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FATIGUE OF CONCRETE

by
John P. Lloyd
James L. Lott
Clyde E. Kesler

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ABSTRACT

PROGRESSIVE, PERMANENT STRUCTURAL DAMAGE OCCURS IN PLAIN CONCRETE WHICH IS SUBJECTED TO TIME FLUCTUATING STRESSES AND STRAINS. THIS FATIGUE PROCESS HAS BEEN UNDER INVESTIGATION SINCE ABOUT 1900 WITH MOST STUDIES BEING PHENOMENALISTIC. STUDIES OF A MORE FUNDAMENTAL NATURE WHICH EMPHASIZE THE INITIATION AND DEVELOPMENT OF FATIGUE DAMAGE ARE REVIEWED.

INVESTIGATIONS OF REPEATING LOADS, STATIC LOADS, FRACTURE MECHANICS, DRYING SHRINKAGE, AND EFFECT OF TIME RATE OF APPLIED STRESS MODULUS OF RUPTURE ARE CORRELATED WITH A PROPOSED FAILURE MECHANISM. FATIGUE FAILURE OF PLAIN CONCRETE IS RELATED TO THE PRESENCE OF DISCONTINUITIES, THE PRESENCE OF STRESSES, SOME OF A REPEATING NATURE, AND THE RESISTANCE OF CONCRETE TO FRACTURE OR GROWTH OF DISCONTINUITIES.

THE PRESENT STATE OF KNOWLEDGE OF FLEXURAL FATIGUE IS REVIEWED WITH EMPHASIS ON THOSE ASPECTS WHICH HAVE PRACTICAL SIGNIFICANCE ON CONCRETE PAVEMENT DESIGN.

ACKNOWLEDGMENTS

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The general administrative supervision was provided by W. L. Everitt, Dean of the College of Engineering, R. J. Martin, Director of the Engineering Experiment Station, T. J. Dolan, Head, Department of Theoretical and Applied Mechanics, and Ellis Danner, Director, Illinois Cooperative Highway Research Program and Professor of Civil Engineering.

The administrative supervision by the Division of Highways, State of Illinois was provided by Virden Staff, Chief Highway Engineer, John E. Burke, Engineer of Research and Development, and John McKay, Engineer of Research Coordination.

The Project Advisory Committee which served the project comprised the following members:

Representing the Illinois Division of Highways:

John E. Burke, Engineer of Research and Development
W. E. Baumann, Assistant Engineer of Design
Robert E. Dellert, Liaison Engineer

Representing the Bureau of Public Roads:

J. L. Hirsch, Bridge Engineer

Representing the University of Illinois:

Chester P. Siess, Professor of Civil Engineering
George M. Sinclair, Professor of Theoretical and Applied Mechanics.

Clyde E. Kesler, Professor of Theoretical and Applied Mechanics and of Civil Engineering, was Project Supervisor and served as Chairman of the Advisory Committee; John P. Lloyd, Research Associate, Department of Theoretical and Applied Mechanics, served as Secretary of the Project Advisory Committee and as Project Investigator; and James L. Lott, Assistant Professor of Theoretical and Applied Mechanics, served as consultant to the Project.

Appreciation is also extended to the reviewers of this Bulletin: John E. Burke, Illinois Division of Highways, Professor John F. McLaughlin, Purdue University, and Professor John W. Murdock, University of Illinois.

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FIGURES

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I. INTRODUCTION

1.1 GENERAL

Fatigue is a process of progressive, permanent structural change occurring in a material which is subjected to conditions which produce time fluctuating stresses and strains; the structural changes may culminate in cracks or complete fracture after a sufficient number of fluctuations.

The fatigue process occurring in plain concrete has been under investigation since about 1900 with the majority of the significant work having been done during the past twenty years. This process has been observed in concrete under repeated compressive and repeated flexural loading, and small amounts of experimental work show that it also occurs under reversed flexural loading and repeated tensile loading. This paper will consider recent research and the present state of knowledge for plain concrete subjected to repeated flexural loading, a loading condition of particular importance in the design of many highway structures. Emphasis is placed on initiation and propagation of fatigue damage, which are summarized in Chapter IV, Proposed Mechanism of Failure. This approach to fatigue of concrete was the major reason for undertaking the study.

Because of large amounts of data considered but only briefly referred to herein, this must be considered a summary report.

1.2 STRUCTURE OF CONCRETE

To understand the fatigue mechanism by which damage and failure occur in concrete which is subjected to repeated loading, it is necessary to relate the physical occurrences which produce fatigue damage to the internal structure of concrete.

From the gross or macroscopic level of observation, the structure of concrete is relatively easy to describe. It is observed that concrete is a mixture of coarse aggregate material and mortar, having the characteristic that each aggregate particle is separated from one another by a layer of mortar. Viewed on this scale, the strength of concrete derives from the cohesion of the mortar and aggregate phases and from the adhesion between them.

Examination of the mortar reveals what may be considered small scale concrete. It is a mixture of various sized sand grains and hydrated portland cement paste and has the characteristics that the aggregate particles are separated from one another by the paste or binding matrix. Direct observation of the tensile or flexural failure surface of mortar reveals that fracture occurs through the paste or at the interface between the sand and paste with very few failures occurring through the sand particles. Once again the strength derives from the cohesion of the aggregate and the paste and from the adhesion between them.

The solid constituents of the paste are derived from chemical reactions between water and portland cement. At the microscopic level, the products of these reactions may be divided into two major phases, crystalline calcium hydroxide and microscopically amorphous cement gel. The gel phase, which is structurally submicroscopic and heterogenous, can be divided into a solid phase, designated as gel particles, and interstices, designated as gel pores. The gel particles are colloidal in size and have a crystalline structure which is similar to that of the natural mineral tobermorite. The gel particles may be plate-, foil-, or needle-like in shape.

While the strength of the paste can be reasonably attributed to the gel phase and the magnitude of the strength to the gel particle concentration, Powers(1)* points out that no acceptable theory presently exists as to the source of strength. He speculates that the strength arises from two kinds of cohesive bonds: physical attraction between solid surfaces and chemical bonds. The presence of significant surface or van der Waal forces is suggested by the extremely small size of the gel particles; the existence of chemical or primary bonds may be surmised because of the limited swelling nature of the gel, i.e., the inability of water to disperse the gel particles. It appears that the majority of the internal forces which enable the gel to resist tensile stresses are of the van der Waal type.

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*Superscript numbers in parentheses refer to entries in References, Chapter VII.

II. ILLINOIS STUDIES

Murdock(2) has prepared a comprehensive and critical review of research conducted in the area of conrete fatigue through the early 1960's, which covers the historical background, problems encountered, results obtained, and the state of the art at that time. Murdock noted that the fatigue of concrete can be studied from two viewpoints. The essential differences between these approaches which can be designated as fundamental and phenomenological is that the former is concerned with the actual happenings in the material, the mechanism of failure in this case, while the latter is concerned only with the gross behavior of the material or, for example, the fatigue response of a particular type of specimen. It should be recognized that these two approaches are different in philosophy, but sometimes similar or even identical in experimental technique because of the present limitations of the experimental art. Furthermore, it is true that the ultimate goal of each of the two types of studies is the same, namely a more efficient use of concrete as a construction material. The principal difference between fundamental and phenomenological research is that the results of a phenomenological study can usually be applied directly and immediately to some particular practical problem, while the results of a fundamental study may be of a type which would increase the total store of knowledge about the behavior of the material without necessarily being immediately applicable to any specific problem.

The investigations which have been conducted at the University of Illinois in recent years have been primarily fundamental in orientation. The extent of these studies is described below.

Fatigue tests were conducted with mortar beams containing various types of preshaped aggregate inclusions. These beams which were assumed to be simplified models of concrete were used to study aggregate-mortar bond, drying shrinkage, and the influence of the elastic modulus of the aggregate.

Two fatigue studies investigated a fracture mechanics approach to fatigue failure. In support of these studies, two series of beams were tested statically to define necessary relationships between flexibility and flaw depth.

The remaining phases of the program dealth with restrained shrinkage, stress rate, and the relationship between the modulus of rupture and the dynamic properties of concrete.

Because of the diversity of the various studies, this chapter will consider the findings of each study separately; the following chapter will tie the results together and discuss them in light of other recent investigations.

2.1 FATIGUE STUDIES

2.1.1 General

Eight series of mortar beams containing various arrangements of preshaped aggregate inclusions and one control series of plain mortar beams were tested under fatigue loading. The beams containing the inclusions were designated as models of concrete. All specimens had a 2-in. width and a 4-in. depth; specimens were simply supported on a 40-in. span and subjected to symmetrical two-point loading supplied by a constant displacement fatigue machine. The loads were applied 15 in. from either support resulting in a 10-in. constant moment region.

The first phase of the model investigation(3) consisted of four series of beams. This portion of the model investigation was conducted to evaluate the influences of bond failure on fatigue response, which Murdock and Kesler(4) had earlier suggested might be critical. With the exception of one series of plain mortar beams, all specimens contained a single aggregate inclusion within the constant moment region. The inclusions were limestone having an elastic modulus similar to the mortar; because the inclusions extended the entire width of a beam and because of the similarity in elastic properties of the mortar and limestone, the models were assumed to be two-dimensional. Figure 1a shows the size, shape, location, and orientation of the inclusions in the beams. The four series are designated as follows:

I - plain mortar beams
II - beams with a circular inclusion
III - beams with a square inclusion
IV - beams with a diamond inclusion

The second phase of the model study(5) was designed to investigate the influence of shrinkage stresses on fatigue behavior. Three types of inclusions were used to provide varying restraint to shrinkage deformations in the mortar. Two of the inclusions, an unbonded aluminum cylinder and a bonded granite cylinder, had high elastic moduli with respect to the mortar and, therefore, provided a high degree of restraint; the other "inclusion" was actually a cylindrical void providing no restraint but permitting comparison with results obtained from specimens with reduced cross sections. Figure 1b shows the location of the various inclusions in the specimens of the three series which are designated as follows:

V - beams with unbonded aluminum cylinders
VI - beams with bonded granite cylinders
VII - beams with cylindrical voids

The third and final phase of the model investigation(6) studied the influence of aggregate modulus on fatigue behavior. The beams of one series which were subjected to drying shrinkage contained a limestone and a granite cylindrical inclusion in the constant moment region. The granite inclusion was much more rigid than the limestone. A preference for failure to occur at either the rigid or flexible inclusion would have been attributed to the influence of the aggregate's elastic modulus. The specimens of the other series were kept continuously wet, thus minimizing the influence of shrinkage. The specimens contained three bonded granite cylindrical inclusions in the constant moment region. The clear spacing between the inclusions was one diameter. Figure 1c shows the location of the inclusions in beams for the two series which are designated as follows:

VIII - beams with both limestone and granite cylindrical inclusions

IX - beams with three granite cylindrical inclusions

2.1.2 Results

The relationship between maximum applied stress and cycles to failure can be determined by conducting fatigue tests on a group of identical specimens. Unfortunately, it is very difficult to fabricate concrete specimens which are sufficiently alike to be considered identical. Concrete fatigue data are usually normalized for this reason; i.e., the maximum applied stress is divided by the beam's modulus of rupture which is determined from static tests conducted on "halves" of fatigue specimens. This ratio, designated as stress level, has been used in the analysis of data from virtually all plain concrete fatigue studies in recent years.

The plain mortar beams of Series I were air dried in a laboratory environment a minimum of five months prior to testing. The test data yielded a fatigue strength at 10 million cycles equal to 61 percent of the static ultimate strength. This fatigue strength agrees with earlier results obtained from similar specimens and loading conditions. Fracture occured at random locations in the constant moment region with no cracking visible prior to failure.

Specimens of Series II were air dried in a laboratory environment for a minimum of six months prior to testing. The fatigue strength at 10 million cycles was equal to 66 percent of the static ultimate strength. The average static ultimate strength of these beams with circular inclusions was 87 percent of the static ultimate strength of the mortar beams of Series I. The aggregate prism restricted the location of fracture, and three general types of failure were obtained: (a) "Socket" type of failure in which the aggregate prism remained intact; (b) fracture within the prism approximately through a vertical diametrical plane; (c) a failure partially through and partially around the aggregate prism. However, only one failure of each of the last two types occurred in the 23 fatigue specimens. As in Series I, no cracking could be detected visually prior to fatigue failure.

Specimens of Series III had a minimum age of one month at the time of the test. Unlike Series I and II, in which runout was defined to be 10 million repetitions of load, runout in Series III was defined to be five million repetitions. The fatigue strength extrapolated to 10 million cycles was 62 percent of the static strength. The average static ultimate strength of these beams with square inclusions was 63 percent of the static ultimate strength of the mortar beams of Series I. The failure section always included one of the vertical faces of the square aggregate prism. Although no visible cracking was observed prior to failure, some specimens did not completely fracture at failure; this may be explained by the fact that load was applied by a constant displacement machine. At failure, sufficient cracking and associated deflection occurred to remove practically all load from the specimen.

Specimens of Series IV were also at least one month old at the time of testing. Like Series III, runout was defined to be five million repetitions of load. The fatigue strength extrapolated to 10 million cycles was 63 percent of the static strength. The average static ultimate strength of these beams with diamond inclusions was 53 percent of the static ultimate strength of the mortar beams of Series I. The plane of failure always occurredin the region of the aggregate prism and often was displaced laterally from the aggregate's vertical diagonal plane by approximately 3/8 in. The beams of this series showed visible cracking at the extreme tensile surface prior to failure. When a crack was detected early in the fatigue like of a specimen, little growth could be detected until approximately mid-life in the fatigue history. After mid-life, the crack lengthened and widened and the width of the crack showed increasing pulsation as failure became imminent.

The lower load was 25 percent of the maximum load for all specimens in this phase of the model study. The fatigue strength at 10 million cycles varied between 61 and 66 percent of the static strength which is not a statistically significant variation. Although the fatigue strength in terms of stress level was not statistically influenced by the aggregate inclusions, it was found that the presence of the aggregate inclusions influenced both the location of fracture and the static strength of the beams. Murdock and Kesler(3) noted that the nominal stresses occurring in the specimens with aggregate inclusions were sufficient to cause failure of the bond between the mortar and the aggregate material.

The results from the second phase of the model investigation indicated that there were no statistical differences in the fatigue strength for the various conditions of restrained shrinkage, providing results were analyzed in terms of the stress level. However, the static strengths were definitely influenced by shrinkage. The modulus of rupture of beams without any inclusion was approximately 660 psi. The beams of Series V which contained an unbonded aluminum cylinder had an average modulus of rupture of 520 psi. Beams of Series VI, containing bonded granite inclusions, had an average modulus of rupture of 570 psi and the beams of Series VII, containing cylindrical voids, had an average modulus of rupture of 460 psi.

The third and final phase of model investigation provided fatigue data which, when analyzed in terms of stress level, yielded fatigue strengths in agreement with the other two phases of the program. Of 71 breaks which occurred in the beams of Series VIII, 55 percent of the breaks occurred at the granite inclusion. Since 50 percent would have represented a random selection of the two inclusion locations, no strong preference for the aggregate with the higher modulus was shown. While the aggregate modulus may in fact have little influence on the restrained shrinkage, it is also possible that any clear trend was masked by different bond strengths associated with the two aggregate materials. Of 87 breaks which occurred in the beams of Series IX, 37 percent of the breaks occurred at the middle inclusion. Since 33 percent would have represented a random selection of the three inclusion locations, no clearly defined trend existed. However, the clear spacing of one diameter, which is greater than would occur in actual concrete, may have minimized the interaction between the inclusions.

2.2 STATIC TESTS

2.2.1 General

The fracture mechanics parameter, flaw depth, must be considered in certain methods of analysis as will be discussed in Section 2.3. In fatigue tests, cracks can propagate slowly to a significant depth before the onset of rapid crack propagation. A common approach to estimate the amount of stable crack growth has been to determine a relationship between a measure of the distortion or flexibility of a beam and the measured flaw depth.(7,8,9) Two static test series(10,11) were conducted with the primary goal of determining the suitability of such an approach. A secondary objective was to obtain general information concerning the flexural behavior and strength of plain concrete members containing various types of flaws.

2.2.2 Results

The following results and conclusions were obtained from the two series of static flexural tests:

(a) Sawn notches or load-induced cracks in a tensile stress region of a beam influenced the measured flexibility in distinctly different fashions; therefore the development of a relationship between flexibility and flaw depth should consider the influences produced by notches and load-induced cracks separately.

(b) A plurality of load-induced macrocracks can develop in a constant moment region of an unnotched beam. This suggests that the accuracy of a relationship between flexibility and flaw depth for unnotched beams will be compromised by the number and severity of flaws present in a particular beam.

(c) Flaw depths present in fatigue specimens can be predicted with greater accuracy if both the flexibility of the fatigue specimen and the flexibility-flaw depth relationship are based upon the relative change in flexibility. This normalization tends to minimize the influence of flexibility parameters which are not related to the flaw depth of a specimen.

(d) For a given specimen geometry, the normalized flexibility-flaw depth relationships appear to be dependent on certain, and presently undetermined, concrete parameters.

(e) Stable crack growth in notched beams initiates at loads significantly less than the ultimate capacity of the beams.

2.3 FRACTURE MECHANICS

2.3.1 General

Fracture mechanics is an analysis of the stress and displacement fields in the region of a flaw at the onset of fracture. An energy concept of fracture mechanics was developed for an ideally elastic material. Griffith(12,13) suggested that the fracture of glass resulted from the propagation of flaws inherent to glass; the energy release accompanying a small extension of a flaw would furnish the energy demands for flaw extension. Since glass exhibits little ductile behavior, the only energy requirement considered was surface energy. The Griffith method was found to be unsuitable for most real materials, and consequently received little attention until over 200 catastrophic brittle fractures occurred in welded ships during World War II. Since that time other cases of brittle fracture such as in pressurized airplane cabins and pressure vessels have caused much attention to be devoted to the method.

Irwin(14,15) and Orowan(16,17) modified the Griffith theory of fracture for materials that exhibit ductile behavior. In ductile materials the energy demands associated with plastic deformations near a crack tip often exceeds the surface energy by several orders of magnitude. Although the fracture mechanics concept was extended to material having some ductility, no procedure was proposed to extend the concept to polyphase materials such as concrete.

Presently the use of fracture mechanics normally involves the use of a stress intensity factor concept. It is not a failure criterion; rather, it involves the description of the stress field present at a crack tip at the onset of fracture. The concept recognizes that inelastic deformations occur at the crack tip; however, if the inelastic region is small compared to the flaw size, the elastic stress field is a good approximation. When the inelastic region becomes large, the inelastic deformations must be considered in this analysis.

Because of the apparent brittleness of portland cement paste, mortar, and concrete, it is natural that fracture mechanics concepts have been used to study failure in these materials. It appears that the first use of fracture mechanics with mortars and concretes was made by Kaplan,(18) who investigated one mortar and two concretes. The two concretes differed in water-cement ratio, coarse aggregate percentage, and coarse aggregate material. Kaplan compared the critical strain energy release rate, G_c,* calculated by an analytical and by a direct method for a number of test conditions. Slow crack propagation prior to failure was neglected.

In a more extensive investigation than that of Kaplan, Lott and Kesler(19) studied influences of various parameters on the value of critical stress intensity factor which is proportional to $\sqrt{G_c}$.

In the analytical treatment of the problem it was shown that the critical stress intensity factor for plain concrete is derived from two sources, the fracture resistance of the mortar and the crack arresting mechanism due to the coarse aggregate material in the concrete. A mechanism of fracture in concrete was proposed which gave four reasonably distinct parts to the load-deformation curve for any given specimen: the linear stage, where the material simply resists load in a reversible elastic manner; the microcracking stage, where a region of microcracks form around the tip of the crack, which is trying to propagate through the material, thus consuming the energy which would otherwise drive the main crack; the slow cracking stage, where the main crack begins to grow and the crack tip moves through the microcrack region causing the formation of new microcracking regions which in general grow in size; and the fracture stage where the microcracking region has reached a limiting size and any additional energy supplied to the specimen by the load causes the main crack to grow spontaneously to complete fracture. These four stages are shown schematically in Figure 2.

Final fatigue failure is a case of rapid crack propagation. Therefore consideration of G_c or the critical stress intensity factor might reveal if fracture mechanics is applicable to fatigue failure. Glucklich(7) extended fracture mechanics to mortar under repeated loading, and Neal, _et al._,(8,9) and Lloyd, _et al._,(11) applied Glucklich's approach to concrete. Glucklich suggested the study of the parameter $\sigma^2 c$ (σ is the nominal stress and c is the critical flaw depth). This parameter is proportional to G_c which, if a material constant, would be independent of the fatigue life or the stress applied to a specimen. Unlike the use of fracture mechanics with static loading where the amount of slow crack growth is small, repeated loading results in appreciable slow crack growth which cannot be ignored. For this reason relationships between flexibility and flaw depth were determined experimentally and used to estimate the critical flaw depth indirectly.

2.3.2 Results

The results obtained by Kaplan indicated that G_c was influenced by the mix proportions, beam cross section,

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*G is the release of energy ΔE supplied by a virtual extension ΔC necessary to furnish the energy requirements of the crack surface ΔC as ΔC approaches zero; i.e.,

$$G = \lim_{\Delta C \to 0} \frac{\Delta E}{\Delta C};$$

G_c is the determination of G at the onset of rapid crack propagation.

and the location of the applied loads. Kaplan recognized that some slow crack propagation often occurred prior to instability, but no method existed to determine the extent of the crack growth. The neglect of the slow crack propagation resulted in smaller values of G_c and might have had a significant effect on Kaplan's results.

Certain analytical difficulties existed at the time of the Kaplan study. An analytical expression developed for a beam loaded with pure couples was applied to beams with center- and third-point loading. While the expression was a good approximation for the case of third-point loading it was not accurate for the center-point loading. Better techniques of analysis for both loading conditions have been developed recently and their use reduces the apparent variations in G_c. Kaplan's direct method of analysis for beams loaded at the third-points was invalid. The method requires that compliance be related to the magnitude and deflection of applied loads. However, Kaplan related compliance to the magnitude of applied loads and the deflection of the beams at midspan. Nevertheless, Kaplan's findings are especially significant in that they represent the first application of fracture mechanics to concrete.

The critical stress intensity factor investigation conducted by Lott and Kesler yielded, in addition to load deformation curves of the type described above, the following observations:

(a) For the ranges of the variables investigated, the critical stress intensity factor was independent of water-cement ratio for three mortars and also for various concretes where the aggregate percentages were the same.

(b) The critical stress intensity factor was independent of fine aggregate percentage for three mortars with the same water-cement ratio.

(c) The critical stress intensity factor varied directly with the coarse aggregate content for concretes with the same water-cement ratio and fine aggregate content. The critical stress intensity factor for concrete was found to be about 20 percent greater than that for a mortar with the same water-cement ratio and fine aggregate content.

Glucklich obtained results indicating close agreement between G_c for notched and unnotched static beams. If slow crack growth is ignored, the results are also in good agreement with those of Kaplan. However, values of G_c for unnotched beams subjected to repeated loading were about 7.5 percent lower than those obtained from static loading, while the notched beams subjected to repeated loading yielded G_c values 10 percent higher than those obtained from the static tests.

The results obtained by Neal, _et al._,(9) indicated a large discrepancy between the G_c values for notched and unnotched beams. Later static tests (Section 2.2) revealed that this discrepancy arose from additional parameters that were found to affect the flaw depth-flexibility relationship.

Lloyd, _et al._,(11) found that when a more accurate expression was applied to the data from a final series of notched beams, the results contained appreciable scatter. Much of the scatter can be attributed to the difficulty in determining the flexibility at

SUPPORTING DATA

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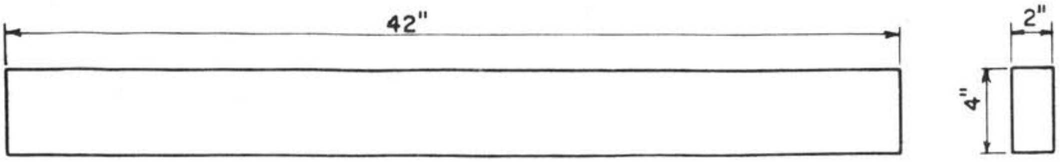
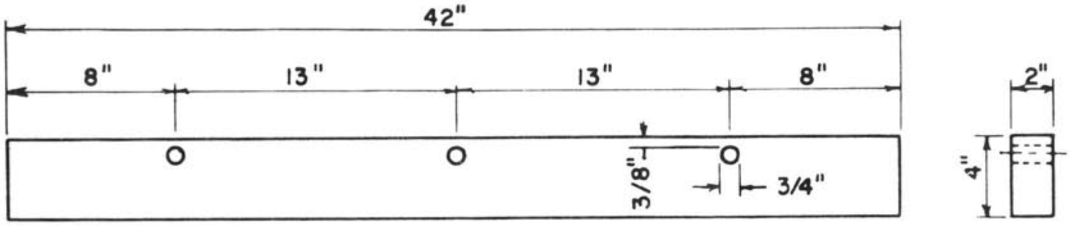


Series I



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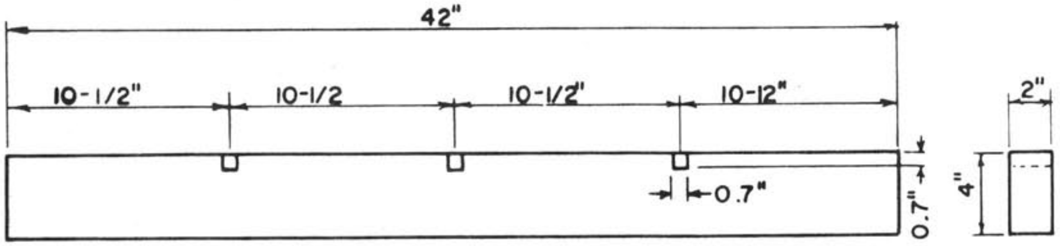

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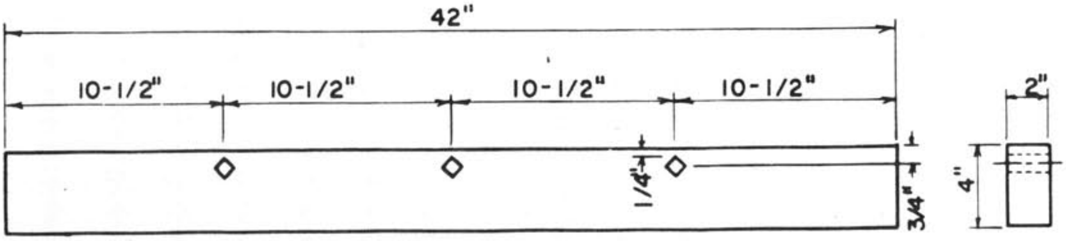
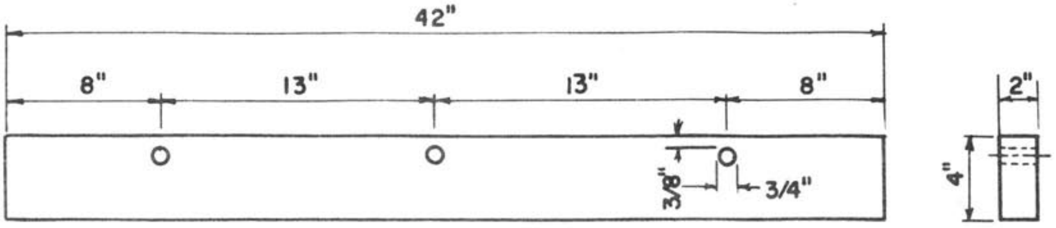


Series IV

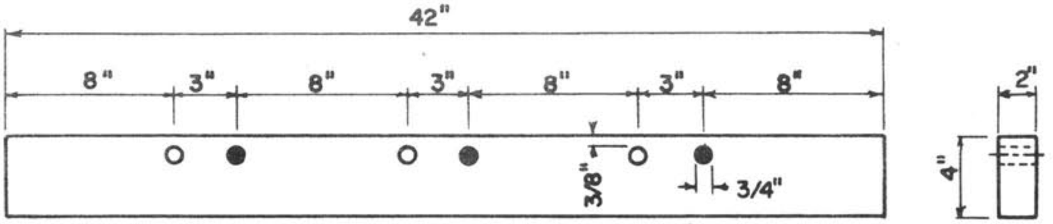
(a) Series I – IV

FIGURE 1. DETAILS OF MORTAR BEAMS USED

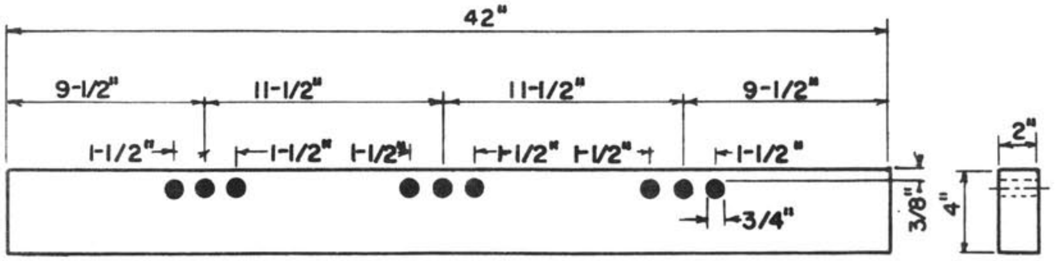


Types of Inclusions:	Series:
Unbonded Aluminun	V
Bonded Granite	VI
Void	VII

(b) Series V–VII



Series VIII



Series IX

Types of Inclusions: Granite ● Limestone ○

(c) Series VIII & IX

TO MODEL THE STRUCTURE OF CONCRETE.

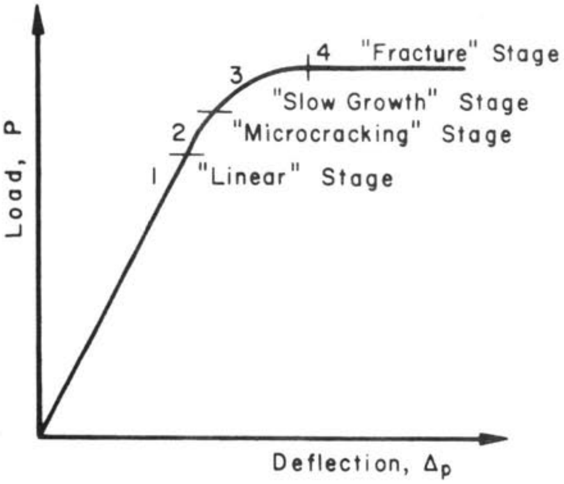


FIGURE 2. QUALITATIVE LOAD-DEFLECTION CURVE FOR TEST SPECIMENS (REFERENCE 19).

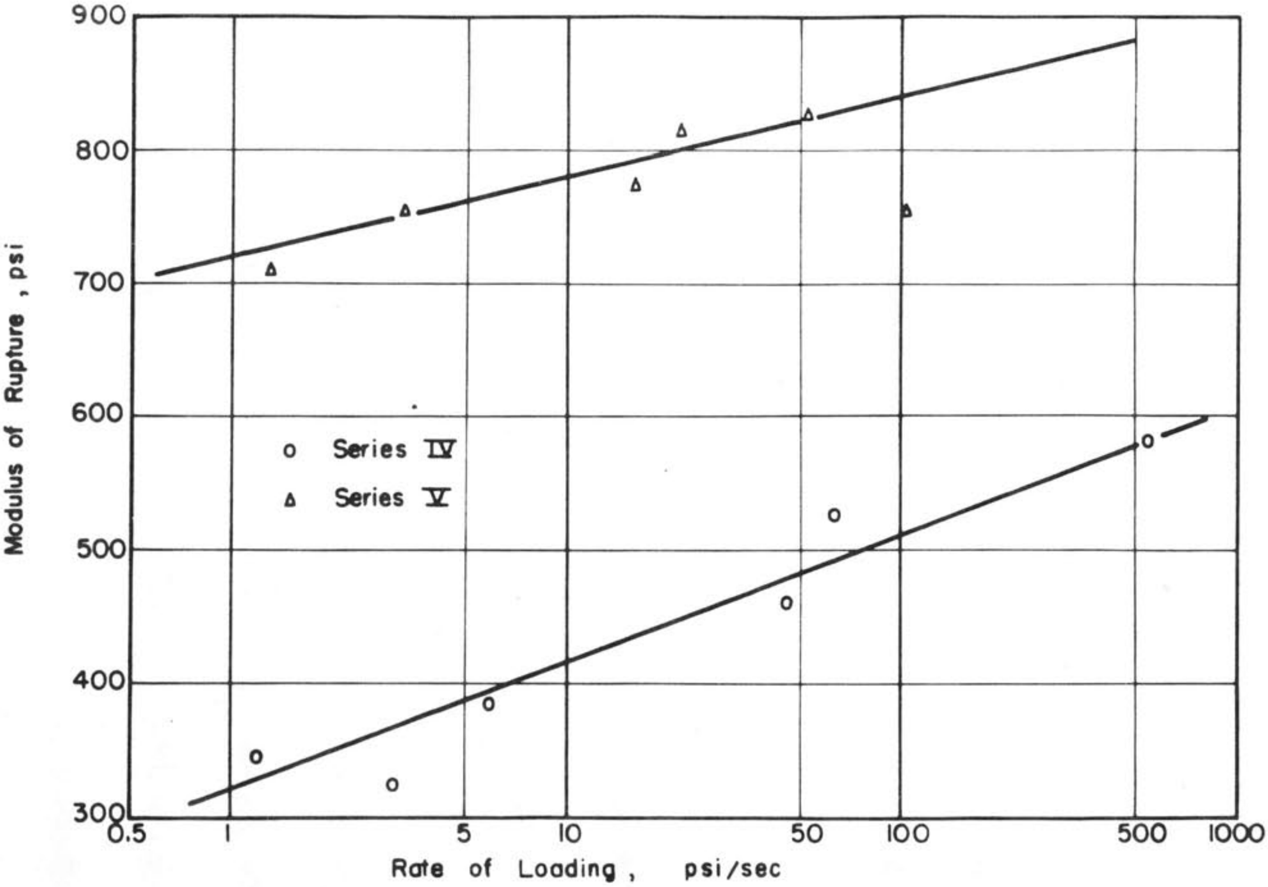


FIGURE 3. RELATIONSHIP BETWEEN RATE OF APPLIED LOAD AND INDICATED MODULUS OF RUPTURE FOR PLAIN CONCRETE SPECIMENS (REFERENCE 6).

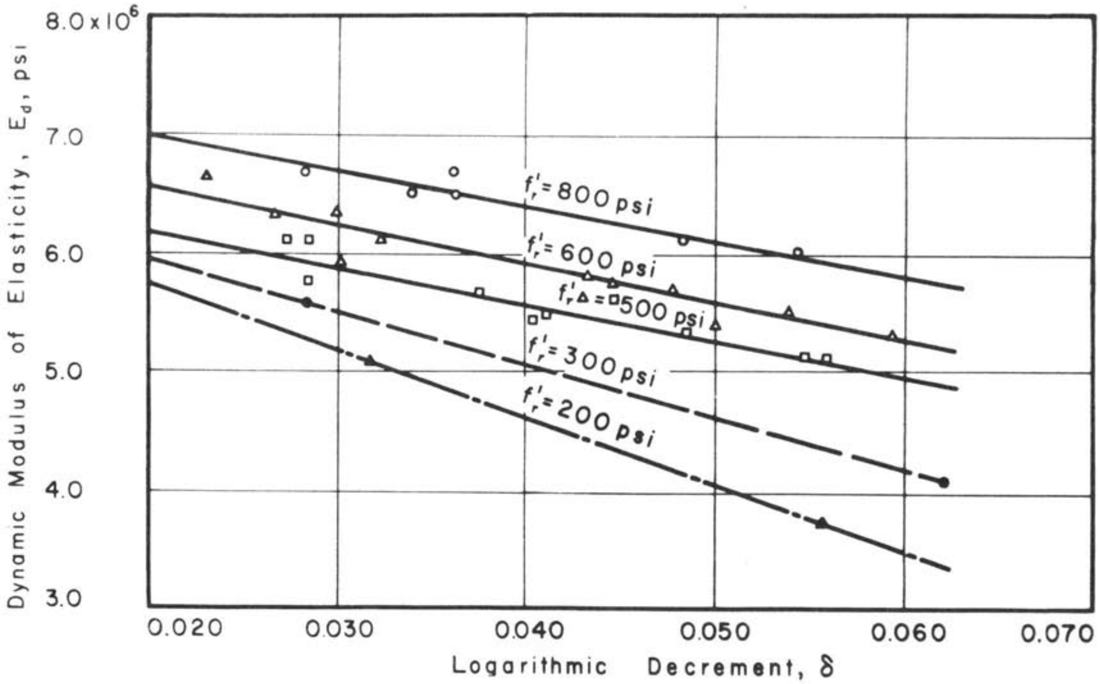


FIGURE 4. RELATIONSHIP BETWEEN DYNAMIC MODULUS OF ELASTICITY, LOGARITHMIC DECREMENT, AND MODULUS OF RUPTURE (REFERENCE 5).

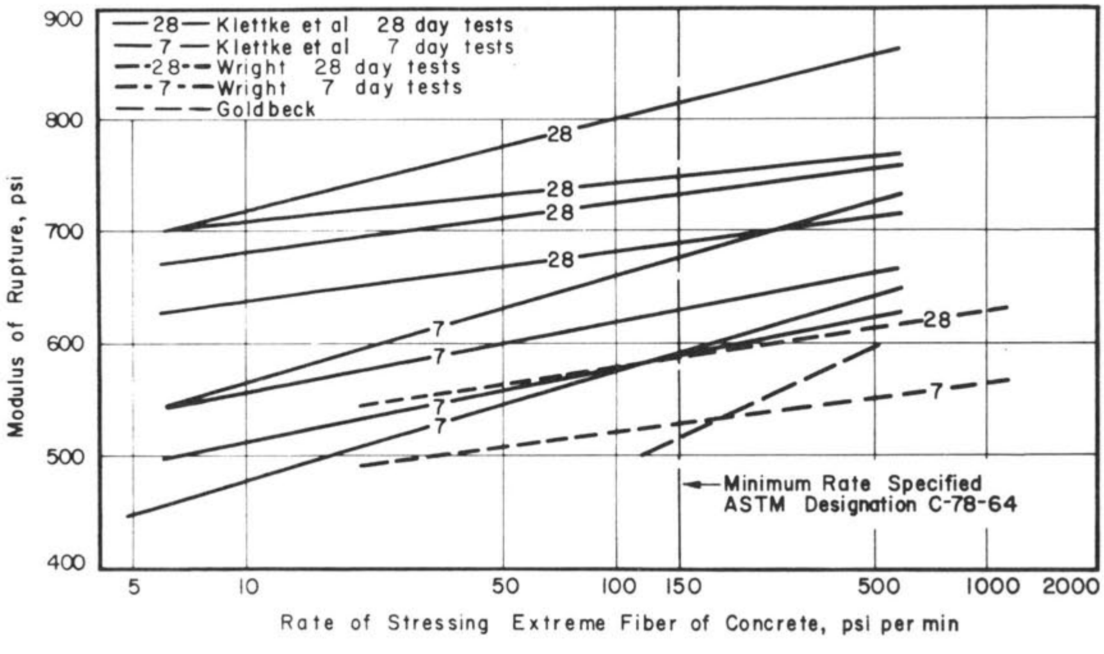


FIGURE 5. EFFECT OF RATE OF LOADING ON THE MODULUS OF RUPTURE OF CONCRETE (REFERENCE 25).

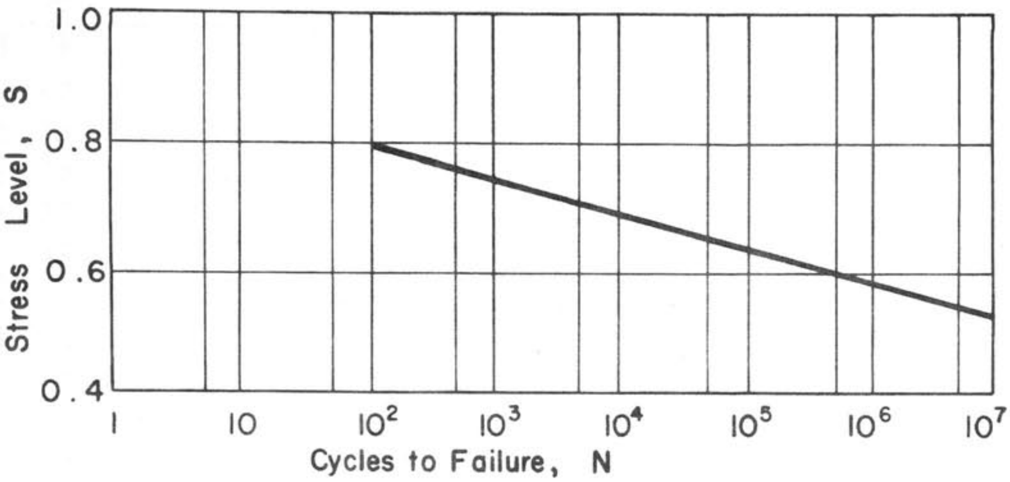


FIGURE 6. TYPICAL FATIGUE CURVE FOR CONCRETE SUBJECTED TO REPEATED FLEXURAL LOADING.

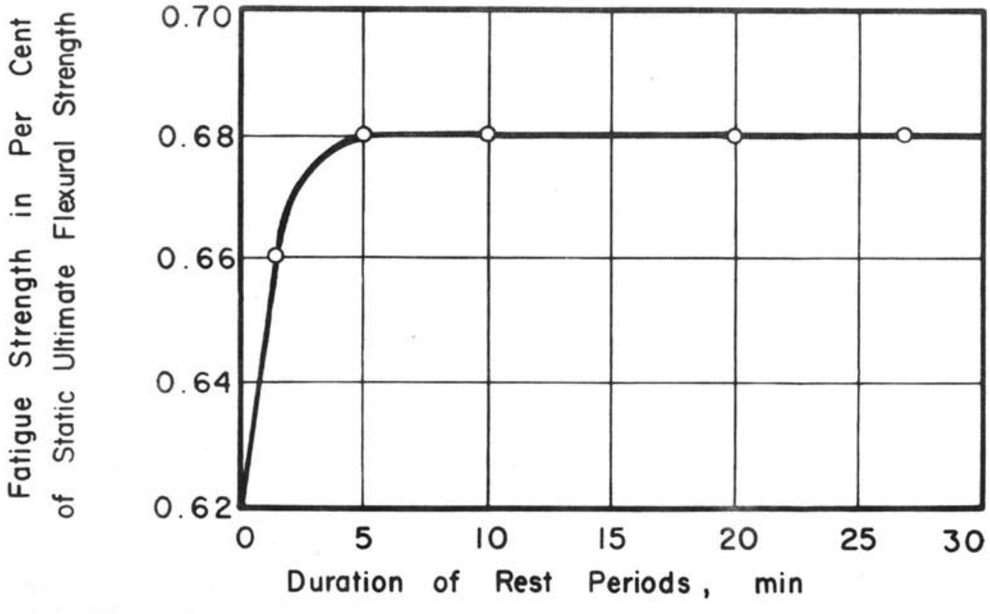


FIGURE 7. EFFECT OF REST PERIODS ON FATIGUE STRENGTH AT 10 MILLION CYCLES (REFERENCE 28).

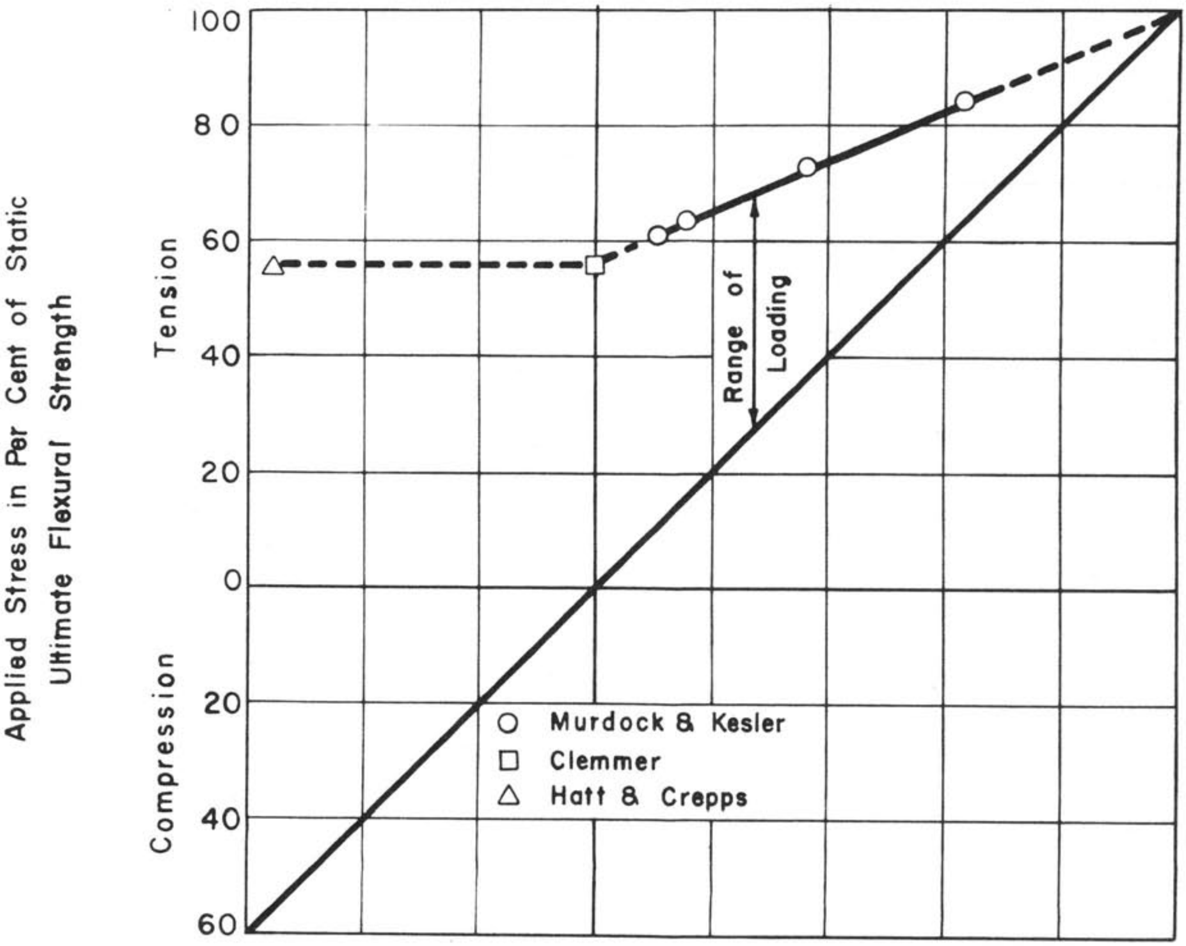
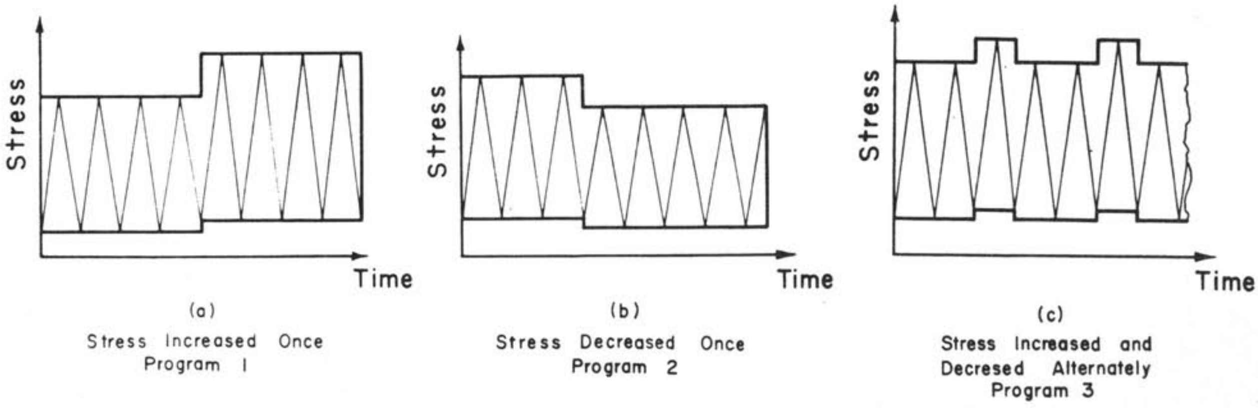


FIGURE 8. MODIFIED GOODMAN DIAGRAM SHOWING THE EFFECT OF THE RANGE OF STRESS ON THE FATIGUE STRENGTH OF PLAIN CONCRETE UNDER 10 MILLION CYCLES OF REPEATED LOADING (REFERENCE 4).



(a)
Stress Increased Once
Program 1

(b)
Stress Decreased Once
Program 2

(c)
Stress Increased and
Decresed Alternately
Program 3

FIGURE 9. STRESS HISTORIES INVESTIGATED BY HILSDORF AND KESLER (REFERENCE 28).

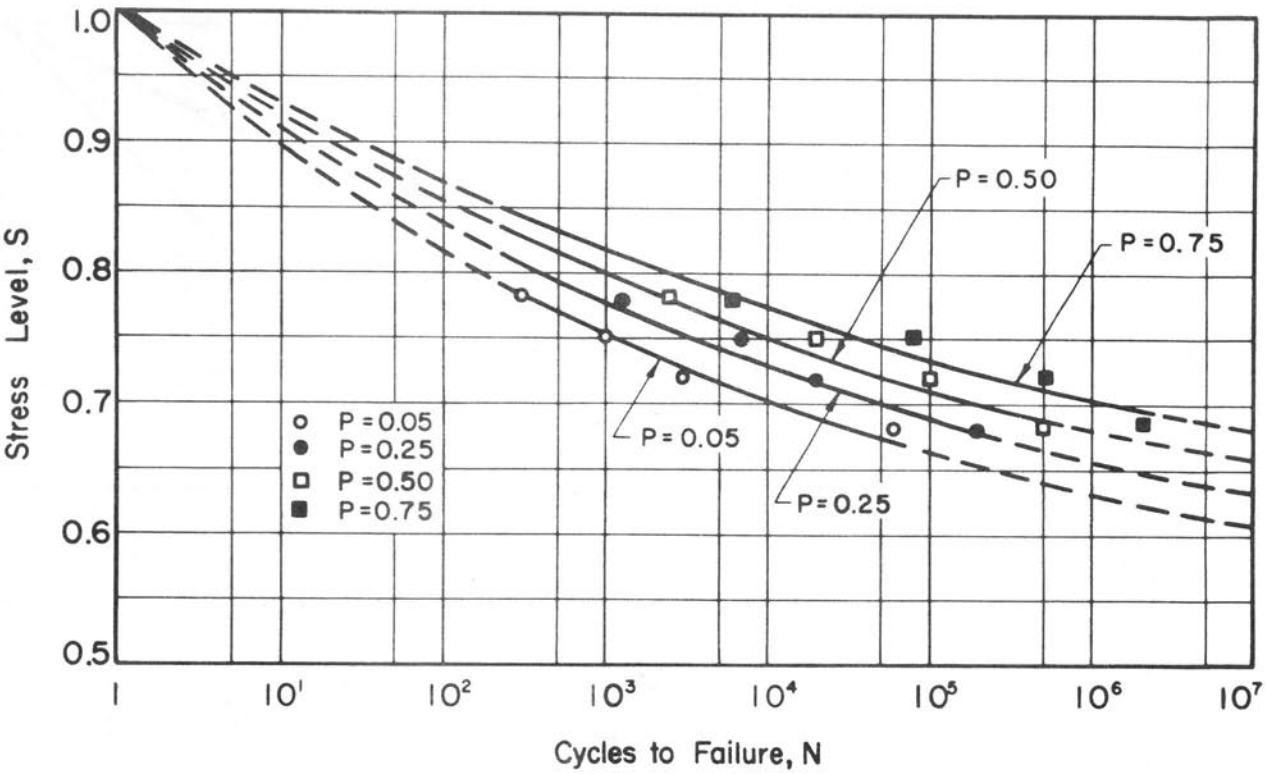


FIGURE 10. S-N RELATIONSHIPS FOR CONSTANT PROBABILITIES OF FAILURE (SPECIMENS SUBJECTED TO PROGRAM 3 WITH LOWER LOAD 17 PERCENT OF UPPER LOAD) (REFERENCE 28).

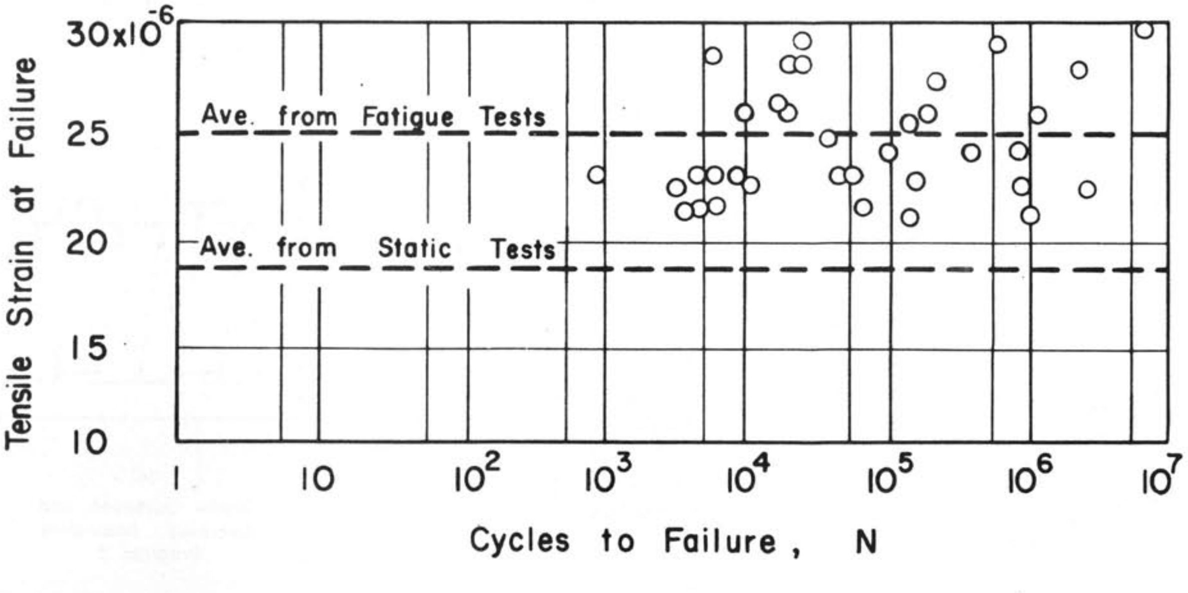


FIGURE 11. MAXIMUM TENSILE STRAINS AT FAILURE (REFERENCE 35).

incipient failure.

The applicability of fracture mechanics techniques to concrete under service conditions is limited by the present state of the art. The necessity of obtaining the size of internal flaws is a present difficulty which must be resolved before fracture mechanics can supply quantitative answers.

2.4 DRYING SHRINKAGE

2.4.1 General

The time-dependent deformations of creep and shrinkage of the paste matrix induce complex and varying stresses and strains in the aggregate and binding matrix. Only a qualitative description of the phenomena is possible at the microscopic level. Studies at the macroscopic level have supplied methods(20) of quantitatively predicting the average, time-dependent deformations produced by drying, applied loads, or both with reasonable accuracy. Since the quantitative description of stress is not available, there is no direct application of these methods to failure. However, a study of the deformations and cracking associated with drying shrinkage gives an insight into the phenomena.

Drying shrinkage is produced by loss of water in the paste matrix of concrete. The shrinkage deformations of paste are restrained by aggregate particles and reinforcement. In addition, large internal relative humidity gradients produce differential shrinkage strains which allow moist regions of a member to restrain adjacent regions which have experienced greater shrinkage. Creep acts to mitigate shrinkage deformations resulting in a complex interaction between the two phenomena.

In a recent investigation of the free and restrained shrinkage of mortar, Kung and Kesler(21) considered a number of parameters: two water-cement ratios, three times of curing, two degrees of restraint, and three relative humidities. Steel rods cast along the longitudinal axis of 1-by 1-by 11-3/4-in. mortar prisms provided restraint.

As shrinkage occurred, the measurement of the shortening of the restraining rod which was assumed equal to the restrained shrinkage, permitted the calculation of stress in the rod. From static equilibrium it was thus possible to determine the average stress in the mortar. Free shrinkage specimens provided the potential shrinkage of the mortar. The difference between the free shrinkage and restrained shrinkage was thus the total strain due to stress. It was assumed that this total strain due to stress could be separated into elastic and inelastic components. The elastic component of strain was assumed equal to the average stress in the mortar divided by the sonic modulus of elasticity.

2.4.2 Results

It was found that for specimens cured a minimum of seven days prior to drying, the elastic cracking strain, which was assumed critical, was approximately 130 μ in./in. Test results indicated that the time of cracking for the weaker mortar mix was influenced by the period of wet curing; such a trend was not found for the stronger mix. Lower relative humidities produced higher elastic strain at any given time. It is not surprising that restrained shrinkage is a very complex phenomenon, since it involves the interaction of two phenomena, shrinkage and creep, each complex in their own right.

2.5 RELATIONSHIP BETWEEN THE MODULUS OF RUPTURE AND THE TIME RATE OF APPLIED STRESS

Because of the possibility that the rate of stress application could be a meaningful parameter in the interpretation of experimental fatigue data as well as in the rational application of results to concrete under service conditions, studies(6,8) were conducted to investigate the relationship between the time rate of applied stress and the modulus of rupture. For stress rates between 1 and 500 psi per second which were used in the investigation, it was found that higher stress rates produced higher indicated strength, Figure 3.

2.6 RELATIONSHIP OF DYNAMIC PROPERTIES TO FATIGUE

2.6.1 General

The dynamic properties of concrete, sonic modulus of elasticity, and logarithmic decrement were related to the modulus of rupture;(5,22) separate fatigue investigations had previously defined the relationship between modulus of rupture and fatigue behavior. The nondestructive sonic method has the immediate advantage of yielding a direct estimate of strength thus minimizing or eliminating the need for control specimens which are normally used to provide an estimate of strength. Additional work with nondestructive testing might make possible the evaluation or prediction of the fatigue response of a member under the ambient service conditions. Thus it might be possible to determine the adequacy of an existing structure to a proposed loading condition.

2.6.2 Results

The results of these investigations are shown in Figure 4. The modulus of rupture is obtained by entering the figure with the dynamic modulus and the logarithmic decrement on the ordinate and abscissa, respectively, and by projecting these two parameters into a family of strength curves. The static strength increases with dynamic modulus and decreasing logarithmic decrement. This is as expected since the classical water-cement theory of strength states that the greater the volumetric concentration of hydrated portland cement in the paste, the greater the strength, and an increase in the volumetric concentration results in an increase in the dynamic modulus and a decrease in the logarithmic decrement. It was found that with only a few exceptions, the strength could be predicted within 10 percent regardless of age, mix, or curing.

III. SIGNIFICANCE OF ILLINOIS STUDIES

3.1 MECHANICS OF FATIGUE FRACTURE

The objectives of this investigation were fundamental, but a large portion of the results have practical significance. By approaching the fatigue of concrete with mortar-inclusion models, it was possible to study the influence of a number of concrete parameters on fatigue behavior.

In the first phase of the investigation, single preshaped limestone inclusions placed in the tensile region of the constant moment span of mortar beams reduced the static strength of specimens 13 to 47 percent and severely restricted the plane of failure. However, when the fatigue behavior was expressed in terms of the static strength, no statistically significant difference in behavior existed between plain mortar beams and mortar beams with inclusions. On the basis of the results of this phase of the study, the hypothesis was developed that fatigue failure initiated in the bond between the coarse aggregate material and the mortar matrix and then propagated through the mortar matrix to produce failure of the specimen.

The second and third phases of the study considered the influence of aggregate modulus and the interaction of the modulus and drying shrinkage on fatigue behavior. Although the plane of failure was restricted by the presence of inclusion and the static strengths were lower than the strength of plain mortar, the fatigue behavior expressed in terms of the static strength was the same for the plain mortar beams and the various models of concrete. The investigators concluded that when fatigue results were expressed in terms of static strength there were no noticeable effects which could be attributed to the shrinkage stresses in the mortar or variations in the elastic modulus of the inclusion; the results did not contradict the failure mechanism hypothesis that failure initiates in the bond between the coarse aggregate material and the mortar matrix.

The static test results permit a critical study of the use of a fracture mechanics approach to fatigue failure. The results indicate that the fatigue damage exists on the macroscopic level as slow and stable propagation of one or more cracks. There was indirect evidence that failure could initiate at a surface crack produced by drying shrinkage. The results had a direct bearing on the interpretation of the fatigue studies employing a fracture mechanics concept.

Several series of concrete beams demonstrated that a fracture mechanics analysis can provide both an alternate method of predicting the resistance of concrete to fracture and a method for studying the influence of various concrete parameters and environmental conditions.

Largely as a result of the various fatigue studies, "fatigue damage" is no longer a simple phrase for a phenomenon causing the weakening of concrete under repeated loading. It is now known that fatigue damage occurs at the structural level in the form of stable crack growth. Less is known at the microscopic level, where the description of damage and strength remains largely a matter of conjecture. Bond between coarse aggregate and the mortar matrix affects the response of concrete to static and repeated loading.

It is of practical significance that even though some of the variables considered presented more severe conditions than would normally be encountered in service the fatigue behavior expressed in terms of the static strength was the same as had been determined in earlier investigations.

3.2 DRYING SHRINKAGE

While drying shrinkage is important in its own right, consideration is limited here to its influence on crack formation. The observations of Kung and Kesler(21) considered the restrained shrinkage in a mortar. Their tests were primarily designed to study the restraining action of a coarse aggregate particle on the shrinkage deformations and cracking occurring in the surrounding mortar matrix. It is also possible to consider the extent to which the self-restraining action of a member can contribute to crack formation.

In a study of restrained shrinkage of concrete, Polivka(23) utilized specimens 4-7/8 by 6 by 40 in. The specimens received seven days of wet curing followed by drying in a 50 percent relative humidity, 70°F environment. Cracking occurred most often within two weeks of drying and at elastic strains of 60 to 105 μ in./in. The existence of shrinkage cracking prior to loading is also substantiated by Blakey.(24) Shrinkage cracking occurred within approximately 10 days in mortar beams cured at 50 to 65 percent relative humidity at 70°F without wet curing. The sonic elastic modulus of the mortar specimens was a maximum at an early age. Thereafter the modulus decreased for a period of time and then started to increase again. Blakey attributed early reduction in the modulus to the formation of microcracks in the paste and the subsequent recovery to autogeneous healing and speculated that the load induced cracks may originate as a reopening of shrinkage microcracking.

The detection of shrinkage cracks in specimens subjected to "free" shrinkage suggests such damage may have also occurred in the fatigue specimens used to model the structure of concrete or in mortar or concrete specimens used in most fatigue studies conducted to date. It is probable such damage did exist in many fatigue studies; however, the presence of shrinkage cracks or stresses apparently reduced the strength of both the static control specimens and the fatigue specimens to a similar degree and masked the influence of shrinkage.

3.3 RELATIONSHIP BETWEEN THE MODULUS AND RUPTURE AND THE TIME RATE OF APPLIED STRESS

It was found that the measured flexural strength is sensitive to the rate of applied loading. The results obtained agree qualitatively with those presented by McHenry and Shideler,(25) Figure 5. The sensitivity to load rate suggests that it would be more accurate to express fatigue strength as a function of the static strength obtained with a monotonically increasing load having a stress rate equal to the repeated loading stress rate, which would result in lower fatigue strengths than are currently reported in the literature.

Kesler(26) found no influence on the fatigue strength for cyclic load frequencies between 70 and 440 cycles per minute. This observation, which was made for a range of loading rates of less than one order of magnitude, might be invalid for several orders of magnitude; however, it would not be of practical significance in highway structures. Since the rate of load application is closely linked with the speed of vehicular traffic, there are practical limits to the rate of loading, which generally fall in the range studied.

The effect of stress rate may also be reduced in a fatigue investigation by the nature of the repeated loading. It is known that the load-deflection curves for concrete are approximately linear up to about 40 to 60 percent of the ultimate strength. Rüsch(27) has shown that the effect of load rate in the initial portión of these curves is far less than in the latter portion. Since the magnitude of repeated loading used in fatigue studies would normally strain the concrete only slightly beyond the linear response portion of the curve, it is not likely that the fatigue strength is seriously affected by the rate of loading.

3.4 RELATIONSHIP OF DYNAMIC PROPERTIES TO FATIGUE

The nondestructive sonic techniques considered in this phase provide the most reliable prediction of flexural strength available at the present time. The fabrication of flexural fatigue specimens which are nearly identical is extremely difficult even under careful laboratory control; the prediction of strength with the method considered here will permit screening and rejection of unsuitable specimens in future fatigue studies.

IV. PROPOSED MECHANISM OF FAILURE

The fatigue failure of concrete is complex and is strongly related to four parameters: the presence of stress regardless of origin or time variations; the repeating nature of some stresses; the presence of discontinuities such as microcracks, macrocracks, and structural heterogeniety; and the resistance of concrete to fracture. Other parameters such as environment are probably important to fatigue behavior; however, they directly affect the above parameters and have secondary effects on fatigue.

Internal flaws in concrete are large compared to the gel structure of the matrix. These flaws act as stress raisers, and minute volumes of the matrix are stressed to the ultimate strength of the paste while the average stress resulting from shrinkage, temperature differentials or applied load is much less than this potential strength. The critical regions of high stress occur near the tip of macrocracks. These stresses can form microcracks, which then relieve the high stresses at the macrocrack tip. The stability of existing macrocracks cannot always be accomplished by the formation of microcracks alone; then one or more macrocracks will form or propagate.

The resistance of concrete to fracture is developed through the ability of microcracks to stabilize macrocracks in a stress field. Repeating stresses modify the formation of microcracks and cause a slow, stable growth of macrocracks until the unstable condition is reached.

Under the action of a static load concrete undergoes time-dependent deformations known as creep. If the load is 70 percent or more of the short-time static strength, the material can creep to failure. Such load levels are within the range of fatigue failures. The significance of creep on the failure mechanism is at present unclear. Perhaps creep is related to the influence of the mean load level which was not considered of great importance although the importance of the range in stress was recognized.(4) However, Glucklich(7) has pointed out that for cases where stress reversal occurs, the presence of a stress concentration factor associated with a flaw in a tension field will result in a mean stress much larger than would be calculated simply from the nominal stress.

One may with confidence conclude that the mechanism of fracture at the macroscopic level involves the growth of a major crack or cracks through the paste matrix and paste-aggregate interfaces with the crack or cracks feeding upon internal damage found ahead of a crack tip. Less confidence can be placed on a mechanism hypothesis based at the microscopic level simply because of a lack of knowledge of the structure and behavior at that level. The mechanism of failure at the microscopic level appears to be directly related to the interaction of the primary and secondary bonds present in the paste structure. Upon application of load to a specimen, some secondary bonds would tend to fail -- and possibly reform immediately -- in an effort to arrive at an energy state most compatible with the imposed loading. The primary bonds which are at least an order of magnitude stronger than the surface bonds would become increasingly stressed with the movement permitted by the secondary bond failures. With the removal of load, the primary bonds which remained intact would, with the passage of time, tend to restore the internal structure to its original configuration and energy state. This recovery would explain the beneficial effects of rest periods. The fluctuating nature of a repeated loading would allow a more extensive and rapid readjustment of internal bonds than would be possible under static loading.

V. PRACTICAL SIGNIFICANCE OF EXISTING KNOWLEDGE

Up to this point major emphasis has been given to studies of a fundamental nature -- particularly those which have been conducted at the University of Illinois. The present state of knowledge will be briefly reviewed, considering in most detail certain aspects which have a practical significance. Because of the importance of flexural fatigue behavior in concrete pavement design, the following discussion will frequently refer to such structures for purposes of illustration; this is not intended to imply that the application of results are restricted to such structures.

5.1 MAGNITUDE OF FATIGUE STRENGTH

Under repeated loading the strength of a specimen is reduced and the strength at failure may be much less than the static capacity. If a group of similar flexural specimens are subjected to repeated loading, the strength reduction is normally found to be proportional to the logarithm of the cycles to failure. For example, if the repeated loading continuously fluctuates between zero and a maximum value, the fatigue data expressed in terms of stress level will result in a graphic relationship similar to Figure 6. Investigations conducted in recent years have not shown the existence of a fatigue limit at less than 10 million cycles of load, the maximum length of most laboratory tests. The fatigue strength at 10 million cycles is approximately 55 percent of the static strength, a value often used in design, and independent of virtually any concrete parameter.

5.2 REST PERIODS

Service conditions are often characterized by rest periods interspersed in the loading history. Hilsdorf and Kesler (28) were the first investigators to describe the influence of rest periods in quantitative terms. The length of rest periods was varied from one to 27 minutes. No difference was found between the effect of a five-minute rest period and the effect of any longer rest period, but the one-minute rest periods did not have as great effect as longer rest periods. The rest period raised the fatigue strength at 10 million cycles from 62 to 68 percent of the static strength as shown in Figure 7. The fatigue strength for no rest periods was 62 percent in this case because the minimum load was an appreciable percentage of the maximum load. A design based on test results obtained with continuously repeated loading is conservative.

5.3 RANGE OF LOADING

In 1934 Graf and Brenner(29) considered the effects of range of stress on the compressive fatigue strength of concrete and established a modified Goodman diagram which graphically expresses combinations of mean and fluctuating stresses a material can withstand for a specific fatigue life.

Murdock and Kesler(4) conducted several series of flexural fatigue tests. With each series of tests, a repeated loading pattern was used in which the ratio of the minimum load to the maximum load was constant. It was found that the fatigue strength at 10 million cycles was influenced by this ratio. These results and the earlier results of Clemmer,(30) Hatt,(31,32) and Crepps(33) permit the construction of the modified Goodman diagram shown in Figure 8.

This diagram provides a standard basis of comparison for various results which are expressed in terms of static strength. With the use of this diagram it has been shown that the flexural fatigue strength at 10 million cycles is 55 percent of the static strength if the minimum stress level is equal to zero. It has been shown that virtually no concrete parameter influences the fatigue behavior at 10 million cycles -- providing, of course, that results are interpreted in terms of static strength. The benefits of this diagram as a design aid are fairly obvious; the diagram graphically presents all combinations of mean stress and fluctuating stress under which concrete will withstand the number of load repetitions for which the diagram is constructed.

5.4 RATE OF LOADING

The influence of the rate of loading was investigated by Kesler.(26) Results of these tests indicated that for frequencies of loading between 70 and 440 cycles per minute the rate of applied load has negligible effect on the fatigue strength of concrete. It should be made clear that the stress rate occurring in a pavement during any one cycle of load will be similar to the stress rates used in laboratory studies, and the low frequency of loads on pavements, normally less than one cycle per minute, results from rest periods after each cycle of load.

5.5 VARYING MAXIMUM FLEXURAL STRESSES AND THE MINER HYPOTHESIS

Hilsdorf and Kesler(28) used three types of tests in a study of varying load levels. The load was either increased once during the test (Program 1), decreased once during the test (Program 2), or repeatedly changed so as to form "blocks" of stress cycles with each block having the same number of cycles at the low and high levels (Program 3). The various loading histories are illustrated in Figure 9.

The results of Programs 1 and 2 indicated that specimens subjected to a very brief period of high maximum stress level followed by a lower stress level to failure have a greater fatigue life than specimens subjected to a lower stress level continuously. Conversely, a brief period of low maximum stress level followed by a higher stress level until failure occurs can result in shorter fatigue lives than if the higher stress level had been applied continuously.

The results of Programs 1 and 2 were compared to the commonly used Miner hypothesis,(34) which assumes linear accumulation of damage. The Miner hypothesis was found to be nonconservative for specimens subjected to Program 1 and conservative for specimens subjected to Program 2.

With Program 3, where the load was repeatedly varied, the fatigue strength was found to decrease as the ratio of the number cycles at the high stress level to those at the low stress level increased. The fatigue strength also decreased for a maximum stress level as the difference between the two stress levels was increased.

These results indicated the complexity of fatigue behavior under even a semi-random variation in maximum stress levels and demonstrate conclusively that the assumption of linear accumulation of fatigue damage in concrete in fatigue is false.

Hilsdorf and Kesler illustrated a method of adjusting their results which permitted the use of the Miner hypothesis. An S-N diagram based on a probability of failure concept was also constructed from the average results of Program 3 loading, Figure 10.

This diagram was based on tests in which the lower load was 17 percent of the upper load. For this reason and because of the limited range of their study, Hilsdorf and Kesler were careful to point out that their results were not suggested as being directly applicable to design.

5.6 INITIATION OF FATIGUE FAILURE

The results of the investigation using mortar beam with preshaped inclusions(3,5,6) supported an earlier hypothesis(4) that failure originates as a failure of bond between coarse aggregate particles and the binding matrix. Lloyd, et al.,(11) suggested that shrinkage cracks contributed to the formation of muitiple cracking detected in beams under static loading, a hypothesis also suggested by Blakey.(24) A general hypothesis of the initiation of fatigue failure should consider the importance of bond deterioration and internal flaws such as shrinkage cracks.

5.7 RELATIVE STRAINS IN STATIC AND REPEATED LOADING

Numerous unsuccessful attempts to measure a limiting tensile strain under static and repeated loading have been made. However, it has been shown that the failure strain for concrete under repeated loading is significantly greater than that under static loading. Hilsdorf and Kesler(35) also noted that the tensile strain at failure was independent of the fatigue life of a specimen, Figure 11.

VI. NECESSARY RESEARCH

The preceeding discussions point out that although considerable research has been performed which has been useful from a phenomenological as well as a fundamental point of view, accurate assessment of the results from basic studies has been hampered to varying degrees by incomplete knowledge of the structure of cement gel, the origins of bond, and the nature and propagation of internal damage. During the past few years the use of the electron microscope has furnished considerable information about the structure of cement gel. Additional work in this area may permit the detailed study of gel structures with reference to parameters normally considered important to strength. It is also highly desirable to study the structure of gel at the interface of aggregate particles. Such work would be of value in developing a rational explanation of the origins of bond strength.

Results obtained in the future with electron microscopes will probably be of most benefit to those interested in the very basic aspects of behavior of concrete. However, if this work is performed in conjunction with studies using high-power light microscopes and X-ray techniques, the practical benefits would be greatly increased. Although a number of technical difficulties associated with specimen preparation and interpretation of results currently hinder the use of microscopic examination, further work may resolve some of these difficulties. It appears that present techniques would suffice to qualitatively examine flaws induced during setting, curing, and loading and correlate the damage to mix parameters and ambient conditions. Study of the microcracking zone ahead of macrocracks produced by drying shrinkage or applied loading would be of special interest.

Much attention has been given to various sonic tests, but present methods obtain only an average characteristic of the material. The ability to evaluate a specific property in a small region of the material is needed.

To researchers interested in cracking or failure produced by drying shrinkage or applied loading, there is a discouraging lack of study of the phenomenon of stress rupture -- especially for conditions of flexure and direct tension. Further study in this area could provide much basic information about elastic and inelastic strains, influences of loading history, and the mechanism of fracture.

The recent development of testing equipment capable of applying virtually any desired load to a specimen will allow investigations to be conducted in which the range of fatigue stress is held constant but the average stress is varied independently. This research is necessary to understand the influence of the mean stress on fatigue behavior.

A number of phenomenological investigations might profitably be conducted to determine the influences of biaxial stress, severe moisture and temperature gradients, curing conditions, corrosive environments, and random loading on the fatigue behavior of concrete.

The research suggested above is almost entirely experimental in nature; one area of study, fracture mechanics, would profit from a combined experimental and analytical approach. Although the basic concepts of fracture mechanics appear rational in nature, the application of the method to a polyphase material such as concrete requires many approximations. This approach makes application of results difficult. The development of new techniques of analysis would require some compromise between concrete as a real material and an analytical model but would help the researcher in basic parametric studies and also bridge the gap between experimental results and practical applications.

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