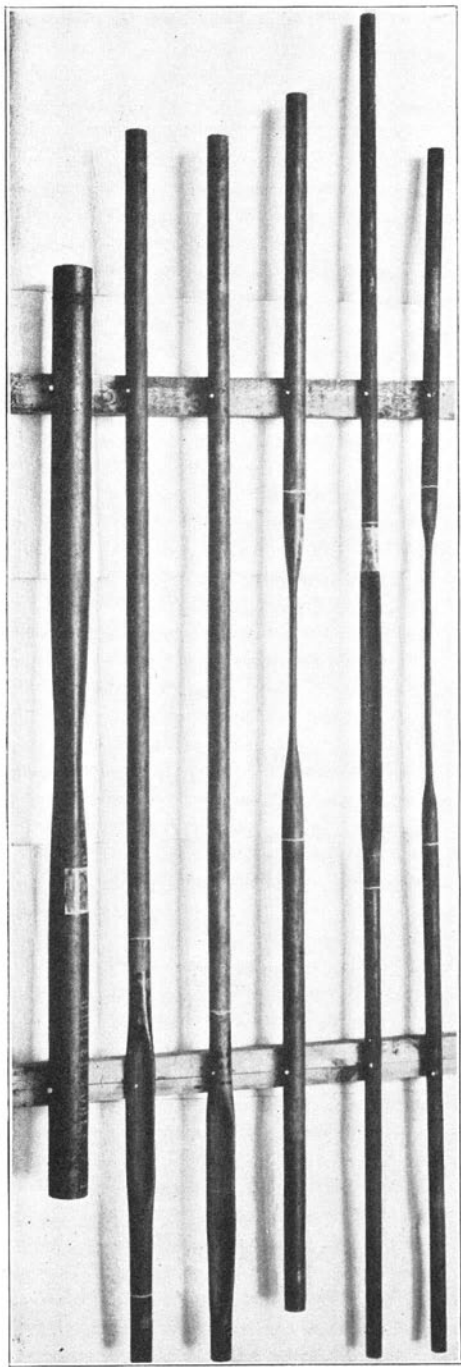


FIG. 8 TUBES WHICH HAVE BEEN COLLAPSED, SHOWING SHAPE OF COLLAPSE

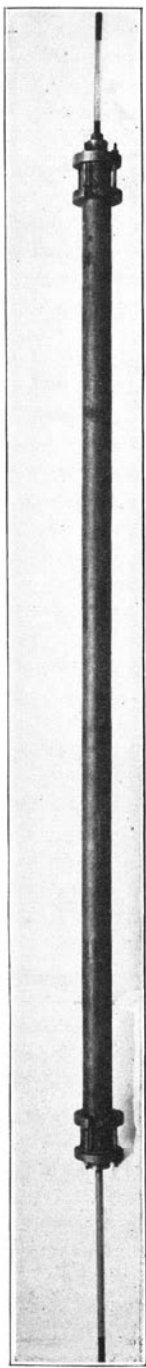


FIG. 9 TUBE WITH END STOPPED AND WITH STAY RODS, READY TO BE PUT INTO CYLINDER FOR TEST

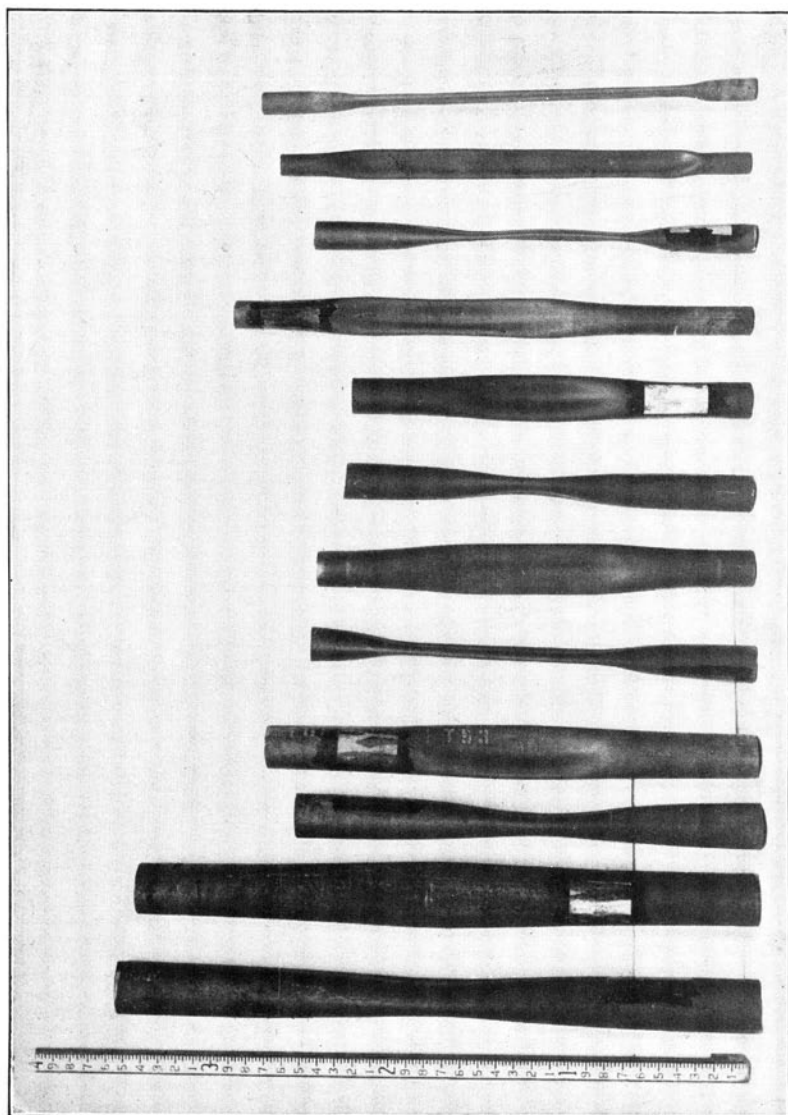


FIG. 10 COLLAPSED PORTIONS OF SOME BOILER TUBES

Gauges Nos. 1 and 3 had maximum pointers, and all had check valves to protect the springs from the shock at collapse. The drawing of the cylinder, Fig. 6 and the photograph, Fig. 7, of the assembled apparatus show other features not easily described.

When a test was to be made, the prepared tube was placed in the cylinder, the heavy head bolted on, and the cylinder filled with water from the hydrant. All the openings were then closed, and the pressure pump started. Several minutes of pumping were usually required before the gauges began to record. Except for a few thin tubes of large diameter, failure, which was sudden in all cases, was accompanied by a sound much like that accompanying the failure of a specimen in a testing machine. Failure was also indicated by the dropping of the gauges and the rise of the water in the manometer connected with the inside of the tube. Each specimen was carefully measured after collapse, and in the case of most of the tested tubes, the collapsed portion was sawed across and the actual average thickness of the tube was obtained. Nearly all of the tubes tested were ten feet long at the start. In many cases, three or more collapses were made by cutting off the collapsed portion after each failure, and then testing the remainder of the tube. In the tables of data, these are noted as sections of the flue, e. g., flue No. 1 was divided into three sections, 1, 2 and 3. The tubes tested were commercial steel boiler tubes, both lap-welded and cold drawn seamless. We have to thank J. T. Ryerson and Son, Chicago, for the gift of four tubes, and the Scully Steel and Iron Company, Chicago, for the gift of twenty-five tubes for these tests. The other tubes used were bought in the open market. In order to have data on entirely different material, tests were made on a set of brass tubes similar in size to the steel tubes. The advantage of the brass tube tests was that the dimensions and the material were much more uniform.

## CHEMICAL TESTS OF SAMPLES OF BOILER FLUES COLLAPSED

Description of Sample of Flue	CHEMICAL ANALYSIS				
	Si	S	P	Mn	C
Lap-welded steel.....	.....	.021	.137	.286	.080
Lap-welded steel.....	.....	.038	.099	.330	.080
Seamless cold drawn steel....	.....	.001	.018	.462	.170
Seamless cold drawn steel....	.....	.002	.013	.525	.220

TABLE III  
DATA ON SEAMLESS COLD DRAWN FLUES

FLUE		Length	Diameter, inches	Nominal Thickness, inches	Actual Av. Thickness, inches	Collapsing Pressure, lb. per sq. in.	Length of Collapse, inches	Distance of Collapse from Vented End
No.	Sec.							
25	.....	9 ft. 6 in.	1.5	.095	.097	4220	32	5 ft. 2 in.
26	1	9 ft. 7 in.	1.5	.095	.098	4260	30	8 ft. 2 in.
26	2	6 ft. 11 in.	1.5	.095	.....	4260	38	5 ft. 6 in.
26	3	2 ft. 8 in.	1.5	.095	.....	3930	25	1 ft. 6 in.
27	.....	9 ft. 7 in.	1.5	.095	.....	5200	3	At extreme end of flue.
28	.....	8 ft. 4 in.	1.5	.095	.100	4330	32	4 ft. 6 in.
29	.....	8 ft. 4 in.	1.5	.095	.102	4240	29	1 ft. 4 in.
45	.....	9 ft. 2 in.	1.5	.095	.095	3930	29	6 ft. 4 in.
Mean	.....	.....	.....	.....	.098	4200	30.4	.....
42	.....	9 ft. 7 in.	2	.095	.102	3050	27	1 ft. 0 in.
43	.....	9 ft. 7 in.	2	.....	.092	2420	20	6 ft. 4 in.
44	.....	9 ft. 7 in.	2	.....	.103	3390	36	6 ft. 4 in.
Mean	.....	.....	.....	.....	.099	2950	28	.....
36	.....	9 ft. 8 in.	2.5	.083	.084	1280	27	5 ft. 0 in.
37	.....	9 ft. 7 in.	2.5	.083	.084	1100	26	6 ft. 8 in.
Mean	.....	.....	.....	.....	.084	1190	26.5	.....
39	.....	9 ft. 7 in.	2.5	.095	.103	1820	35	.....
40	.....	9 ft. 6 in.	2.5	.095	.098	1730	27	4 ft. 6 in.
41	.....	9 ft. 7 in.	2.5	.095	.098	1890	22	9 ft. 0 in.
Mean	.....	.....	.....	.....	.099	1810	28	.....
3	1	10 ft. 0 in.	2.5	.109	.114	1720	28	1 ft. 2 in.
3	2	6 ft. 10 in.	2.5	.109	.....	1920	27	2 ft. 2 in.
3	3	3 ft. 4 in.	2.5	.109	.....	2200	24	1 ft. 10 in.
4	1	9 ft. 8 in.	2.5	.109	.116	1980	25	.....
4	2	5 ft. 10 in.	2.5	.109	.....	2470	27	2 ft. 3 in.
4	3	.....	2.5	.109	.....	2610	.....	.....
38	.....	9 ft. 7 in.	2.5	.109	.109	1580	25	1 ft. 9 in.
Mean	.....	.....	.....	.....	.113	1960	27	.....
1	1	9 ft. 0 in.	2.5	.120	.120	1830	27	.....
1	2	4 ft. 11 in.	2.5	.120	.....	1850	26	.....
1	3	1 ft. 9 in.	2.5	.120	.....	2190	21	.....
2	1	7 ft. 9 in.	2.5	.120	.121	2320	28	5 ft. 6 in.
2	2	3 ft. 6 in.	2.5	.120	.....	2650	23	1 ft. 9 in.
Mean	.....	.....	.....	.....	.120	2070	25	.....
33	.....	9 ft. 7 in.	3	.095	.094	1190	28	4 ft. 0 in.
34	.....	9 ft. 7 in.	3	.095	.102	1160	33	8 ft. 0 in.
35	.....	9 ft. 7 in.	3	.095	.097	1130	32	8 ft. 0 in.
Mean	.....	.....	.....	.....	.098	1160	31	.....
30	.....	9 ft. 7 in.	3.5	.095	.093	990	37	6 ft. 0 in.
31	.....	9 ft. 7 in.	3.5	.095	.096	900	55	7 ft. 0 in.
32	.....	9 ft. 7 in.	3.5	.095	.093	970	42	6 ft. 7 in.
Mean	.....	.....	.....	.....	.094	950	45	.....



TABLE IV

DATA ON LAP-WELDED STEEL BOILER FLUES

FLUE		Length	Diameter, inches	Nominal Thickness, inches	Actual Av. Thickness, inches	Collapsing Pressure, lb. per sq. in.	Length of Collapse, inches	Distance of Collapse from Vented End
No.	Sec.							
5	1	8 ft. 10 in.	2.5	.109	.114	2850	29	.....
5	2	5 ft. 10 in.	2.5	.109	.....	3040	31	.....
5	3	3 ft. 0 in.	2.5	.109	.....	3400	29	1 ft. 1 in.
6	1	8 ft. 8 in.	2.5	.109	.116	2810	.....	.....
6	2	6 ft. 0 in.	2.5	.109	.....	2930	27	4 ft. 11 in.
6	3	3 ft. 7 in.	2.5	.109	.....	3080	21	0 ft. 7 in.
7	1	8 ft. 8 in.	2.5	.109	.120	2700	37	.....
7	2	2 ft. 4 in.	2.5	.109	.....	2850	27	1 ft. 1 in.
8	1	9 ft. 0 in.	2.5	.109	.112	2900	25	4 ft. 6 in.
8	2	3 ft. 3 in.	2.5	.109	.....	2950	25	2 ft. 0 in.
8	3	2 ft. 6 in.	2.5	.109	.....	3170	24	1 ft. 6 in.
9	1	9 ft. 0 in.	2.5	.109	.120	3160	20	.....
9	2	5 ft. 10 in.	2.5	.109	.....	3450	27	3 ft. 1 in.
10	1	9 ft. 0 in.	3	.109	.105	2080	30	1 ft. 9 in.
10	2	4 ft. 11 in.	3	.109	.....	2150	29	3 ft. 5 in.
10	3	2 ft. 10 in.	3	.109	.....	2350	29	0 ft. 8 in.
11	1	9 ft. 0 in.	3	.109	.111	2190	23	8 ft. 6 in.
11	2	6 ft. 6 in.	3	.109	.....	2430	30	1 ft. 7 in.
11	3	3 ft. 8 in.	3	.109	.....	2620	26	2 ft. 6 in.
11	4	1 ft. 8 in.	3	.109	.....	3150	20	0 ft. 10 in.
12	1	9 ft. 0 in.	3	.109	.105	1910	4	At extreme end of flue.
12	2	8 ft. 8 in.	3	.109	.....	2020	27	7 ft. 6 in.
12	3	5 ft. 1 in.	3	.109	.....	1960	30	1 ft. 4 in.
12	4	2 ft. 6 in.	3	.109	.....	1920	26	1 ft. 9 in.
13	1	9 ft. 0 in.	3	.109	.106	2140	32	6 ft. 8 in.
13	2	5 ft. 1 in.	3	.109	.....	2220	33	2 ft. 10 in.
14	1	9 ft. 0 in.	3	.109	.113	2080	32	4 ft. 3 in.
14	2	2 ft. 11 in.	3	.109	.....	2000	27	1 ft. 9 in.
14	3	2 ft. 6 in.	3	.109	.....	2230	28	1 ft. 2 in.
15	1	9 ft. 5 in.	3.5	.120	.124	1940	31	1 ft. 4 in.
15	2	6 ft. 8 in.	3.5	.120	.....	2310	30	1 ft. 2 in.
15	3	4 ft. 1 in.	3.5	.120	.....	2470	30	2 ft. 11 in.
15	4	1 ft. 4 in.	3.5	.120	.....	2800	16	0 ft. 8 in.
16	1	9 ft. 6 in.	3.5	.120	.129	2180	31	1 ft. 1 in.
16	2	6 ft. 9 in.	3.5	.120	.....	2500	30	5 ft. 7 in.
16	3	4 ft. 1 in.	3.5	.120	.....	2790	27	2 ft. 11 in.
17	1	9 ft. 8 in.	3.5	.120	.129	2150	41	.....
17	2	6 ft. 8 in.	3.5	.120	.....	2240	36	1 ft. 5 in.
17	3	3 ft. 4 in.	3.5	.120	.....	2660	36	2 ft. 0 in.
18	1	9 ft. 4 in.	3.5	.120	.128	1890	39	.....
18	2	6 ft. 4 in.	3.5	.120	.....	1870	.....	.....
19	1	9 ft. 4 in.	3.5	.120	.127	2200	32	1 ft. 4 in.
19	2	6 ft. 8 in.	3.5	.120	.....	2290	42	4 ft. 4 in.
19	3	2 ft. 6 in.	3.5	.120	.....	2440	30	1 ft. 2 in.
20	1	8 ft. 5 in.	2	.095	.095	3100	33	7 ft. 2 in.
20	2	5 ft. 5 in.	2	.095	.....	3410	29	3 ft. 2 in.
20	3	2 ft. 3 in.	2	.095	.....	3640	27	1 ft. 1 in.
21	.....	8 ft. 4 in.	2	.095	.098	2650	32	6 ft. 7 in.
22	1	8 ft. 2 in.	2	.095	.106	3180	30	6 ft. 6 in.
22	2	5 ft. 8 in.	2	.095	.....	3500	32	4 ft. 1 in.

TABLE IV—*Concluded*

FLUE		Length	Diameter, inches	Nominal Thickness, inches	Actual Av. Thickness, inches	Collapsing Pressure, lb. per sq. in.	Length of Collapse, inches	Distance of Collapse from Vented End
No.	Sec.							
22	3	2 ft. 8 in.	2	.095	.....	3700	26	1 ft. 1 in.
23	1	8 ft. 4 in.	2	.095	.102	3260	28	.....
23	2	6 ft. 0 in.	2	.095	.....	3960	37	1 ft. 8 in.
23	3	3 ft. 0 in.	2	.095	.....	4030	27	2 ft. 0 in.
24	.....	8 ft. 5 in.	2	.095	.109	3470	31	4 ft. 2 in.

TABLE V

DATA ON BRASS TUBES

TUBE		Length	Diameter, inches	THICKNESS		Collapsing Pressure, lb. per sq. in.	Length of Collapse, inches
No.	Sec.			Gauge	Inches		
1	1	8 ft. 8 in.	3	14	.083	450	34
1	2	3 ft. 7 in.	3	14	.083	490	24
1	3	1 ft. 9 in.	3	14	.083	555	21
2	1	3 ft. 11 in.	3	14	.083	465	26
2	2	0 ft. 8 in.	3	14	.083	725	8
3	1	8 ft. 7 in.	3	19	.042	75	41.5
4	1	8 ft. 7 in.	2	14	.083	1400	23.5
4	2	5 ft. 11 in.	2	14	.083	1440	22
4	3	2 ft. 7 in.	2	14	.083	1440	21
5	1	2 ft. 3 in.	2	14	.083	1400	23
6	.....	6 ft. 6.5 in.	2.5	10	.134	2530	24
7	.....	6 ft. 0.0 in.	2	12	.109	2440	27.5
8	.....	6 ft. 6.5 in.	2	10	.134	3840	31
9	1	6 ft. 5.5 in.	2	19	.042	275	24.5
9	2	3 ft. 4 in.	2	19	.042	325	20
10	.....	5 ft. 6.5 in.	2.5	14	.083	710	23
11	.....	4 ft. 7 in.	1.5	14	.083	2600	31

## OTHER TESTS AND DATA

The present paper was practically completed when a paper appeared by Professor R. T. Stewart, entitled, "Collapsing Pressures of Bessemer Steel Lap-welded Tubes, Three to Ten Inches in Diameter". This paper was read by Professor Stewart before the American Society of Mechanical Engineers at the Chattanooga meeting, May 1 to 4, 1906, and at the present writing is available only in the advance papers of the meeting. It describes a very complete and valuable investigation made at the McKeesport

Works of the National Tube Company on the lap-welded steel tubes of that company, "the Tube Company generously providing every needful facility for carrying on the research in a most thorough manner". Tests on over five hundred tubes are recorded, so that every advantage of averages is gained. Professor Stewart's tests are certain to play an important part in future discussions of tube collapse on account of the completeness and carefulness of the work. The great number of different facts noted will be invaluable to students, although possibly at first confusing to some readers. The method of experimenting is substantially the same as described in this paper, and also the same as was used by Fairbairn. The ends of the tested tubes were not stayed, and much

TABLE VI  
DATA ON LAP-WELDED BESSEMER STEEL TUBES—STEWART<sup>1</sup>

Thickness, inches ( <i>t</i> )	Diameter, inches ( <i>d</i> )	$\frac{t}{d}$	$\left[\frac{t}{d}\right]^2$	$\left[\frac{t}{d}\right]^3$	Collapsing Pressure, lb. per sq. in.	Source of Data
.194	10.026	.0193	.000372	.00000718	383	Table No. 37
.354	8.673	.0408	.001670	.00006800	2028	Table No. 37
.279	6.987	.0400	.001600	.00006400	2147	Table No. 36
.250	6.677	.0374	.001400	.00005240	1879	Table No. 36
.186	6.024	.0309	.000950	.00002940	1251	Table No. 35
.167	6.028	.0277	.000768	.00002120	928	Table No. 35
.327	4.014	.0815	.006640	.00054000	5560	Table No. 34
.212	4.026	.0527	.002780	.00014600	3170	Table No. 34
.175	4.014	.0436	.001900	.00008300	2280	Table No. 34
.112	2.997	.0374	.001400	.00005240	1860	Table No. 34
.165	10.041	.0164	.000269	.00000440	225	Table No. 30

longer specimens were used, particularly in the first experiments. Most of the experiments were on tubes of larger diameter than ours, Stewart's smaller tubes being indeed the same as our larger tubes. Stewart's experiments are confined to lap-welded tubes, while the tests described in the present paper include also cold drawn seamless steel and brass tubes. The experiments thus supplement each other. Professor Stewart sums up the results of his work as follows: "The principal conclusions to be drawn from the results of the present research may be stated briefly as follows:

(1) The length of tube, between transverse joints tending to hold it to a circular form, has no practical influence upon the

<sup>1</sup>Data taken from a paper read by Professor R. T. Stewart before the American Society of Mechanical Engineers, at Chattanooga, May, 1906. Paper No. 091.

collapsing pressure of a commercial lap-welded steel tube so long as this length is not less than about six diameters of tube.

(2) The formulæ, as based upon the present research, for the collapsing pressure of modern lap-welded Bessemer steel tubes are as follows:

$$(a) \quad P = 1,000 \left( 1 - \sqrt{1 - 1600 \frac{t^2}{d^2}} \right)$$

$$(b) \quad P = 86,670 \frac{t}{d} - 1386$$

Where  $P$  = collapsing pressure, pounds per square inch;  $d$  = outside diameter of tube in inches;  $t$  = thickness of wall in inches.

Formula (a) is for values of  $P$  less than 581 pounds, or for values of  $\frac{t}{d}$  less than 0.023, while formula (b) is for values greater than these.

(3) The apparent fiber stress under which the different tubes failed, varied from about 7000 pounds for the relatively thinnest to 35,000 pounds per square inch for the relatively thickest walls. Since the average yield point of the material was 37,000 and the tensile strength 58,000 pounds per square inch, it would appear that the strength of a tube subjected to a fluid collapsing pressure is not dependent alone upon either the elastic limit or ultimate strength of the material constituting it".

It will, of course, be impossible to make even an abstract of Professor Stewart's voluminous data, but the substantial agreement of our curves of results with his will be shown in the discussion. A considerable amount of experimental work was done by Professor Stewart in reaching his first conclusion, viz., that the law of inverse lengths did not hold for long tubes. As mentioned above, we had reached this conclusion in experiments described and published before the present experiments were begun.

#### RATIONAL FORMULÆ

We turn now to the theoretical discussion of tube collapses. The problem involved is one of unstable equilibrium, and as A. E. H. Love, the author of one of our best theoretical treatises on elasticity, says, belongs to one of the most difficult chapters in the theory of elasticity. Within the last eighteen years, three mathematicians of the University of Cambridge, England, have

discussed the problem of collapse, basing the discussion on the general equations of mechanics and elasticity.

Professor G. H. Bryan<sup>1</sup> seems to have been the first to deduce a formula. Mr. A. B. Basset<sup>2</sup> soon after discussed the subject and proved the same formula in a slightly different form and by other methods. Mr. A. E. H. Love,<sup>3</sup> in his elaborate treatise on elasticity, gives Bryan's formula with a new discussion and with some additions. As stated above, these discussions are all based on the general mathematical equations of elasticity, and are too long and complicated to be presented here. The results, however, have been summed up in excellent form by Love. He says: "Combining the results of this and the previous article, we conclude:

(1) That no flue, however long, can collapse unless the pressure exceed

$$\left[ \frac{2 E}{1 - \sigma^2} \right] \frac{h^3}{a^3}$$

(2) That when the pressure exceeds this limit, any flue will collapse if its length exceed a certain multiple of the mean proportional between the diameter and the thickness".

In the above formula,  $E$  is the coefficient of elasticity;  $h$  is one-half of the thickness;  $a$  is the radius of the mean section; and  $\sigma$  is Poisson's ratio. So far as known, no use of this formula has been made in experiments or in engineering practice. It has been developed wholly from the general theory of elasticity, and is distinctly limited by the assumptions, to thin tubes, i. e., where "the thickness  $2 h$  is small compared with the mean radius  $a$ ".

Previous to this work of the three Cambridge mathematicians, we know of only two published attempts to deduce a rational formula for tube collapses, one by Professor W. C. Unwin and the other by Dr. F. Grashof. Professor Unwin was an assistant of Fairbairn in the original experiments, and nearly twenty years later, in 1876, gave a new discussion of these experiments. He says that the formulæ which have been based on Fairbairn's experiments have no relation to ordinary formulæ of applied me-

<sup>1</sup> G. H. Bryan, Application of the Energy Test to the Collapse of a Long Thin Pipe under External Pressure, Proceedings of the Cambridge Philosophical Society, October 29, 1888. Vol. 6.

<sup>2</sup> A. B. Basset, On the Difficulties of Constructing a Theory of the Collapse of Boiler Flues, Philosophical Magazine, London, 1892, Vol. 200, pp. 221-233.

<sup>3</sup> A. E. H. Love, Mathematical Theory of Elasticity, Cambridge, 1903, Vol. II, pp. 308-316.

chanics, and notes the advantages if "the collapse of tubes could be expressed by the ordinary laws of the resistance of materials". Unwin then discusses tube collapse on the analogy of this to the failure of long thin columns, and deduces from Euler's formula for columns a similar expression for the collapsing pressures of tubes. After making certain assumptions, he concludes, "The lowest collapsing pressure for long tubes is:

$$P_{\min.} = \frac{8}{3} E \frac{t^3}{d^3}$$

If  $l$  is less than  $28 t$ , the collapsing pressure at the limit is:

$$P_{\max.} = \left[ \frac{\pi^2 E}{4704} \right] \frac{t}{d}$$

If  $t$  is greater than  $\frac{1}{36} d$ , the formulæ cease to be applicable".<sup>1</sup> The above formulæ are thus proposed only for very thin tubes. Unwin then modified his theory to make it agree with Fairbairn's experiments, and reached as an approximation the formula already quoted on page 5. Like other writers, he did not appreciate the very limited range of Fairbairn's experiments as regards length of tubes. Grashof's attempt to deduce a rational expression for tube collapses was made much earlier, but seems generally to have been overlooked. It is added at the end of his paper on Fairbairn's experiments, in which he gives the empirical formula already noted on page 5. Grashof deduces

$$P = \frac{2k \frac{t}{d}}{1 + \frac{3}{2} a \frac{d}{t}}$$

the formula where  $P$  = collapsing pressure;  $t$  = thickness of wall;  $d$  = diameter of tube;  $k$  = yielding strength of the material; and  $a = \frac{a-b}{a}$ , where  $a$  is the maximum diameter, and  $b$  the minimum diameter of the distorted tube at the instant of failure.<sup>2</sup>

<sup>1</sup> Unwin, Institution of Civil Engineers, 1876, Part IV, p. 232.

<sup>2</sup> F. Grashof, Zeitschrift des Vereines deutscher Ingenieure, Vol. III, p. 241.

## CURVES AND FORMULÆ

An inspection of the data of our experiments shows that the portion of a long tube affected by the collapse from hydraulic pressure is generally not longer than twelve times the diameter, and that for greater lengths the collapsing pressure is independent of the length. The law according to which the collapsing pressure varies inversely as the length, is true only for very short tubes, i. e., tubes shorter than a certain "critical minimum length", which in most cases is from four to six times the diameter. We can thus omit further consideration of all formulæ of the Fairbairn type in which the length appears in the denominator.

All considerations show that the collapsing pressure of a tube is a function of  $t$ , the thickness of the tube wall, and also of  $d$ , the diameter, varying directly as some function of  $t$ , and inversely as some function of  $d$ . Further, all the theoretical discussions indicate that this collapsing pressure varies as a function of the

ratio  $\frac{t}{d}$ , i. e., that  $t$  and  $d$  have the same exponents. The sim-

plest method of showing and studying the relation between  $p$ , the

collapsing pressure, and the ratio,  $\frac{t}{d}$ , is the graphic one, construct-

ing curves from the experimental data. In Figs. 11, 12 and 13 such curves are drawn. In all these curves, the ordinates represent the values of  $P$ , and the abscissas represent the cor-

responding values of  $\frac{t}{d}$ ,  $\frac{t^2}{d^2}$  and  $\frac{t^3}{d^3}$ . Fig. 11 is for drawn brass

tubes, Fig. 12 for cold drawn seamless steel tubes, and Fig. 13 was made by taking the numerical results from Professor Stewart's paper on lap-welded steel tubes and calculating the ratios. While our tables contain results of a large number of tests of lap-welded tubes, these tubes were all thick and also of about the same thickness, so that they gave but few points of a curve. We relied on the cold drawn steel and the brass tubes for the form of the complete curve, as these tubes were more uniform in thickness, diameter, and probably in material also than the lap-welded tubes. The agreement in the general shape of these curves for tubes of different materials and from independent observations is good evidence of the general reliability of the experimental data. An examination of these curves shows the following:

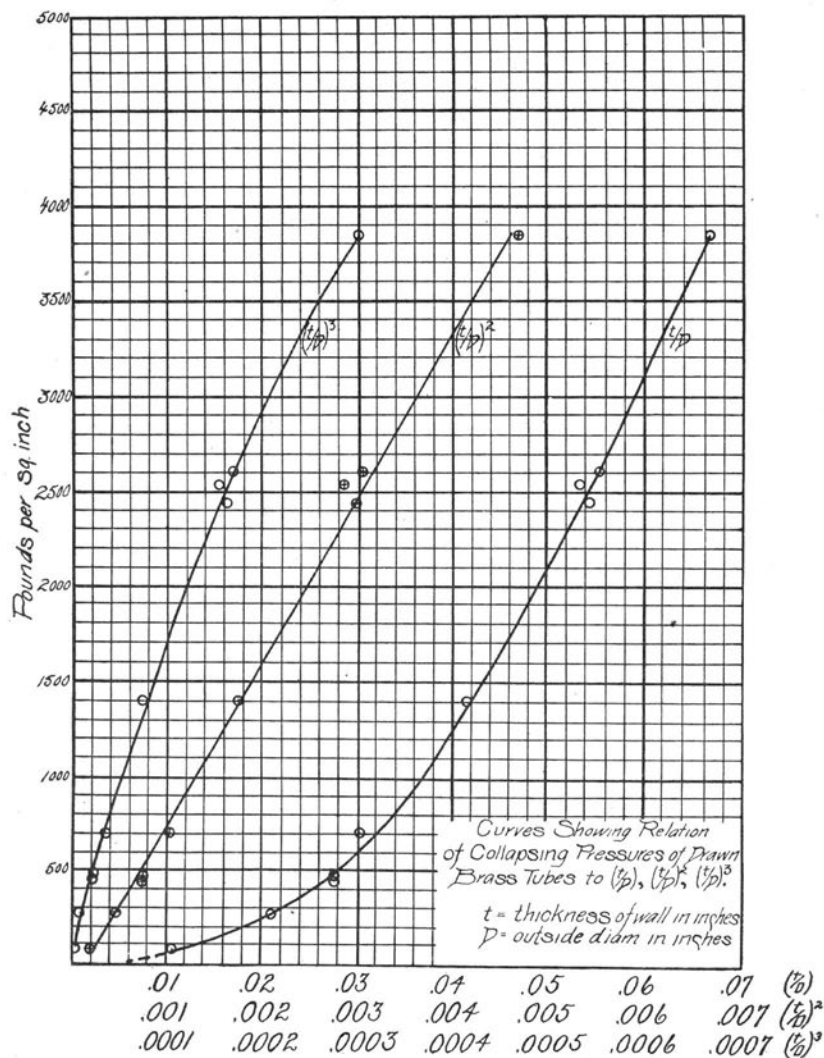


FIG. 11 CURVES SHOWING RELATION OF COLLAPSING PRESSURES OF DRAWN BRASS TUBES TO

$$\left[ \frac{t}{d} \right], \quad \left[ \frac{t}{d} \right]^2 \quad \text{and} \quad \left[ \frac{t}{d} \right]^3$$



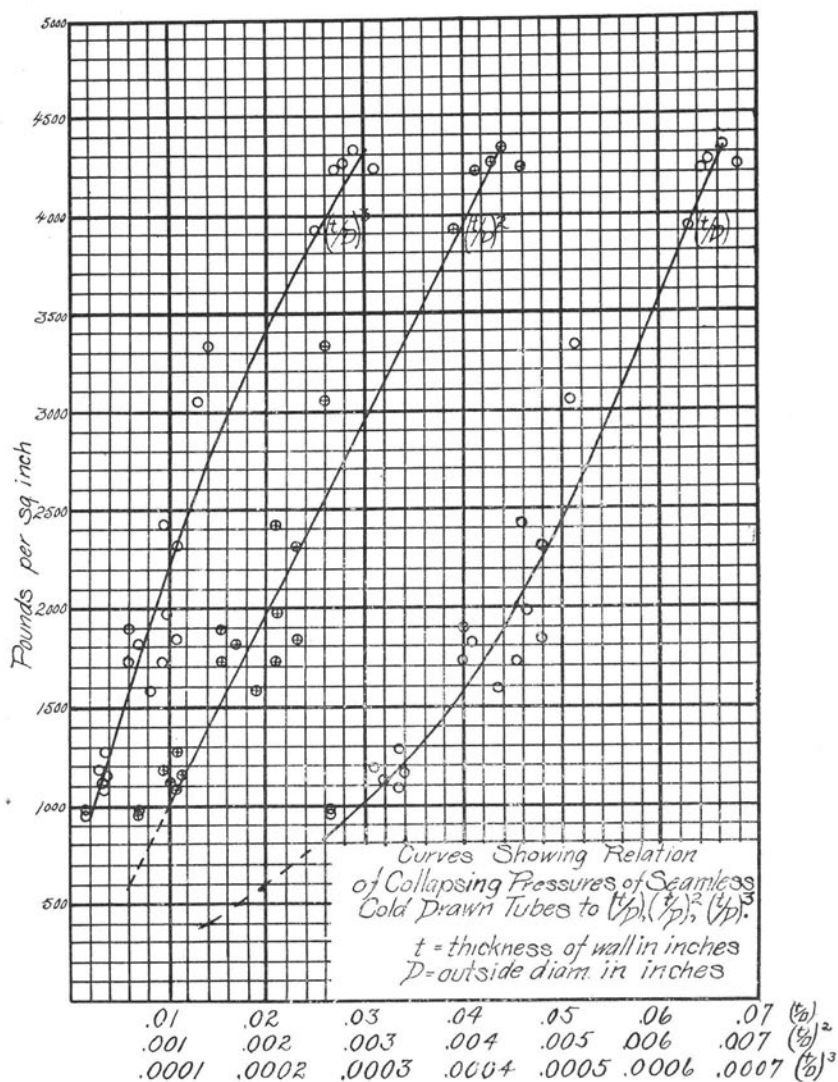


FIG. 12 CURVES SHOWING RELATION OF COLLAPSING PRESSURES OF SEAMLESS COLD DRAWN STEEL TUBES TO

$$\left[ \frac{t}{d} \right], \quad \left[ \frac{t}{d} \right]^2 \quad \text{and} \quad \left[ \frac{t}{d} \right]^3$$

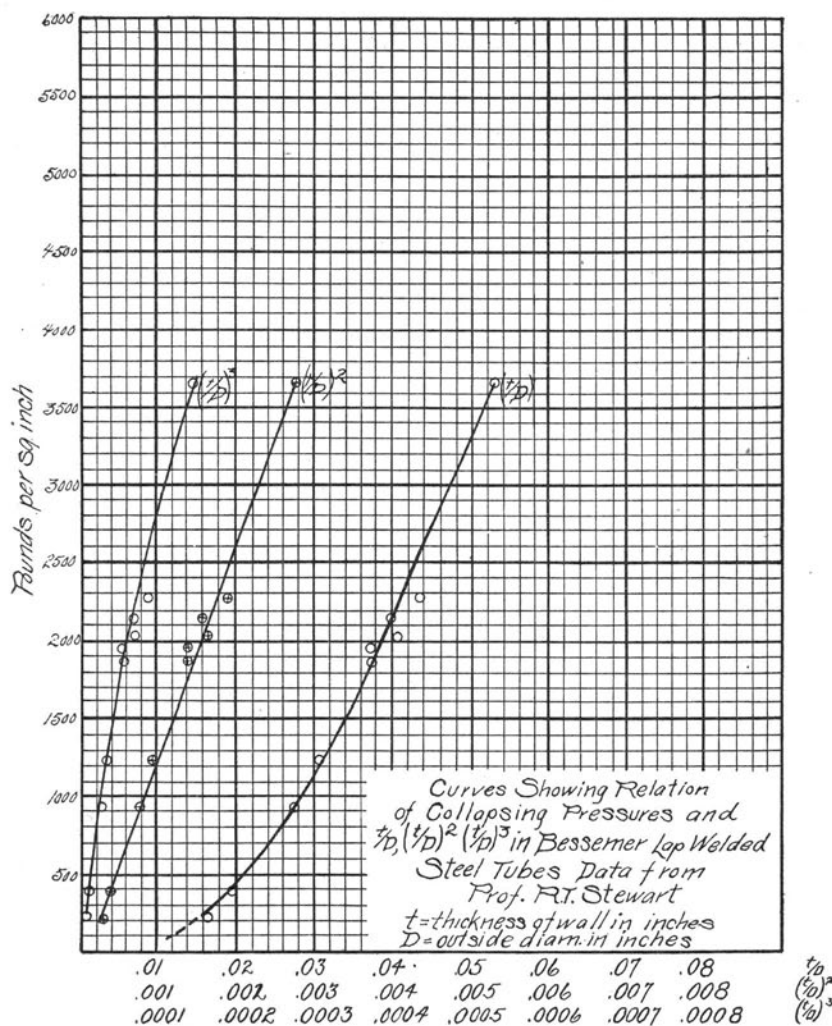


FIG. 13 CURVES SHOWING RELATION OF COLLAPSING

PRESSURES AND  $\left[ \frac{t}{d} \right], \left[ \frac{t}{d} \right]^2$  and  $\left[ \frac{t}{d} \right]^3$  IN

BESSEMER STEEL TUBES, DATA FROM PROF. R. T. STEWART

(1) For thin tubes, i. e., for values of  $\frac{t}{d}$  below about .025, the formula,  $P = k \left[ \frac{t}{d} \right]^3$ , is very nearly true. This assumes that for this portion of the curve of  $P$  and  $\left[ \frac{t}{d} \right]^3$ , the curve is practically a straight line. So far, this is in agreement with the theoretical conclusions of Bryan and Unwin. The limit agrees with the value of  $t = \frac{d}{36}$ , given by Unwin. The values of  $k$  are also of the same order as the coefficient of Bryan's formula, although no very close agreement need be expected, as it is difficult to get extremely accurate readings of the small collapsing pressures. The constants have been calculated and the formulæ are as follows:

(a) For thin brass tubes:

$$P = 25,150,000 \left[ \frac{t}{d} \right]^3$$

(b) For thin cold drawn seamless steel tubes:

$$P = 50,200,000 \left[ \frac{t}{d} \right]^3$$

All of the lap-welded tubes tested by us were thick tubes. This formula with practically the same numerical coefficient applies to the thin tubes given in Stewart's tables of results.

(2) The curve of  $P$  and  $\frac{t}{d}$  is nearly straight for the thick tubes, i. e., for tubes having a value of  $\frac{t}{d}$  greater than about .03.

For thinner tubes, the curve bends rapidly toward the axis. The straight part of the curve can evidently be represented by an equation,  $P = k \left[ \frac{t}{d} \right] - c$ , where  $k$  and  $c$  are constants. We have calculated these constants from our data, and find the following formulæ for tubes having a ratio  $\frac{t}{d}$  greater than .03:

(a) For brass:

$$P = 93,365 \frac{t}{d} - 2474;$$

(b) For seamless cold drawn steel:

$$P = 95,520 \frac{t}{d} - 2090;$$

(c) For lap-welded steel:

$$P = 83,270 \frac{t}{d} - 1025.$$

Professor Stewart found for his lap-welded tubes the formula

$$P = 86,670 \frac{t}{d} - 1386.$$

This formula, as thus stated, is purely empirical, and its lower limit is entirely arbitrary. From a suggestion of Professor A. N. Talbot, department of Theoretical and Applied Mechanics, University of Illinois, attention was called to Grashof's rational formula:

$$P = \frac{2k \frac{t}{d}}{1 + \frac{3}{2} a \frac{d}{t}}$$

as a possible solution. When plotted for  $P$  and  $\frac{t}{d}$ , assigning constant values to  $k$  and  $a$ , this gives a curve suggesting the experimental curves but no possible constant values of  $k$  and  $a$  satisfy the data. It is probable that  $a$  is not a constant, being approximately so for thick tubes, but having quite different values for thin tubes. The formula thus becomes too complicated for use. The above formulæ and their limits are suggestive in showing where the stiffness of the material is the important factor, and where the effect of the strength of the material comes in as the controlling factor. Bryan's formula for thin tubes involves the modulus of elasticity, not the strength of the material. If we suppose that such a formula as Grashof's rational form expresses the facts for thick tubes, the yield point of the material is the factor.

(3) An approximate formula of the form  $P = k \left[ \frac{t}{d} \right]^2$  is suggested by the curves of  $P$  and  $\left[ \frac{t}{d} \right]^2$ , particularly for the steel tubes. For cold drawn seamless steel tubes, this approximate formula is

$$P = 1,000,000 \left[ \frac{t}{d} \right]^2$$

and this can be used for tubes for which  $\frac{t}{d}$  is less than .06. For lap-welded steel tubes, the same formula becomes

$$P = 1,250,000 \left[ \frac{t}{d} \right]^2$$

and this can also be used for values of  $\frac{t}{d}$  less than .06. This approximate formula has been useful to us in getting probable collapsing pressures, and gives satisfactory rough values for tubes of the most common commercial thickness.

In applying any formula to calculate the collapsing pressure of a particular tube, a considerable factor of safety should be used. The constants in all these formulæ are large, and  $\frac{t}{d}$  (or a power of  $\frac{t}{d}$ ) is a comparatively small quantity, so that a small change in the numerical value of  $\frac{t}{d}$  greatly affects the result.

Lack of uniformity in the material, and slight deformations are also very important factors. It is to the credit of modern manufacturers of tubes that their product is as uniform as these tests show. With the knowledge which this discussion gives of the law of tube collapse, the user of tubes is in a position to calculate with fair approximation the collapsing pressure, particularly if he can get tests made of one or more sample tubes of the material so as to fix the constants.

While the advance in this field of investigation is thus considerable, yet much work remains to be done especially in connecting the subject more closely with the ordinary equations of elasticity.

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