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TESTS OF REINFORCED CONCRETE BEAMS SERIES OF 1906.

BY ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING AND IN CHARGE OF THEORETICAL AND APPLIED MECHANICS.

CONTENTS.

I. Introduction.

1. *Preliminary.*—The tests on reinforced concrete in the Laboratory of Applied Mechanics of the University of Illinois were continued during the college year of 1905-6. The results on shear and bond were reported in Bulletin No. 8 of the University of Illinois Engineering Experiment Station; those on columns, in Bulletin No. 10; those on T-beams, in Bulletin No. 12. The tests on rectangular beams will be described in this bulletin. The analytical theory of reinforced concrete beams was quite fully treated in Bulletin No. 4, and the methods and nomenclature used in this bulletin will follow those there given.

2. *Scope of Tests.*—The discussion given in Bulletin No. 4 suggested many topics for investigation. Several of these were taken up and considerable information obtained, though it was not feasible to make the investigations complete enough to be fully conclusive. Effect of quality of concrete, effect of method of loading, effect of repetitive loading, and diagonal tension failures were among the topics considered. The beams were of the standard width and depth adopted by the Joint Committee on Concrete and Reinforced Concrete, but in the investigation of resistance to diagonal tension failure the length was varied and shortened to ensure this form of failure. Plain round rods of mild steel were generally used for reinforcement, but a deformed bar was used in some of the beams. The concrete was varied in quality both in the richness of the mixture and in the conditions of fabrication. These tests have since been followed with others bearing on some of the same topics.

3. *Acknowledgment.*—These tests were part of the work of the University of Illinois Engineering Experiment Station. The tests on effect of quality of concrete and effect of method of loading were made in co-operation with the Joint Committee on Concrete and Reinforced Concrete through the sub-committee on tests, of which Mr. Richard L. Humphrey is chairman. The work of testing the beams was done principally as thesis work. The students conducted the tests in a careful and skillful manner, and showed considerable discrimination in making observations and in drawing conclusions. The following members of the class of 1906 in Civil Engineering were connected with the work:

E. W. Sanford, Effect of Quality of Concrete.

H. R. Armeling, Comparison of Methods of Loading.

C. E. Andrew and J. L. Bannon, Effect of Repetition of Load.

T. E. Phipps and R. H. Whipple, A Study of Diagonal Tension Failures.

The work was under the direct supervision of D. A. Abrams, Assistant in the Engineering Experiment Station, and to him and to W. R. Robinson, Assistant in the Engineering Experiment Station, acknowledgment is made for aid in the interpretation of results and in the preparation of this bulletin.

II. Materials, Test Pieces, and Method of Testing.

4. *Materials.*—Materials for the tests on "Effect of Method of Loading" and "Effect of Quality of Concrete" were furnished by the Joint Committee on Concrete and Reinforced Concrete through Mr. Richard L. Humphrey, chairman of the Committee on Tests. Materials for the tests on "Effect of Repetition of Load" and "Diagonal Tension Failure" were furnished by the Engineering Experiment Station. The terms "Joint Committee tests" and "Experiment Station tests" will be used to designate this difference of work and materials.

Stone.— The stone for the Joint Committee tests was a good quality of limestone from Kankakee, Illinois, ordered screened through a 1-in. screen and over a ¼-in. screen. It contained from 45% to 50% voids and weighed 85 lb. per cu. ft. loose. The stone for the Experiment Station tests was also a Kankakee limestone somewhat softer than the other, screened as above, and contained 50% to 54% voids. It was somewhat finer than the Joint Committee stone. In the determination of the voids in both stone and sand, the material was poured slowly into the water so that the voids became filled with water and no air was caught.

Sand.—The sand used was the same for all tests. It came from near the Wabash river at Attica, Indiana. It was of good quality, well graded, and fairly clean. It weighed 115 lb. per cu. ft. loose, and contained 28% voids. Table 1 gives the result of a mechanical analysis of this sand.

Cement.—The cement furnished by the Joint Committee was made up of a mixture of five standard American portland cements, selected and mixed by the manufacturers, and was of excellent quality. The cement furnished by the Experiment Station was Chicago AA portland, purchased in the open market of a local

TABLE 1.

MECHANICAL ANALYSIS OF SAND.

Sieve No.	Size of Mesh inches	Per cent passing
4	.208	100
10	.073	73
20	.034	36
50	.011	12
74	.0078	5
100	.0045	2

TABLE 2.

TENSILE STRENGTH OF CHICAGO AA PORTLAND CEMENT.

Ref. No.	Ultimate Strength, lb. per sq. in.			
	Age 7 days		Age 60 days	
	Neat	1–3	Neat	1–3
1	634	283	890	443
2	717	281	916	440
3	732	275	840	422
4	687	217	942	365
5	580	206	872	352
6	731	189	885	....
Av.	680	242	891	404

dealer. The tensile strength of this last cement, as determined from briquettes made by standard methods, is given in Table 2.

Concrete.—Men accustomed to making concrete mixed the materials and made the test beams. Care was taken in measuring, mixing, and tamping, to secure as uniform a concrete as possible. All materials were measured by loose volume. The mixing was done with shovels by hand. The sand and cement were first mixed dry. The stone was then added and the mass mixed until uniform in appearance. Water was added in such proportion as to give a slightly wet concrete.

Steel.—The longitudinal reinforcement consisted generally of $\frac{1}{2}$-in. or $\frac{3}{4}$-in. mild-steel plain round rods. In a few beams $\frac{1}{2}$-in. high-steel Johnson corrugated bars were used. The results of tensile tests of the steel used are given in Table 3. The plain round bars had an average yield point of 40 500 lb. per sq. in. and an ultimate strength of 60 000 lb. per sq. in. The corrugated bars developed an average yield point of 57 300 lb. per sq. in. with an ultimate strength of 87 400 lb. per sq. in. In general, the yield points of the various bars in one beam varied less than 3% from one another.

TABLE 3.

TENSION TESTS OF STEEL USED IN BEAMS.

Values given are in general the average of results of tests of specimens cut from four different rods.

Specimens Taken from Beams No.	Nominal Size inches	Diameter inches	Per cent Elongation in 8 inches	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.
2	$\frac{3}{4}$	.750	31.0	34000	56300
13	$\frac{3}{4}$	.747	33.0	38800	56900
14	$\frac{3}{4}$	.750	29.7	38700	58000
15	$\frac{1}{2}$	.501	29.3	41400	61700
16	$\frac{1}{2}$	.500	28.5	37700	58100
17	$\frac{1}{2}$	.500	28.0	36700	57900
18	$\frac{1}{2}$	.497	29.0	40300	60200
20	$\frac{1}{2}$	.502	28.0	44400	62500
21	$\frac{1}{2}$	.502	31.2	41800	58000
23	$\frac{3}{4}$	.750	31.5	35000	56900
26	$\frac{1}{2}$	.500	29.0	37200	58600
27	$\frac{3}{4}$	.748	31.2	39400	57100
28	$\frac{1}{2}$	.503	29.1	42700	60800
30	$\frac{1}{2}$	.503	29.1	41000	61100
31	$\frac{1}{2}$	.504	28.5	41800	60900
34	$\frac{3}{4}$	.749	32.5	38800	56000
35	$\frac{1}{2}$	.502	29.5	40800	58400
36	$\frac{1}{2}$	.500	28.0	36300	58100
38	$\frac{3}{4}$	.750	29.0	35800	54700
39	$\frac{3}{4}$	.749	32.5	38800	56200
40	$\frac{1}{2}$	.502	28.1	42800	62200
42	$\frac{1}{2}$	.502	29.9	42500	61800
43	$\frac{1}{2}$	.501	28.4	42400	62100
44	$\frac{1}{2}$	.503	28.7	42100	61800
45	$\frac{1}{2}$	.503	29.6	42000	61900
50	$\frac{1}{2}$	.503	29.5	42800	61700

TABLE 3—*Concluded.*

Specimens Taken from Beams No.	Nominal Size inches	Diameter inches	Per cent Elongation in 8 inches	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.
51	$\frac{1}{2}$	.502	28.5	41800	60900
52	$\frac{1}{2}$	.501	27.9	44200	63100
53	$\frac{1}{2}$	.504	29.2	42500	62800
54	$\frac{1}{2}$	.505	28.0	42100	61200
55	$\frac{1}{2}$	.503	27.8	42000	62300
56	$\frac{5}{8}$	.622	30.7	37800	60600
58	$\frac{3}{4}$	.753	29.5	40300	62900
59	$\frac{3}{4}$	.753	26.0	41000	65300
60	$\frac{3}{4}$	.750	28.5	39200	63000
61	$\frac{3}{4}$	.752	30.5	39000	59100
62	$\frac{3}{4}$	.750	32.3	40600	58100
63	$\frac{3}{4}$	.748	30.3	42350	62600
64	$\frac{3}{4}$	.748	33.5	37800	54400
65	$\frac{3}{4}$	.747	29.5	41700	61600
68	$\frac{3}{4}$	.754	29.0	44200	61900
69	$\frac{3}{4}$	.747	29.5	42200	62700
70	$\frac{3}{4}$	.752	29.5	44500	60700
71	$\frac{3}{4}$	.749	32.2	40100	59200
72	$\frac{3}{4}$	.748	31.5	39900	58900
73	$\frac{3}{4}$	.750	31.2	40700	56900
74	$\frac{1}{2}$	.499	27.6	43900	60100
66*	$\frac{1}{2}$ in. (net section = 0.25 sq. in.)		16.1	58800	91100
67*			17.4	57300	87300

* Corrugated bars.

5. *Test Beams.*—In all of the tests herein discussed the cross-section of the beams was 8 in. x 11 in., the center of the steel being placed 10 in. below the top surface except in some cases where the ends of the bars were bent up. In the tests on "Diagonal Tension Failure" the test span varied from 6 ft. to 12 ft. In all the other series of tests a 12-ft. span was used. Unless otherwise specified the reinforcing bars were straight and were placed horizontally throughout the beam. In the beams marked "Bars bent up" the bars were bent up at a point about 3 in. outside the load points and passed diagonally either in a straight line or in a slightly curved line to a point within 2 in. or 3 in. of the top of the beam near its ends. In several beams stirrups of $\frac{1}{2}$-in. plain

round bars were used. These stirrups were placed 12 in. apart longitudinally throughout the outer thirds of the span length, beginning at the load points. They were U-shaped and passed under all the reinforcing bars and extended nearly to the top of the beam. The stirrups were left very close to the sides of the beam. For general data on all beams see Table 5.

TABLE 4.

COMPRESSION TESTS OF CUBES AND CYLINDERS.

Concrete as in Beam No.	Kind of Concrete	Compressive Strength lb. per sq. in.	
		Cubes	Cylinders
59	1-2-4	2030	1700
61	1-2-4	3500	....
69	1-2-4	1510	....
Av.	....	2350	1700
51	1-3-5½	....	1470
58	1-3-5½	1580	1060
60	1-3-5½	1690	1060
65	1-3-5½	2360	....
68	1-3-5½	1850	885
70*	1-3-5½	2250	1100
72	1-3-5½	1770	540
73*	1-3-5½	....	770
Av.	....	1920	980
5†	1-3-6	1390	
6†	1-3-6	1420	
7†	1-3-6	1050	
12†	1-3-6	1080	
14	1-3-6	1600	
18	1-3-6	1380	
19	1-3-6	1170	
Av.	....	1300	
71	1-5-10	1230	

* Poorly mixed.
† Poorly made.

TABLE 5.

GENERAL DATA ON BEAMS.

Beam No.	Kind of Concrete	Reinforcement		Span, ft.	Age at Test days	Classification	
		Per cent	Amount and Disposition			Table No.	Kind
3	1-3-6	0.98	4 $\frac{1}{2}$-in. round.	12	72	10	E. S.
4	1-3-6	0.98	4 $\frac{1}{2}$-in. round.	12	74	12	E. S.
5	1-3-6	0.98	4 $\frac{1}{2}$-in. round.	12	71	10	E. S.
6	1-3-6	0.98	4 $\frac{1}{2}$-in. round.	10	71	10	E. S.
7	1-3-6	0.98	4 $\frac{1}{2}$-in. round. Bars bent up.	10	71	11	E. S.
8	1-3-6	2.21	4 $\frac{3}{4}$-in. round.	10	76	10	E. S.
9	1-3-6	2.21	4 $\frac{3}{4}$-in. round.	10	76	10	E. S.
10	1-3-6	0.98	4 $\frac{1}{2}$-in. round.	12	73	12	E. S.
11	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	73	10	E. S.
12	1-3-6	2.21	4 $\frac{3}{4}$-in. round.	10	70	12	E. S.
13	1-3-6	2.21	4 $\frac{3}{4}$-in. round. Bars curved up.	10	69	11	E. S.
14	1-3-6	2.21	4 $\frac{3}{4}$-in. round. Bars curved up.	10	68	11	E. S.
15	1-3-6	0.98	4 $\frac{1}{2}$-in. round.	8	69	10	E. S.
16	1-3-6	0.98	4 $\frac{1}{2}$-in. round.	8	69	10	E. S.
17	1-3-6	0.98	4 $\frac{1}{2}$-in. round. Bars bent up.	8	70	11	E. S.
18	1-3-6	0.98	4 $\frac{1}{2}$-in. round.	6	71	10	E. S.
19	1-3-6	0.98	4 $\frac{1}{2}$-in. round.	6	67	10	E. S.
20	1-3-6	0.98	4 $\frac{1}{2}$-in. round. Bars bent up.	6	67	11	E. S.
21	1-3-6	0.98	4 $\frac{1}{2}$-in. round. Bars bent up.	6	68	11	E. S.
22	1-3-6	0.98	4 $\frac{1}{2}$-in. round. 2 bars bent up.	6	70	11	E. S.
23	1-3-5$\frac{1}{2}$	2.21	4 $\frac{3}{4}$-in. round.	10	67	10	E. S.
25	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	6	70	10	E. S.
26	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round. Bars curved up.	8	68	12	E. S.
27	1-3-5$\frac{1}{2}$	2.21	4 $\frac{3}{4}$-in. round.	8	68	10	E. S.
28	1-2-4	0.98	4 $\frac{1}{2}$-in. round.	12	70	8, 9, 12	E. S.
29	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	70	8, 9, 10	E. S.
30	1-4-7$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	71	8, 9, 10	E. S.
31	1-2-4	0.98	4 $\frac{1}{2}$-in. round.	12	78	8, 9, 12	E. S.
32	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	79	8, 9, 12	E. S.
33	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	6	73	10	E. S.
34	1-3-5$\frac{1}{2}$	2.21	4 $\frac{3}{4}$-in. round.	8	67	10	E. S.
35	1-4-7$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	75	8, 9, 12	E. S.
36	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round. Bars bent up.	10	61	11	E. S.
37	1-3-5$\frac{1}{2}$	2.21	4 $\frac{3}{4}$-in. round.	8	70	10	E. S.
38	1-3-5$\frac{1}{2}$	2.21	4 $\frac{3}{4}$-in. round. Bars bent up.	8	64	11	E. S.
39	1-3-5$\frac{1}{2}$	2.21	4 $\frac{3}{4}$-in. round. Bars bent up.	8	64	11	E. S.
40	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	62	7	J. C.
42	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	60	7	J. C.
43	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	60	7	J. C.
44	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	60	7	J. C.
45	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	60	7	J. C.
46	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	6	85	10	E. S.
47	1-2-4	0.98	4 $\frac{1}{2}$-in. round.	6	88	10	E. S.
48	1-2-4	0.98	4 $\frac{1}{2}$-in. round.	6	84	10	E. S.
49	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	6	82	10	E. S.
50	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	60	7	J. C.
51	1-3-5$\frac{1}{2}$	0.98	4 $\frac{1}{2}$-in. round.	12	60	7	J. C.

TABLE 5.—*Concluded*

GENERAL DATA ON BEAMS.

Beam No.	Kind of Concrete	Reinforcement		Span, ft.	Age at Test days	Classification	
		Per cent	Amount and Disposition			Table No.	Kind
52	1-3-5½	0.98	4 ½-in. round.	12	60	7	J. C.
53	1-3-5½	0.98	4 ½-in. round.	12	60	7	J. C.
54	1-3-5½	0.98	4 ½-in. round.	12	60	7	J. C.
55	1-3-5½	0.98	4 ½-in. round.	12	60	7	J. C.
56	1-3-5½	0.98	4 ½-in. round.	12	60	7	J. C.
57	1-2-4	1.15	3 ⅝-in. round.	12	76	6	J. C.
58	1-3-5½	1.10	2 ¾-in. round.	12	61	6	J. C.
59	1-2-4	1.10	2 ¾-in. round.	12	62	6	J. C.
60	1-3-5½	1.10	2 ¾-in. round.	12	61	6	J. C.
61	1-2-4	1.10	2 ¾-in. round.	12	61	6	J. C.
62	1-5-10	1.10	2 ¾-in. round.	12	62	6, 10	J. C.
63	1-5-10	1.10	2 ¾-in. round.	12	61	6, 10	J. C.
64	1-3-5½	1.10	2 ¾-in. round.	12	61	6	J. C.
65	1-3-5½	1.10	2 ¾-in. round.	12	60	6	J. C.
66	1-5-10	1.25	4 ½-in. corrugated bars.	12	59	6, 10	J. C.
67	1-5-10	1.25	4 ½-in. corrugated bars.	12	60	6, 10	J. C.
68	1-3-5½	1.65	3 ¾-in. round. 10 ½-in. stirrups.	12	60	6, 10	J. C.
69	1-2-4	1.65	3 ¾-in. round. 10 ½-in. stirrups.	12	61	6	J. C.
70	1-3-5½	1.65	3 ¾-in. round. 10 ½-in. stirrups.	12	60	6	J. C.
71	1-5-10	1.65	3 ¾-in. round. 10 ½-in. stirrups.	12	60	6, 10	J. C.
72	1-3-5½	1.10	2 ¾-in. round. 10 ½-in. stirrups.	12	60	6	J. C.
73	1-3-5½	1.10	2 ¾-in. round. 10 ½-in. stirrups.	12	60	6	J. C.
74	1-3-5½	0.98	4 ½-in. round.	10	62	12	E. S.

NOTE:—E. S. and J. C. refer to Engineering Experiment Station and Joint Committee respectively. For explanation of terms, see "4. Materials," p. 3.

6. *Making of the Beams.*—The beams were made directly on the concrete floor of the laboratory, a strip of building paper being laid beneath the forms. Several proportions of concrete were used, varying from 1-2-4 to 1-5-10 by loose volume. A number of the first beams were made of 1-3-6 mixture, and the later ones of 1-3-5½, as it was thought that the voids in the stone were not properly filled in the 1-3-6 mixture. The forms, which were of the ordinary wooden knock-down type, were removed four days after making the beams and the beams were not moved in any way for 14 days. Generally the stone for the concrete was dampened and the concrete well mixed and wet enough to secure proper hardening. The making of the beams

was skillfully done. In Beams No. 3 to 13 inclusive, however, through some oversight the stone (a porous material), was not dampened, insufficient water was used in mixing, the making and

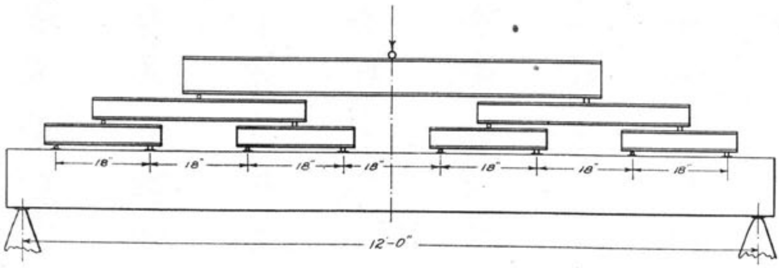

FIG. 1. 8-POINT LOADING.

tamping were not properly done, and the concrete was allowed to become too dry. The beams so made proved to be of inferior concrete and are referred to as poorly made concrete. The low results obtained are of interest in showing the effect of improper methods even if enough cement is used.

7. *Minor Test Pieces.*—Tests were made on 6-in. cubes and on 8-in. cylinders 16 in. high taken from concrete used in some of the beams. The results of these tests are given in Table 4. The values given for the cubes are the averages of three test specimens. For the cylinders, a single test specimen was used.

8. *Storage.*—The beams were stored in a room the temperature of which was from 60° F. to 70° F. They were tested at the age of about 60 days.

9. *Method of Testing.*—The usual method of testing was by loads applied at the one-third points as described in Bulletin No. 4, page 34. The beams were all tested in the 200 000-lb. Olsen testing machine, and in all cases except the tests on "Effect of Method of Loading," were loaded at the one-third points. The method used for loading at eight points is shown in Fig. 1. The supports of the beam allowed longitudinal movement, the bottom of the rocker being an arc of 12-in. radius, and the top, on which cast-iron blocks rested, having a radius of 1½ in. Turned steel rollers, 2 in. in diameter, were used for applying the load at the third points. The blocks at the supports and load points were bedded in plaster of paris which was allowed to harden under the weight of the beam and the apparatus used in loading before the

load was applied. The cubes, cylinders, and steel were tested in the 100 000-lb. Riehlé and 200 000-lb. Olsen testing machines.

Center deflections were read on all the beams. Deformations of the upper fiber and steel were measured by means of four extensometers. The methods of measuring deflections and deformations were fully described in Bulletin No. 4.

TABLE 6.

EFFECT OF QUALITY OF CONCRETE.

Span 12 ft. Loaded at one-third points.

Beam No.	Maximum Load, lb.	k	Per cent Reinforcement	Stress in Steel lb. per sq. in.	Vertical Shearing Stress $v = \frac{V}{bd'}$ lb. per sq. in.	Manner of Failure
			1-2-4 Concrete.			
57	11730	.43	1.15	39800	96	Tension.
59	12000	.53	1.10	44000	97	Tension.
61	10960	.47	1.10	39700	90	Tension.
69*	16000	.57	1.65	39000	129	Tension.
			1-3-5½ Concrete.			
58	9860	.52	1.10	36900	82	Tension.
60	10360	.57	1.10	39400	85	Tension.
64	9850	.55	1.10	37300	82	Tension.
65	10000	.54	1.10	37600	83	Tension.
68*	14220	.57	1.65	35000	115	Diagonal Tension. [crete]
70*	13710	.69	1.65	35800	112	Compression (Poor con-
72*	10270	.47	1.10	37400	85	Tension.
73*	9900	.52	1.10	37000	82	Tension.
			1-5-10 Concrete.			
62	8850	.60	1.10	34600	74	Diagonal Tension.
63	7360	.48	1.10	28100	63	Diagonal Tension.
66†	8000	.63	1.25	29400	69	Diagonal Tension.
67†	7850	.57	1.25	27100	68	Diagonal Tension.
71*	7600	.59	1.65	20200	67	Diagonal Tension.

* Stirrups.

† Reinforced with corrugated bars.

III. Experimental Data and Discussion.

10. *Explanation of Tables 6 to 12.*—In Tables 6 to 12, the position of the neutral axis was obtained by the method used in Bulletin No. 4. In calculating the per cent of reinforcement the area of the beam above the center of the reinforcing bars is used. The columns headed "Maximum Applied Load" do not include the weight of the beam loading apparatus, but these weights were considered in calculating the stress in the steel. In determining the amount of the vertical shear, the weight of the beam and loading apparatus was considered, 6 lb. per sq. in. being added to the unit-stress for a 6-ft. beam, 7 for an 8-ft. beam, 8 for a 10-ft. beam, and 10 for a 12-ft. beam. In obtaining the vertical shear, the formula $v = \frac{V}{.86bd}$ or $.0145V$ was used for 1% beams and $v = \frac{V}{.81bd}$ or $.0154V$ for 2.2% beams.

11. *Effect of Quality of Concrete.*—In this series, tests were made on beams made of three kinds or grades of concrete,—1-2-4, 1-3-5½, and 1-5-10 mixtures. The purpose of this series was to determine the effect of quality of concrete upon the strength of the beam and upon the manner of failure. The beams were planned to give a variety of manners of failure,—tension in the steel, compression in the concrete, and diagonal tension in the concrete. Table 6 gives the results of this series. The calculations were made as described, under "10. Explanation of Tables". No attempt is made to calculate the diagonal tensile stresses developed, but the ability to resist diagonal tension will be compared by means of the vertical shearing stresses developed. The load was applied at the one-third points of the beams. From the tables it will be noted that the manner of failure for beams of these proportions depends upon the richness of the concrete. Counting the effect of the weight of the beam and loading apparatus, it is seen that all the beams made with 1-2-4 concrete failed by tension in the steel at calculated stresses somewhat above the elastic limit of the steel. Beam No. 57, (Fig. 4), is typical of the appearance of the beams after failure. Beam No. 69, having 1.65% reinforcement, has a load-compression diagram (Fig.12) which indicates that the stress in the concrete at the maximum load was well within the limits of its ultimate compressive strength. The vertical shearing stress developed in this

beam was 124 lb. per sq. in. and there was no indication of approaching failure by diagonal tension.

Of the test beams of 1-3-5½ concrete all but two failed by tension in the steel. The calculated stresses in the steel, when allowance is made for the weight of the beam, are slightly above the elastic limit of the steel. The two not failing by tension in the steel, when compared by the stresses computed either from the bending moment or the observed deformations, gave stresses in the steel not much lower than the average of the remainder of the series. Both of these beams, therefore, had hardly reached the load which would have been followed by failure by tension in the steel. Beam No. 68 failed by diagonal tension at a calculated vertical shearing stress of 115 lb. per sq. in. No. 70 failed by compression of the concrete. The compression diagram (Fig. 13) shows that the concrete in this beam lacked stiffness, the amount of deformation being more than for the average beam but not much more than that of No. 68, its companion beam, as is seen from the load-deformation diagrams. And yet this beam of 1-3-5½ concrete and 1.65% reinforcement carried a load nearly to the elastic limit of the steel reinforcement.

All the beams made with 1-5-10 concrete failed by diagonal tension at loads which show a rather narrow range regardless of the amount or method of reinforcement. The vertical shearing unit-stress developed averaged 68 lb. per sq. in. As the beams failed by diagonal tension at loads much smaller than those at which failure by compression in the concrete may be expected, there is nothing in these tests upon which to base the limit of the concrete or the amount of reinforcement at which the compressive strength of the concrete and the tensile strength of the steel may be considered to be balanced.

A comparison of beams having 1.1% reinforcement which failed by tension in the steel shows that the 1-2-4 beams carried greater loads than the 1-3-5½ beams, the additional load amounting to 10% or 15%. This increase of load probably is due to the fact that the greater strength of the richer concrete allows the steel to be stretched a greater distance beyond the elastic limit before developing the full compressive strength of the concrete and also that the moment arm of the couple formed by the compressive stresses is somewhat greater with the richer concrete. The added strength of the richer concrete in preventing failure by diagonal tension is apparent. This feature of the series will be further discussed under "14. Diagonal Tension Failures". As will be shown afterward, the arrangement of stirrups used was not well planned and their presence seemed not to add to the strength of the beams, although in Beams No. 68 and 71 it might have been expected that well designed stirrups would prevent failure by diagonal tension.

12. *Effect of Method of Loading.*—It was the purpose of this series to determine the effect of the method of loading upon the resisting moment developed in the beam. With this in view, the beams were so proportioned that failure by tension in the steel was expected in all cases. In Bulletin No. 4, page 54, a discussion of this topic is given. It was there stated that beams loaded at the middle have been found to develop a higher moment of resistance than is to be expected if the distribution of stresses is as assumed in the ordinary theory of flexure. Six methods of loading were used in the tests of this series: (1) load applied at center of the span only; (2) load applied at two points 1½ feet apart; (3) load applied at two points 3 feet apart; (4) load applied at the one-third points; (5) load applied at two points 7½ feet apart; (6) load applied at eight points (to approximate a uniform load). The appliances used for the loading at eight points have been described under "9. Method of Testing".

Table 7 gives the results of these tests, together with the calculated stresses in the steel. All beams failed by tension in the steel, as was clearly shown by the load-deformation diagrams. If the effect of the weight of the beam and loading apparatus is included, it will be seen that the calculated stresses in the steel all lie above the elastic limit. A comparison of the resisting moments of the beams may be made by comparing the calculated stresses in the steel, given in Table 7, since the tests of the steel used in these beams show that there was little variation in the yield point of the test pieces. It will be noted that the highest stress developed was in beams having the loading at the middle, and that when the two loads were close to the middle the results were not much lower. For the other methods of loading the variation in stress developed was not large, no greater than may be expected with the difference in the materials and fabrication in such beams, though the method in which the load was applied at eight points gave a somewhat higher resisting moment. These

TABLE 7.

EFFECT OF METHOD OF LOADING

Reinforcement .98%. Concrete 1-3-5½. Span 12 ft. All failed by tension in the steel.

Beam No.	Method of Loading	Maximum Applied Load lb.	k	Stress in Steel lb. per sq. in.
50	Center.	7400	.45	45200
52	Center.	7650	.38	45400
53	2 points 1½ ft. apart.	7900	.45	42500
54	2 points 1½ ft. apart.	8250	.42	43500
55	2 points 3 ft. apart.	8900	.44	40700
56	2 points 3 ft. apart.	9610	.51	45000
40	One-third points.	10000	.46	41000
42	One-third points.	9420	.47	39000
45	2 points 7½ ft. apart.	17300	.50	40100
51	2 points 7½ ft. apart.	18000	.45	40700
43	8 points 18 in. apart.	14000	.44	42100
44	8 points 18 in. apart.	15000	.41	44300

tests go to show the general applicability of the ordinary beam theory to simple beams without end restraint or horizontal restraint for any of the usual methods of loading, with the exception of center loading, provided, of course, that the proportions of the beam are such that the method of failure is by tension in the steel. It will be seen that the beams loaded at the middle give about 10% greater resistance than the more usual methods of loading. This excess is not so great as has been found in beams having a high percentage of reinforcement. With high reinforcement the resulting moment developed is considerably greater than for loading at the one-third points. Evidently under such conditions loading at the middle gives a distribution of stresses at sections near the center of the beam which is different from that assumed in the ordinary theory of flexure.

The load-deformation curves have the same general characteristics in all of the beams. The position of the neutral axis, as determined by the method used, is nearly the same for the several methods of loading, the variation being as little as may be expected in tests of this character and no characteristic difference being noticeable in any method of loading. Of course, with the load applied in the middle the deformations were taken over so great a gauged length that a discrepancy in the distribution of stress at a section at the middle had little effect on the values found.

13. *Effect of Repetitive Loading.*—Tests were made on six beams to determine the effect of repeatedly applying and releasing the load on the beam, from 24 to 30 applications of a single load being made. Three mixtures of concrete were used, thus permitting a study of the effect of quality of concrete. All beams were reinforced with 1% of steel. The load applied was 5000 lb.

TABLE 8.

EFFECT OF REPETITIVE LOADING.

Span 12 ft. Loaded at one-third points.

Beam No.	Mixture	Age days	Per cent Reinforcement	Repeated Load lb.	No. of Applications	Failed at lb.	Manner of Failure
28	1-2-4	70	0.98	6000	26	9900	Tension.
31	1-2-4	78	0.98	6000	25	9400	Tension.
29	1-3-5½	71	0.98	5000	30	10000	Diagonal tension.
32	1-3-5½	79	0.98	6000	24	9800	Tension.
30	1-4-7½	71	0.98	5000	30	5900	Diagonal tension.
35	1-4-7½	75	0.98	5000	30	7500	Compression.

TABLE 9.

DEFLECTIONS UNDER REPETITIVE LOADING.

Beam No.	Mixture	Center Deflection in inches for 5000-lb. Load.						
		Application						
		1st	5th	10th	15th	20th	25th	30th
28	1-2-4	0.25	0 33	0.34	0.36	0.37	0.38	
31	1-2-4	0.25	0.33	0.35	0.35	0.36	0.37	
29	1-3-5½	0.15	0.25	0.25	0.25	0.25	0.26	0.25
32	1-3-5½	0.24	0.31	0.33	0.34	0.34		
30	1-4-7½	0.40	0.49	0.54	0 57	0.60	0.62	0.65
35	1-4-7½	0.31	0.38	0.40	0.41	0.41	0.41	0.42

in three beams, and 6000 lb. in the other three. These loads were from 50 % to 85 % of the maximum load carried by the beam when the load was finally increased to the point of failure. The deflection at the midpoint of the span was measured, as were the deformations at the top and bottom for a gauged length along the middle of the beam, though the latter measurements were not entirely satisfactory. Table 8 and Table 9 give results of these tests.

The following notes show the principal features of the tests of the several beams.

Beam No. 28. The beam was made of 1-2-4 concrete. The general behavior of this beam during the repetition of the 6000-lb. load is representative of the action of the other beams. Hair cracks appeared on the tension side of the beam in the middle third of the length, one or more of them usually on the first application of the load and others at subsequent applications, as shown in Fig. 2. In this beam hair cracks appeared at the third, eighth, thirteenth, and twenty-sixth applications. In each case the cracks closed upon the removal of the load. Upon increasing the load after the twenty-sixth application of 6000 lb., the cracks opened still further. Finally the crack marked 8 opened much more rapidly, the beam failed by tension in the steel, and this was followed by the load dropping off and the concrete finally crushing at the top at a load much less than the maximum. It may be noted that the deflection of the beam increased with the repetition of the 6000-lb. load, the amount at the twenty-fifth application being 50% more than at the first. The position of the cracks is shown in Fig. 2, the numbers indicating the applications at which the cracks were noted.

Beam No. 31. The beam was of 1-2-4 concrete. Hair cracks appeared at the second, fifth, seventh, tenth, and twenty-fifth applications of the load of 6000 lb. On increasing the load after the twenty-fifth application, the cracks opened, and at 8000 lb. crack No. 2 (Fig. 2) rapidly widened. At a load of 9400 lb., the steel passed its yield point and as usual this was finally followed with the crushing of concrete at the top of the beam. The increase in the deflection of the beam at the twenty-fifth application over that at the initial application of the load was 50%. It should be noted that the diagonal crack marked 10 formed at the 10th application of the load, but, although it widened when the load was increased beyond 6000 lb., it did not cause failure. The vertical shearing unit-stress, as calculated by equation 18 (See page 20, Bulletin No. 4), was 53 lb. per sq. in. for a load of 6000 lb., and 78 lb. per sq. in. at failure.

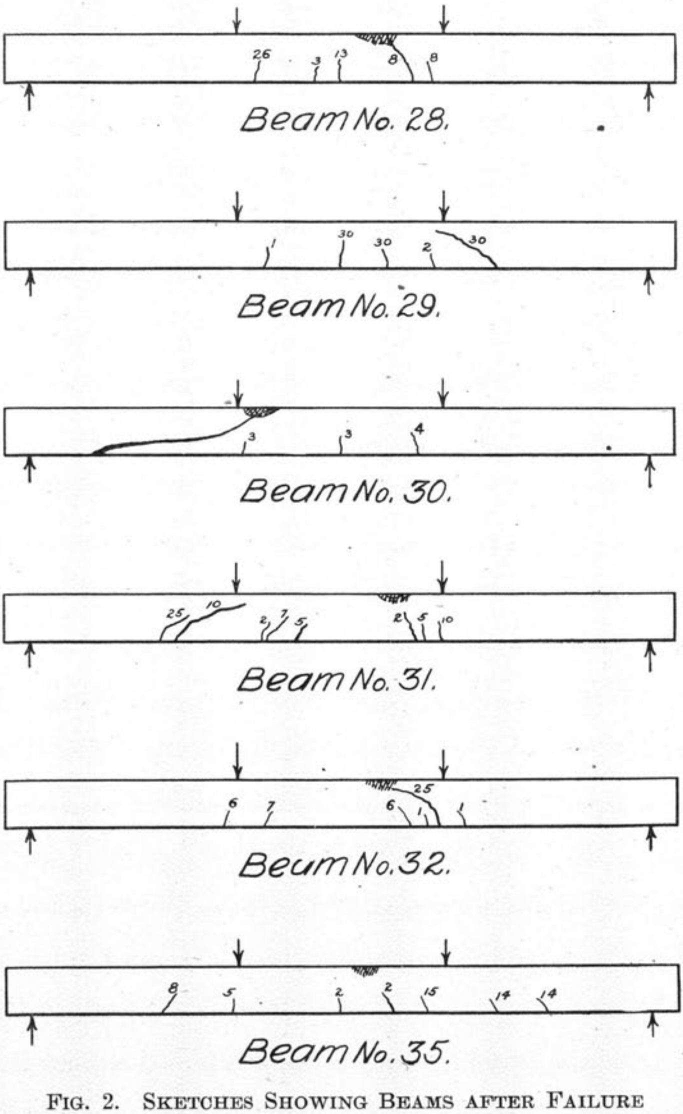

FIG. 2. SKETCHES SHOWING BEAMS AFTER FAILURE UNDER REPETITIVE LOADING.

Beam No. 29. The beam was made of 1-3-5½ concrete. A load of 5000 lb. was applied 30 times. Hair cracks appeared as shown in the sketch in Fig. 2. The deflection for the load of 5000 lb. increased 67% during the repetitions. At a load of 9000 lb. a diagonal crack appeared one foot outside the load point, and the beam finally failed by diagonal tension along this crack. The vertical shearing unit-stress at this load was 76 lb. per sq. in., and at the maximum load 83 lb. per sq. in.

Beam No. 32. The beam was made of 1-3-5½ concrete. A load of 6000 lb. was applied 24 times. Two hair cracks appeared at the first application, and several more at later applications of the load, but no additional effect was observed. As the load was finally increased to an amount near 9400 lb., a crack near the load point appeared, and failure by tension in the steel at this point followed at a maximum load of 9800 lb. The repetition of the load gave a greatly increased deformation in the upper fiber (See Fig. 14) but when the load was increased beyond 6000 lb., the upward direction of the deformation curve for the upper fiber indicates that the concrete had not lost its strength or elasticity. The changes in the deflection are shown in Fig. 10.

Beam No. 30. The beam was made of 1-4-7½ concrete. A load of 5000 lb. was applied 30 times. Hair cracks appeared on the third and fourth applications. On increasing the load a diagonal crack appeared outside the load point, and the beam failed by diagonal tension at a maximum load of 5900 lb., followed by a stripping of the bars for some distance beyond. The vertical shearing unit-stress for this load was 53 lb. per sq. in. The exposed ends of the bars showed a slip in the concrete after the maximum load was reached. Fig. 8 gives the changes in deflection.

Beam No. 35. The beam was made of 1-4-7½ concrete. Thirty applications of the load of 5000 lb. were made. Fine hair cracks appeared at the bottom over the the middle third during the repetition, and a few outside of the load points. The beam finally failed by compression in the upper face of the beam at a load of 7500 lb. It seems possible that the strength of the concrete may have been affected by the repetition of stress, although it is more likely that the test is an example of the effect of poor concrete.

These tests throw light upon the phenomena of repetitive loading and show the need of further investigation in this direction, but they are not at all conclusive. The manner of failure in general is the same as may be expected with beams of the same reinforcement and same quality of concrete loaded progressively to final failure. Whether the maximum load carried in the case of the repetitive loading is less than would have been the case with progressive loading is not known. There are some indications that the maximum load was less than it would have been without repetition.

The increase in the deflection of the beams with repetition of the load is quite apparent. Much of this increase is due to the increased amount of shortening of the concrete in the compression side of the beam with repetition. A part is due to the breaking of the concrete in tension and the transferring of the tensile stress once taken by the concrete to the steel itself. This accounts for part of the set in the deflection curve upon the release of the load. Part of the set must be due to the concrete in the lower fiber not meshing, so to speak, when the load is released after numerous fine cracks have appeared. For this reason some tension remains in the steel reinforcement after the load is taken off. It seems evident that upon the removal of the load the beam does not regain its original shape and a section which was plane before bending will not be plane upon release of the load. The plastic nature of the concrete on the compression side gives a set, and the concrete on the tension side is unable to return to its original position; the two act together to cause the fibers not to return to the original plane section. These several causes operate together to produce the permanent deflection or set.

The load applied in the cases of the leaner concretes was 67% to 85% of the maximum load which the beam finally held. In the case of the better concrete, the repeated load was 50% to 60% of the maximum load. The effect of the quality of the concrete is seen in the manner of failure.

This topic is one of such importance that it merits fuller investigation. The few tests which have been made indicate that the deflection and the deformations increase with repetition. It seems quite probable that the breaking load under a number of repetitions will be smaller than under a single load. It seems to be true also that the amount of reinforcement for which the elastic tensile strength of the steel used for reinforcement may be considered to balance the compressive strength in the concrete of the beam (which the writer calls "the balanced reinforcement") should be taken at a lower percentage in beams subjected to a repetition of load than is found necessary in the case of beams tested by means of a gradually applied load.

The term "balanced reinforcement" referred to above is a convenient term for general use. It should be taken to mean that amount of reinforcement for which the allowable stress in the steel and the allowable stress in the concrete both exist at the same time. The factors of safety for the two materials will not be the same. The determination of the balanced reinforcement for given conditions of materials, fabrication, and use is a matter involving calculation and experimentation, but in any event the judgment of the designer must enter into the choice of the amount.

14. *Diagonal Tension Failures.*—As shown in Bulletin No. 4 (pages 20, 21, and 26), certain secondary stresses or web stresses exist in the concrete of a reinforced concrete beam in addition to the horizontal or longitudinal tensile and compressive stresses which are always considered in the analysis. Strictly speaking, the shearing stresses developed under ordinary conditions are relatively light, and the actual shearing strength of concrete is considerably greater than the shearing stress which exists in ordinary beams at the time of failure. It is quite common, however, to use the term "shearing failure" as a name for a class of failures in the web of a beam, but it must not be understood from this use of terms that the failure necessarily involves actual failure by shear. Generally speaking, such failures are due to the inability of the concrete to resist the tensile stresses developed in the web in a diagonal direction, and the term "diagonal tension failure" is a much more appropriate name for this form of failure. It is a principle of mechanics that where shearing stresses exist tensile and compressive stresses are set up at an angle with the direction of the shearing stresses. If longitudinal tensile stress also exists in the concrete, the diagonal tensile stress induced by the combination of these with the shear is even higher than that due to shear alone. If v represents the horizontal and vertical unit-stress at any point in the web of a beam and s the horizontal tensile unit-stress existing in the concrete at the same

point, then, as shown in Bulletin No. 4, the formula for the maximum diagonal tensile unit-stress is

$$t = \tfrac{1}{2}s + \sqrt{\tfrac{1}{4}s^2 + v^2} \quad \text{.......................} \quad (19)$$

If there is no longitudinal tension in the concrete, this formula reduces to

$$t = v \quad \text{.......................................} \quad (20)$$

and maximum diagonal tension makes an angle of 45° with the horizontal and is equal in intensity to the vertical shearing stress.

It is evident then that the amount of this diagonal tension is dependent upon both the shearing stress and the longitudinal tensile stress in the concrete at the point considered. The amount of longitudinal tension is not easy to determine and hence the actual amount of the diagonal tensile stress is uncertain. The best method for ordinary computation seems to be to compute the vertical shearing unit-stress and make all calculations upon the basis of this value. The value of the vertical shearing unit-stress, where the longitudinal reinforcement is straight (not bent up or inclined), may be computed from the formula given in Bulletin No. 4,

$$v = \frac{V}{bd'} \quad \text{.......................................} \quad (18)$$

where V is the total external vertical shear at the section considered, b is the breadth of the beam, d' is the distance from the center of the steel to the center of the compressive stresses. For beams with 1% reinforcement d' is about 0.86 d, d being the distance from the center of the steel to the upper face of the beam.

The value of v thus calculated for beams which fail by diagonal tension ranges from one-half to one-third of the tensile strength of the concrete. Diagonal tension failures are frequently characterized by sudden failures without much warning, as is the case in the failure of plain concrete beams. A variation from this gives a slower failure, part of the shear being carried through the reinforcing bars, and the ultimate failure involving the splitting and stripping of the bars from the beam above. When the reinforcing bars are bent up or inclined toward the ends of the beams the distribution of the vertical shear is different from that just outlined and the analysis is more complex. However, for purposes of comparison, the use of equation (18) is advantageous, and the values of v given in the tables for beams with bars inclined are calculated by this formula, using d as 10 in., though the amounts as calculated do not represent the actual vertical shear.

Forty test beams were made with a view of studying diagonal tension failure. To make a sufficient variety of conditions the concrete was varied from a fairly rich mixture to a very lean concrete. The first of the 1-3-6 concrete beams numbered up to 13 were very poorly made, as described under "6. Making of the Beams", and the general appearance of the concrete and its action during the tests go to show that the concrete was of a very inferior quality. The beams were made with the same depth, and their length was varied to give a variable relation between depth and span.

Table 10 gives the results of beams having the reinforcing bars horizontal and in which failure occurred by diagonal tension. The beams are grouped according to the quality of the concrete.

TABLE 10.

DIAGONAL TENSION FAILURES.

BARS HORIZONTAL.

All beams loaded at one-third points.

Beam No.	Span ft.	Per cent Reinforcement	k	Maximum Applied Load lb.	Vertical Shearing Stress lb. per sq. in. $v=\frac{V}{bd'}$	Manner of Failure
			1-5-10 Concrete.			
62	12	1.10	.60	8850	74	Diagonal tension.
63	12	1.10	.48	7360	63	Diagonal tension.
66	12	1.25	.71	8000	69	Diagonal tension.
67	12	1.25	.57	7850	68	Diagonal tension.
71	12	1.65	.59	7600	67	Diagonal tension.
Av.	......	....	......	....	68	
			1-4-7½ Concrete.			
30	12	0.98	.71	5900	53	Diagonal tension.

TABLE 10—*Concluded*

Beam No.	Span ft.	Per cent Reinforcement	k	Maximum Applied Load lb.	Vertical Shearing Stress lb. per sq. in. $v = \frac{V}{bd'}$	Manner of Failure
			1-3-6 Concrete.			
3	12	0.98	.48	7380	64	Diagonal tension.
5	12	0.98	.44	6200	55	Diagonal tension.
6	10	0.98	.47	7050	59	Diagonal tension.
8	10	2.21	.56	8470	73	Diagonal tension.
9	10	2.21	.64	9150	79	Diagonal tension.
15	8	0.98	.52	13000	107	Diagonal tension.
16	8	0.98	.41	12420	97	Diagonal tension.
18	6	0.98	.40	13000	101	Diagonal tension.
19	6	0.98	.54	12760	99	Diagonal tension.
Av.	......	....	.....	....	81	
			1-3-5½ Concrete.			
11	12	0.98	.52	10000	83	Diagonal tension.
29	12	0.98	.55	10000	83	Diagonal tension.
68	12	1.65	.57	14220	115	Diagonal tension.
23	10	2.21	.61	13830	115	Diagonal tension.
27	8	2.21	.67	11000	92	Diagonal tension.
34	8	2.21	.76	11230	94	Diagonal tension.
37	8	2.21	.70	15140	114	Diagonal tension.
25	6	0.98	.56	15430	118	Diagonal tension.
33	6	0.98	.57	15670	120	Diagonal tension.
46	6	0.98	.34	13700	106	Diagonal tension.
49	6	0.98	.49	19000	144	Diagonal tension.
Av.	......	....	......	....	108	
			1-2-4 Concrete.			
47	6	0.98	.46	18800	142	Diagonal tension.
48	6	0.98	.38	17940	136	Diagonal tension.
Av.	......	....	......	....	139	

The value of v, calculated by equation (18), offers a means of comparison of the resistance of the concrete to failure by diagonal tension. The effect of lean concrete and of poorly made concrete is quite evident. The results are instructive. The low values for the poor concrete may be helpful as a warning against assuming high web stresses for beams in which the concrete may not be well made.

The range of the values of the vertical shearing unit-stress v found in these tests with beams having the reinforcing rods in a horizontal position may be summarized as follows, the results being obtained with a single application of the load on beams about 60 days old:

1-2-4 concrete..............	136 to 142 lb. per sq. in.	Av. 139 lb. per sq. in.
1-3-6 concrete..............	92 to 115 lb. per sq. in.	Av. 99 lb. per sq. in.
1-3-6 poorly made concrete.	55 to 83 lb. per sq. in.	Av. 69 lb. per sq. in.
1-5-10 concrete...	63 to 74 lb. per sq. in.	Av. 68 lb. per sq. in.

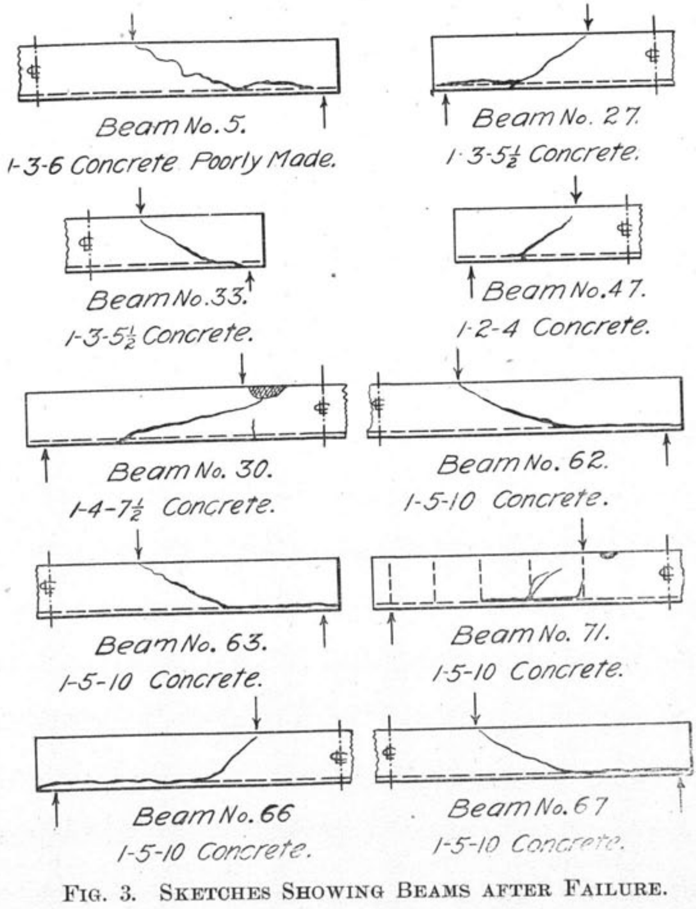

Fig. 3. Sketches Showing Beams after Failure.

The one beam of 1-4-7½ concrete which failed in this way gave 43 lb. per sq. in. This beam was subjected to repetitive loading. These results show the importance of using a rich concrete in the web of reinforced concrete beams which are subjected to any considerable amount of diagonal tension when there is no metallic web reinforcement or when the web reinforcement is not effective. It is probable that not enough attention has been given to this element in the design of short and deep beams.

Fig. 3 gives sketches showing the cracks which were observed in these beams. The sketches represent the position of the cracks after the failure of the beam, or after the load had reached a maximum, and do not indicate the position or extent of the cracks within the maximum load. Fig. 4 and 5 are reproduced from photographs and give the appearance of beams after failure. Generally speaking, the crack when first observed, extended from the bottom of the beam to the steel reinforcement and from the steel reinforcement diagonally a short distance toward a load point, although in some cases the diagonal crack was observed before the vertical crack was visible. Sometimes this diagonal crack was observed before the maximum load was reached and sometimes not until the maximum load had been passed, or even until after the beam had failed quite suddenly. In some of the beams the diagonal crack was seen to extend forward slowly toward the load point before or just after the maximum load was reached and then a horizontal crack grew along the level of the top of the reinforcing bars toward the support. The phenomena of final failure were frequently connected with the slipping of the reinforcing bars or with the stripping of these bars from the concrete above as was described in Bulletin No. 4 for a former series of tests. The slipping of the bars which occurred was not observed until the maximum load had been passed, and generally in these cases the crack also extended along the bars. It seemed evident to the observers that this slip did not occur before the maximum load was applied and before the existence of the diagonal crack had materially modified the conditions in the beam. Beams No. 66 and 67 (See Fig. 3 and 5) throw some light upon this matter. They were made for this purpose with very lean concrete and reinforced with corrugated bars. The condition of the beam at failure showed that the horizontal crack was due to vertical tension and that horizontal

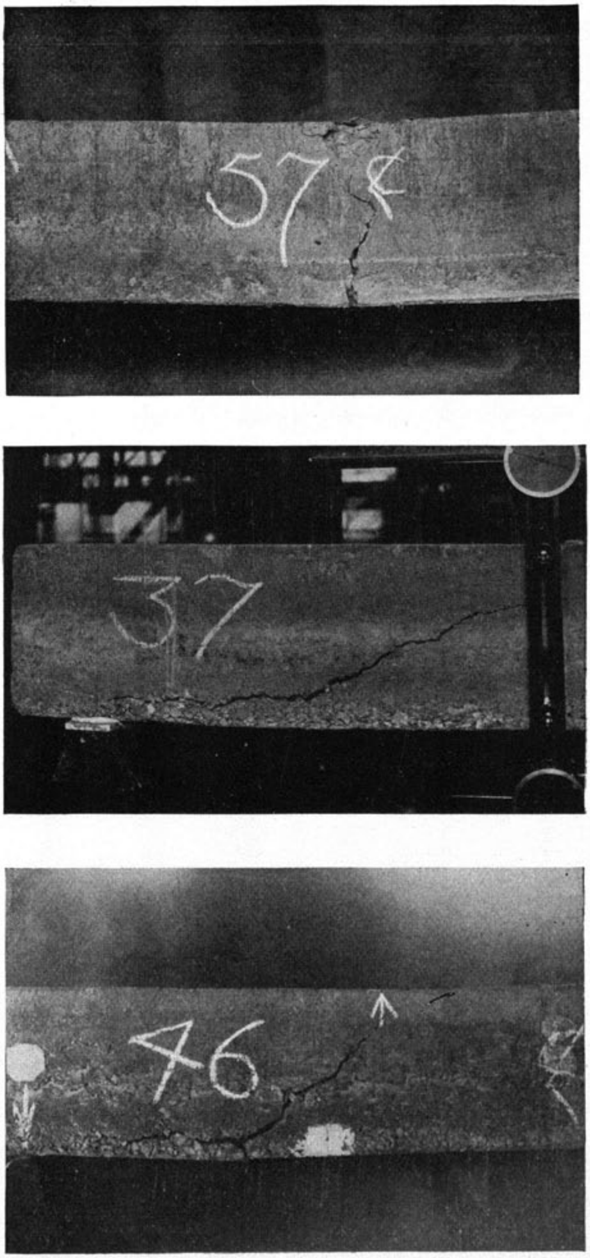

FIG. 4. VIEWS SHOWING BEAMS AFTER FAILURE.

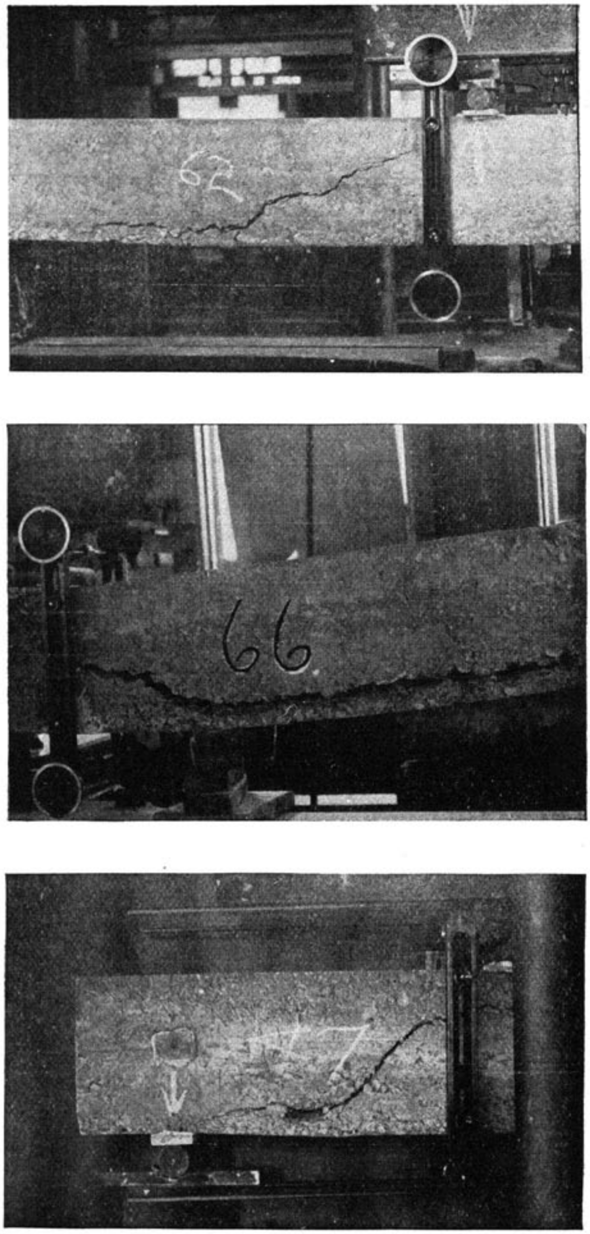

FIG. 5. VIEWS SHOWING BEAMS AFTER FAILURE.

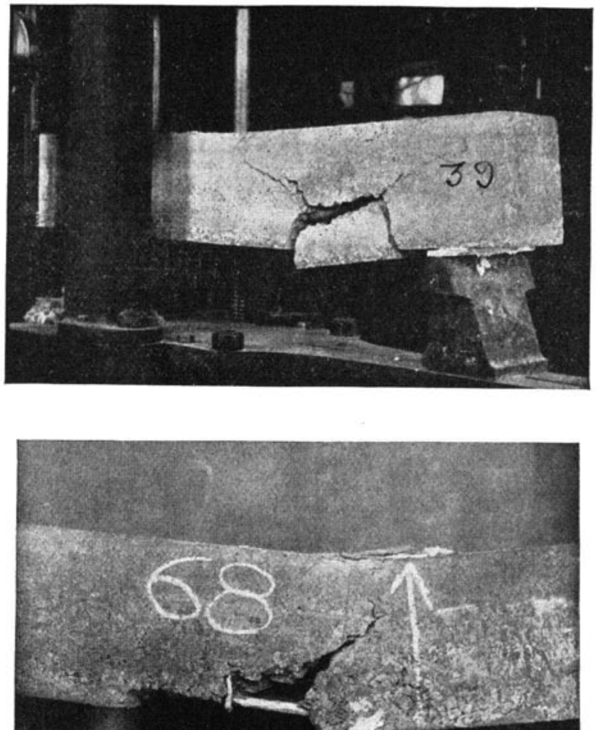

FIG. 7. VIEWS SHOWING BEAMS AFTER FAILURE.

shear or slip did not take place until after this crack had been formed. The indentations in the concrete formed by the corrugations of the bars were left in perfect condition and there was no crushing or tearing at the edges of these indentations. The bar had simply been pulled down and out of the place in which it had rested. Comparisons with the results of beams made with the same concrete and with smooth steel (Beams No. 62, 63, and 71) show values almost identical and go to indicate that slipping had no part in the critical failure of the beams made up with smooth bars. In all the cases where slipping of the bar took place the action extended progressively from the diagonal

TABLE 11.

DIAGONAL TENSION FAILURES.

BARS BENT UP.

All beams loaded at one-third points.

Beam No.	Span ft.	Per cent Reinforcement	k	Maximum Applied Load lb.	Vertical Shearing Stress lb. per sq. in. $v=\frac{V}{bd'}$	Manner of Failure
1-3-6 Concrete.						
7	10	0.98	.44	8370	69	Diagonal tension.
13	10	2.21	.75	9320	80	Diagonal tension.
14	10	2.21	.66	10560	89	Diagonal tension.
17	8	0.98	.46	9150	74	Diagonal tension.
20	6	0.98	.48	11620	90	Diagonal tension.
21	6	0.98	.48	16500	126	Diagonal tension.
22	6	0.98	.49	16760	128	Diagonal tension.
Av.	......	....	....	....	95	
1-3-5½ Concrete.						
36	10	0.98	.58	7910	64	Diagonal tension
38	8	2.21	.70	11920	99	Diagonal tension
39	8	2.21	.73	14000	115	Diagonal tension
Av.	.....	....	.....	....	93	

crack toward the load point, though in some cases this action was quite sudden. From all the information available it seems to be evident that, whatever slip of the bars may have taken place, the slipping did not exist before the time of the maximum load and resulted from the changed conditions incident to the formation of the diagonal crack. It therefore seems evident that the failure of these beams should be credited to diagonal tension.

Table 11 gives results of beams with reinforcing bars bent up, or inclined in the outer thirds of the length of the beam. The values of v are calculated by equation (18) and with d as 10 in., and hence do not give the actual amount of the shear. The amount and position of this bending have been described under "5. Test Beams". Fig. 6 gives the sketches of the appearance of

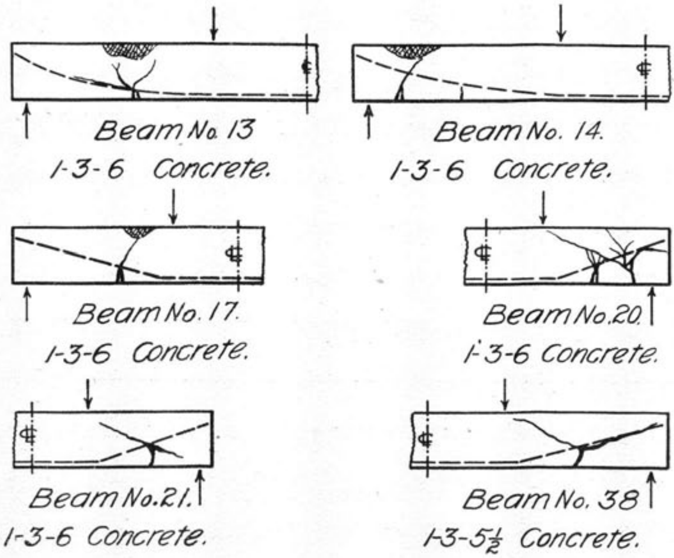

FIG. 6. SKETCHES SHOWING BEAMS AFTER FAILURE.

the crack after final failure or after the maximum load had been passed. The view of Beam No. 39, shown in Fig. 7, is from a photograph of the beam after failure. Generally speaking, the vertical portion of the crack from the bottom of the beam to the reinforcement formed first, and was due to the failure of the concrete in tension. The diagonal crack then grew toward the load point, generally forming before the maximum load was reached, and the growth of the crack along the reinforcing bar generally followed. It was expected that this method of bending up bars would give a higher value for the vertical shear as calculated by equation (18) in beams failing by diagonal tension than that found in beams with the bars horizontal, but in this the results were disappointing. In a few beams the values ran higher. Comparing Beam No. 21 with Beam No. 22 it will be seen that no difference was observed whether part of the bars were bent up and part left horizontal, or all were bent up. The values of v for these beams were among the highest found with this quality of concrete. In the phenomena of failure it appeared that the element of slip was present, though it is not known that this slip occurred before failure. Calculations indicate that the bond stress developed at the end of the bars must have been considerable. It should be noted that none of the bars were anchored at the ends.

Table 12 gives the beams in which failure occurred by tension in the steel or compression of the concrete. It will be seen that the dimensions of these beams were such that the diagonal tension developed (as measured by the vertical shear) at the time of failure was less than was found with beams which failed in diagonal tension and hence that the strength of the concrete in diagonal tension had not been reached. Beam No. 35 which failed in compression, was of 1-4-7½ concrete, a very lean mixture. Beam No. 10 was one of the beams with poorly made concrete and in this case the inferior quality of the concrete was especially noticeable.

In Beams No. 68, 69, 70, 71, 72, and 73, U-shaped stirrups of ½-in. mild-steel round rods were placed in a vertical position and enveloped the horizontal reinforcing bars. The longitudinal spacing was inadvertently made 12 in. Beam No. 69 failed by tension in the steel at a value of v of 129 lb. per sq. in., which is below the resistances developed in the beams of 1-2-4 concrete which failed in diagonal tension, and the efficiency of the stirrups was not determined. Beams No. 72 and 73 also failed by tension in the steel at values of v below what was found in beams of 1-3-5½ concrete of the same quality which failed in diagonal tension. Beam No. 70 failed by compression of the concrete, but a diagonal crack had formed, before the maximum load was applied, at a value of v which is high for 1-3-5½ concrete, and the stirrups seemed to be effective in preventing sudden failure after the

TABLE 12.

MISCELLANEOUS FAILURES.

All beams loaded at one-third points.

Beam No.	Span ft.	Per cent Reinforcement	k	Maximum Applied Load lb.	Vertical Shearing Stress lb. per sq. in. $v=\frac{V}{bd'}$	Manner of Failure
1-4-7½ Concrete.						
35	12	0.98	.68	7500	64	Compression.
1-3-6 Concrete.						
4	12	0.98	.48	9360	78	Tension
10	12	0.98	.64	9430	77	Compression.
12	10	2.21	.79	10200	89	Compression
Av.	......	....	......	....	81	
1-3-5½ Concrete						
32	12	0.98	.57	9800	81	Tension.
74	10	0.98	.52	11950	95	Tension.
26	8	0.98	.51	14030	109	Tension.
Av.	......	....	......	....	95	
1-2-4 Concrete.						
28	12	0.98	.36	9900	82	Tension.
31	12	0.98	.48	9400	79	Tension.
Av.	......	....	......	....	80	

maximum load was passed. Beam No. 68 (Fig. 7) failed in diagonal tension at a value of v of 115 lb. per sq. in. At a load of 13 000 lb. the diagonal crack extended 7 in., and at the maximum load, 14 220 lb., it extended to a point under the load point and was $\frac{1}{32}$ in. wide. The load fell off very slowly, and the stirrups prevented sudden failure. Beam No. 71 (Fig. 7) failed by diagonal tension at a value of v nearly the same as the companion beams of 1-5-10 concrete which did not have stirrups. The load dropped off rapidly after the maximum load was reached and the stirrups seemed to have little effect. It is clear that the spacing of the stirrups in these beams caused them to be inefficient, the distance apart being too great, and besides, the stirrups were not properly placed in the beam. Further tests are now in progress, in which the dimensions of the beam and the size and spacing of the stirrups are expected to bring out the effectiveness of this method of metallic web reinforcement.

15. *Summary.*—The following summary of parts of the foregoing discussion is given:

1. For beams proportioned to give failure by tension in the steel reinforcement (i. e., when neither compression of concrete nor diagonal tension causes failure), those made with the richer concrete carried higher loads. The beams with 1-2-4 concrete carried loads greater by, say, 10%, than those with 1-3-5½ concrete.

2. In beams which failed by tension in the steel, the resisting moment developed was found to be about the same for loads applied at two points more or less far apart and at eight points (approaching a uniform load), thus confirming the general applicability of the ordinary beam theory to simple beams without end restraint or horizontal restraint for any of the ordinary methods of loading. For center loading the resisting moment developed ran 10% higher and in former tests even greater. This excess indicates a different distribution of stresses for center loading from that assumed in the ordinary beam theory.

3. The tests with repetitive loading are not conclusive and show the need of further investigation in this direction. The manner of failure in general was the same as may be expected with beams of the same reinforcement and same quality of concrete loaded progressively to final failure. Whether the maximum load carried in the case of this repetitive loading is less than would have been the case with progressive loading is not known, but there are some indications that the maximum load was less than it would have been without repetition. The in-

crease in the deflections of the beam with repetition of the load was marked, and the set in the beam was considerable. It should be remembered that the repetitive load was a considerable proportion of the maximum load finally applied and much higher than ordinary working loads. It seems evident that the amount of reinforcement for which the elastic tensile strength of the steel may be considered to balance the compressive strength of the concrete, (conveniently called the "balanced reinforcement"), should be taken at a lower percentage in beams subjected to a repetition of load than is found necessary in the case of beams tested by means of a gradually applied load, and that for the ordinary conditions of fabrication and use the "balanced reinforcement" selected should be much less than that determined by test beams.

4. The manner of failure depends not only upon the relative dimensions of depth and length of beam and the amount of reinforcement, but also upon the richness and strength of the concrete.

5. The loads carried by beams failing by diagonal tension depended both upon the richness of the concrete and upon its quality as influenced by the methods of mixing and storing. Poorly made concrete gave a vertical shearing stress averaging 71 lb. per sq. in. as compared with 99 lb. per sq. in. for well made concrete of the same mixture. The value, 138 lb. per sq. in., for the 1-2-4 concrete shows the advantage of the richer mixture.

6. Failure by diagonal tension generally occurs without warning, resembling somewhat in this respect the failure of unreinforced concrete beams. On account of the variability of concrete and its unreliability in resisting tensile stresses, relatively low diagonal tensile stresses (high factor of safety), as measured by the vertical shearing stresses, should be specified, unless there is effective metallic web reinforcement. The values allowed by many building ordinances seem too high to secure safety under the condition of ordinary building operations. Short, deep beams and beams restrained at the ends require that special attention be given to web stresses.

7. Slipping of bars and stripping of bars may accompany final failure of beams which fail by diagonal tension. It appears that slipping or stripping did not take place in the beams having the reinforcing bars horizontal until after the maximum load was

reached and the presence of diagonal cracks had modified the distribution of stresses. At these maximum loads the calculated bond resistance developed was low as compared with the bond strength of the steel and concrete. The beams reinforced with deformed bars carried no higher loads than those with plain bars. The beams with bars bent up or inclined toward the ends gave quite variable results, but in general the values were even lower than those with the bars in a horizontal position. There is some probability that slipping occurred in these tests at or before the maximum load and that anchoring the ends of the bars would have been beneficial. The results for the beams having vertical stirrups showed that the stirrups as used were not efficient in taking web stresses.

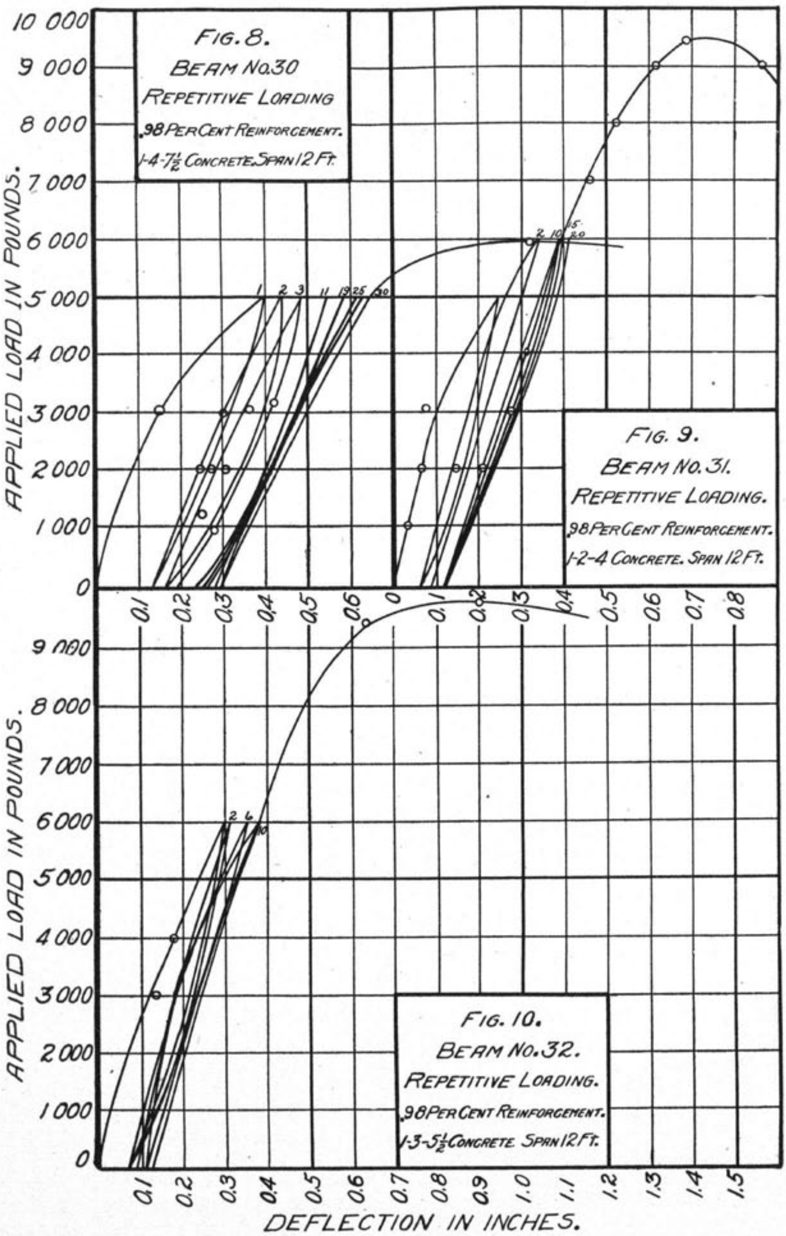
FIG. 8.
BEAM No. 30
REPETITIVE LOADING
.98 PER CENT REINFORCEMENT.
1-4-7½ CONCRETE. SPAN 12 FT.
FIG. 9.
BEAM No. 31.
REPETITIVE LOADING.
.98 PER CENT REINFORCEMENT.
1-2-4 CONCRETE. SPAN 12 FT.
FIG. 10.
BEAM No. 32.
REPETITIVE LOADING.
.98 PER CENT REINFORCEMENT.
1-3-5½ CONCRETE. SPAN 12 FT.
APPLIED LOAD IN POUNDS.
APPLIED LOAD IN POUNDS.
DEFLECTION IN INCHES.

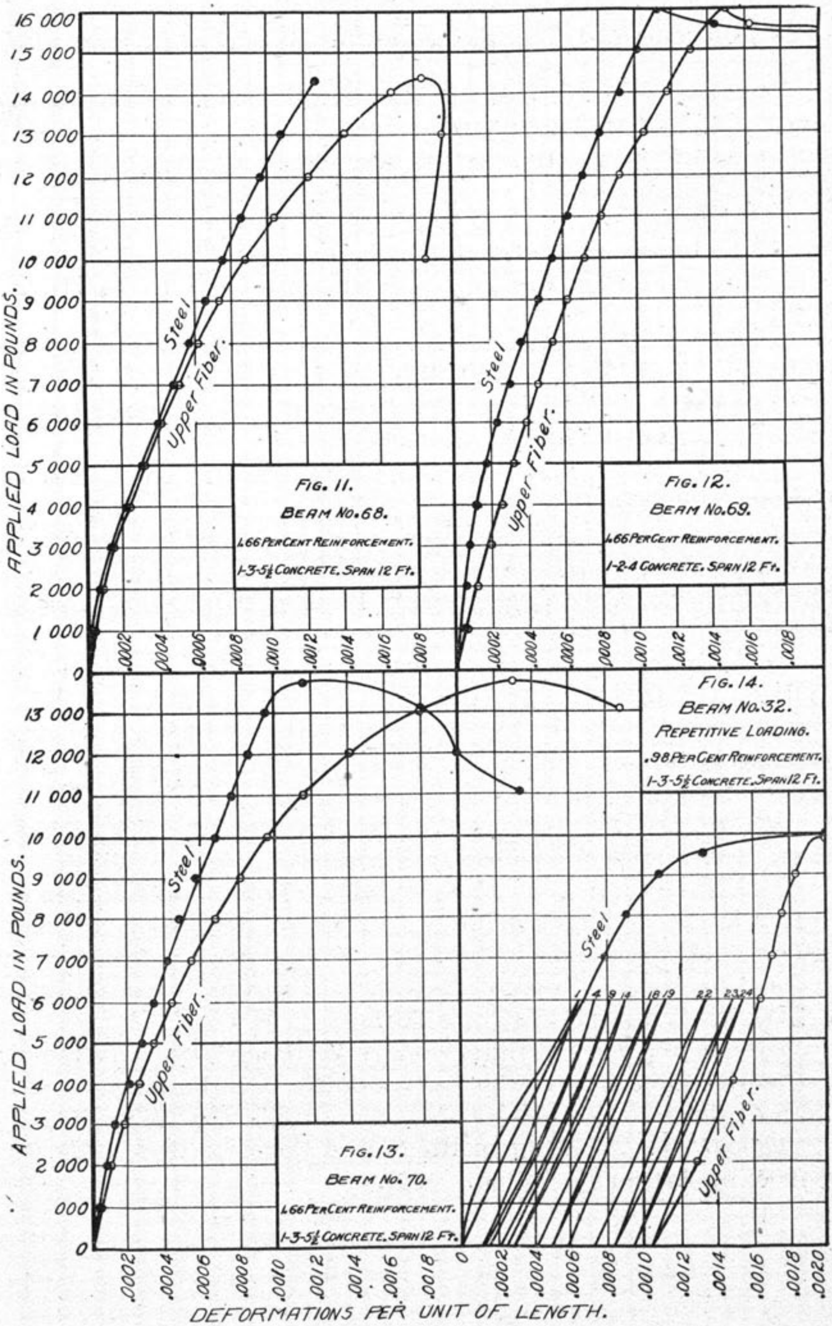

FIG. 11.
BEAM NO. 68.
1.66 PER CENT REINFORCEMENT.
1-3-5½ CONCRETE. SPAN 12 FT.

FIG. 12.
BEAM NO. 69.
1.66 PER CENT REINFORCEMENT.
1-2-4 CONCRETE. SPAN 12 FT.

FIG. 13.
BEAM NO. 70.
1.66 PER CENT REINFORCEMENT.
1-3-5½ CONCRETE. SPAN 12 FT.

FIG. 14.
BEAM NO. 32.
REPETITIVE LOADING.
.98 PER CENT REINFORCEMENT.
1-3-5½ CONCRETE. SPAN 12 FT.

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Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905.

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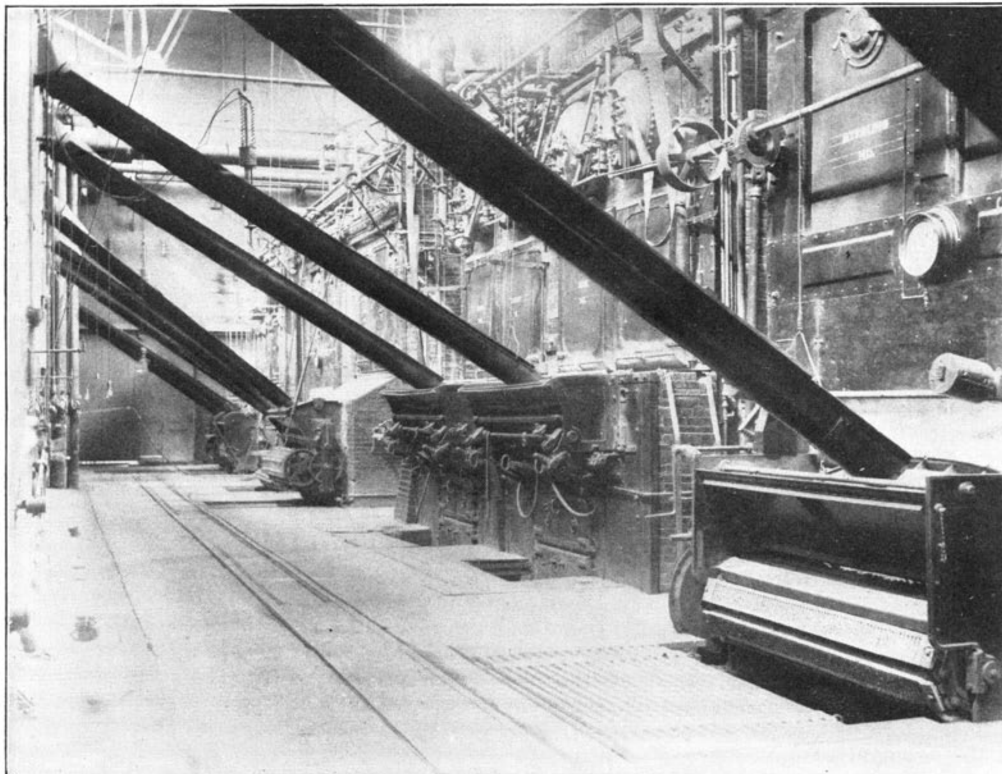

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