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INDUCTANCE OF COILS

BY

MORGAN BROOKS

AND

H. M. TURNER



UNIVERSITY OF ILLINOIS
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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 53

JANUARY, 1912

INDUCTANCE OF COILS

BY MORGAN BROOKS, PROFESSOR OF ELECTRICAL ENGINEERING AND
H. M. TURNER, ASSISTANT IN ELECTRICAL ENGINEERING

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INDUCTANCE OF COILS

I. INTRODUCTION

This bulletin deals with the self-inductance of closely-wound cylindrical coils of wire without iron cores, and presents tables and charts for obtaining without effort, the approximate inductance or reactance of coils of all dimensions. A given length of conductor has a definite resistance, but may have as many different values of inductance as there are different shapes of coils into which it can be wound, although the inductance becomes definite when the dimensions of the winding are fixed. From a large number of calculations and of tests of actual coils of many shapes, material has been gathered from which a universal formula for the self-inductance of coils has been derived, making it a simple matter to compare the relative value of different winding proportions. Regardless of the size of conductor, it is found that the shape for producing the maximum inductance from any length of wire is neither a long solenoid nor a flat disk, but a compact coil not unlike the ordinary wire bundle as received from the factory. While tables showing the resistance of copper wire are found in all electrical reference books, similar data for the inductive reactance of coils, if found at all, are often incorrect, or so presented as to be unavailable for occasional use. It is hoped that the information here given will prove convenient and useful.

II. PRECISION FORMULAS

Valuable information upon the mutual and self-inductance of lines and coils will be found in the bulletins of the Bureau of Standards, Washington, D. C., and special reference is made to Vol. V, No. 1 (1908), where many formulas are compared. For the various shapes of coils, a bewildering choice of formulas is offered; several for short single-layer windings, several for disks and rings, and others for intermediate shapes. The use of one of these precision formulas for a type of coil for which it is not intended may result in serious error, and their delimitation is not always clearly stated, so that it may be necessary to try two or more of them, and compare results.

Most theoretical formulas assume that the winding is composed of a thin conducting tape, whose edges lie close together, though electrically insulated, producing what is mathematically known as a current sheet, although a few are based upon the usual square or round wires employed to produce the ampere

turns of practical work. There are small differences, sometimes amounting to one per cent or more between these conceptions of inductance, and the Bureau of Standards has done an admirable work in reconciling the differences, and in providing correction formulas for bringing the results into strict harmony, establishing the accuracy of certain formulas, as shown by illustrative formal examples to a point of agreement within one part in a million.

The very precision of these elaborate formulas is a barrier to their usefulness to the busy engineer, by whom coil dimensions may not be known accurately. No simple formula of wide range is given, or suggested, and the only formula recommended for long solenoids requires the use of elliptic integrals for its solution, and even then is limited to a single-layer winding. Diversity in units and notation, and the absence, from the examples, of wires of ordinary gauge diameters place further difficulties in the utilization of the formulas, which are often insurmountable.

Uniformity in the winding of commercial coils is impossible, when, in the matter of gauge sizes alone, a variation of one per cent from tabulated diameter is allowable, and when the thickness of insulations is never known definitely. Compactness of winding is ordinarily subject to uncertainty, and the exact pre-determination of the mean radius of a coil extremely difficult. The cumulative effect of these uncertainties is such that for many purposes an approximate formula is quite accurate enough.

III. ADVANTAGE OF COILS WITHOUT IRON CORES

There is a demand for coreless reactance coils, since they eliminate the uncertainty of action incident to coils with iron cores. If reactance is needed for protecting alternating current apparatus against the dangers of short circuits, the immediate response of a coil, not dependent upon the dilatory magnetizing of iron, is required. While a few turns of wire around iron seem to give the same reactance as many turns without iron, the coil with iron is relatively sluggish. Two coils of same measured reactance, but differing in design, may behave very differently because one has a closed, the other an open magnetic circuit; and even the same ferric coil may behave differently under similar conditions because of a difference in the residual magnetism. There is a further advantage in non-ferric coils, in the absence of core loss. Such coils also find an increasing use in telephony and in wireless telegraphy, where certainty of action, and alertness at

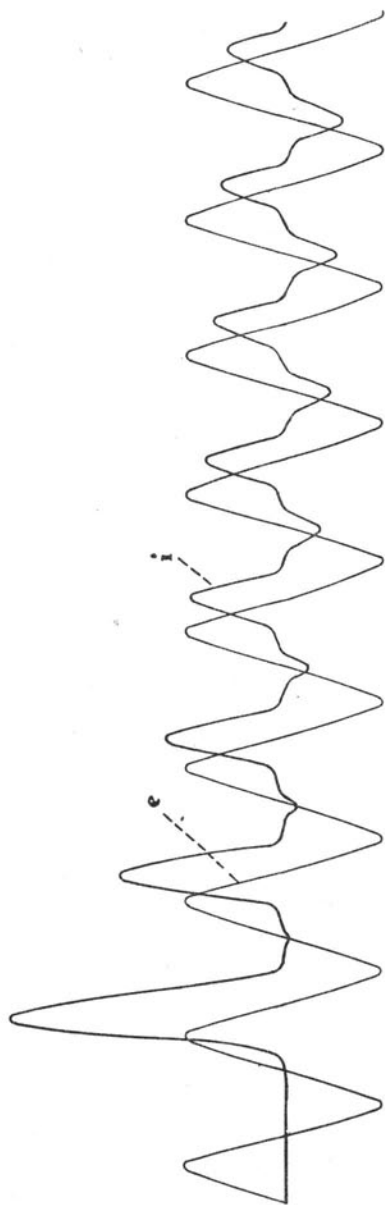


FIG 1a

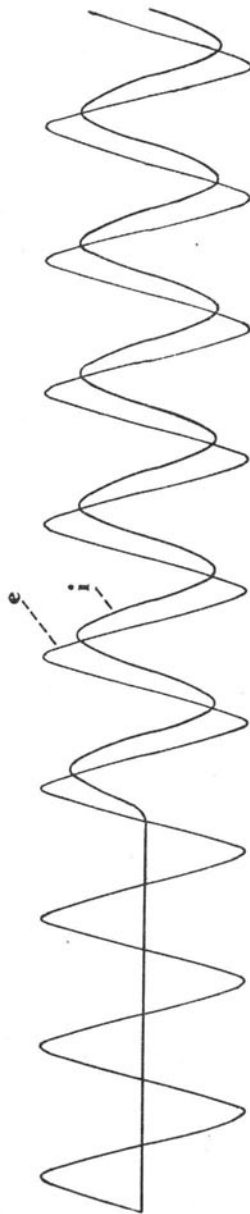


FIG. 1b

high frequencies are important. The data here given are just as applicable to small coils for telephone use as to those of large size for power use.

IV. COMPARISON WITH IRON-CORED COILS

The reactive difference between iron-cored and coreless coils, when thrown suddenly into action, is well shown by the instantaneous photographic record of the oscillograph, as illustrated in Fig. 1*a*, 1*b*, 2*a*, and 2*b*. The same alternating electromotive force is shown in all these reproductions; therefore, the scale of the figures is to be understood as a relative one merely. Fig. 1*a* shows a considerable current-surge, the first wave having many times the amplitude of those of normal current after a few cycles.

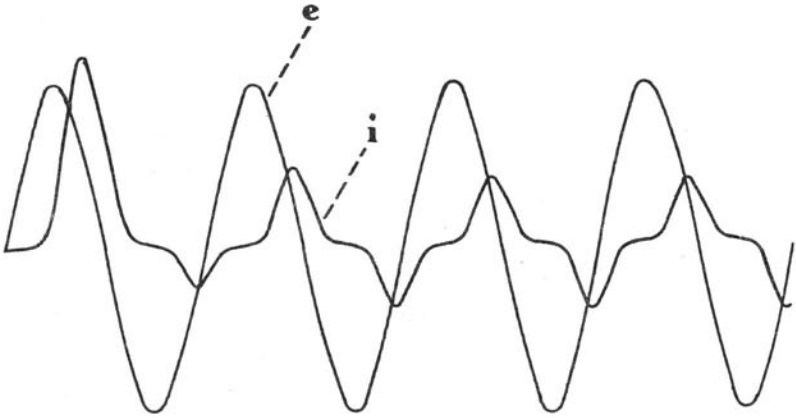


FIG. 2*a*

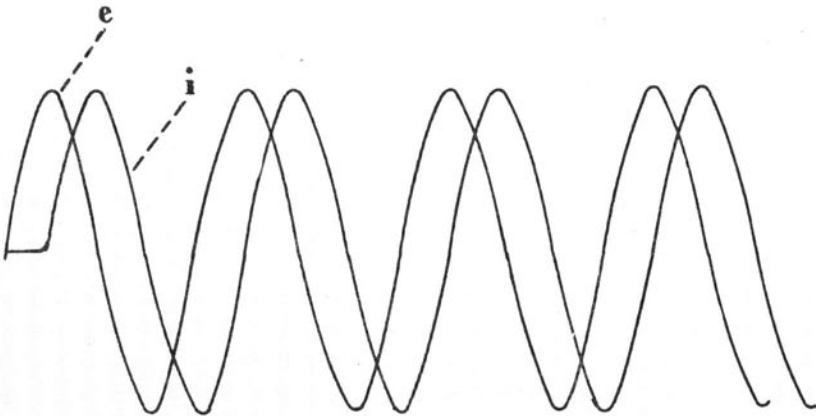


FIG. 2*b*

This records the action in a coil with iron core, to which the electromotive force was applied intentionally at the instant of its most rapid change, the moment when the normal current would naturally be a maximum. Fig. 1*b* represents the same action in a coreless coil, and it is seen that the surge, though present, is limited, the maximum effect never exceeding twice normal current wave values.

Fig. 2*a* and 2*b* represent the lesser action occurring in cored and coreless coils, respectively, when the electromotive force is applied at the instant of maximum, when its value is not changing, the very moment when the current is normally passing through zero, so that the current starts on its normal wave at once. In this case, there is little or no difference in the action of the coils, except that due to residual magnetism in the cored coil; but under the extreme condition of Fig. 1*a* and 1*b*, the advantage of the coreless coil is manifest, since there is relatively little difference between the best and the worst conditions. Non-ferric coils have an increasing use for electrical work of all kinds, as their advantages are better appreciated.

V. INDUCTANCE AND REACTANCE

A current in a coil of wire gives it the properties of a magnet, even when an iron core is not present. Magnetic properties of non-ferric coils are greater than is generally supposed. Within such a coil is produced a magnetic field or stress, called flux, (ϕ), proportional to the magneto-motive-force, (mmf), of the current, (i), flowing through the turns, (N), of the coil, and limited by the reluctance, (\mathcal{R}), of the flux path. Flux is expressed in maxwells, or lines of force, commonly called "lines". Magneto-motive-force is expressed in gilberts, where one gilbert is 4π current turns, ($4\pi Ni$), in absolute units, or as 0.4π ampere-turns ($0.4\pi Ni$) when i is in amperes, the practical unit of current, since the ampere is one-tenth the absolute unit; reluctance is expressed in oersteds, these units being in absolute values of the centimeter-gram-second system. Reluctance varies directly with length of the flux path, and inversely with its area, and also with its permeability when iron is included in the path. Length and area of flux path in a normal closed magnetic circuit of iron are easily determined with considerable precision, but in air or in any non-magnetic material, the dimensions of the flux path, while definite, cannot be expressed by any simple rule that applies to a variety of cases. In iron, the path has

a high permeability, varying with magnetization; in air and most substances, the permeability is low, but constant, and does not appear in equations for non-ferric coils, its value being taken as unity.

A steady current in a coreless coil produces a definite magnetic flux, since the magneto-motive-force and the reluctance are constants. If the current varies, the flux varies proportionally. In this case, another action arises, since experiment shows that a varying flux induces an electromotive-force, (e), in every turn of wire interlinking or surrounding the flux path. This electromotive-force is constant only while the flux is varying at a constant rate, being proportional to the rate-of-change of the flux, ($d\phi/dt$). Flux cannot long continue to change at a constant rate, but can pulsate or alternate and thus produce alternating electromotive-forces. An alternating current in a coil must be accompanied by an alternating magnetic flux, which will induce, in the turns of the coil, the counter-electromotive-force of self-induction, and in any other coil, an electromotive-force of induction, of equal value per turn, if similarly situated in respect to the flux path.

The coefficient of self-induction, or, briefly, the inductance, (L), of a coil is the constant ratio existing in non-ferric coils between the counter-electromotive-force, (e), and the rate of change of the current, (di/dt), on which it depends. It may also be expressed by the ratio of flux-turns, ($N\phi$), to inducing current, (i), as shown by the following equations expressing these laws;

$$\phi \text{ (in maxwells or lines)} = \frac{(\text{mmf in gilberts})}{\mathcal{R} \text{ (in oersteds)}} = \frac{4\pi Ni}{\mathcal{R}} \dots\dots\dots (1)$$

$$e \text{ (in absolute units)} = -\frac{N d\phi}{dt} = -\frac{4\pi N^2 di}{\mathcal{R} dt} = -\frac{L di}{dt} \dots\dots\dots (2)$$

$$\mathcal{R} \text{ (reluctance)} = \frac{\text{length of flux path}}{\text{area of flux path}} = \frac{b+y}{\pi a^2} \dots\dots\dots (3)$$

$$\text{(from (2) and (3)) } L = \frac{N\phi}{i} = \frac{4\pi N^2}{\mathcal{R}} = \frac{4\pi N^2 \pi a^2}{b+y} = \frac{(2\pi aN)^2}{b+y} \dots\dots (4)$$

in which

- a = mean radius of the coil in centimeters;
- b = length of coil, also in centimeters;
- πa^2 = the definite area of flux-path within the coil;
- y = a quantity to be added to b to get equivalent return length of flux-path, outside the coil, as explained under "Flux-path".

The inductance coefficient, L , is seen to be a constant, depending upon constants of the coil winding, its linear dimensions and turns. The numerator, $(2\pi a N)^2$ is the square of the length of conductor of which the coil is made, and the denominator, $(b + y)$, is a coil length. If y could readily be defined for coils of different shapes, the determination of L would be a simple matter. It is evident that the square of a length divided by a length is of the dimension of length, and the inductance L , is, in absolute units, the centimeter. The practical unit is the henry, which is 10^9 centimeters, the length of the earth quadrant. The mil-henry, the thousandth, and the microhenry, the millionth of this unit, are much used for the inductance of small coils.

VI. FLUX-PATH

The dimensions of the flux-path, if known, determine the reluctance. In a particular coil of four turns, the flux-path is graphically indicated by iron filings, as shown in a photograph reproduced from Simmons' "Outlines of Electrical Engineering," as Fig. 3, in which it may be seen how iron filings obey the ampere-

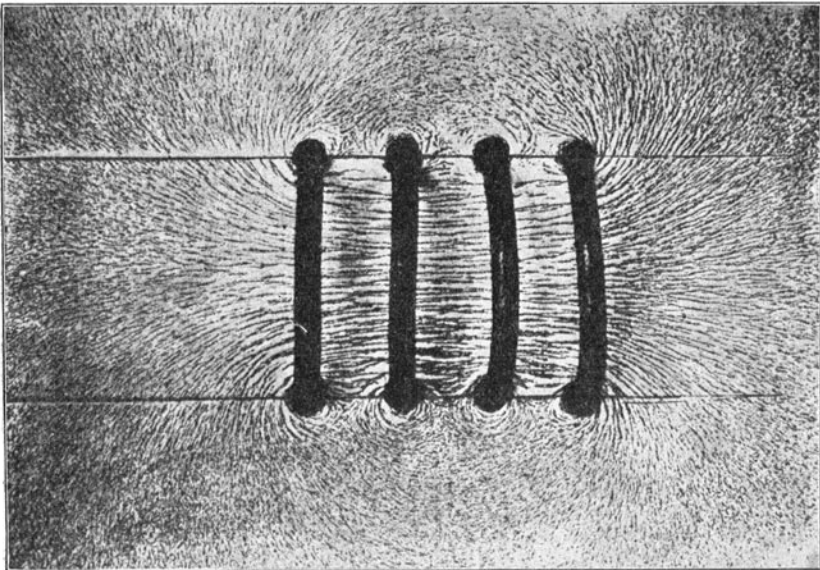


FIG. 3. MAGNETIC FLUX DUE TO AMPERE-TURNS OF SOLENOID

turns of a steady current. The flux appears to be uniformly strong within the coil and to diminish rapidly without. The flux-path can be traced some distance beyond the coil before it be-

comes indefinite, just as smoke can be traced for some distance from a chimney mouth. As the path increases in area, its reluctance decreases; therefore, the reluctance of the return path outside the coil is relatively small. It is not zero, however, as sometimes assumed, since the path does not become indefinitely extended immediately upon leaving the coil. The flux-path within the coil may be assumed to have the area of an average turn of the coil, a definite value, and a length the same as the coil length. If the same area is assumed for the return path, evidently much too small, it is fair to assume a return length, y , also much too small, to correspond. It is found by trial that if $c + R$, the sum of the thickness of the winding and the outside radius of the coil, be substituted for y in the equation (4), a close approximation of L is obtained for long coils.

In a coil of but a single turn, the flux distribution must be somewhat as shown in Fig. 4, reproduced from Nipher's "Electricity and Magnetism". The flux is confined to a definite area only

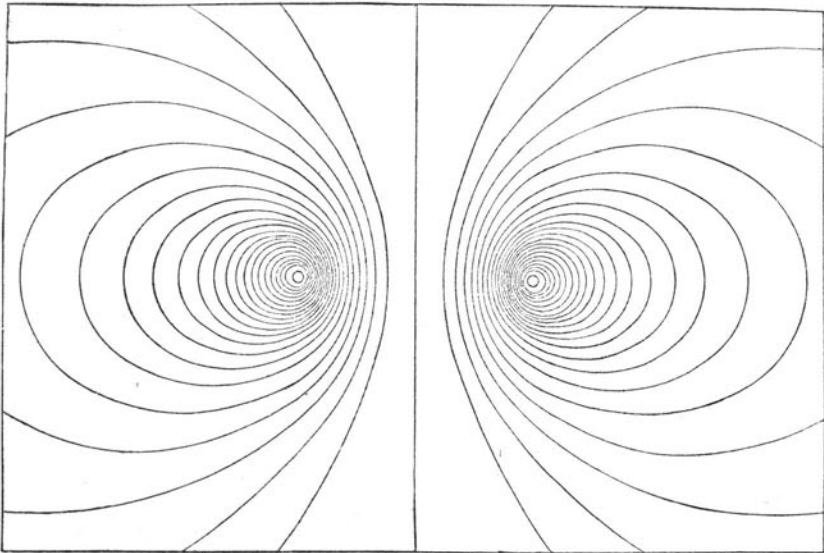


FIG. 4

where it passes through the ring, shown in section, but is not uniformly distributed even there, the lines being crowded more closely together within or near the conductor than toward the center of the ring. Flux paths may be represented by the smallest circle just around the conductor, or by any of the longer curves farther away. Indeed with reference to that portion of the

current energy at the conductor's center, the flux-path may lie wholly or partially within the metal of the conductor, and have a length that is less than the smallest circumference of Fig. 4. At very high frequencies, where "skin-effect" forces the current away from the center of a large conductor, increasing its absolute resistance, this internal flux path may be eliminated with consequent diminution of absolute inductance, an effect quite negligible for ordinary frequencies. In such a formula as (4), the definition of γ , the equivalent length of the average flux line outside the coil is elusive for single turns, and other forms of equations have generally been derived for the inductance of rings.

In long coils, γ is evidently subordinate to b , the coil length, and a slight error in its evaluation makes a smaller error in the value of inductance, but in ring coils such is not the case. In the general formula presented, (5) or (6), page 13, equation (4) is modified by factors, $F' F''$, which virtually modify $b + \gamma$, giving an approximate value for all shapes of coils including rings.

VII. DEVELOPMENT OF BROOKS' UNIVERSAL FORMULA

There is an evident advantage in having a single formula apply to all forms of coils, provided it is reliable and approximately accurate. Where different formulas have to be used for long and for short coils, as has heretofore been the case, there is an uncertainty as to intermediate shapes, due to the discontinuity of the separate formulas. Professor Morgan Brooks, one of the writers of this bulletin, has developed such a universal formula, applicable to all shapes of closely-wound cylindrical coils. It is in agreement with the theoretical formula for very long solenoids, and represents approximately the best formulas for coils of other shapes of one or many layers, even including the most extreme cases of single turns of fine wire. Values calculated from precision formulas, as well as measured values for a great variety of coil shapes, indicate a possible variation not exceeding three per cent for this universal formula.

To the engineer who uses resistance tables without allowance for temperature, this degree of accuracy will be found sufficient, for it must be remembered that a difference of but 8°C . will change the resistance of copper wire three per cent. In fact, it is difficult to duplicate a reactance coil, and get results within three per cent without adjustment. In case greater precision is required, the universal formula will be found useful in

detecting any errors of calculation in using more elaborate precision formulas.

The universal formula is partly empirical, but is based upon the theoretically derived equation (4) which is directly applicable to long solenoids. The empirical factors are so devised that they cannot cause gross errors, no matter what the extreme of dimensions may be. It indicates, as no discontinuous formula can, the approximate percentage of increase or decrease of inductance that any modification of coil dimensions will produce. It points directly to the proportions for obtaining the maximum inductance in coreless coils. By its use the tables and charts accompanying this bulletin have been prepared; and by means of these any coil problem involving inductance values can be rapidly solved.

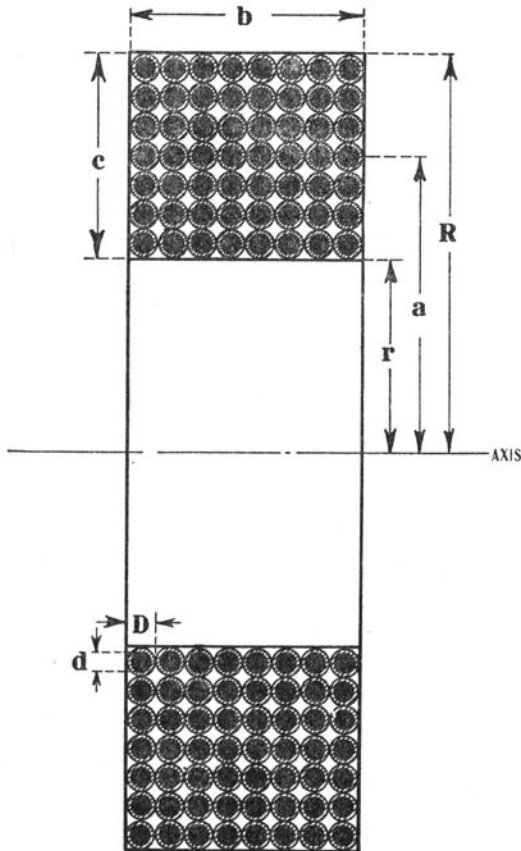


FIG. 5. COIL SECTION ILLUSTRATING NOTATION USED IN BULLETIN

VIII. NOTATION

Fig. 5 illustrates the notation used throughout this bulletin.

- a = the mean radius of the winding;
- b = the axial length of the coil; for single turns use d ;
- c = the thickness of the winding; for single turns use d ;
- r = the inner radius of the winding;
- R = the outer radius of the winding;
- d = the diameter of the bare wire;
- D = the diameter of the wire over insulation.

The above dimensions are in centimeters, or in inches, as may be indicated. Mils are not used.

Cm indicates the length of the conductor in centimeters;
 Ft indicates the length of the conductor in feet, and $Ft/1000$
 = thousands of feet;

N is the total number of turns in the winding, whence
 $Cm = 2\pi a N$, when a is in centimeters, and

$$\frac{Ft}{1000} = \frac{2\pi a N}{12000}, \text{ when } a \text{ is in inches;}$$

N' is number of turns for coils of prescribed maximum shape;

f = frequency of alternating current in cycles per second;

L = inductance, generally in henries, sometimes in millihenries or microhenries;

$X = 2\pi fL$ = ohms of reactance.

R = ohms resistance; for use on pp. 36 and 37 only.

UNIVERSAL FORMULA FOR COIL INDUCTANCE

The formula derived by Professor Morgan Brooks is given in two forms, one (5) for dimensions in centimeters, the other (6) for dimensions in English units. Both give results in henries.

$$L \text{ (in henries)} = \frac{Cm^2}{b+c+R} \times \frac{F' F''}{10^9} \dots\dots\dots (5)$$

$$L \text{ (in henries)} = \frac{0.366 \left(\frac{Ft}{1000}\right)^2}{b+c+R} \times F' F'' \dots\dots\dots (6)$$

In (6) the conductor length is in thousands of feet, and the coil dimensions in inches. 0.366 is the conversion factor. F' and F'' are empirical coil-shape factors dependent upon the relative, and independent of the absolute dimensions of the winding. Values of F' and F'' are as follows:

$$F' = \frac{10b+12c+2R}{10b+10c+1.4R}; \quad F'' = 0.5 \log_{10} \left(100 + \frac{14R}{2b+3c}\right)$$

These factors F' and F'' may appear formidable, but they are

easy to use. F' varies from unity for very long coils to 1.43 for the shortest of coils, the single turn. It is a factor that takes account of the varying thickness of coil windings, as well as of the relative importance of radius and length of coil in their influence upon inductance. F'' is negligible for long coils, falling below 1.01 for all coils whose axial length is the greatest dimension. F''' becomes active for short coils, especially for those of but a single turn.

TABLE 1
SHAPE-FACTORS, F' AND F'' FOR CERTAIN COIL PROPORTIONS

| <i>a</i> | <i>b</i> | <i>c</i> | <i>R</i> | F' | | F'' | | F''' | |
|----------|----------|----------|----------|-------|-------|-------|-------|--------|-------|
| | | | | | | | | | |
| 19 | 200 | 2 | 20 | 1.008 | | 1.001 | | 1.009 | |
| 15 | 200 | 10 | 20 | | 1.015 | | 1.001 | | 1.016 |
| 19 | 100 | 2 | 20 | 1.015 | | 1.003 | | 1.018 | |
| 15 | 100 | 10 | 20 | | 1.028 | | 1.002 | | 1.030 |
| 19 | 50 | 2 | 20 | 1.029 | | 1.008 | | 1.035 | |
| 15 | 50 | 10 | 20 | | 1.051 | | 1.004 | | 1.055 |
| 19 | 20 | 2 | 20 | 1.064 | | 1.013 | | 1.078 | |
| 15 | 20 | 10 | 20 | | 1.097 | | 1.008 | | 1.106 |
| 19 | 12 | 2 | 20 | 1.095 | | 1.019 | | 1.116 | |
| 15 | 12 | 10 | 20 | | 1.129 | | 1.011 | | 1.141 |
| 19 | 8 | 2 | 20 | 1.125 | | 1.026 | | 1.154 | |
| 15 | 8 | 10 | 20 | | 1.154 | | 1.013 | | 1.169 |
| 19 | 5 | 2 | 20 | 1.163 | | 1.035 | | 1.204 | |
| 15 | 5 | 10 | 20 | | 1.182 | | 1.015 | | 1.200 |
| 19 | 4 | 2 | 20 | 1.182 | | 1.040 | | 1.229 | |
| 15 | 4 | 10 | 20 | | 1.190 | | 1.016 | | 1.209 |
| 19 | 3 | 2 | 20 | 1.205 | | 1.045 | | 1.259 | |
| 15 | 3 | 10 | 20 | | 1.203 | | 1.017 | | 1.223 |
| 19 | 2 | 2 | 20 | 1.235 | | 1.054 | | 1.301 | |
| 15 | 2 | 10 | 20 | | 1.216 | | 1.017 | | 1.237 |
| 19 | 1 | 2 | 20 | 1.277 | | 1.065 | | 1.360 | |
| 15 | 1 | 10 | 20 | | 1.232 | | 1.018 | | 1.254 |
| 19 | 0.5 | 2 | 20 | 1.302 | | 1.073 | | 1.397 | |
| 15 | 0.5 | 10 | 20 | | 1.241 | | 1.018 | | 1.263 |
| 19 | 0.2 | 2 | 20 | 1.320 | | 1.079 | | 1.424 | |
| 15 | 0.2 | 10 | 20 | | 1.246 | | 1.019 | | 1.270 |
| 19 | 0.1 | 2 | 20 | 1.326 | | 1.081 | | 1.433 | |
| 15 | 0.1 | 10 | 20 | | 1.249 | | 1.019 | | 1.273 |

SQUARE WINDING-SECTION COILS

| | | | | | | |
|-------|-----|-----|----|-------|-------|-------|
| 17.5 | 5 | 5 | 20 | 1.172 | 1.023 | 1.199 |
| 19. | 2 | 2 | 20 | 1.235 | 1.054 | 1.301 |
| 19.5 | 1 | 1 | 20 | 1.292 | 1.097 | 1.418 |
| 19.75 | 0.5 | 0.5 | 20 | 1.342 | 1.163 | 1.561 |
| 19.9 | 0.2 | 0.2 | 20 | 1.387 | 1.29 | 1.79 |
| 19.95 | 0.1 | 0.1 | 20 | 1.407 | 1.42 | 2.00 |

The trend of values of F' and of F'' and of their product $F' F''$ may be seen from Table 1, in which will be found values for a variety of coil proportions. Approximate values for intermediate shapes may be estimated by interpolation.

Equations (5) and (6) may be relied upon for giving a close

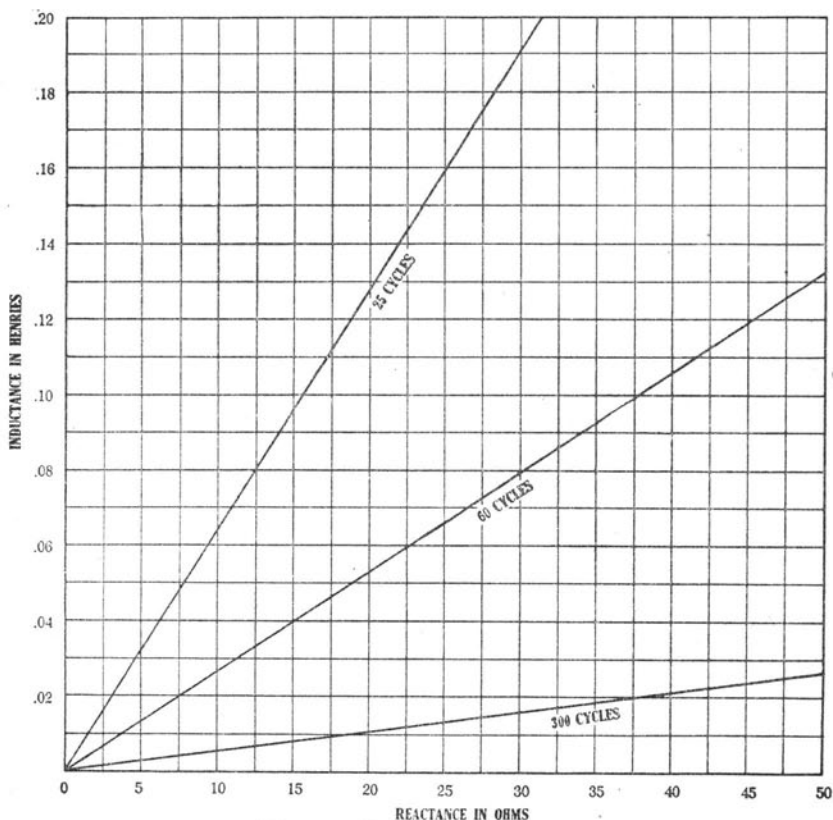


FIG. 6. REACTANCE CHARTS

approximation to the inductance of any closely-wound cylindrical coil, long or short, thick or thin, from the long solenoid to the single turn of fine wire. For spaced windings, they are not recommended, especially when the space between turns is great as compared with the diameter of the bare conductor. Equation (4), upon which these general formulas are based, assumes a perfectly smooth flux-path of uniform density within the coil. The photographic record of the iron filings, Fig. 3, shows some irregulari-

ties along the sides of the coil, as if the magnetic lines were trying to escape between the turns. If the turns were spaced farther apart, this effect would be increased, introducing another element into the equation.

IX. REACTANCE

When sine-wave alternating current is used, the value of the reactance in ohms that any coil will give is obtained from its inductance, L , in henries, by multiplying by $2\pi f$, which is 157, 377, and 1885 for frequencies of 25, 60, and 300 cycles per second, respectively. 300 frequency is often assumed in telephone circuits, although telephonic waves include many higher frequency harmonics. Fig. 6 is a chart for giving at once values of reactance, when inductance is known, for the frequencies above mentioned.

X. APPLICATION OF FORMULA TO LONG COILS

For long coils, F' and F'' may be neglected for first approximations, when formula (5) becomes reduced to its first term,

$$L \text{ (in henries)} = \frac{Cm^2}{(b+c+R)} 10^9 \dots\dots\dots (7)$$

This corresponds to equation (4) with $c + R$ substituted for y as explained. The substitution is empirical, justified by the results obtained.

Even for approximations, (7) is not recommended unless the length, b , is at least twice the outside diameter, $2R$, when the error involved will scarcely exceed 4 per cent with a multilayer winding, and will be within 2 per cent for a single layer solenoid, becoming more accurate as the length of the coil increases.

In extremely long coils, c and R are so small as compared with b , that they too may be neglected, when the formula becomes reduced to $L \text{ (in henries)} = \frac{Cm^2}{b \times 10^9}$, the equivalent of the formula often given in text-books without reservation as suitable for solenoids. The Bureau of Standards refers to this "approximate formula for long solenoids" as having "a considerable error", but offers no substitute except the Lorenz single-layer formula, which involves elliptic integrals. It is, in fact, in error from 20 to 40 per cent according to thickness of winding, if the coil is twice as long as the outside diameter, and more than 5 per cent when coil length is as much as $20R$, its error being roughly ten times as great as that of (7), which is therefore recommended as a substitute for this common form.

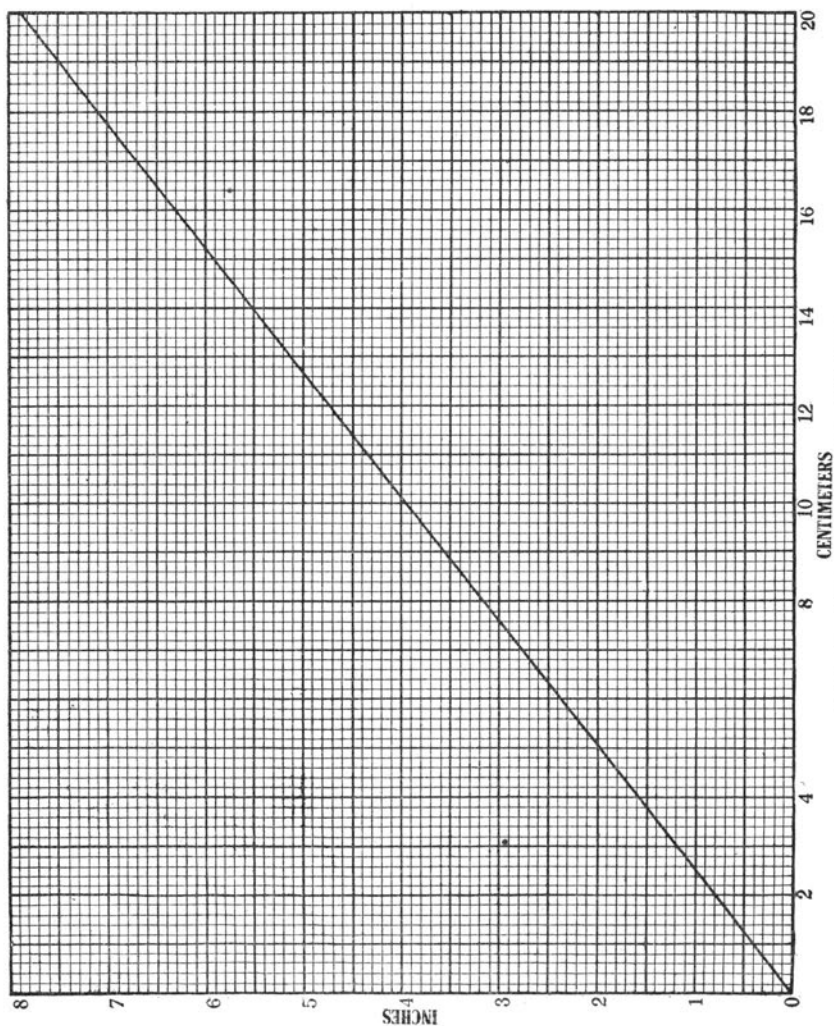


FIG. 7. CENTIMETER-INCH CHART

XI. APPLICATION OF FORMULA TO SHORT COILS

There is no sharp distinction between long and short coils, but coils whose axial length is less than their outside diameter may be classed as short. With a given length of conductor short solenoids yield more inductance than long, but equation (7) shows, as the common objectionable form does not, that this gain has a limit where the value of R begins to predominate, for the radius of the coil must increase with a material shortening of the winding. Experiment shows also that disks and rings have less inductance than ordinary compact shapes, using the same length of wire. However, equation (7) does not place enough emphasis upon the radius for short coils, and the empirical factors F' and F'' were introduced to correct this, and cause the general formula (5) to follow closely the varying influences of radius, length and thickness. A comparison with precision formulas for the several types of coils shows how well they perform their part.

XII. STEFAN'S AND KIRCHHOFF'S PRECISION FORMULAS

Stefan's formula for multilayer short coils has proved serviceable for comparisons, since it is stated to be precise without correction for very short coils, and within one per cent for coils whose length does not exceed radius. This formula, together with its table of constants, is reproduced from the Bureau of Standards bulletin. The examples of its use there given are confined to coils of relatively few layers.

TABLE 2
CONSTANTS FOR STEFAN'S FORMULA

| $\frac{b}{c}$ or $\frac{c}{b}$ | y_1 | y_2 | $\frac{b}{c}$ or $\frac{c}{b}$ | y_1 | y_2 |
|--------------------------------|--------|-------|--------------------------------|--------|-------|
| .00 | .50000 | .1250 | .55 | .80815 | .3437 |
| .05 | .54899 | .1269 | .60 | .81823 | .3839 |
| .10 | .59243 | .1325 | .65 | .82648 | .4274 |
| .15 | .63102 | .1418 | .70 | .83311 | .4739 |
| .20 | .66520 | .1548 | .75 | .83831 | .5234 |
| .25 | .69532 | .1714 | .80 | .84225 | .5760 |
| .30 | .72172 | .1916 | .85 | .84509 | .6317 |
| .35 | .74469 | .2152 | .90 | .84697 | .6902 |
| .40 | .76454 | .2423 | .95 | .84801 | .7518 |
| .45 | .78154 | .2728 | 1.00 | .84834 | .8162 |
| .50 | .79600 | .3066 | | | |

Stefan's formula for multilayer short coils is as follows:

L (in centimeters) =

$$4\pi a N^2 \left[\left(1 + \frac{3b^2 + c^2}{96a^2} \right) \log_e \frac{8a}{\sqrt{(b^2 + c^2)}} - y_1 + \frac{b^2}{16a^2} y_2 \right] \dots \dots (8)$$

All dimensions are in centimeters. Result divided by 10^9 gives henries. For notation see page 13, except for values of γ_1 and γ_2 , given in the table for certain ratios of b to c , or of c to b , so taken as to make the ratio a fraction. Napierian logarithms are employed in this formula. Multiply common logs by 2.3026 to get Napierian logs.

An excellent formula for the inductance of a single turn is that of Kirchhoff, which the Bureau of Standards says is extremely accurate, where $\frac{a}{d}$ is large, the error being less than one part in a million where $\frac{a}{d} = 250$, and is no more than one in 500 where $\frac{a}{d} = 5$. The formula is evidently more exact than any possible construction of a single turn in practice.

Kirchhoff's formula for single turns is as follows:

$$L \text{ (in centimeters)} = 4\pi a \left(\log_e \frac{16a}{d} - 1.75 \right) \dots \dots \dots (9)$$

Transformed into a form to employ common logarithms this becomes,

$$L \text{ (in centimeters)} = 28.9 \times a \times \log_{10} \frac{2.8a}{d} \dots \dots \dots (10)$$

The above have a , the mean radius, in centimeters. For English measure, it becomes

$$L \text{ (in microhenries)} = .0734 \times a \times \log_{10} \frac{2.8a}{d} \dots \dots \dots (11)$$

where a is in inches. The logarithmic term being a ratio, values are in any units.

XIII. MODIFIED KIRCHHOFF FORMULA FOR MANY TURNS

As an extremely simple formula, good for approximate work with short coils, the following modification of Kirchhoff's is submitted:

$$L \text{ (in milhenries)} = 0.29 \times a \times \left(\frac{N}{100} \right)^2 \times \log_{10} \left(\frac{5a}{b+c} \right) \dots \dots \dots (12)$$

where coil dimensions are in centimeters, and

$$L \text{ (in milhenries)} = 0.736 \times a \times \left(\frac{N}{100} \right)^2 \times \log_{10} \left(\frac{5a}{b+c} \right) \dots \dots \dots (13)$$

where coil dimensions are in inches.

These formulas (12) and (13) are not recommended for use where the value of the logarithmic term is less than unity; i. e., where $\frac{5a}{(b+c)}$ is less than 10, although it gives fair results

when this ratio is between 5 and 10. They apply equally well to single and multi-layer coils and to disks and single turns, covering substantially the field of both Stefan's and Kirchoff's. It is discontinuous, as it would become negative for coils of relatively small radius. For single turns $b+c = 2d$.

XIV. COMPARISONS

Table 3 gives the specifications of nineteen coils of widely varying proportions, of which a comparison of the value of induc-

TABLE 3

DESCRIPTION OF COILS*

| Coil | Layers | Turns | Length | Thickness | Mean Radius | Length of Conductor |
|------|--------|-------|--------|-----------|-------------|---------------------|
| 1 | 28 | 784 | 3.2 | 3.20 | 6.19 | 30480 |
| 2 | 10 | 330 | 3.0 | 0.79 | 2.90 | 6003 |
| 3 | 14 | 462 | 3.0 | 1.15 | 3.08 | 8926 |
| 4 | 18 | 594 | 3.0 | 1.59 | 3.25 | 12111 |
| 5 | 20 | 660 | 3.0 | 1.65 | 3.28 | 13511 |
| 6 | 1 | 1 | 0.2 | 0.20 | 25.00 | 157 |
| 7 | 1 | 1 | 0.1 | 0.10 | 25.00 | 157 |
| 8 | 1 | 1 | 1.0 | 1.00 | 25.00 | 157 |
| 9 | 1 | 2 | 0.2 | 0.10 | 99.85 | 627 |
| 10 | 2 | 4 | 0.2 | 0.20 | 99.90 | 2507 |
| 11 | 1 | 4 | 0.4 | 0.10 | 100.00 | 2513 |
| 12 | 1 | 10 | 1.0 | 0.10 | 25.00 | 1571 |
| 13 | 1 | 20 | 2.0 | 0.10 | 25.00 | 3142 |
| 14 | 4 | 16 | 0.4 | 0.40 | 100.00 | 10053 |
| 15 | 1 | 50 | 5.0 | 0.10 | 20.00 | 6283 |
| 16 | 10 | 100 | 1.0 | 1.00 | 4.00 | 1257 |
| 17 | 1 | 1 | 2.0 | 2.00 | 10.00 | 63 |
| 18 | 1 | 1 | 1.0 | 1.00 | 25.00 | 157 |
| 19 | 20 | 400 | 1.0 | 1.00 | 10.00 | 25133 |

* All dimensions are in centimeters.

TABLE 4

COMPARISON

| Coil | By Experiment | Prof. Brooks' Formula | Error per cent |
|------|---------------|-----------------------|----------------|
| 1 | 75.754 | 76.10 | 0.4 |
| 2 | 5.543 | 5.578 | 0.64 |
| 4 | 18.476 | 18.989 | 2.77 |
| 5 | 24.110 | 23.511 | -2.49 |

BUREAU OF STANDARDS

| | | | |
|----|---------|----------|-------|
| 6† | .02058 | .00204 | -0.88 |
| 7* | .00202 | .00204 | 1.22 |
| 8 | .00041 | .000416 | -1.39 |
| 9 | .0374 | .0372 | -0.53 |
| 10 | .1435 | .1410 | -1.75 |
| 11 | .1394 | .1397 | 0.31 |
| 12 | .1490 | .1506 | 0.89 |
| 13 | .5150 | .5220 | 1.64 |
| 14 | 2.070 | 2.050 | -0.97 |
| 15 | 1.860 | 1.900 | 1.93 |
| 16 | 1.147 | 1.176 | 2.53 |
| 17 | .000331 | .000326 | -1.51 |
| 18 | .00133 | .00131 | -1.50 |
| 19 | 64.158 | 65.80000 | 2.55 |

† (round)

* (square)

tance obtained by use of the universal formula with those given by the Bureau of Standards is given in Table 4. It will be observed that there is no error greater than 3 per cent.

It will perhaps give greater confidence in the universal formula to show its fair agreement in the most extreme case, that of a single turn of large radius, even if such a winding is not required commercially. For this purpose, a comparison with the exact Kirchhoff formula (10) is invited, as illustrated in the tabular statement below. The nearly uniform variation of the approximate formula (5) from the standard for such single turns is 2.5 per cent, and the error cannot exceed the 3 per cent limit in the most extreme case, as proved in the foot-note on this page*. The comparative values derived from the modified Kirchhoff formula (12) are also set down, to indicate that its extension to coils of many turns does not prevent its use for single turns. It is seen that the empirical factors F' and F'' are so held in check in the shortest of coils, the single turns, as well as the extremely long coils, where they become unity, that it is not possible for them to cause the errors that often arise from empirical formulas when used beyond usual limits.

Comparison of formulas (5) and (12) with (10) for large single turns:

| a | (b=c=d) | R | L (in microhenries) by | | | | Variation from (10) in per cent | |
|-------|---------|---------|------------------------|---------|---------|-----|---------------------------------|--|
| | | | (10) | (5) | (12) | (5) | (12) | |
| 100 | 1 | 100.5 | 7.07 | 6.88 | 6.96 | 2.7 | 1.6 | |
| 1000 | 1 | 1000.5 | 99.60 | 96.90 | 98.50 | 2.7 | 1.1 | |
| 10000 | 1 | 10000.5 | 1285.00 | 1253.00 | 1275.00 | 2.5 | 0.8 | |

*For extremely large circles of but a single turn equation (5) reduces rapidly to a form similar to Kirchhoff's formula (10) as follows: The transformation is obtained by using $2\pi a$ for Cm , its equivalent; by neglecting b and c in the denominator of the first term, and by using a for R ; by omitting b and c as negligible in comparison with R , or a in the F' term, making the value of that term 1.428; and lastly by omitting the 100 in the F'' term, as negligible in comparison with the ratio of $14 \frac{a}{5d}$, as it will be in the most extreme cases. (5) then becomes transformed as follows:

$$L \text{ (in centimeter)} = \frac{(2\pi a)^2}{a} \times 1.428 \times 0.5 \log 14 \frac{a}{5d}$$

$$= 28.2 \times a \times \log 2.8 \frac{a}{d}$$

This differs from (12) only in the coefficient, which is 28.9 in that equation, showing a difference of 2.5 per cent in the most extreme case possible, such as the third line of the tabular comparison above.

XV. EFFECT OF SHAPE OF COIL ON INDUCTANCE

Fig. 8 is a photograph of two coils wound with the same length, 50 ft. of No. 8 wire, the larger being wound in 7 turns loosely upon a bicycle rim, the other wound into a small coil of 56 turns. The small coil has about 2.5 times the inductance that the larger has. The measured values are .239 milhenries and .085 milhenries for the small and large coils. Calculated values are not very accurate for these coils, since they are not closely wound.

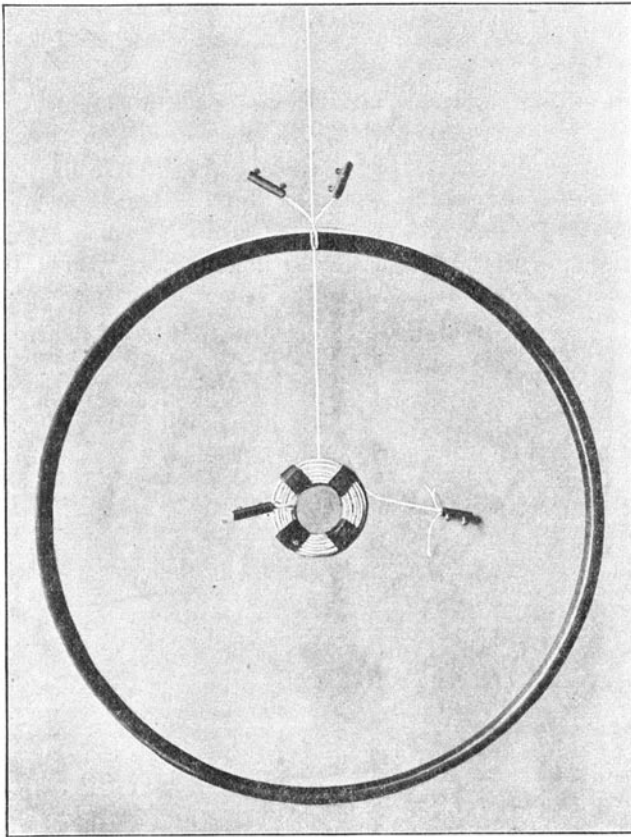


FIG. 8. PHOTOGRAPH OF TWO COILS WOUND TO ILLUSTRATE INFLUENCE OF SHAPE ON INDUCTANCE. (See Sec. XV.)

Table 5 has been prepared to illustrate the variation of inductance due entirely to the shape of the coil into which a given conductor may be wound. The length of wire chosen, 52.4 ft.

is that required to produce 80 turns of an average diameter of 2.5 in. These 80 turns are shown disposed in various ways, as a single layer, as two layers, as four, eight and sixteen layers, this last being a very thick, short coil, with a very small hole. Any shorter coil with 80 turns would fill up the hole completely, so the succeeding lines show coils of less turns and greater mean radius, until the final stage of a single turn of the entire 52 ft. is reached. The values were calculated by the general formula 6 and, so far as applicable, by formula (13), which is easier to use for very short coils, such as disks and rings. An additional line in the table gives the calculated inductance for the same conductor in a straight line, showing that the inductance of a single turn is less than the straight value.

TABLE 5

ILLUSTRATING THE EFFECTS OF COIL SHAPE UPON INDUCTANCE

Inductance in milhenries of 52.4 ft. of magnet wire, 0.1 in. diameter outside (small No. 11 d. c. c.) wound into cylindrical coils of various shapes as indicated.

| No. | Description | No. of Layers | Total Turns <i>N</i> | Mean Radius <i>a</i> | Length <i>b</i> | Thick-ness <i>c</i> | Outer Radius <i>R</i> | <i>F'</i> <i>F''</i> | Induc. <i>L</i> . Milhenries (6) (13) | Per cent of Max. |
|-----|-----------------------|---------------|----------------------|----------------------|-----------------|---------------------|-----------------------|----------------------|---------------------------------------|------------------|
| 1 | Spaced solenoid.... | 1 | 80 | 1.25 | 16. | 0.1 | 1.3 | 1.007 | .058 | 17 |
| 2 | Solenoid..... | 1 | 80 | 1.25 | 8. | .1 | 1.3 | 1.015 | .108 | 33 |
| 3 | Double layer ditto | 2 | 80 | 1.25 | 4. | .2 | 1.35 | 1.032 | .186 | 56 |
| 4 | Thick tube..... | 4 | 80 | 1.25 | 2. | .4 | 1.45 | 1.072 | .279 | 84 |
| 5 | Compact MAXI-MUM..... | 8 | 80 | 1.25 | 1. | .8 | 1.65 | 1.141 | .331 | 100 |
| 6 | Thick disk..... | 16 | 80 | 1.25 | 0.5 | 1.6 | 2.05 | 1.2 | .289 | 87 |
| 7 | Thick disk..... | 10 | 50 | 2. | .5 | 1. | 2.5 | 1.20 | .301 .303 | 91 |
| 8 | Sq. section ring... | 5 | 25 | 4. | .5 | .5 | 4.25 | 1.28 | .244 .239 | 74 |
| 9 | Sq. section ring... | 4 | 16 | 6.25 | .4 | .4 | 6.45 | 1.38 | .190 .188 | 57 |
| 10 | Flat ring..... | 4 | 8 | 12.50 | .2 | .4 | 12.7 | 1.57 | .119 .119 | 36 |
| 11 | Thin disk..... | 4 | 4 | 25. | .1 | .4 | 25.2 | 1.78 | .069 .070 | 21 |
| 12 | Thin disk..... | 2 | 2 | 50. | .1 | .2 | 50.1 | 2.13 | .042 .043 | 13 |
| 13 | Single turn..... | 1 | 1 | 100. | .1 | .1 | 100.05 | 2.46 | .025 .025 | 8 |
| 14 | Straight line..... | | 0 | Inf. | | | | | .030(XVI) | 9 |

XVI. INDUCTANCE OF A STRAIGHT WIRE

L (in milhenries) = $Ft. \times 0.00014 (1.35 + \log_{10} \frac{Ft.}{d})$, where $Ft.$ is the length of the conductor in feet, and d its diameter bare in inches. It is assumed that the return conductor is at infinite distance.

XVII. ILLUSTRATIVE CALCULATIONS

Complete calculations are here given for the coil represented by line 10 of Table 5, values being taken from the table.

By (6) L (in henries)

$$\begin{aligned}
 &= \frac{.366 (.0524)^2}{.2+.4+12.7} \times \frac{2+4.8+25.4}{2+4. +17.8} \times .5 \log \left(100 + \frac{178}{.4+1.2}\right) \\
 &= \frac{.366 \times .00275}{13.3} \times \frac{32.2}{23.8} \times .5 \log 211 \\
 &= .0000757 \times 1.35 \times 1.162 \\
 &= .000119 \text{ or } .119 \text{ milhenries.}
 \end{aligned}$$

By (13) L (in milhenries)

$$\begin{aligned}
 &= 0.736 \times 12.5 \times (.08)^2 \times \log \frac{62.5}{.6} \\
 &= .0589 \times 2.018 = .119
 \end{aligned}$$

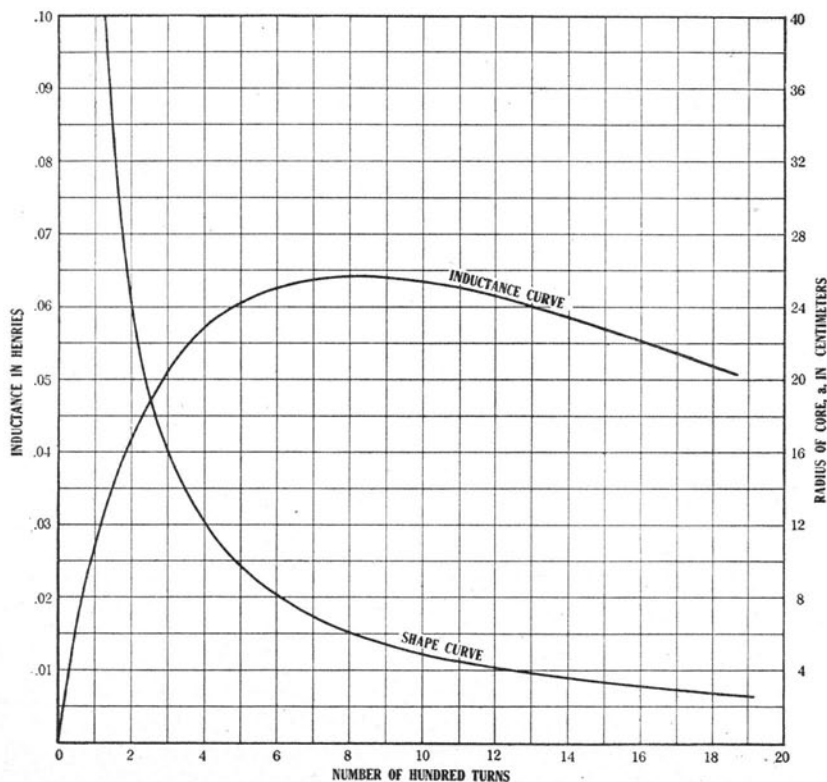


FIG. 9. CHART SHOWING THE VARYING INDUCTANCE OF 1000 FEET OF NO. 16 D. C. C. WIRE. When wound into coils of varying mean radius, a , while the ratio of $\frac{b}{c}$ is maintained equal to 1.2.

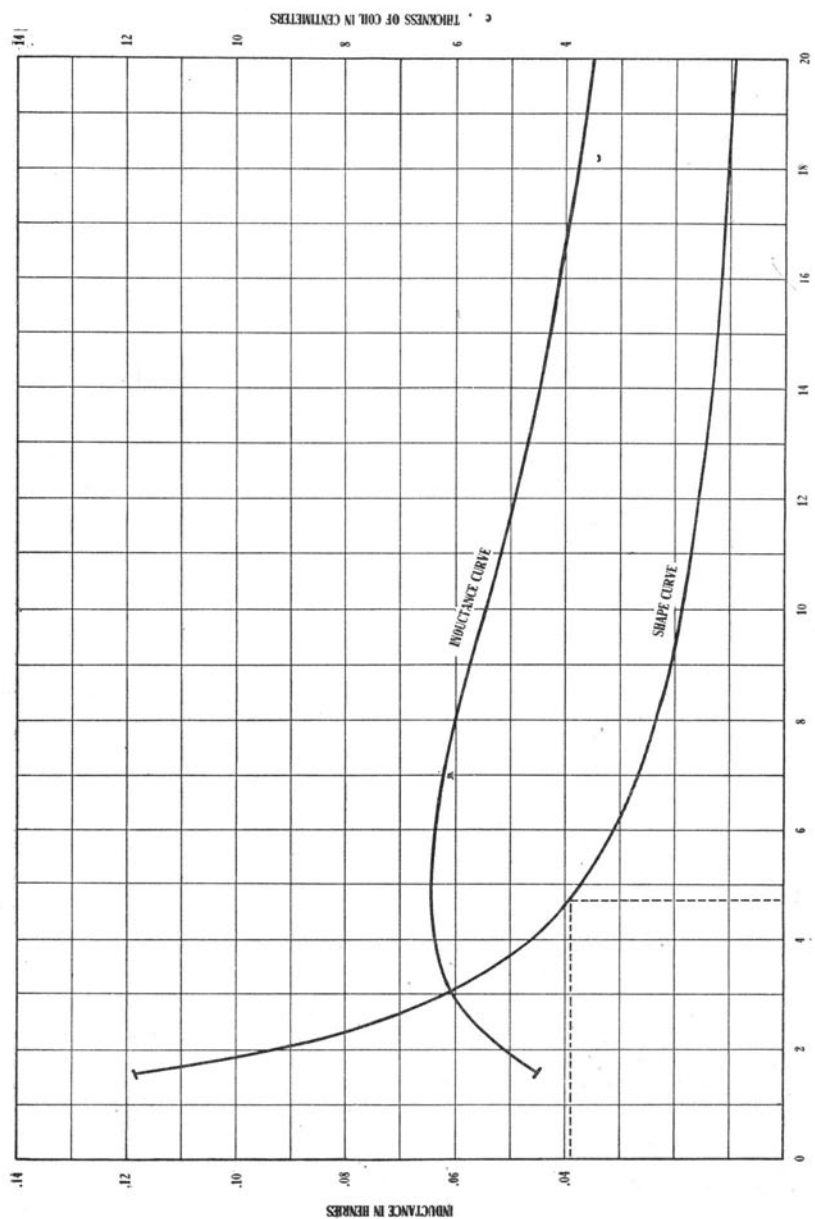


FIG. 10. CHART SHOWING THE VARYING INDUCTANCE OF 1000 FEET No. 16 D. C. C. WIRE

When wound into coils of mean radius, $a = 5.85$ cm., length b , and thickness, c , varying reciprocally as indicated by Shape Curve. The inductance values correspond to values of length of coil b .

Values are calculated by equations (6) and (13), except line 14, for which see page 23. Line 1, spaced solenoid, is calculated by (6), since the spacing is not sufficient to vitiate the result. The formula is not designed for spaced windings. Line 5 is the maximum inductance, the coil being practically the prescribed maximum shape. Equation (18) gives .3315 milhenries as the possible maximum. Lines 4, 6 and 7 show that a considerable alteration of coil proportions does not seriously affect the inductance. Equation (13) is applicable only to short coils.

Fig. 9 and 10 show how the inductances of a given length of wire vary with certain factors. In Fig. 9, the ratio of b to c was kept constant ($b = 1.2 c$), while the number of turns was varied. The shape curve in Fig. 9 gives the mean radius of the coil for a given inductance under the conditions imposed. In Fig. 10, the mean radius a was kept constant, while b and c were varied through a wide range; since their product was kept constant, an equilateral hyperbola (shape curve) will give shape of cross section of coil, i. e., the abscissa to any point on the inductance curve gives the axial dimension b and the ordinate to the shape curve at this point will give the thickness of the winding.

XVIII. MAXIMUM INDUCTANCE IN CORELESS COILS, WITH EQUATIONS

For a given conductor, the numerator of the first term of equations (5) or (6), the metric and English forms of the universal formula, is constant, and the variation of inductance is determined by the shape of the coil, as indicated by the denominator of the first term, and by the shape factors, F' F'' . In closely-wound coils, the values of a , b and c must vary, so that their product remains constant, since $2\pi a \times bc$ is the volume of the winding. The denominator, $b + c + R$, may be written $a + b + 1.5c$, since $R = a + .5c$, and its minimum value occurs when $a = b = 1.5c$ under the law stated. This would indicate the coil proportions for maximum inductance but for the varying values of F' and F'' . These factors both increase if the coil is shortened and its radius slightly increased, while the denominator is but little affected. It is found by the method of approximations that the relative proportions of the coil which will produce the maximum inductance are in fact nearly as follows: $a : b : c = 1.5 : 1.2 : 1.0$. To the same scale, $r = 1$ and $R = 2$, showing that the outside diameter of the coil is twice its inner diameter, and the axial

length 0.3 its outside diameter, a compact shape, shown in Fig. 5.

A slight variation from these proportions, such as may be necessitated by the integral number of layers and of turns per layer that must exist in a smooth winding, will cause little loss from maximum inductance, while a great variation will reduce the inductance materially. Line 5 of Table 5, already referred to, is so nearly of prescribed proportions that the inductance is indistinguishable from maximum. Line 7 shows a reduction of inductance by 10 per cent due to a moderate change in the coil shape. Lines 4 and 6 show a still greater reduction of inductance although the change in shape would not appear to be greater than that of line 7 from the maximum. It will be noticed that in all the coils of this table, except line 1, which is a spaced coil, the product abc is unity. The effect of further modification of coil shape is graphically shown in Fig. 24 and 25, described under "XIX. Charts".

Since economy in the use of material for the production of inductive reactance requires the employment of approximately the maximum shape, the general formulas may be reduced to a much simpler form if the prescribed shape, where $a:b:c = 1.5:1.2:1.0$ is used. For these proportions $F' \times F'' = 1.14$, and $b + c + R = 4.2c$.

Equation (5) then becomes for maximum inductance:

$$L_{\max} \text{ (in henries)} = 0.2714 \frac{Cm^2}{c \times 10^9} \dots\dots\dots (14)$$

where all dimensions are in centimeters. See also equation (17). The value of c is chosen as the dimension unit, since it is equal to r , the radius of the spool on which the coil must be wound. Its absolute value may be determined from the dimensions of the given conductor as follows:

The coil volume, $2\pi a \times bc$ becomes for maximum shape $3.6\pi c^3$, using the relative proportions given above. This volume in closely-wound coils must equal the conductor length multiplied by its outside diameter squared, or $Cm D^2$, the units being taken in centimeters. From this equality, c is determined.

$$c = \left(\frac{Cm D^2}{3.6 \pi}\right)^{\frac{1}{3}} = 0.4455 (Cm D^2)^{\frac{1}{3}} \dots\dots\dots (15)$$

all values in centimeters; or

$$c \text{ (in inches)} = 1.02 (Ft. \times D^2)^{\frac{1}{3}} \dots\dots\dots (16)$$

where conductor length is in feet, and outside diameter in inches.

Substituting (15) in (14), the maximum inductance is found in terms of conductor dimensions alone:

$$L_{\max} \text{ (in henries)} = 0.609 Cm \left(\frac{Cm}{D}\right)^{\frac{2}{3}} \times 10^{-9}, \dots\dots (17)$$

the conductor dimensions being in centimeters. In English units this becomes:

$$L_{\max} \text{ (in henries)} = 97.3 \text{ Ft. } \left(\frac{\text{Ft.}}{D \text{ (inches)}} \right)^{\frac{3}{2}} \times 10^{-9} \text{ (18)}$$

Ft. is the length of conductor in feet, and *D* is its outside diameter in inches.

A simple equation for maximum inductance, avoiding exponents, is obtained from (14). In every coil, $C_m = 2\pi a N$. Writing N' for the turns and $3c$ for $2a$ in a coil of maximum shape,

$$C_m = 3\pi c N' \text{ or } \frac{C_m}{c} = 3\pi N'$$

Substituting in (14)

$$L_{\max} \text{ (in henries)} = .2714 \times C_m \times 3\pi N' 10^{-9} = 2.56 C_m N' 10^{-9} \text{ (19)}$$

and

$$L_{\max} \text{ (in henries)} = 78 \times \text{Ft. } N' 10^{-9} \dots \dots \dots \text{ (20)}$$

for English units.

N' is the number of turns for the prescribed shape of coil only, and a small variation from that shape may affect the number of turns more than it does the inductance. Putting (17) = (19) N' is defined by

$$N' = 0.238 \left(\frac{C_m}{D} \right)^{\frac{2}{3}} \dots \dots \dots \text{ (21)}$$

This shows that the number of turns in a coil, wound for maximum inductance, is dependent upon the ratio of length to outside diameter of conductor, and independent of absolute dimensions, hence any units may be used in equation (21), providing the same units are used for numerator and denominator. The number of turns derived from (21) should be used in equations (19) and (20), since this number may differ from actual count of turns, unless the winding is very exact.

XIX. CHARTS

Since curves often present information that is to be obtained only with effort from formulas, it is a feature of this bulletin to offer charts for the determination of the inductance of any coreless cylindrical coil.

Unlike resistance, inductance depends not only upon the conductor dimensions but upon the thickness of insulation, and the shape of the coil winding, therefore to exhibit in simple chart form the information desired requires a comprehensive scheme. Since the inductance of any conductor will reach a maximum when wound into the best shape, and has a definite proportion of this maximum value when wound into any other specified shape, the use of the two curves may be made to determine the inductance of any cylindrical winding whatever; one giving the maximum value, the other the percentage of that maximum that the conditions prescribe. Their product is the desired inductance.

A curve from which the maximum may be taken will be found upon one of the several inductance charts covering the principal commercial sizes and insulations of magnet wire, as explained fully under section XX. A curve for the proportion of maximum value inherent in various possible shapes of coil will be found in one of the two shape-coefficient charts for long and short coils, as explained in detail in section XXII. These charts are so bound as to permit the use of the two selected charts simultaneously.

The charts may seem to be drawn on too small a scale for definite readings, but the use of a more open scale might lead to the assumption of greater accuracy than the method warrants. There should be no difficulty in deriving values accurate to 5%, with perhaps a little greater uncertainty for interpolated conditions and furnishing an excellent check upon calculated values. The use of logarithmic section paper will be seen to give the same proportionate accuracy throughout the range of the charts, and facilitates the extension of this range to any desired degree.

XX. MAXIMUM INDUCTANCE CHARTS

Charts herewith presented permit direct readings of the maximum self-inductance that may be obtained from any commercial magnet wire for lengths between 100 and 10 000 ft., wound into coils without iron cores. They also furnish information as to the necessary dimensions of the coil, and of the number of its turns. Other charts give shape-factors in percentage of the maximum inductance that coils of other than the prescribed maximum shape will produce from a certain length of conductor.

The equations of the preceding section furnish means of calculating the information needed to produce these charts. When a definite size of wire is chosen with known insulation, inductance

is seen to vary as the $\frac{5}{3}$ power of the conductor length, as shown by equation (17) for closely wound coils of the prescribed maximum shape. Upon ordinary coordinate paper, such an exponential function produces a curve difficult to plot with exactness, while with logarithmic coordinate paper, the curves become straight lines, whose slope is determined by the exponent. Two points determine a line, and single points then determine other lines, since they must be parallel, having the same exponent.

The logarithmic paper chosen for the inductance charts has four "squares", whose horizontal and vertical divisions are similar to the upper scale of the ordinary slide rule, starting from unity and going to 100. In all charts, except the shape factor charts, horizontal divisions, or abscissas, represent thousands of feet of conductor length, the scale running from 0.1, or 100 ft., to 10, or 10 000 ft. Vertical divisions represent different quantities on the several series of charts.

The first series, Fig. 11 to 14, gives inductance directly in henries on its vertical divisions, or ordinates, the bottom line indicating .01 henry, the middle line, .1 henry, and the upper line 1 henry with intermediate readings between as on a slide rule.

Since inductance depends upon the compactness of the winding, the thinner the insulation upon the conductor the higher the inductance. Commercial magnet wires have various insulations, and different charts were found to be necessary for the most important of these. Fig. 11*a* and 11*b* are for double-cotton-covered wires; Fig 12*a* and 12*b* for double-silk-covered wires; other pairs of charts are 13*a* and 13*b* for single silk, and 14*a* and 14*b* for enamel insulated wire. In each pair of charts, one has lines representing even gauge numbers (B. & S. gauge) that are divisible by four; the other those even numbers that are not divisible by four. Odd numbers are not directly represented, but may be closely estimated as between the adjacent even sizes.

It will be noticed that the parallel lines representing sizes, 4, 8, 12, 16, etc., are not always in normal space relation. This is not due to an error in making charts, but to the fact that insulations are not progressive like gauge numbers. The thickness of insulation upon the several wire sizes was taken from the tables of the Standard Handbook, reproduced as Tables VI, VII, VIII, IX, X and XI, and the same thickness of cotton or silk is used for several sizes of wire, when there is a sudden change to another

thread, causing the apparent discontinuity in the charts. For paper-insulated magnet wire, the charts for single-silk suffice.

XXI. CHARTS FOR DIMENSIONS AND NUMBER OF TURNS

To wind a coil, it is necessary to know the spool dimensions, and as the calculation involves cube-root, as shown by equation (16), it is believed that the series of dimension charts, Fig. 15*a* to 18*b*, will prove useful. These have horizontal dimensions representing lengths of wire in feet as before, and vertical dimensions, or abscissas, giving the value of c directly in inches. c is the thickness of the winding; $1.2c$ its length; $2c$ its inner diameter, these dimensions giving all spool dimensions. If any other shape than these maximum proportions be used, it will be necessary to provide somewhat more space for the winding, since more wire will be required for a given inductance.

It is often desirable to know the number of turns that a coil will have, and a third series of charts, Fig. 19*a* to 22*b*, gives this information for coils of prescribed maximum shape, employing the different insulations as before. Conductor lengths are abscissas as usual, and ordinates give directly the number of hundreds of turns, the range of the charts being from 100 to 10000. For large conductors, it may be necessary to adjust dimensions slightly to accommodate an integral number of turns in the length or thickness of the winding. A moderate change in the number of turns, owing to such adjustment, makes little change in the inductance. Values of turns were determined by equation (21).

Fig. 23 has two curves upon it, to illustrate the use of the charts both for inductance and number of turns. The two lines concern No. 24 single-silk-covered wire. Thus to get 0.1 henry of inductance, 890 ft. of this wire are required (1.15 lb.), and the coil will have 1420 turns. The coil dimension, c , is obtained from Fig. 17*a*.

It may be necessary to find values beyond the range of the charts. The lines, being straight, may be extended as indicated upon Fig. 23. In the lower right-hand corner, is the extension of the inductance line, so that, for example, 8000 ft. will produce 4 henries of inductance. For the extension, inductance values are multiplied by 100, and readings are therefore in the whole number henries, hence 4 in this case. Equation (18) shows that L varies with the $\frac{5}{3}$ power of length of any given conductor, and it is convenient to know that $4^{\frac{3}{5}} = 10$ almost exactly, so that

the value for 8000 ft. could have been obtained from the chart unextended, by reading the inductance for 2000 ft. as 0.4 henry, and multiplying it by 10.

In a similar way the number-of-turns line can be extended. A backward extension is shown in the lower left-hand corner. Turns vary with the $\frac{2}{3}$ power of length, hence 8 times the length gives 4 times the turns for same shape of coil.

XXII. SHAPE-FACTOR CHARTS

Two charts, Fig. 24 and 25, as well as small scale duplicates, Fig. 24*a* and 25*a*, are provided to indicate the percentage of maximum inductance that a coil of any shape will give. As there are three variables involved in a winding: radius, length and thickness, it is difficult to show this in a simple way. Parallel curves represent different winding sections. On Fig. 24, these are marked 1.2, 2, 5, 10, 20, 50, 100, and 500, for values of $\frac{b}{c}$. The larger numbers, such as 100, and 500 will generally represent single-layer windings of 100, or 500 turns; the smaller figures will generally be multilayer coils; thus, 5 might represent two layers of 10 turns each, 5 layers of 25 turns each, or any such ratio. The curve for $\frac{b}{c} = 1.2$ includes the maximum value 100 per cent where $\frac{a}{b} = 1.25$. Fig. 24 includes all coils whose axial length, b , exceeds thickness, c , while Fig. 25 includes square section coils, $\frac{c}{b} = 1$, and all coils where c exceeds b . Whole numbers are attached to the curves, the ratio being inverted to $\frac{c}{b}$ to avoid fractions in Fig. 25, which includes all disk-shaped coils. The curve $\frac{c}{b} = 1$ is for all single turns, as well as for those whose turns per layer are equal to the number of layers, assuming that round wire is used, and no allowance made for bedding.

Logarithmic paper is again used, as it is easy to read, and covers a wide range. It also avoids the employment of reversed curves. The shape-factor charts have horizontal divisions, or abscissas, representing ratios of $\frac{a}{b}$ throughout. In Fig. 24 $\frac{a}{b}$ runs from .01 to 100, and in Fig. 25 from .1 to 1,000. Ordinates

represent percentages from 1 to 100, most coil portions giving more than 10 per cent of the maximum inductance as would be expected. To find the percentage ratio of a coil whose section is not represented, such as $\frac{4}{1}$; its value may be estimated from the curve representing $\frac{5}{1}$.

The abrupt ending of the lines at the left indicates the point where the coil space is entirely occupied with wire, or where the hole ceases to exist. For example, in Fig. 24 the line for $\frac{b}{c} = 5$ ends at $\frac{a}{b} = 0.1$. These proportions are all relative, and value $c = 2$; $b = 10$; $a = 1$, fit the case at this terminal point. In order to have the mean radius, a , half the thickness of the winding, since 1 is half of 2, the winding must come down to the axis of the coil, and any further decrease of radius, which could be represented by a continuation of the curve to the left, is impossible. At this terminal point, any length of wire wound into a solid coil would have 62 per cent of the maximum inductance, and if the 5 to 1 section were retained, the same length of wire would have 86 per cent at $\frac{a}{b} = .8$, for which ratios the proportions $c = 1$, $b = 5$, $a = 4$, in which the coil has a central hole of diameter 7, give the same volume of winding as the solid coil above mentioned.

In Fig. 25, the chart for disk-shaped coils, the lines representing various winding sections intersect. If the abscissas were based upon the ratio of a to c , there would be no intersections. In disk coils, the value of c , the thickness is more important than that of b , the axial length, but it was thought best to use the same ratio for the abscissas in both charts, Fig. 24 and 25, to avoid confusion in readings.

To find the inductance of any coreless coil, the maximum inductance charts give direct readings of the inductance that it is possible to obtain from a given length of wire, while charts 24 and 25 show the percentage of that maximum that the actual shape of the coil permits; hence the product of the two readings gives the inductance of any coil whatever, whose dimensions are known. The same information, without calculation, is thus available, that the universal formulas (5) or (6) give.

XXIII. ILLUSTRATIVE EXAMPLES

To find the inductance, L , of 320 turns of No. 11 d. c. c. magnet wire, wound into a four-layer solenoid, whose dimensions are $a = 2.5$, $b = 8$, and $c = 0.4$ in. The length of conductor is therefore 419 ft., and since the charts do not directly indicate odd sizes, find the maximum inductance of 419 ft. of No. 10 wire from Fig. 11*a*, as 0.0095 henry, and of the same length of No. 12 from Fig. 11*b* as 0.0115, and assume the mean, 0.0105, as the value for No. 11 when wound for maximum inductance.

For the shape indicated, find the shape-factor by referring to Fig. 24 and as indicated by curve marked 20, which is $\frac{b}{c} = \frac{8}{0.4}$, and read the factor .57 as the ordinate of this curve at value $\frac{a}{b} = 0.31$. 57 per cent of .0105 is .006, or 6 milhenries, the inductance value sought.

It may be noted that this assumed coil has twice the linear dimensions of the coil of line 3 of Table 5, and therefore 8 times the weight and conductor length. Now for the same coil shape, inductance varies as the $\frac{5}{3}$ power of conductor length. The $\frac{5}{3}$ power of 8 is 32, and the tabular value of inductance of No. 3 coil, .186, multiplied by 32, gives 5.95 milhenries, agreeing with the chart value already found, and checking its essential accuracy.

As another example, let it be desired to design a coreless coil to produce 30 ohms reactance at 60 cycles, employing No. 10 d. c. c. magnet wire. Fig. 6 shows that 30 ohms at 60 cycles is obtained from .08 henry of inductance; Fig. 11*a* gives 1500 ft. of No. 10 for .08 henry, if wound into the maximum-inductance shape; and Fig. 15*a* gives the dimensions: $r = 2.75$, $a = 4.12$, $b = 3.30$ and $c = 2.75$ in., prescribing the coil and conductor dimensions. If the number of turns is desired, Fig. 19*a* gives $N' = 730$ as the number of turns.

If there is a restriction upon the coil dimensions, for example, that the inner radius, r , of the coil cannot be less than 8 in., and that the axial length of coil must be 1 in., the following approximation method will determine the greater amount of wire required. The restrictions make the coil disk-shaped, and Fig. 25 will indicate the percentage factor for the less advantageous shape. To use Fig. 25, a is required, and $a = r + .5c$, but c is to be determined also. As a first approximation, take $c = 4$, when $a = 10$, with b

given = 1; then $\frac{a}{b} = 10$, and $\frac{c}{b} = 4$. The curves of Fig. 25 represent directly $\frac{c}{b} = 2$ and $\frac{c}{b} = 5$. The values for $\frac{c}{b} = 4$ will fall between these curves, and as they intersect near the abscissa 10, no interpolation is required, and the shape-factor is read off as .75.

To obtain 0.08 henry will therefore require as much wire as would produce $\frac{.08}{.75} = .1067$ henry inductance if wound into the prescribed shape. Fig. 11a shows that 1750 ft. of No. 10 wire will be required, and Fig. 15a gives its coil dimensions as $a' = 4.35$, $b' = 3.48$, and $c' = 2.9$, from which $a' b' c' = 44$. The volume constant, abc , for the approximate dimensions assumed, is 40, showing that the assumed dimensions are 10 per cent too small to contain the 1750 ft. of wire. Since r and b are restricted, c alone can be increased, but as increasing c also affects a to a less degree, it is not required to increase c 10 per cent, but 8 per cent only, making the revised values as follows: $a'' = 10.16$, $b'' = 1$, $c'' = 4.32$, and their product, $a'' b'' c'' = 44$.

Referring again to Fig. 25, for determining any change in the shape-factor for $\frac{c}{b} = 4.32$, and $\frac{a}{b} = 10.16$, it is seen that no appreciable change has occurred, and the 1750 ft. of No. 10 wire are sufficient. If a noticeable difference has arisen, a second approximation, and further use of the charts as before would be advisable.

In using logarithmic charts by interpolation, it is well to note that it should be done by logarithmic differences, which vary slightly from ordinary proportion.

XXIV. WEIGHTS AND REACTANCE-RESISTANCE RATIOS

If it be assumed that equal weights of wire occupy the same space, true of bare wire, and approximately true of insulated wire, then equation (14) shows that equal weights of wire produce inductance in proportion to the square of the conductor length. Thus a given weight of No. 13 magnet wire would have twice the length and yield four times the inductance that the same weight of No. 10 would give. It is seen that inductance is more cheaply obtained from small wires, and that the smallest wire that it is safe to use should be employed

For equal weights, resistance also varies as the square of the length of the conductor, so the ratio of inductance to resistance is practically constant for a given weight of wire, regardless of its size, provided the coil is wound for maximum inductance. Therefore the time constant, the value of $\frac{L}{R}$, depends upon the weight of the coil for maximum shaped coils.

From another equation (17), it is seen that with definite sized wire, inductance increases with the $\frac{5}{3}$ power of the conductor length. Resistance increases directly with length, therefore the ratio of the two increases with the $\frac{2}{3}$ power of the increase in length, or of weight, since weight and length increase together. A definite ratio of reactance to resistance requires a nearly definite weight of wire, independent of the size of the conductor. Thus 0.5 lb. of wire will produce a reactance at 60 cycles equal to its resistance in ohms, while a coil weighing 1000 times as much, or 500 lb. will produce a reactance that is 100 times as great as its resistance, since $100 = 1000^{\frac{2}{3}}$. As a specific example, 0.5 lb. of No. 14 wire has a reactance and a resistance of about 0.1 ohm, while the 500 lb. will give 10 000 ohms of reactance, and 100 ohms of resistance. This ratio of reactance to resistance of 100, giving a lag angle of 89.5° if connected at its terminals to an alternating electromotive-force may be considered nearly pure reactance, as the resistance is negligible in comparison.

TABLE 6.

ESTIMATE OF COIL WEIGHTS FOR CERTAIN RATIOS OF REACTANCE TO RESISTANCE COILS TO BE CLOSE-WOUND IN THE PRESCRIBED SHAPE FOR PRODUCING MAXIMUM INDUCTANCE.

| $\frac{X}{R}$ | Lag Angle | WEIGHT OF COPPER IN LB. | | |
|---------------|-----------|-------------------------|-----------|------------|
| | | 25 Cycles | 60 Cycles | 300 Cycles |
| 1. | 45 | 2. | 0.5 | .045 |
| 2.5 | 68 | 7.5 | 2. | .18 |
| 6. | 80 | 30. | 8. | .72 |
| 10. | 84 | 60. | 16. | 1.44 |
| 100. | 89.5 | 1800. | 500. | 45. |

This table is not applicable to spaced coils, which it may be necessary to use in order to obtain sufficient insulation between turns.

The table above gives the results of a rough estimate of the weights of wire required for the various ratios of reactance, $X = 2\pi fL$, to resistance, R , of maximum inductance coreless coils wound with wire of any size. The estimate is necessarily rough, since the weights will vary slightly, increasing for the finer sizes of wire.

XXV. RATINGS OF COILS

There is no established rating for inductance coils. Since the object of such coils is usually to offer as much reactance as possible, the volt-ampere or kilovolt-ampere rating should be adopted. The rating is then the value of I^2X , and the indefinite factor is the current, I , since it is a matter of judgment how much current a certain conductor may safely carry, depending upon the conditions of operation. If a definite density is selected, the rating is determined when the coil dimensions are known. This rating corresponds to the resistance rating in I^2R units, which are watts. For a definite current density, I^2R varies directly with weight, while I^2X varies with the $\frac{5}{3}$ power of weight, as explained in the preceding paragraph, the current being fixed by the size of the conductor. Hence the rating of a reactance coil increases much more rapidly than a resistance coil, as conductor length increases.

The cost per kilovolt-ampere diminishes with increase in size of coil. Thus, a coil of No. 14 wire having a weight of 16 lb. has a reactance of about 32 ohms at 60 cycles, and costs, at 20 cents per lb. for copper, \$3.20. The rating, I^2X , is 3.2 kilovolt-amperes, if 10 amperes be employed, making the cost \$1 per kilovolt-ampere. Using the same size wire in the 500 lb. coil, the reactance is 10 000 ohms, the I^2X rating 1000 kilovolt-amperes, and the copper costs but \$100, or 10 cents per kilovolt-ampere. At 25 cycles, the weight of coil is nearly one ton before the cost per kilovolt-ampere is reduced to 10 cents.

The diminishing cost of copper per kilovolt-ampere for reactance may be compared with the constant cost of copper per kilowatt for resistance. Using the same current density and price of copper per pound, the cost is uniformly \$10 per kilowatt of I^2R . The extravagance of resistance control of alternating current is evident.

The rapid gain in the rating of coreless reactance coils with weight thus points to their increased use in engineering work,

since in very large sizes they are relatively cheap, and have the great advantage of absence of core losses as compared with ferric reactances.

XXVI. MUTUAL INDUCTANCE

Mutual inductance may sometimes be ascertained through self inductance calculations. Mutual inductance may be defined as the inductive influence of one coil or circuit upon another. Thus if a coil be thought of as consisting of two half-coils, each half has mutual inductance with respect to the other half as well as its own self-inductance. The self inductance of the whole coil will then be equal to the sum of the mutual inductances of the half coils added to their own self inductances. If, however, the two halves be connected in opposition, the mutual inductances are opposed to the self-inductances, and the combined self-inductance is equal to the difference between the self and mutual inductances. Expressed in symbols these facts are as follows:—

$$(22) \quad L = L_1 + L_2 + 2M, \text{ when the coils are in series.}$$

$$(23) \quad L_0 = L_1 + L_2 - 2M, \text{ when the coils are in opposition.}$$

As thus expressed, these formulas are not confined to equal halves of a single coil, but are general, where L_1 and L_2 are the self-inductances of the two parts, and M their mutual inductance. L_0 is the combined inductance of the parts in opposition, while L is their self-inductance in series.

When L_1 and L_2 are such parts of one coil that together they make a close-winding, the formulas of this bulletin or the charts may be used in obtaining their values, when the above equations will give the mutual inductance of one part upon the other.

If a winding have a square section, the mutual inductance of real or imagined halves, side by side, is substantially equal to that of two halves one within the other. Mutual inductance can never exceed 25 per cent of L , the self-inductance of the whole coil. Highest values of mutual inductance occur when the two parts make nearly the maximum prescribed shape; although if a disk-shaped coil where $\frac{b}{c} = \frac{2}{4}$ is divided into two thinner disks, side by side, each having a section $\frac{b}{c} = \frac{1}{4}$; or a short thick tube having the section proportions $\frac{b}{c} = \frac{4}{2}$, and the two half coils are the inner and outer layers of this short tube,

the value of M will be found to be slightly greater than if a square section coil be divided into two halves either transversely or longitudinally.

It should be noted that 15 per cent to 20 per cent of a large value of L may give a greater value of M than 25 per cent of a small value of L ; and that a minimum value of L_c does not coincide with the high values of M . It is possible to extend the calculation of mutual inductances by self-inductance formulas to coils not contiguous by having or assuming a coil to fill the gap between,—a method described in the Bureau of Standards, Bulletin Vol. V. No. 1, page 20.

It seems to be advantageous in transformers to make the two coils constituting the primary and secondary windings have as high a mutual inductance as possible, independent of the action of the core, in order that the weight of the core may be reduced, especially in the very large transformers that are now being built.

In very large reactance coils without cores, where the actual flux density within the coil attains values approaching those with weak iron magnetic circuits, the mechanical stresses due to sudden short-circuits are but little appreciated. It will be necessary to build such coils substantially, and to use spaced windings for this purpose, even if not required for insulation. That such coils will have an increasing use is probable, especially when it is realized that the cost per kilovolt-ampere decreases so rapidly with increase of rating, making the cost of very large coils comparable with cored coils, to which they are in many respects so superior.

Owing to the increasing use of non-ferric coils, there is a demand for convenient data concerning their characteristics, for definite knowledge of their most effective shape, and for a simple method of determining their inductive reactance. It is hoped that the information herein presented will facilitate the construction and utilization of such coils in every branch of electrical engineering.

XXVII. TABLES AND CHARTS

TABLE 7
DIAMETERS OF COTTON COVERED WIRE*
(Standard Underground Cable Co.)

| Size B & S | Diameter in Mils | | | | | |
|---------------|------------------|----------|----------|----------|----------|----------|
| | Bare | S. C. C. | D. C. C. | T. C. C. | S. S. C. | D. S. C. |
| 0000 | 460 | 469 | 478 | 487 | 462 | 464 |
| 000 | 410 | 419 | 428 | 437 | 412 | 414 |
| 00 | 365 | 374 | 383 | 392 | 367 | 369 |
| 0 | 325 | 334 | 343 | 352 | 327 | 329 |
| 1 | 289 | 298 | 307 | 316 | 391 | 293 |
| 2 | 258 | 267 | 276 | 285 | 260 | 262 |
| 3 | 229 | 238 | 247 | 256 | 231 | 233 |
| 4 | 204 | 213 | 222 | 231 | 206 | 208 |
| 5 | 182 | 191 | 200 | 209 | 184 | 186 |
| 6 | 162 | 170 | 178 | 186 | 164 | 166 |
| 7 | 144 | 152 | 160 | 168 | 146 | 148 |
| 8 | 128 | 135 | 142 | 149 | 130 | 132 |
| 9 | 114 | 120 | 126 | 132 | 116 | 118 |
| 10 | 102 | 107 | 112 | 117 | 104 | 106 |
| 11 | 91 | 96 | 101 | 106 | 93 | 95 |
| 12 | 81 | 86 | 91 | 96 | 83 | 85 |
| 13 | 72 | 77 | 81 | 87 | 74 | 76 |
| 14 | 64 | 69 | 74 | 79 | 66 | 68 |

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TABLE 8
THICKNESS OF COTTON AND SILK INSULATION*

| Size | Thickness of Insulation in Mils | | | | | |
|--------|---------------------------------|--------|--------|--------|--------|--------|
| | B & S | S.C.C. | D.C.C. | T.C.C. | S.S.C. | D.S.C. |
| 0000-5 | 4.5 | 9.0 | 13.5 | 1.0 | 2.0 | R.S.U. |
| 6-7 | 4.0 | 8.0 | 12.0 | 1.0 | 2.0 | " |
| 8 | 3.5 | 7.0 | 10.5 | 1.0 | 2.0 | " |
| 9 | 3.0 | 6.0 | 9.0 | 1.0 | 2.0 | " |
| 10-12 | 2.5 | 5.0 | 7.5 | 1.0 | 2.0 | " |
| 13-19 | 2.25 | 4.5 | 7.75 | 1.0 | 2.0 | R |
| 13-32 | 2.5 | 4.5 | 7.0 | 1.0 | 2.0 | S.U. |
| 14-15 | 3.0 | 5.0 | 7.0 | ... | ... | G.E. |
| 16-18 | 2.5 | 4.0 | ... | ... | ... | " |
| 19-22 | 2.0 | 4.0 | ... | ... | ... | " |
| 23-25 | 2.0 | 4.0 | ... | 1.5 | 3.0 | " |
| 20-40 | 2.0 | 4.0 | 6.0 | 1.0 | 2.0 | R |
| 26-28 | 2.0 | 4.0 | ... | 1.5 | 3.0 | G.E. |
| 29-34 | 2.0 | 4.0 | ... | 1.2 | 2.5 | " |
| 32-36 | ... | ... | ... | 0.87 | 1.75 | S.U. |
| 35-40 | 2.0 | 4.0 | ... | 1.0 | 2.0 | G.E. |

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†R=Roebing's Sons Company.

S.U.=Standard Underground Cable Company.

G.E.=General Electric Company.

NOTE:—Diameter overall = diameter bare + 2 (thickness of insulation).

TABLE 9
DIAMETER OF SMALL SIZES OF MAGNET WIRE*
(GENERAL ELECTRIC CO.)

| Size | Diameter in Mils | | | | | | |
|------|------------------|------|--------|--------|--------|--------|--------|
| | B & S | Bare | S.C.C. | D.C.C. | S.S.C. | D.S.C. | Enamel |
| 14 | 64 | 70 | 64 | | | | 67 |
| 15 | 57 | 63 | 67 | | | | 60 |
| 16 | 51 | 56 | 59 | | | | 53.5 |
| 17 | 45 | 50 | 53.0 | | | | 47.5 |
| 18 | 40 | 46 | 48 | | | | 42 |
| 19 | 36 | 40 | 44 | | | | 37 |
| 20 | 32 | 36 | 40 | | | | 34 |
| 21 | 28 | 32.5 | 36.5 | | | | 30.5 |
| 22 | 25 | 29.4 | 33.4 | | | | 27.5 |
| 23 | 23 | 26.5 | 30.5 | 26 | | 29 | 25 |
| 24 | 20 | 24.1 | 28 | 23 | | 26 | 22 |
| 25 | 18 | 22 | 26 | 21 | | 24 | 20 |
| 26 | 16 | 20 | 24 | 19 | | 22 | 17.5 |
| 27 | 14 | 18 | 22 | 17 | | 20 | 15.5 |
| 28 | 12.6 | 16.6 | 20.6 | 15.6 | | 18.6 | 14 |
| 29 | 11 | 15.3 | 19.3 | 14 | | 17 | 12.3 |
| 30 | 10 | 14 | 18 | 12.5 | | 15 | 11.3 |
| 31 | 9 | 13 | 17 | 11.4 | | 13.9 | 10.2 |
| 32 | 8 | 11.9 | 15.9 | 10.5 | | 13 | 9.2 |
| 33 | 7 | 11.0 | 15 | 9.5 | | 12 | 8.2 |
| 34 | 6.3 | 10.3 | 14.3 | 8.8 | | 11.3 | 7.3 |
| 35 | 5.6 | 9.6 | 13.6 | 7.6 | | 9.6 | 6.8 |
| 36 | 5 | 8.5 | 12 | 7 | | 9.0 | 6.2 |
| 38 | 4.5 | | | 6 | | 8 | 5.2 |
| 40 | 4 | | | 5 | | 7 | 4.2 |

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TABLE 10
 WEIGHTS OF SMALL SIZES OF MAGNET WIRE*
 (GENERAL ELECTRIC CO.)

| Size | Weight in Pounds per 1000 Feet. | | | | |
|-------|---------------------------------|----------|----------|----------|--------|
| B & S | S. C. C. | D. C. C. | S. S. C. | D. S. C. | Enamel |
| 14 | 12.684 | 12.918 | | | 12.684 |
| 15 | 10.082 | 10.274 | | | 10.053 |
| 16 | 8.012 | 8.176 | | | 7.973 |
| 17 | 6.375 | 6.510 | | | 6.322 |
| 18 | 5.081 | 5.188 | | | 5.009 |
| 19 | 4.043 | 4.130 | | | 3.966 |
| 20 | 3.215 | 3.289 | | | 3.136 |
| 21 | 2.569 | 2.628 | | | 2.475 |
| 22 | 2.055 | 2.106 | | | 1.970 |
| 23 | 1.630 | 1.676 | 1.57 | 1.604 | 1.555 |
| 24 | 1.297 | 1.344 | 1.241 | 1.298 | 1.232 |
| 25 | 1.036 | 1.082 | .991 | 1.040 | .980 |
| 26 | .828 | .873 | .791 | .833 | .777 |
| 27 | .661 | .703 | .631 | .666 | .616 |
| 28 | .524 | .562 | .499 | .521 | .485 |
| 29 | .421 | .457 | .397 | .416 | .384 |
| 30 | .336 | .372 | .315 | .332 | .303 |
| 31 | .271 | .307 | .254 | .267 | .242 |
| 32 | .215 | .248 | .203 | .214 | .192 |
| 33 | .174 | .201 | .161 | .172 | .152 |
| 34 | .141 | .161 | .130 | .140 | .121 |
| 35 | .12 | .137 | .110 | .119 | .101 |
| 36 | .099 | .112 | .089 | .096 | .081 |
| 38 | | | .058 | .065 | .051 |
| 40 | | | .037 | .040 | .031 |

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TABLE 11
SPACE OCCUPIED BY MAGNET WIRES*

| Turns per Inch | | | | | |
|----------------|----------|----------|----------|----------|--------|
| B. & S. | S. C. C. | D. C. C. | S. S. C. | D. S. C. | Enamel |
| 14 | 14 | 13 | | | 14 |
| 15 | 15 | 14 | | | 16 |
| 16 | 17 | 16 | | | 18 |
| 17 | 20 | 18 | | | 21 |
| 18 | 22 | 20 | | | 23 |
| 19 | 25 | 22 | | | 27 |
| 20 | 27 | 25 | | | 29 |
| 21 | 30 | 27 | | | 32 |
| 22 | 34 | 30 | | | 36 |
| 23 | 37 | 32 | 38 | 34 | 40 |
| 24 | 41 | 35 | 43 | 38 | 45 |
| 25 | 45 | 38 | 47 | 41 | 50 |
| 26 | 50 | 41 | 52 | 45 | 57 |
| 27 | 55 | 45 | 58 | 50 | 64 |
| 28 | 60 | 48 | 64 | 53 | 71 |
| 29 | 65 | 51 | 71 | 58 | 81 |
| 30 | 71 | 55 | 80 | 66 | 88 |
| 31 | 76 | 58 | 87 | 71 | 104 |
| 32 | 84 | 62 | 95 | 76 | 120 |
| 33 | 90 | 66 | 105 | 83 | 130 |
| 34 | 97 | 69 | 110 | 88 | 140 |
| 35 | 104 | 73 | 130 | 104 | 160 |
| 36 | 117 | 82 | 140 | 110 | 190 |
| 38 | | | 160 | 120 | |
| 40 | | | 200 | 140 | 230 |

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TABLE 12
SPACE OCCUPIED BY MAGNET WIRES*

| Turns per Sq. In. | | | | | |
|-------------------|--------|--------|--------|--------|--------|
| B.&S. | S.C.C. | D.C.C. | S.S.C. | D.S.C. | Enamel |
| 14 | 204 | 182 | | | 222 |
| 15 | 252 | 223 | | | 278 |
| 16 | 318 | 281 | | | 350 |
| 17 | 400 | 356 | | | 443 |
| 18 | 495 | 435 | | | 567 |
| 19 | 625 | 516 | | | 692 |
| 20 | 775 | 625 | | | 865 |
| 21 | 948 | 752 | | | 1070 |
| 22 | 1150 | 896 | | | 1320 |
| 23 | 1420 | 1070 | 1480 | 1190 | 1600 |
| 24 | 1720 | 1270 | 1830 | 1480 | 2060 |
| 25 | 2060 | 1480 | 2270 | 1740 | 2500 |
| 26 | 2500 | 1740 | 2770 | 2060 | 3260 |
| 27 | 3080 | 2060 | 3460 | 2500 | 4160 |
| 28 | 3620 | 2360 | 4100 | 2890 | 5100 |
| 29 | 4270 | 2680 | 5100 | 3460 | 6600 |
| 30 | 5100 | 3080 | 6410 | 4440 | 7830 |
| 31 | 5920 | 3460 | 7690 | 5180 | 9610 |
| 32 | 7070 | 3970 | 9090 | 5920 | 11800 |
| 33 | 8260 | 4440 | 11000 | 6940 | 14800 |
| 34 | 9440 | 4900 | 12900 | 7820 | 17700 |
| 35 | 10800 | 5400 | 17380 | 10800 | 21600 |
| 36 | 13000 | 6940 | 20400 | 12300 | 26000 |
| 38 | | | 27700 | 15600 | 37000 |
| 40 | | | 40000 | 20400 | 54000 |

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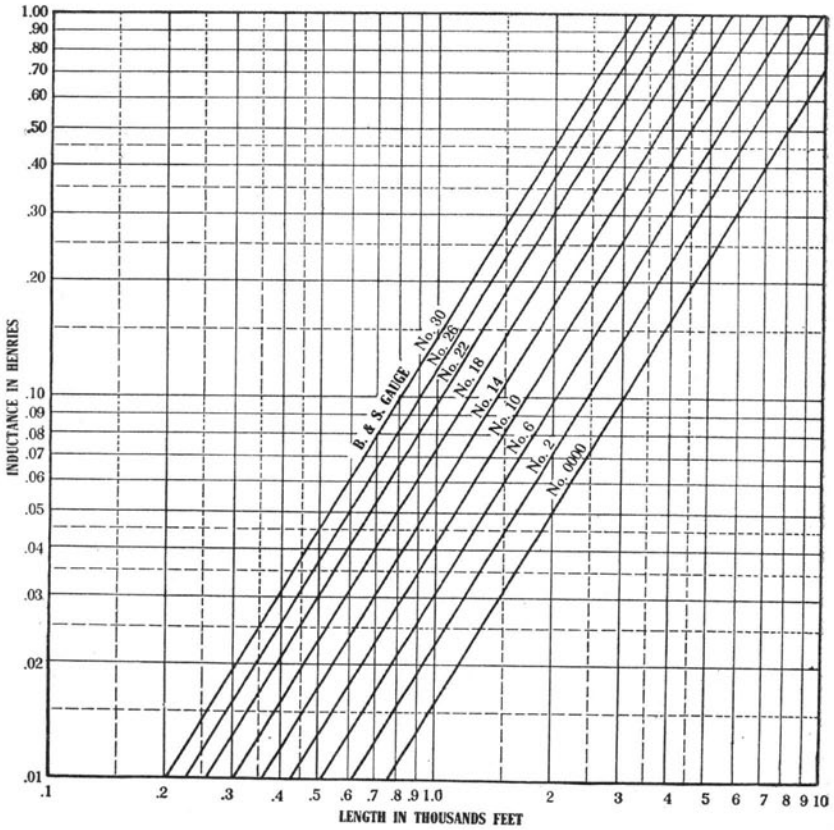


FIG. 11a

FIG. 11a AND 11b APPLY TO DOUBLE-COTTON-COVERED

Charts for Determining the Length of Magnet Wire When Wound into the Most Economical Shape for Producing a Given Inductance, L , in a

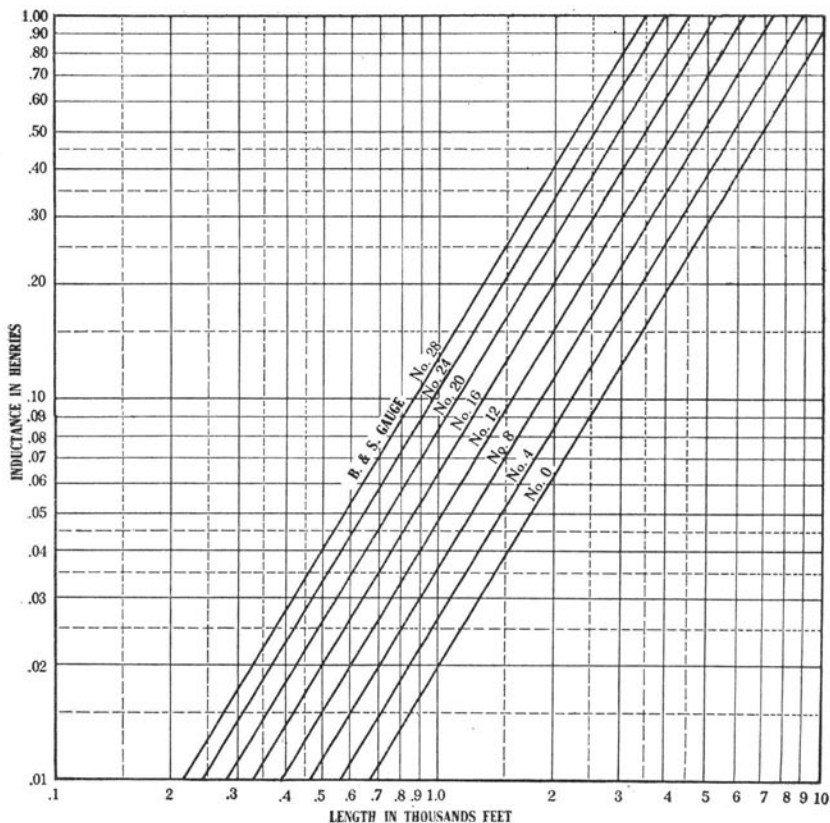


FIG. 11b

(D. C. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

Coil without Iron; or for Finding the Inductance in Henries of Any Length of Wire When Wound into the Prescribed Maximum-Inductance Shape. (See Section XX).

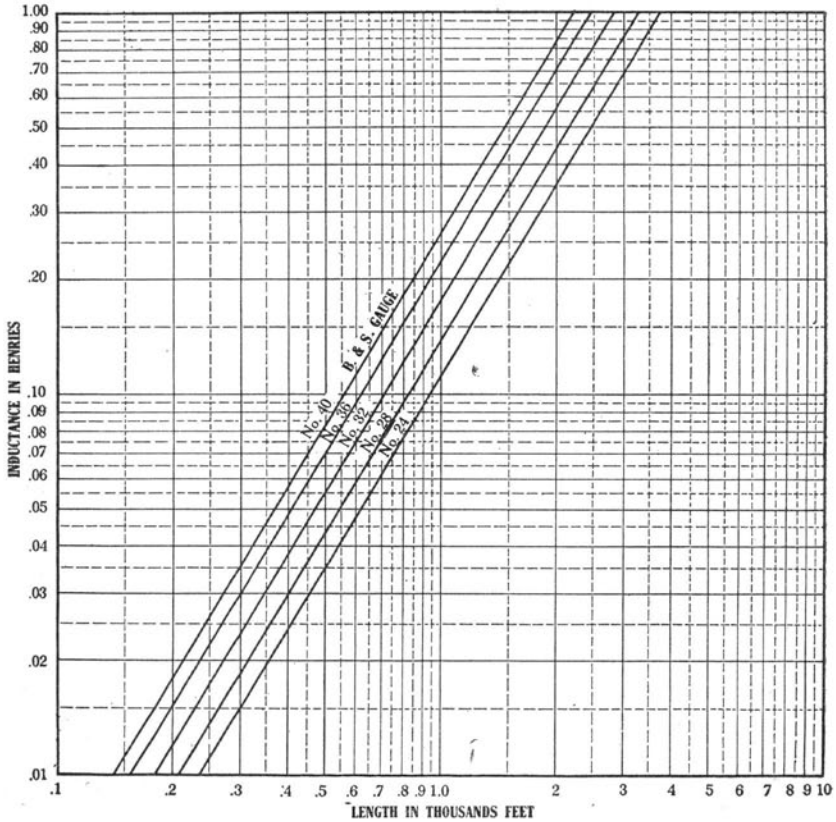


FIG. 12a

FIG. 12a AND 12b APPLY TO DOUBLE-SILK-COVERED

Charts for Determining the Length of Magnet Wire When Wound into the Most Economical Shape for Producing a Given Inductance, L , in a

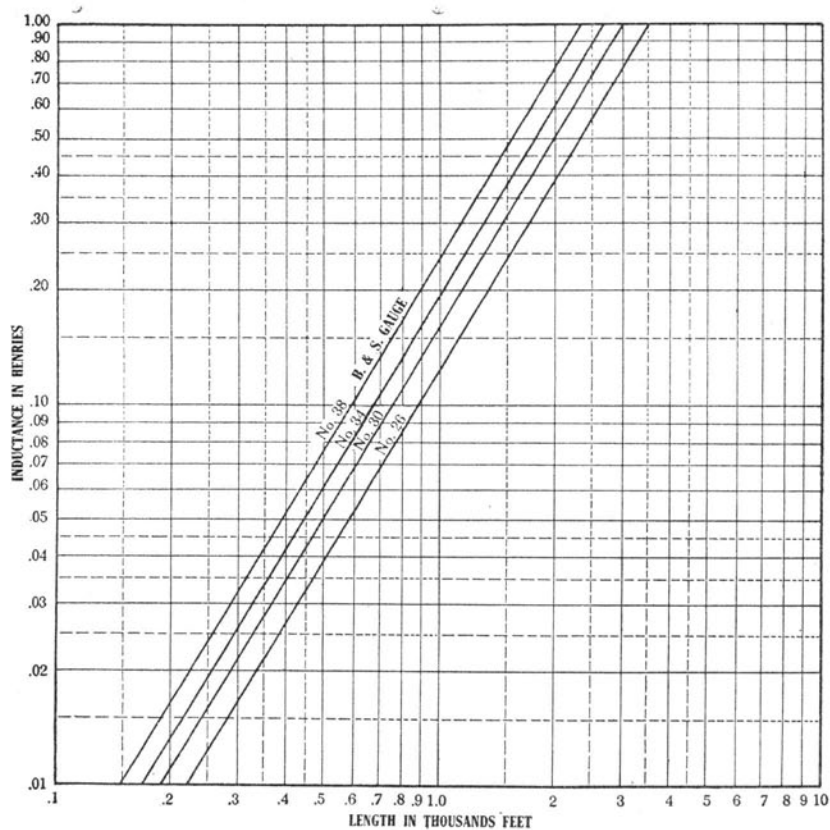


FIG. 12b

(D. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

Coil without Iron; or for Finding the Inductance in Henries of any Length of Wire When Wound into the Prescribed Maximum-Inductance Shape.

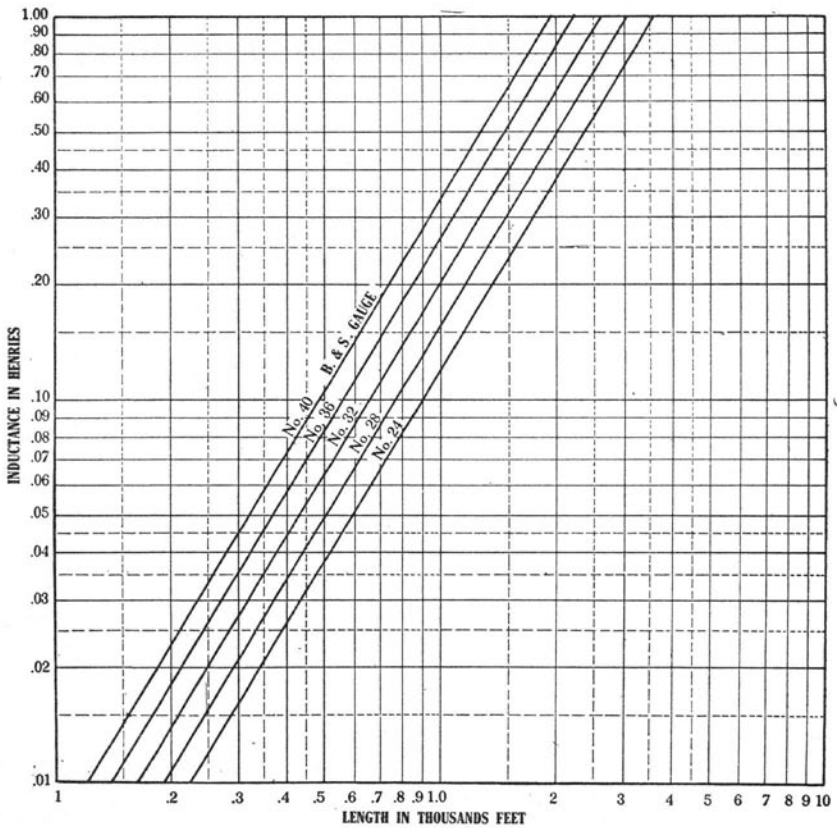


FIG. 13a

FIG. 13a AND 13b APPLY TO SINGLE-SILK-COVERED
 Charts for Determining the Length of Magnet Wire When Wound
 into the Most Economical Shape for Producing a Given Inductance, L , in a

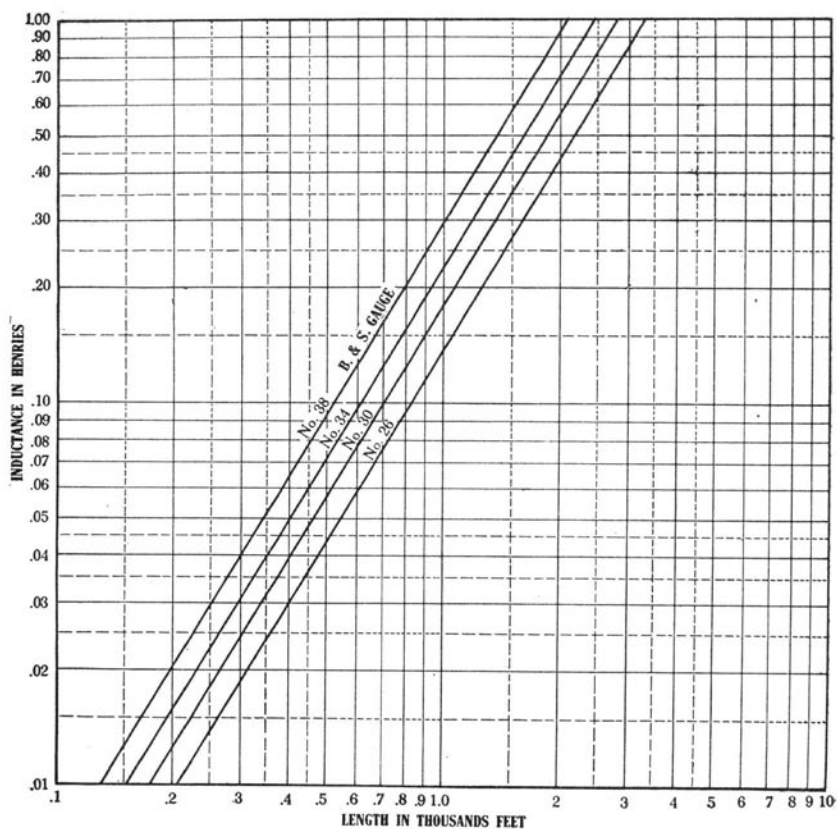


FIG. 13b

(S. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

Coil without Iron; or for Finding the Inductance in Henries of Any Length of Wire When Wound into the Prescribed Maximum-Inductance Shape.

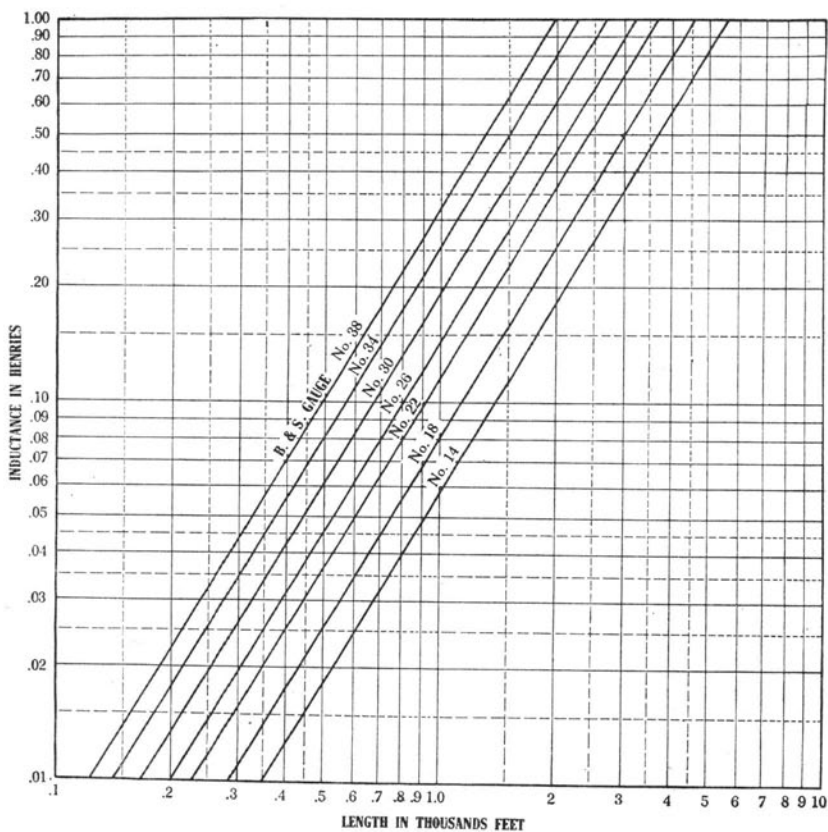


FIG. 14a

FIG. 14a AND 14b APPLY TO ENAMEL-COVERED

Charts for Determining the Length of Magnet Wire When Wound into the Most Economical Shape for Producing a Given Inductance, L , in a

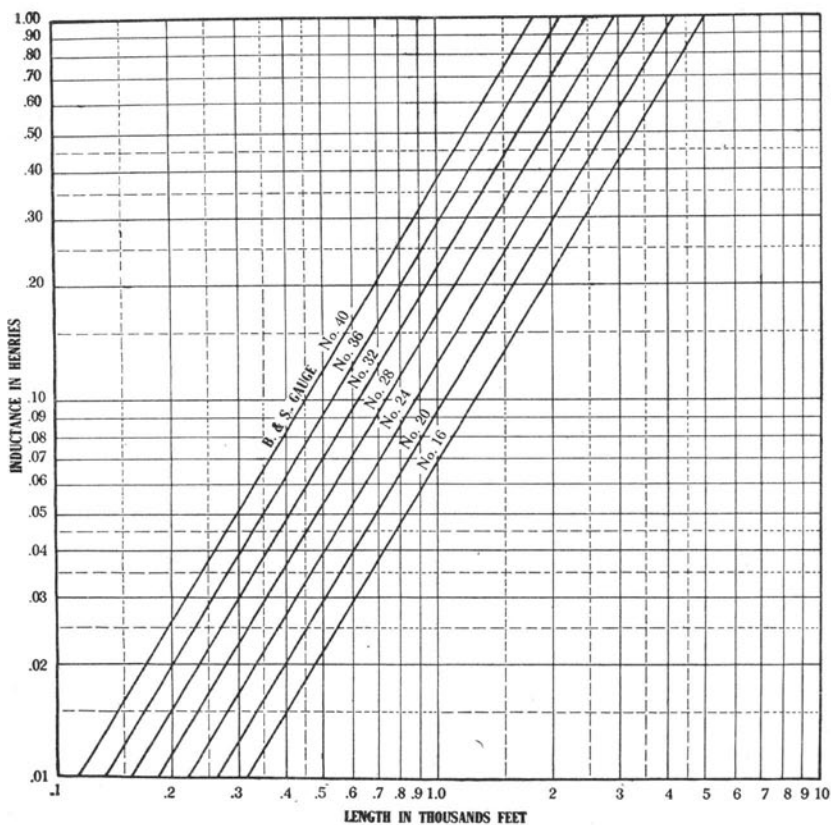


FIG. 14b

MAGNET WIRE OF EVEN GAUGE NUMBERS

Coil without Iron; or for Finding the Inductance in Henries of any Length of Wire When Wound into the Prescribed Maximum-Inductance Shape.

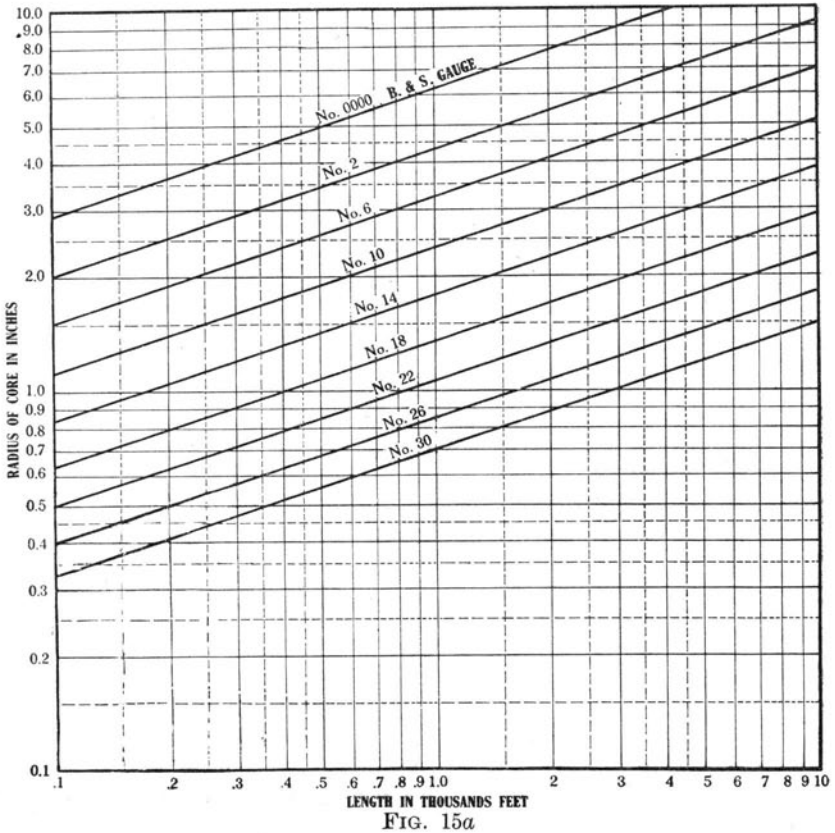


FIG. 15a AND 15b ARE CHARTS FOR INDICATING THE INNER RADIUS, r , OF COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE WOUND WITH Other Coil Dimensions Are Then Proportional:

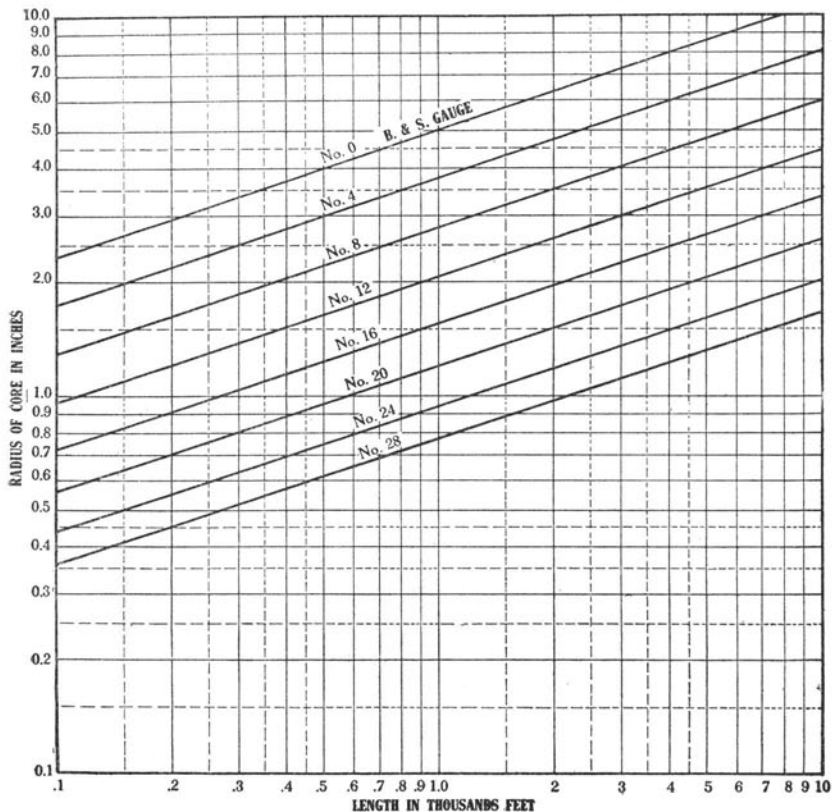


FIG. 15b

ANY LENGTH OF DOUBLE-COTTON-COVERED (D. C. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

$a = 1.5 r$; $b = 1.2 r$; $c = r$. (See section XXI.)

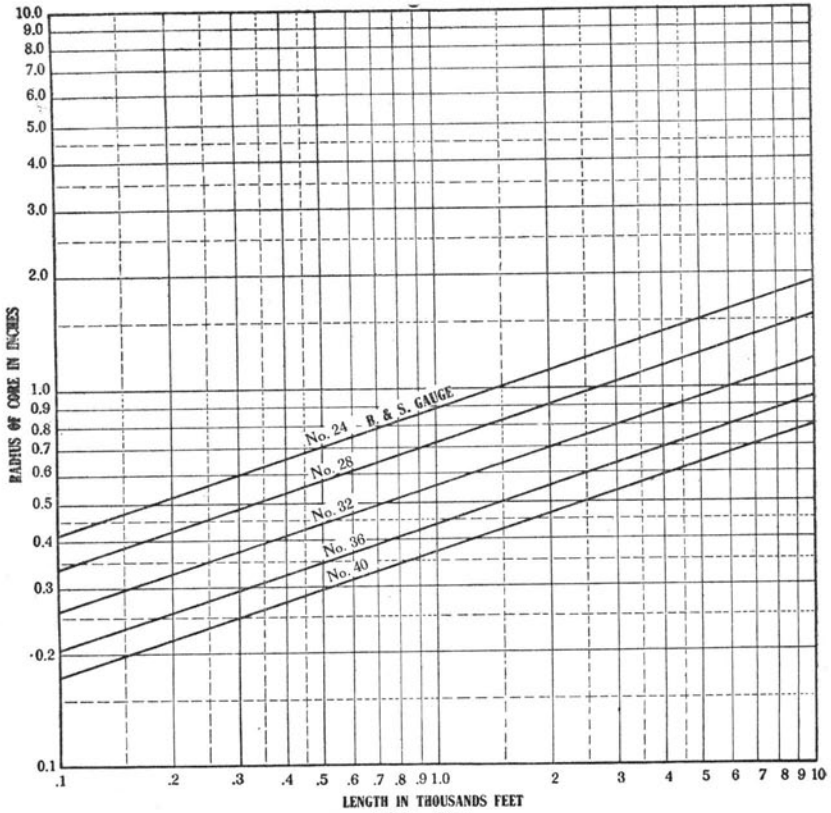


FIG. 16a

FIG. 16a AND 16b ARE CHARTS FOR INDICATING THE INNER RADIUS, r , OF COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE WOUND WITH ANY OTHER COIL DIMENSIONS ARE THEN PROPORTIONAL:

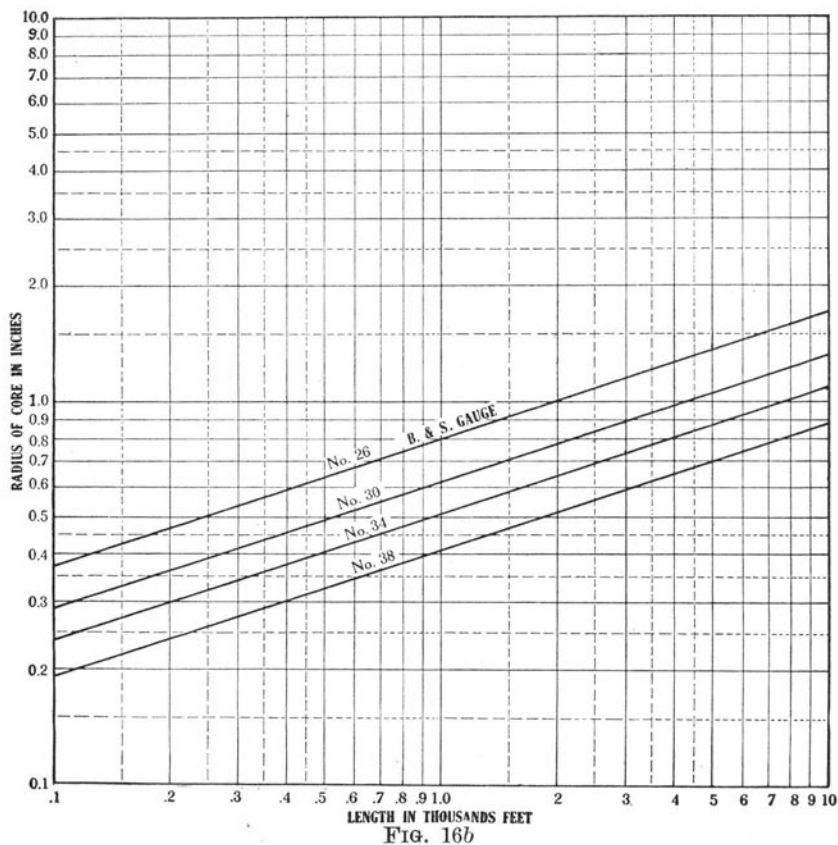


FIG. 16b
 LENGTH OF DOUBLE-SILK-COVERED (D. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

$a = 1.5 r; b = 1.2 r; c = r.$ (See section XXI.)

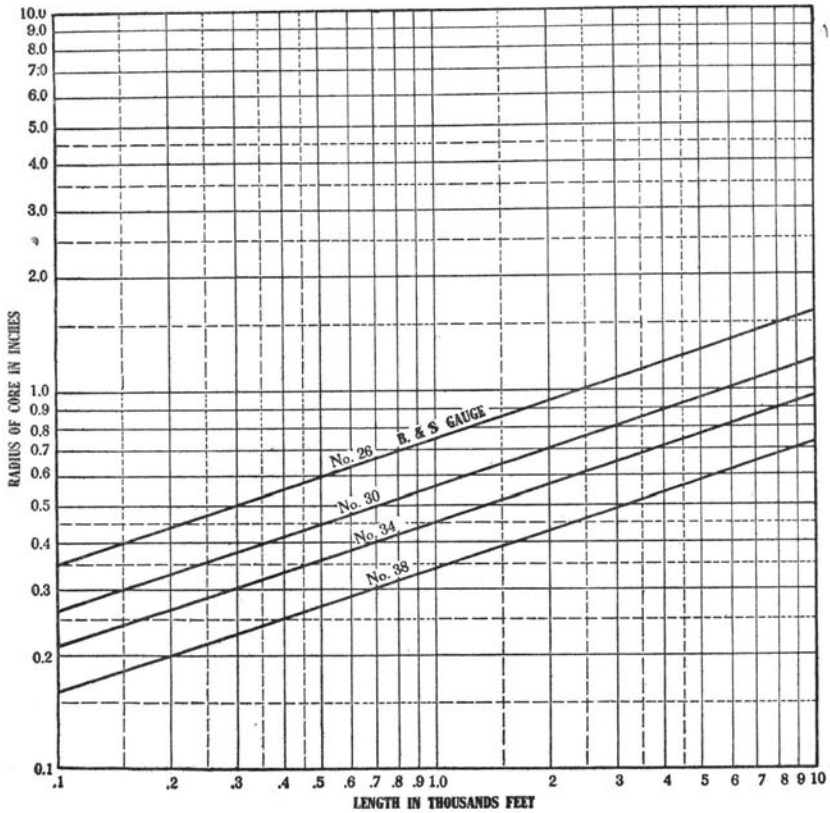


FIG. 17a

FIG. 17a AND 17b ARE CHARTS FOR INDICATING THE INNER RADIUS, r , OF COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE WOUND WITH ANY OTHER COIL DIMENSIONS ARE THEN PROPORTIONAL:

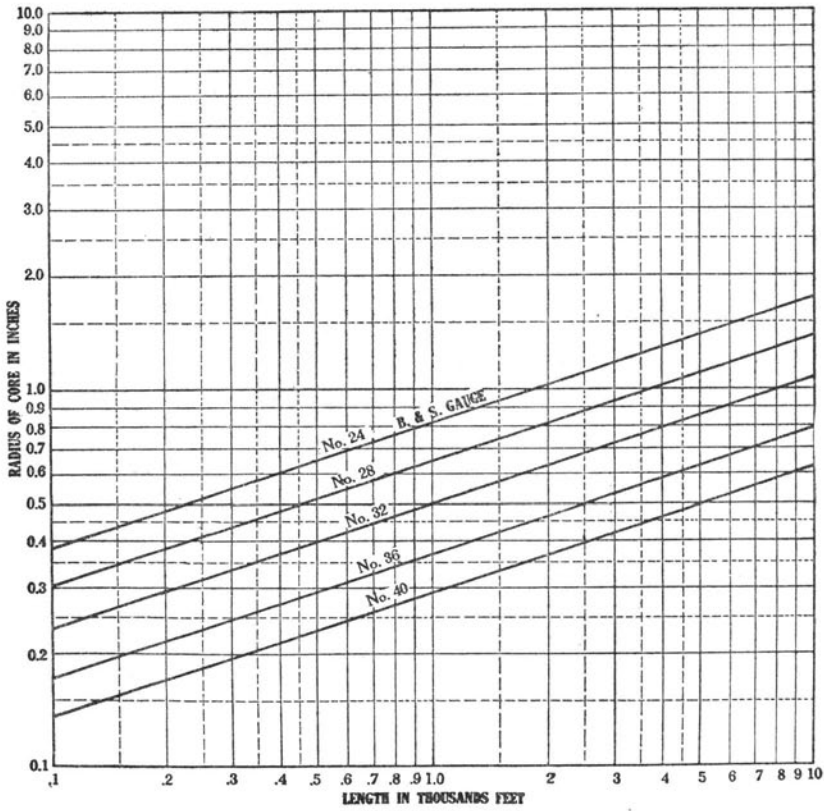


FIG. 17b

LENGTH OF SINGLE-SILK-COVERED (S. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

$a = 1.5 r$; $b = 1.2 r$; $c = r$. See section (XXI.)

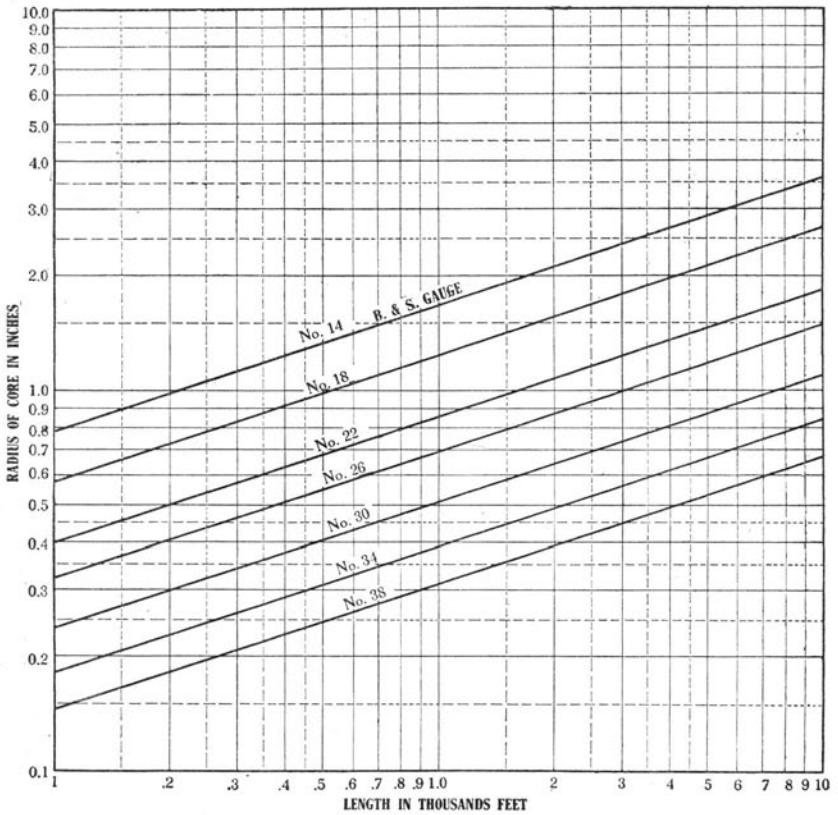


FIG. 18a

FIG. 18a AND 18b ARE CHARTS FOR INDICATING THE INNER RADIUS, r , OF COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE WOUND WITH ANY OTHER COIL DIMENSIONS ARE THEN PROPORTIONAL:

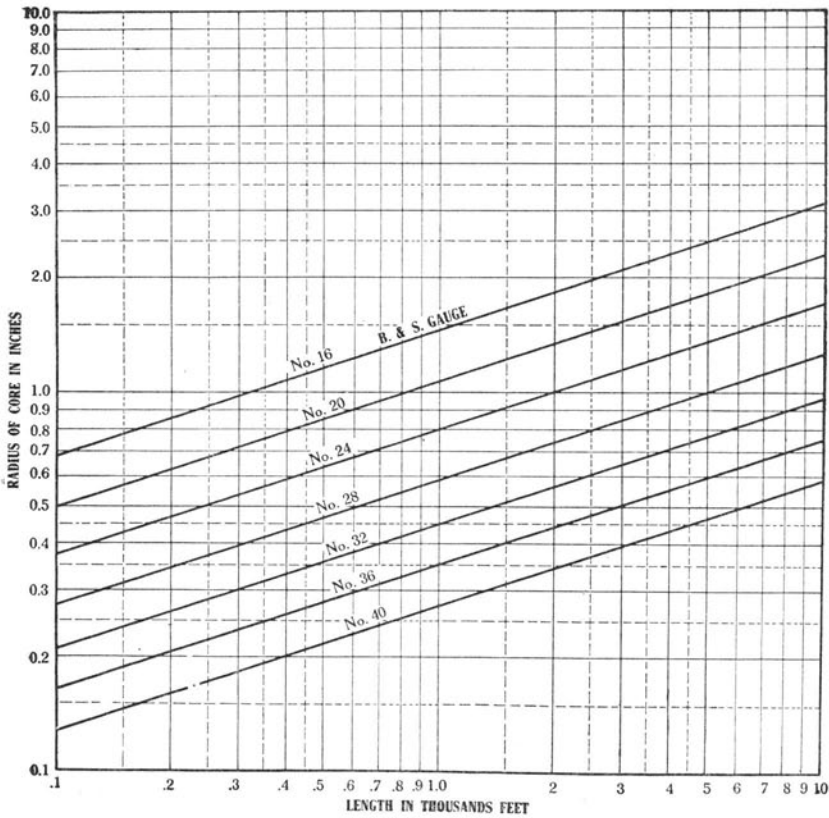


Fig. 18b

LENGTH OF ENAMEL-COVERED MAGNET WIRE OF EVEN GAUGE NUMBERS

$a = 1.5 r$; $b = 1.2 r$; $c = r$. (See section XXI.)

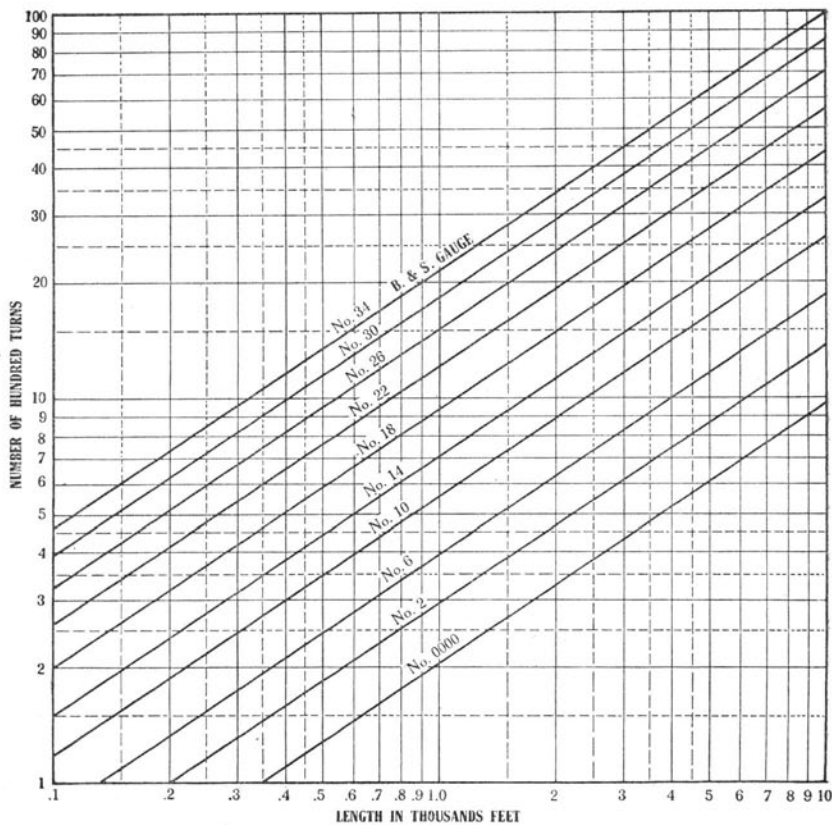


FIG. 19a

FIG. 19a AND 19b ARE CHARTS FOR INDICATING THE NUMBER OF HUNDREDS OF TURNS FOR COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE

Chart Values, Multiplied by 100, Give N' in Formula (20).

The Indicated Number of Turns is Subject to Adjustment for Ob-

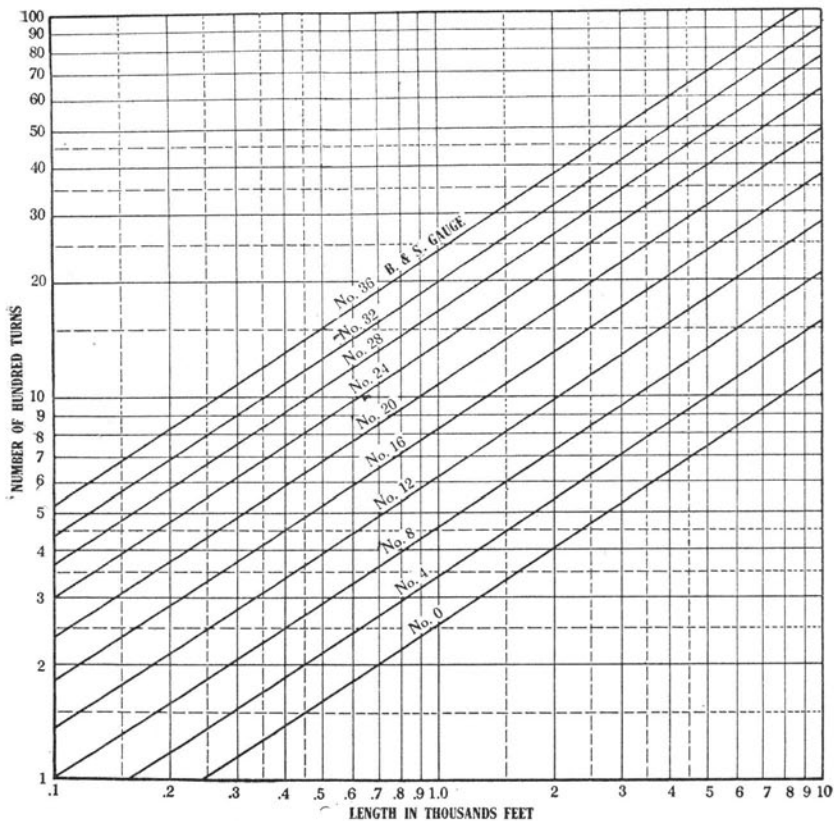


FIG. 19b

WOUND WITH ANY LENGTH OF DOUBLE-COTTON-COVERED (D.C. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS
 taining Complete Layers of the Winding, and Is Not Directly Serviceable for Coils of Other Shapes.

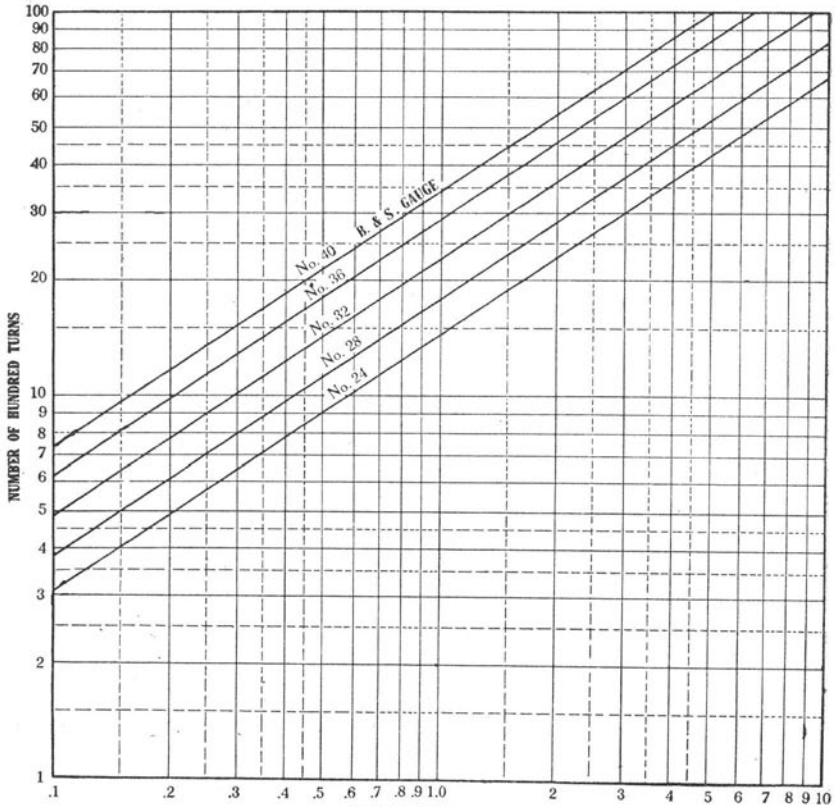


FIG. 20a

FIG. 20a AND 20b ARE CHARTS FOR INDICATING THE NUMBER OF HUNDREDS OF TURNS FOR COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE

Chart Values, Multiplied by 100, Give N' in Formula (20).

The Indicated Number of Turns Is Subject to Adjustment for Ob-

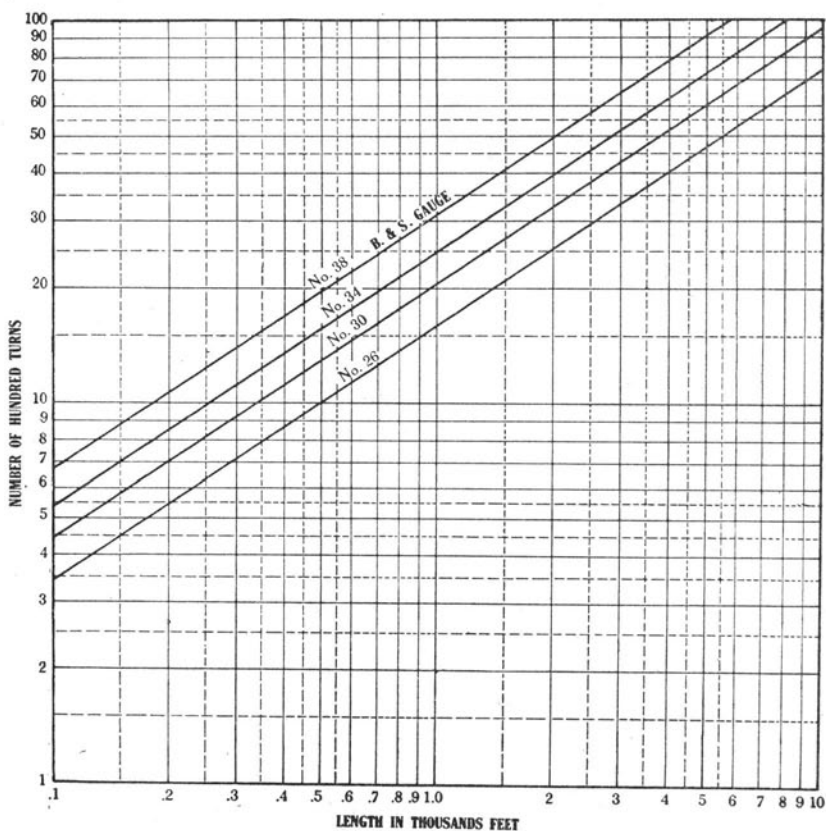


FIG. 20b

WOUND WITH ANY LENGTH OF DOUBLE-SILK-COVERED (D. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

taining Complete Layers of the Winding, and Is Not Directly Serviceable for Coils of Other Shapes.

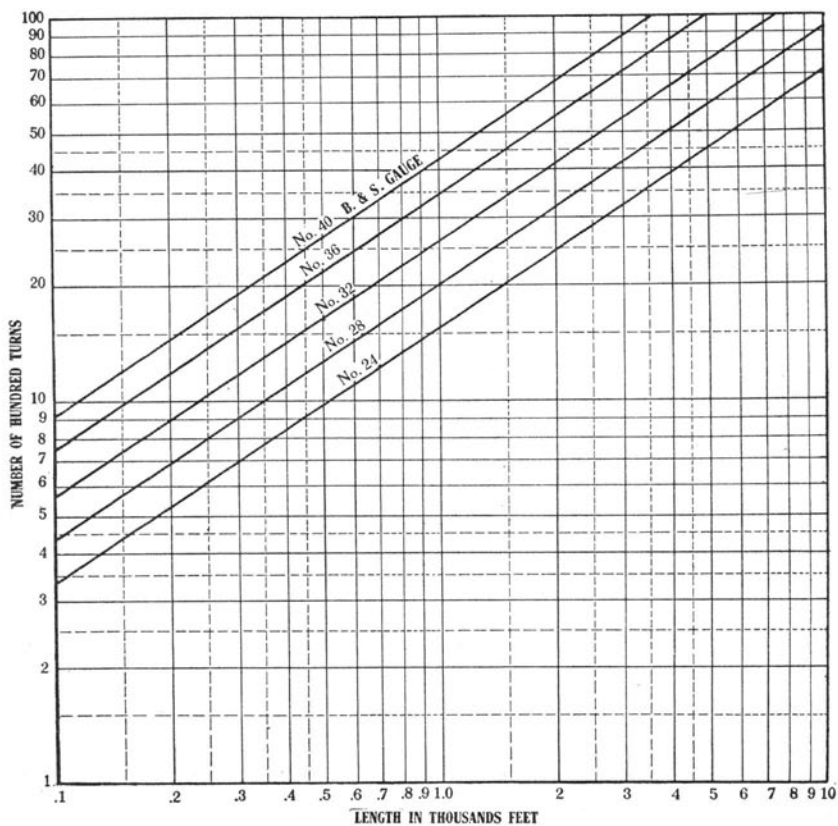


FIG. 21a

FIG. 21a AND 21b ARE CHARTS FOR INDICATING THE NUMBER OF HUNDREDS OF TURNS FOR COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE

Chart Values, Multiplied by 100, give N' in Formula (20).

The Indicated Number of Turns Is Subject to Adjustment for Ob-

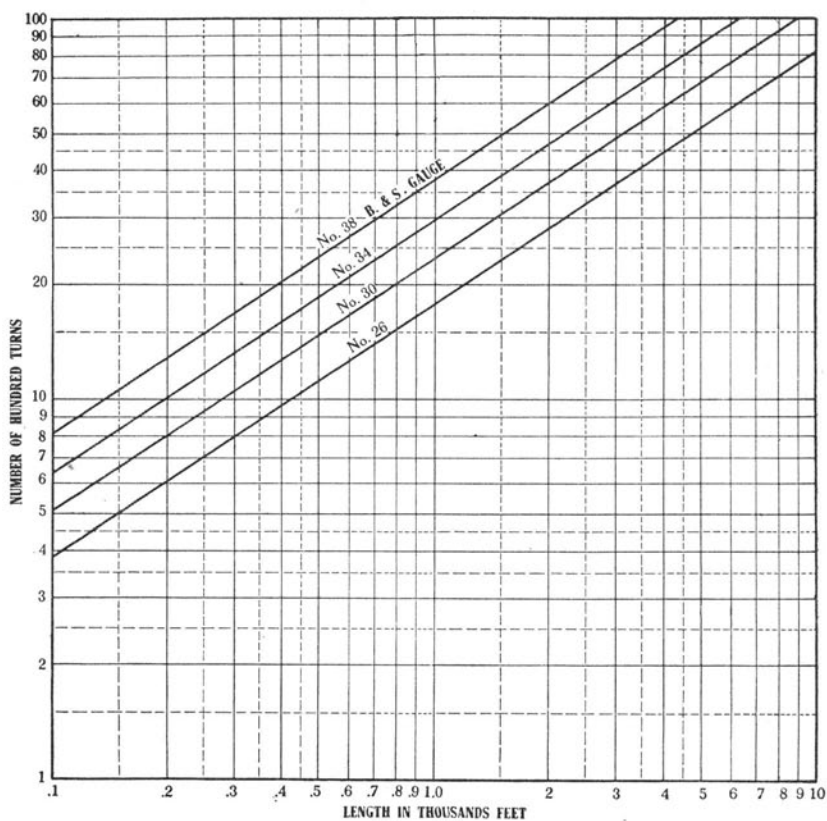


FIG. 21b

WOUND WITH ANY LENGTH OF SINGLE-SILK-COVERED (S. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

taining Complete Layers of the Winding, and Is Not Directly Serviceable for Coils of Other Shapes.

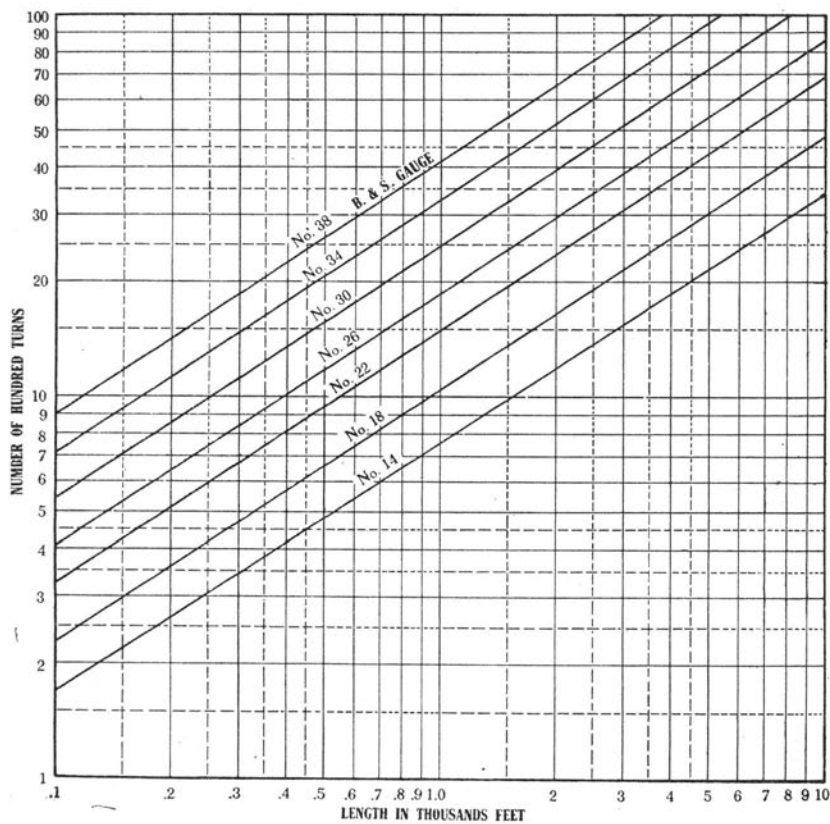


FIG. 22a

FIG. 22a AND 22b ARE CHARTS FOR INDICATING THE NUMBER OF HUNDREDS OF TURNS FOR COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE

Chart Values, Multiplied by 100, Give N' in Formula (20).

The Indicated Number of Turns Is Subject to Adjustment for Ob-

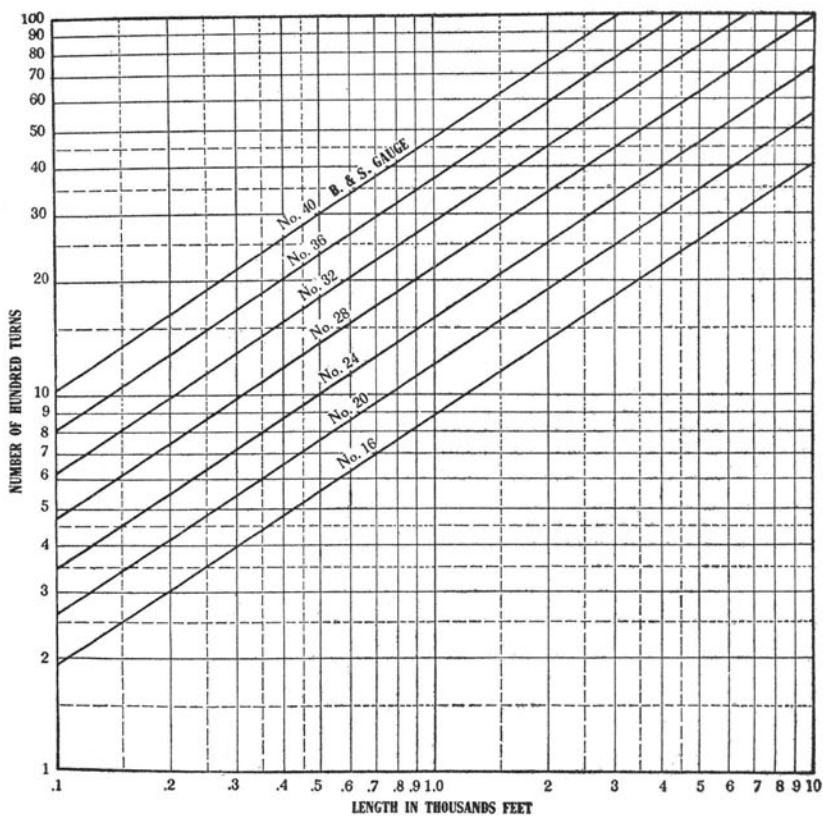


FIG. 22b

WOUND WITH ANY LENGTH OF ENAMEL-COVERED MAGNET WIRE OF EVEN GAUGE NUMBERS

taining Complete Layers of the Winding and Is Not Directly Serviceable for Coils of Other Shapes.

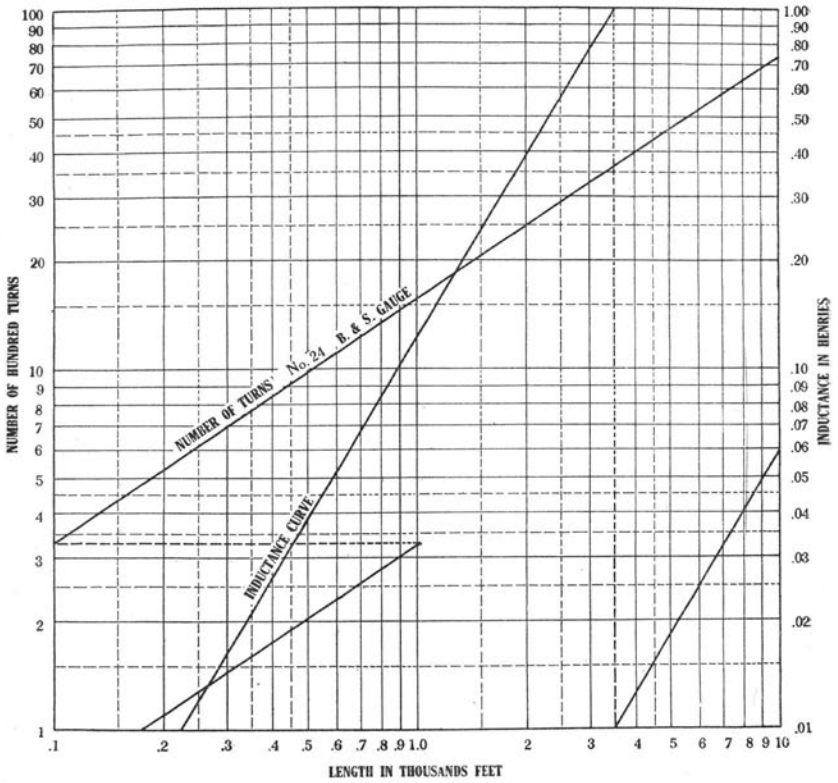


FIG. 23

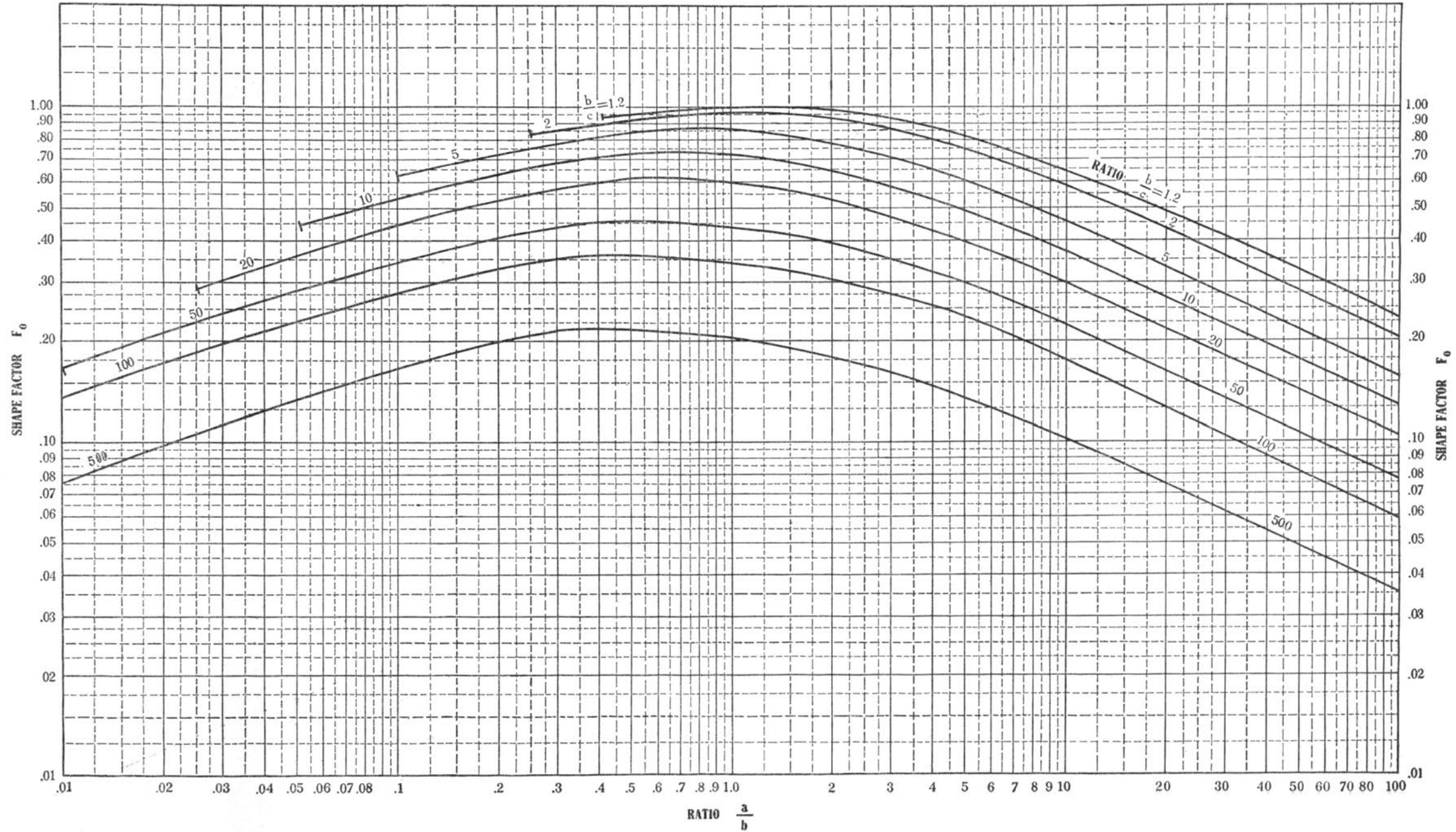
FIG. 24

SHAPE-FACTOR CHART
SOLENOID TYPE

In coreless cylindrical coils, only those having the relative proportions, $a=1.5$, $b=1.2$, $c=1$, (see Fig. 5) give the maximum inductance, while other shapes employ the conductor less effectively.

Fig. 24 and 25 indicate the shape factor, or coefficient for reducing the maximum inductance possible from any conductor to its actual value when wound into other than the prescribed maximum shape.

Fig. 24 includes all cylindrical windings, whose axial length, b , exceeds radial thickness, c , mostly solenoids. Curves apply to specified values of the ratio, $\frac{b}{c}$, from 1.2 (which includes the maximum shape) to 500 for very thin shell windings. Points at left indicate long solenoids and thick tubes of relatively small radius; while points at the right give values for short tubes or bands of large radius. For ratios of $\frac{b}{c}$, not represented by curves, points may be interpolated. For given ratio of $\frac{a}{b}$ as abscissa, the desired factor is the ordinate of the curve or interpolated point for the given ratio of $\frac{b}{c}$. See section XXIII for illustrative examples.



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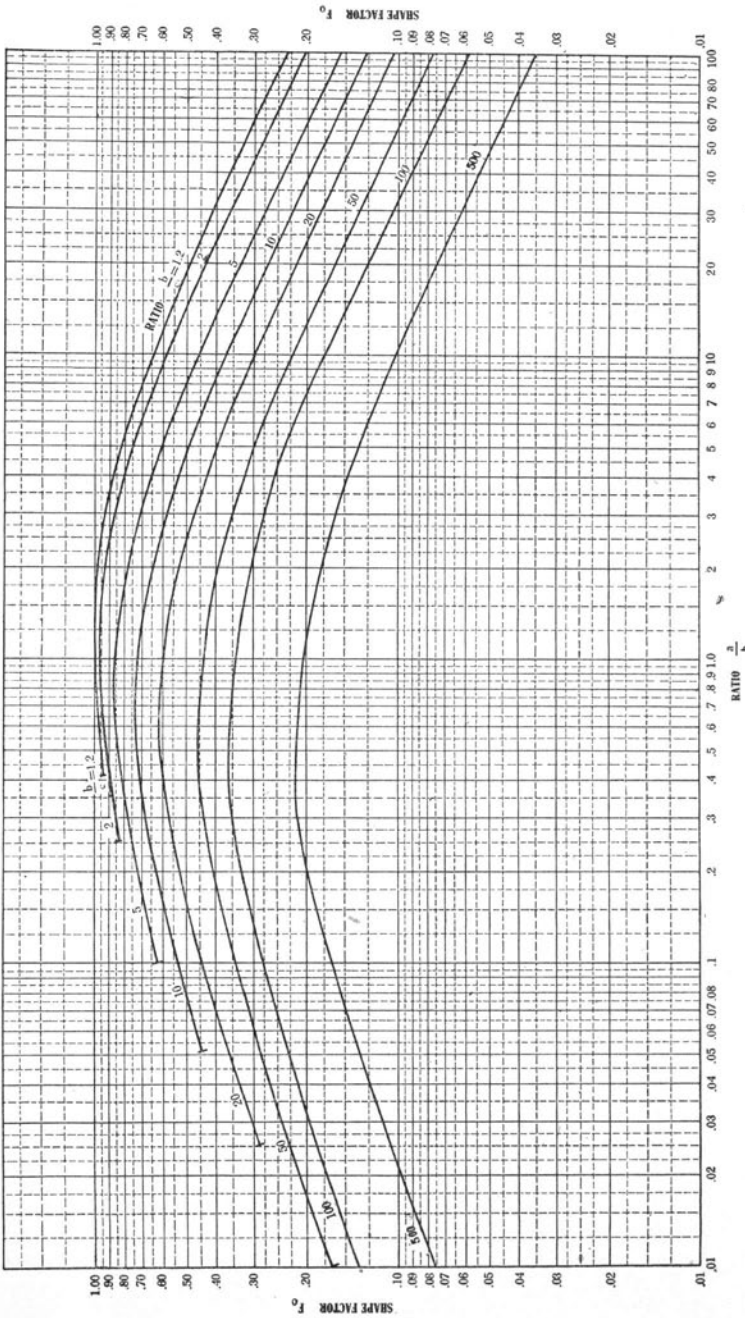


Fig. 24a See Fig. 24

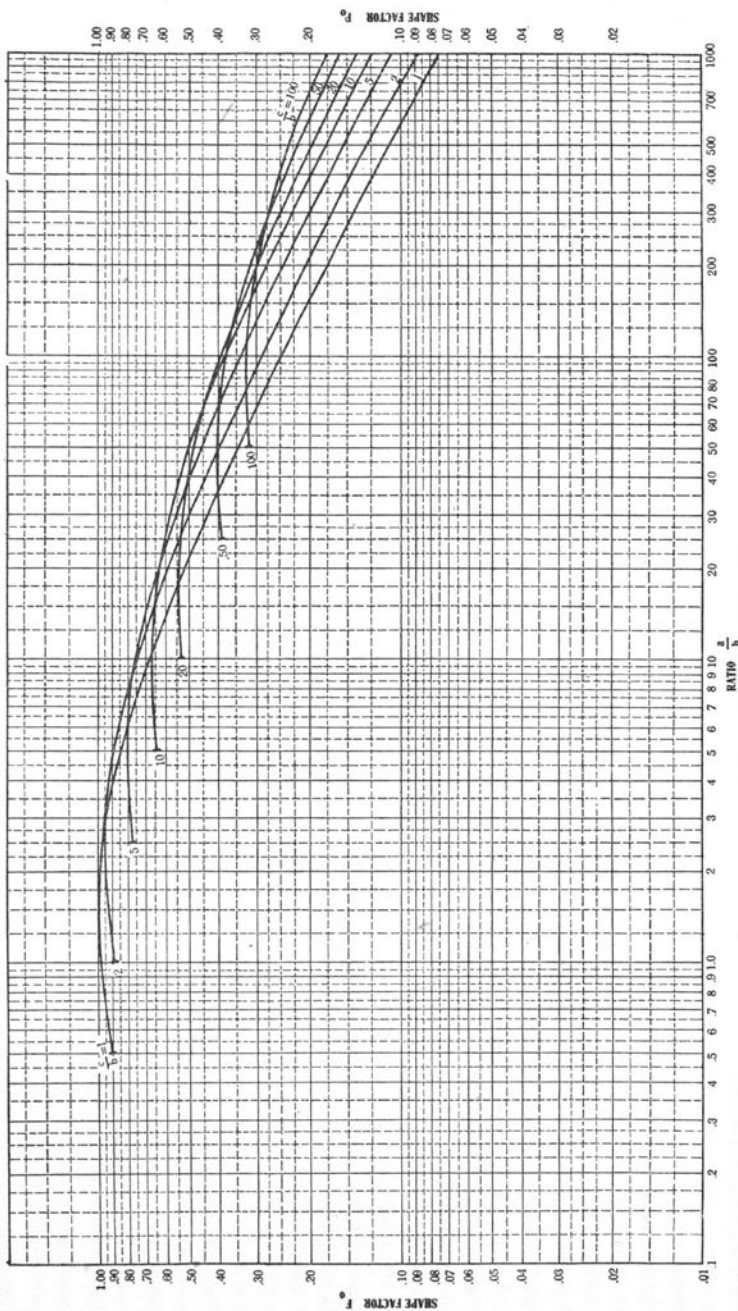


FIG. 25a See FIG. 25

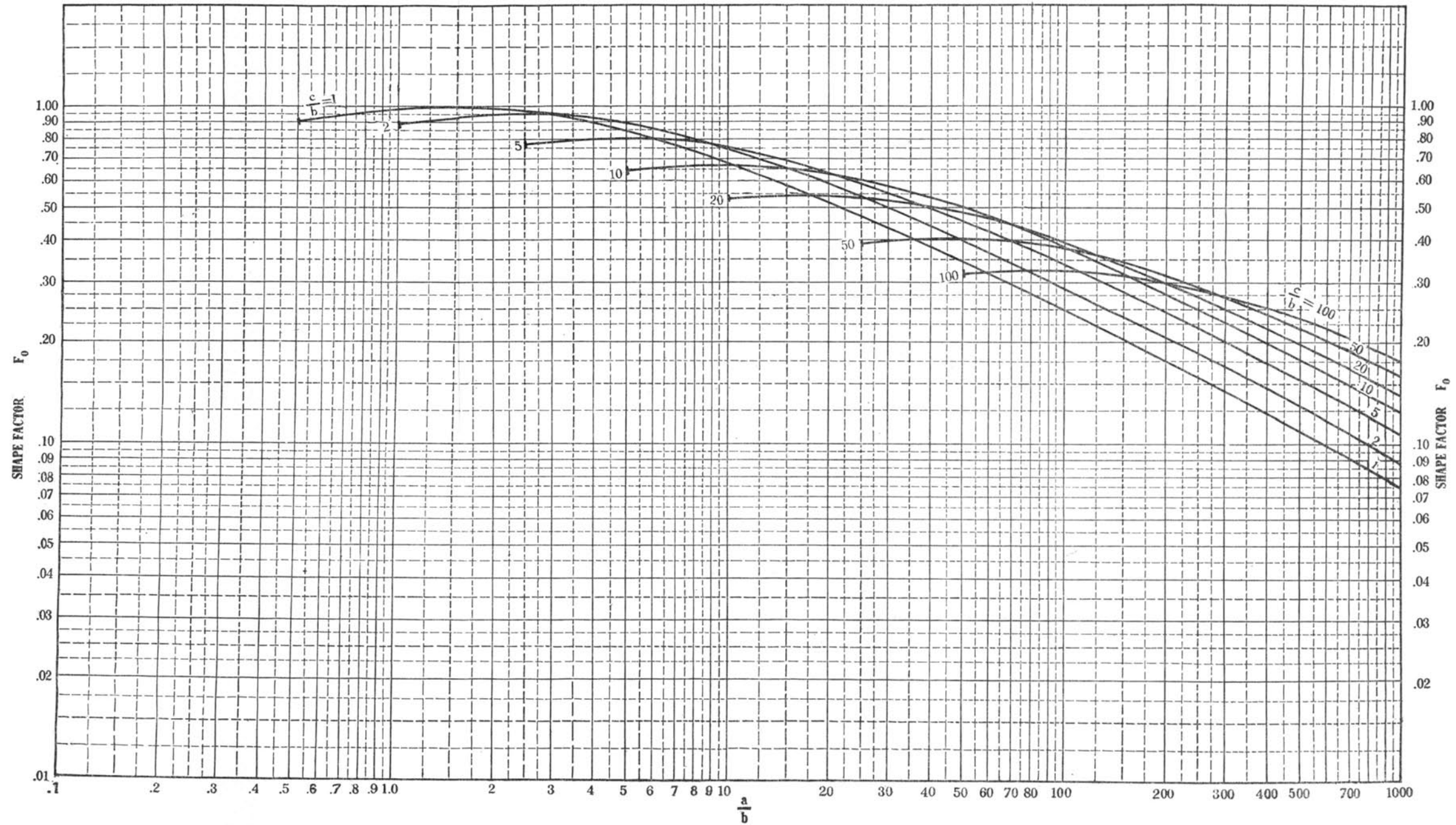
FIG. 25

SHAPE-FACTOR CHART
DISK TYPE

In coreless cylindrical coils, only those having the relative proportions $a = 1.5$, $b = 1.2$, and $c = 1$, (see Fig. 5) give the maximum inductance, while other shapes employ the conductor less effectively.

Fig. 24 and 25 indicate the shape factor, or coefficient for reducing the maximum inductance possible from any conductor to its actual value when wound into other than the prescribed maximum shape.

Fig. 25 includes all cylindrical windings, whose radial thickness, c , equals or exceeds axial length, b , mostly rings or "pancake" coils. Curves apply to integral values of $\frac{c}{b}$ from unity for single turns and rings of square section, to 500 for very flat washer-shaped coils. Points at left indicate relatively small radius, or coils of little or no hole; at right indicate large radius and usually large hole. For ratios $\frac{c}{b}$ not represented by curves, points may be interpolated. For given ratio of $\frac{a}{b}$ as abscissa, the desired factor is the ordinate of the curve or interpolated point for the given ratio of $\frac{c}{b}$. See section XXIII for illustrative examples.



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PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

ALL BULLETINS, THE TITLES OF WHICH ARE NOT STARRED, WILL BE SENT FREE UPON APPLICATION.

- **Bulletin No. 1.* Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*
- **Circular No. 1.* High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available*
- **Bulletin No. 2.* Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*
- **Circular No. 2.* Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*
- **Circular No. 3.* Fuel Tests with Illinois Coal. (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907.) by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*
- **Bulletin No. 3.* The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*
- **Bulletin No. 4.* Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*
- **Bulletin No. 5.* Resistance of Tubes to Collapse, by Albert P. Carman. 1906. *Fifteen cents.*
- **Bulletin No. 6.* Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *Thirty-five cents.*
- **Bulletin No. 7.* Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr and Henry B. Dirks. 1906. *Thirty-five cents.*
- **Bulletin No. 8.* Tests of Concrete: I. Shear; II. Bond, by Arthur N. Talbot. 1906. *None available.*
- **Bulletin No. 9.* An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. *None available.*
- **Bulletin No. 10.* Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
- **Bulletin No. 11.* The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *Fifteen cents.*
- **Bulletin No. 12.* Tests of Reinforced Concrete T-beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
- **Bulletin No. 13.* An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *Fifty cents.*
- **Bulletin No. 14.* Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
- **Bulletin No. 15.* How to Burn Illinois Coal without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*
- **Bulletin No. 16.* A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*
- **Bulletin No. 17.* The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *Twenty cents.*
- **Bulletin No. 18.* The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*
- **Bulletin No. 19.* Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *Twenty-five cents.*
- **Bulletin No. 20.* Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*
- **Bulletin No. 21.* Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*
- **Bulletin No. 22.* Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *Thirty-five cents.*
- **Bulletin No. 23.* Voids, Settlement and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*
- Bulletin No. 24.* The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*
- Bulletin No. 25.* Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Twenty cents.*
- Bulletin No. 26.* High Steam-Pressures in Locomotive Service. A Review of a Report to the Carnegie Institution of Washington, by W. F. M. Goss. 1908. *Twenty-five cents.*

* Out of print; price attached.

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- Bulletin No. 27.* Tests of Brick Columns and Terra Cotta Block Columns, by Arthur N. Talbot and Duff A. Abrams. 1909. *Thirty cents.*
- Bulletin No. 28.* A Test of Three Large Reinforced Concrete Beams, by Arthur N. Talbot. 1909. *Fifteen cents.*
- Bulletin No. 29.* Tests of Reinforced Concrete Beams: Resistance to Web Stresses, Series of 1907 and 1908, by Arthur N. Talbot. 1909. *Forty-five cents.*
- Bulletin No. 30.* On the Rate of Formation of Carbon Monoxide in Gas Producers, by J. K. Clement, L. H. Adams, and C. N. Haskins. 1909. *Twenty-five cents.*
- Bulletin No. 31.* Fuel Tests with House-heating Boilers, by J. M. Snodgrass. 1909. *Fifty-five cents.*
- Bulletin No. 32.* The Occluded Gases in Coal, by S. W. Parr and Perry Barker. 1909. *Fifteen cents.*
- Bulletin No. 33.* Tests of Tungsten Lamps, by T. H. Amrine and A. Guell. 1909. *Twenty cents.*
- Bulletin No. 34.* Tests of Two Types of Tile Roof Furnaces under a Water-tube Boiler by J. M. Snodgrass. 1909. *Fifteen cents.*
- **Bulletin No. 35.* A Study of Base and Bearing Plates for Columns and Beams, by N. Clifford Ricker. 1909. *Twenty cents.*
- Bulletin No. 36.* The Thermal Conductivity of Fire-Clay at High Temperatures, by J. K. Clement and W. L. Egy. 1909. *Twenty cents.*
- **Bulletin No. 37.* Unit Coal and the Composition of Coal Ash, by S. W. Parr and W. F. Wheeler. 1909. *Thirty-five cents.*
- Bulletin No. 38.* The Weathering of Coal, by S. W. Parr and W. F. Wheeler. 1909. *Twenty-five cents.*
- Bulletin No. 39.* Tests of Washed Grades of Illinois Coal, by C. S. McGovney. 1909. *Seventy-five cents.*
- Bulletin No. 40.* A Study in Heat Transmission, by J. K. Clement and C. M. Garland. 1910. *Ten cents.*
- Bulletin No. 41.* Tests of Timber Beams, by Arthur N. Talbot. 1910. *Twenty cents.*
- Bulletin No. 42.* The Effect of Keyways on the Strength of Shafts, by Herbert F. Moore. 1910. *Ten cents.*
- Bulletin No. 43.* Freight Train Resistance, by Edward C. Schmidt. 1910. *Eighty cents.*
- Bulletin No. 44.* An Investigation of Built-up Columns under Load, by Arthur N. Talbot and Herbert F. Moore. 1911. *Thirty-five cents.*
- Bulletin No. 45.* The Strength of Oxyacetylene Welds in Steel, by Herbert L. Whittemore. 1911. *Thirty-five cents.*
- Bulletin No. 46.* The Spontaneous Combustion of Coal, by S. W. Parr and F. W. Kressmann. 1911. *Forty-five cents.*
- Bulletin No. 47.* Magnetic Properties of Heusler Alloys, by Edward B. Stephenson. 1911. *Twenty-five cents.*
- Bulletin No. 48.* Resistance to Flow through Locomotive Water Columns, by Arthur N. Talbot and Melvin L. Enger. 1911. *Forty cents.*
- Bulletin No. 49.* Tests of Nickel-Steel Riveted Joints, by Arthur N. Talbot and Herbert F. Moore. 1911. *Thirty cents.*
- Bulletin No. 50.* Tests of a Suction Gas Producer, by C. M. Garland and A. P. Kratz. 1911. *Fifty cents.*
- Bulletin No. 51.* Street Lighting, by J. M. Bryant and H. G. Hake. 1911. *Thirty-five cents.*
- Bulletin No. 52.* An Investigation of the Strength of Rolled Zinc, by Herbert F. Moore. 1911. *Fifteen cents.*
- Bulletin No. 53.* Inductance of Coils, by Morgan Brooks and H. M. Turner. 1912. *Forty cents.*

* Out of print; price attached.

ANNOUNCEMENT CONCERNING A MODIFICATION IN
THE RULES GOVERNING THE DISTRIBUTION OF BULLETINS

The Board of Trustees of the University of Illinois voted, December 3, 1910, that a price should be affixed to certain University publications, among them being the Bulletin of the Engineering Experiment Station.

This action, so far as it concerns the bulletins of the Engineering Experiment Station, has for its purpose a threefold object:

- (1) To provide a greater degree of control in the distribution of bulletins.
- (2) To make possible the establishment and maintenance of a trade circulation through the regular publishing houses.
- (3) To regulate the distribution of the reserve or "out-of-print" supply.

IT IS NOT INTENDED THAT THIS ACTION SHALL OPERATE IN ANY WAY TO ABRIDGE THE PRIVILEGES OF THOSE WHO HAVE HITHERTO RECEIVED THE BULLETINS OF THE STATION GRATUITOUSLY, OR TO PREVENT REASONABLE EXTENSIONS OF THE EXISTING MAILING LISTS.

Practice under the new procedure will be as follows:

- (1) All bulletins, hereafter issued, will have a price printed upon the cover, together with the name of the London sales agent.
- (2) EACH BULLETIN, HOWEVER, WILL STILL BE SUBJECT TO A FREE INITIAL DISTRIBUTION, AS HERETOFORE, ON THE BASIS OF EXISTING MAILING LISTS.

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