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THE STRENGTH OF CONCRETE ITS RELATION TO THE CEMENT AGGREGATES AND WATER

BY

ARTHUR N. TALBOT

AND

FRANK E. RICHART

BULLETIN No. 137

ENGINEERING EXPERIMENT STATION

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THE STRENGTH OF CONCRETE ITS RELATION TO THE CEMENT AGGREGATES AND WATER

BY

ARTHUR N. TALBOT

PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING
AND IN CHARGE OF THEORETICAL AND APPLIED MECHANICS

AND

FRANK E. RICHART

RESEARCH ASSISTANT PROFESSOR OF THEORETICAL
AND APPLIED MECHANICS

ENGINEERING EXPERIMENT STATION

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THE STRENGTH OF CONCRETE: ITS RELATION TO THE CEMENT, AGGREGATES, AND WATER

I. Introduction

1. *The Problem.*—How to proportion the ingredients in designing concrete mixtures, is an important question. How much cement, how much sand, how much gravel or broken stone, and how much water must be used to secure a specified strength in the concrete or a specified workability with the particular materials that are to be used? What is the weight or the bulk of each ingredient required to produce a certain volume of concrete in place? How may the concrete-making properties of various sands and other fine aggregates be compared, and how may acceptance tests of aggregates be specified? For what sort of mixtures is a given sand suitable? As compared with some standard mix how much more cement is necessary when an inferiorly graded fine aggregate is used? In the effort to increase the mobility or the workability of the freshly mixed concrete what effect do increased amounts of mixing water, or decreased amounts of coarse materials, or changes in the gradation of the aggregate, have upon the strength and other properties of the concrete? What is the upper limit of the amount of coarse aggregate which may be used when the particles are all well rounded particles and also when there is a preponderance of elongated particles? What is this upper limit of the amount of coarse aggregate when the pouring and placing are around reinforcing bars, and also when the concrete is placed in larger masses? These are some of the questions connected with the problem of designing concrete mixtures for definite purposes.

Many views have been put forth concerning the factors which govern the strength of concrete, and quite divergent opinions have been advanced. A variety of methods of dealing with the problem have been proposed—the use of the sieve analysis of the fine and coarse aggregates, either separately or in combination; limitations in the amount of the fine aggregate coming within a given range of sizes; conformity to a given form of sieve-analysis curve; properties based on the gradation of the particles of the aggregates as indicated by sieve analyses, such as fineness modulus and surface area; the amount of mixing water used and its relation to the amounts of the cement and the aggregate; percentages of voids in the aggregate and in the concrete, etc. Most authorities on concrete are in accord with the principle that, other things being similar, within certain limits the strength of concrete increases with the quantity of cement used and with the density or solidity of the resulting concrete, though in recent years statements have been made quite frequently which seem to throw doubt on the truth of this principle and to discredit the earlier writers. Altogether, there has resulted considerable confusion in the minds of engineers as to what constitute the real influences in the makeup of concrete, and as to proper ways for specifying the quality of the aggregates and for proportioning the mixtures.

2. *Acknowledgment.*—The experimental work reported in this bulletin was done in the Laboratory of Applied Mechanics as research problems of the Engineering Experiment Station of the University of Illinois. The tests termed Series 2G were undertaken in 1919 in coöperation with Committee C-9 of the American Society for Testing Materials, having been outlined by a sub-committee of which SANFORD E. THOMPSON was chairman. The making and testing of the specimens were under the direct supervision of PROFESSOR H. F. GONNERMAN, and W. L. SCHWALBE, then Graduate Research Assistant, was responsible for much of the work.

The tests termed Series 211 were undertaken in 1921 to investigate mortar-voids characteristics of fine aggregates and their relation to the concrete-making properties of the materials, together with the development of methods for applying the principles so derived to the designing of mixtures for making concrete. H. J. GILKEY, Research Assistant in the Engineering Experiment Station, REX L. BROWN, Graduate Research Assistant, and M. C. NICHOLS, Graduate Student in Theoretical and Applied Mechanics, gave helpful assistance in developing the methods, maintaining careful performance in the tests, and analyzing the results. From time to time since 1921 auxiliary series of tests and check tests have been made to round out the work.

Acknowledgment is made to PROFESSOR D. A. ABRAMS of the Structural Materials Research Laboratory, Lewis Institute, Chicago, for furnishing a large variety of fine aggregates for use in the tests.

3. *Scope of Bulletin.*—This bulletin enunciates relations between the compressive strength of the concrete and the amount of the cement and voids contained therein. It develops methods for studying the concrete-making properties of fine and coarse aggregates and for the comparison and acceptance of aggregates. It outlines means for designing concrete mixtures for different densities and strengths when the voids in mortars made up with a given cement and fine aggregate have been determined by laboratory tests. Means

are suggested for estimating the effect upon the strength and density of concrete that accompanies an increase in the amount of mixing water beyond that which would give minimum volume to the concrete. Analytical relations are developed and the technique for the proposed tests described. The results of tests of mortars and concretes made up with a variety of fine aggregates are recorded and the accuracy and applicability of the methods discussed. It is found convenient to use the absolute volume of the ingredients in terms of a unit of volume of the concrete in place, and for this purpose the specific gravity of the material should be known. The resulting values may readily be translated into bulk or weight when the voids or bulk weights of the materials in the condition obtaining on the work are known. The method is proposed for use in estimating the density and strength of concrete and in proportioning the materials. The test data cover a considerable range—a variety of aggregates, different proportions of cement, and different amounts of mixing water—and the significance of the results is discussed. The application of the methods and principles deduced to design, specifications, and field use is considered. The usefulness of the method as a means of proportioning mixtures, calculating quantities of materials, making comparisons, and estimating strengths is brought out.

The water content relations of the mortars and concretes are given in terms of that amount of mixing water which results in producing the minimum volume in the concrete, this basis being found to be more definite and satisfactory than others which have been used. Little use has been made of the expression "consistency of concrete" (a term that from its derivation means stiffness or ability to withstand change of form), since this expression is very indefinite. The flow table and the slump test were used to give some measure of mobility and workability of the concrete.

4. *Principles and Methods.*—The following paragraphs are given to outline some principles affecting the relation of the ingredients to the strength and density of the concrete:

(1) Definite quantitative relations exist between the magnitude of the voids in the concrete in place (water and air voids) and the compressive strength of the concrete, the quality and quantity of the cement and the quality and nature of the aggregate remaining the same, but the gradation of the aggregate varying, and other conditions remaining similar. The percentage of voids may, therefore, be taken as an index of the strength of the concrete in such cases.

(2) If concretes be made up with differing amounts of cement in such a way that the voids in the concrete remain the same, the nature of the aggregate and other conditions also remaining the same, the strength of the concrete will vary with the amount of cement used.

(3) For different amounts of cement and different gradations of the aggregate, the quality of the cement and the quality and nature of the aggregate remaining the same and other conditions remaining similar, the strength of the concrete varies with the ratio of the amount of cement used in a unit of volume of concrete to the voids in this volume, and the strength may be taken to be a function of this cement-voids ratio. Instead, the ratio of the voids to the cement may be used. In some respects it is still better to consider the strength of the concrete as a function of the ratio found by dividing the absolute volume of the cement by the sum of the voids in the concrete and the absolute volume of the cement—a ratio that may be termed the cement-space ratio.

(4) If the amount of mixing water be varied in such a way as to change the bulk of the concrete, the strength will still be a function of the cement-voids ratio of the resulting concrete, though the function may differ from that which applies with the smaller water content. When there is much difference in the relative water contents, the value taken from a strength curve based on minimum volume of concrete may be multiplied by a coefficient corresponding to the given relative water content.

(5) For the usual concrete mixtures—at least, for those that make an easily worked concrete, including all mixtures used on construction—the bulk of the coarse aggregate is less than the bulk of the concrete; or, in other words, the bulk of the mortar in a given volume of concrete is greater than the voids in the coarse aggregates alone—the term mortar being used here to include the cement, fine aggregate, and water. For such mixtures the voids in the concrete may then be considered to be made up of the water and air voids in the mortar. The density of the mortar for the consistency used in the concrete is, therefore, an important factor in determining the strength in the concrete.

(6) If the amount of the water and air voids in the mix of mortar to be used in making the concrete is known (the term mortar here meaning the combination of cement, fine aggregate, and water which enters into the concrete), the

voids in a resulting concrete may be calculated and the corresponding cement-voids or cement-space ratio may be used as an index of the strength of the concrete.

(7) By determining the voids in mortars made with a given fine aggregate, but with varying proportions of cement, a characteristic mortar-voids curve giving the relation between the voids in mortar and the ratio of fine aggregate to cement may be made up for any given fine aggregate, and information will then be available on which to base calculations for voids and strength of concrete made with definite proportions of cement, fine aggregate, and water.

(8) By determining the voids in various mortar mixes made with different fine aggregates of the same character and thus obtaining characteristic mortar-voids curves of the given fine aggregates, comparisons may be made of the relative densities and probable resulting strengths of concretes made up with these mortars and with assumed volumes of coarse aggregate.

(9) Knowing the characteristic mortar curve of a fine aggregate, the general nature and quality of the aggregate otherwise being known, the proportions of the mix required to give a specified strength (including the amount of cement, fine aggregate, and coarse aggregate) may be calculated with a fair degree of accuracy.

(10) The foregoing principles and methods are most readily applied by the use of the absolute volumes of the cement, fine aggregate, and coarse aggregate; but if the voids in the fine aggregate and the coarse aggregate in a dry state or in the condition obtaining on the work are known, or the relation between the weights by bulk and by solid volume, the bulk of the fine aggregate and the coarse aggregate required to make a unit of volume of concrete may readily be determined from the absolute volume given by the calculations already described.

(11) If the mixture is such that the voids in the aggregate are not fully filled by the mortar, the mortar voids will not be the measure of the voids in the concrete; but if the voids in the concrete are known, the ratio of the cement to the voids in the concrete may still be an indication of the strength of the concrete, except in the more extreme cases.

(12) The amount of voids in a specified mixture of mortar may be used to determine the acceptability of a given fine aggregate, so far as size and gradation are concerned.

(13) The method may be used to design the mixture of the concrete whatever the amount of cement used and whatever the ratio of coarse aggregate to fine aggregate, provided the mortar fills the voids in the coarse aggregate.

In making a mortar test of a fine aggregate, the voids are determined for increasing amounts of mixing water. The amount of water that gives the minimum volume of the mortar (greatest density and least voids) may be termed the basic water content. The characteristic mortar-voids curve made up from values of the voids for minimum volume of mortar may be termed the basic characteristic mortar-voids curve. The basic water content permits a very good consolidation of the particles of the mass when put into the mold; more water swells the mixture, as it also adds to the mobility. For other water contents, giving the wetter consistencies, such as for example 1.20 times the basic water content, characteristic mortar-voids curves may be made up from the voids data obtained with the specified water-content ratios in the tests already made. If a larger amount of water than the basic water content is to be used in the concrete, the characteristic curve for that proportionate water content should be used in the calculation. For the consistencies generally used in concrete work, the water content will range from 1.10 to 1.40 times the basic water content, though even wetter mixtures are frequently used without regard to consequences.

That there is a relation between cement, voids, and strength of mortars and concrete has been recognized for many years. Feret, the noted French engineer, in discussing his tests* stated that for all series of plastic mortars made with the same cement and of inert sands, the compressive strength after the same time of hardening under identical conditions is solely a function of the ratio of the amount of cement to the space in the mortar outside of the sand particles, and varies with the square of this ratio, whatever be the nature and size of the sand and the proportion of the elements. Taylor and Thompson† give a diagram showing Feret's results and formula. A recent contribution of Feret‡ gives additional information on the mortar-making qualities of a variety of fine aggregates that confirms the earlier statement of the intimate relation of voids, cement, and strength of mortars. Although the tests of Feret referred to are mainly on mortars, inferentially the same principles apply to concretes. Withey¶ has plotted the data from

* Bull. de la Société d'Encouragement pour l'Industrie Nationale. 1897.

† Taylor and Thompson, "Concrete Plain and Reinforced," Third Edition, p. 154.

‡ Sur le Choix des Materiaux pouvant etre employés comme Sable dans les Mortiers Hydrauliques, Revue des Materiaux de Construction et de Travaux Publics, March, April, and May, 1922.

¶ Johnson, "Materials of Construction," Fifth Edition, p. 464.

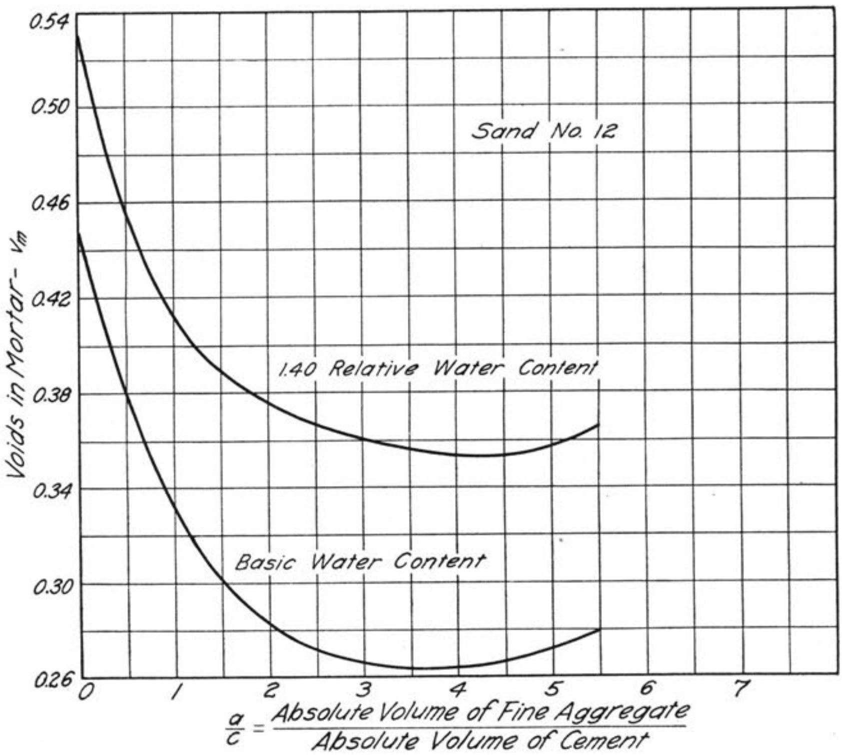

FIG. 1. CHARACTERISTIC MORTAR-VOIDS CURVES FOR ONE SAND

Technological Paper No. 58 of the U. S. Bureau of Standards by Wig, Williams, and Gates, to show the relation between strength of concrete and ratio of amount of cement to voids in concrete. Other writers have pointed out similar relations.*

5. *Density and Voids in Mortars and Concretes.*—By means of the mortar-voids test to be described, the magnitude of the voids in a unit of volume of mortar may be found for a given proportion of cement and fine aggregate. The magnitude of the voids should be determined both for an amount of mixing water which gives a minimum volume of mortar for the given mixture (an amount of water and a condition which herein is called the basic water content) and for one or more other water contents. When such data are available for a number of mixtures (proportions of cement and fine aggregate) the characteristic mortar-voids curve may be drawn. Fig. 1 represents such curves for a given sand, the lines representing basic water content (minimum volume) and a water content 1.40

* A partial outline of the principles and methods used in this bulletin was given in the paper entitled "A Proposed Method of Estimating the Density and Strength of Concrete and of Proportioning the Materials by the Experimental and Analytical Consideration of the Voids in Mortar and Concrete," by Arthur N. Talbot, Proc. A. S. T. M. Vol. 21, p. 940, 1921.

times the basic water content. For this sand with a mixture having twice as much sand as cement by absolute volumes, it will be seen that the voids in the mortar constitute 28.3 per cent of the volume of the mortar when the mixing water is such as to give minimum volume, and 37.5 per cent when 1.40 times this amount of mixing water is used. It will be shown that the characteristic mortar-voids curves for different fine aggregates vary widely, each material showing an individuality.

6. *Analytical Relations.*—If the voids properties of the mortar are known the application to the concrete mixture may be made by analytical considerations through the use of the equations which follow. The following notation will be used:

a = absolute volume of fine aggregate in a unit of volume of freshly placed concrete;

b = absolute volume of coarse aggregate in a unit of volume of freshly placed concrete;

c = absolute volume of cement in a unit of volume of freshly placed concrete;

d = density or solidity ratio of the freshly placed concrete;

v_m = voids (air and water) in a unit of volume of the mortar mixture of cement, fine aggregate, and water as it exists in the concrete;

v = voids in a unit of volume of concrete. This, of course, will be equal to $1 - d$.

It is evident that

$$a + b + c = d = 1 - v \quad \dots\dots\dots\dots\dots\dots\dots (1)$$

Since the mortar and the coarse aggregate together make up the unit of volume, it is also evident that

$$\frac{c + a}{1 - v_m} + b = 1 \quad \dots\dots\dots\dots\dots\dots\dots (2)$$

A third equation derived from equations (1) and (2) will be found useful:

$$v = v_m(1 - b) \text{ or } b = 1 - \frac{v}{v_m} \quad \dots\dots\dots\dots (3)$$

With the characteristic mortar-voids curve of a given fine aggregate known, it will be shown that the analysis may be made to apply to a variety of problems, such as to find the density and thereby the strength of the concrete with a given cement content and a given ratio of fine aggregate to cement; to find the density of the concrete and the amount of fine and coarse aggregate in a unit of volume of concrete when the amount of cement and the ratio of the coarse aggregate to the fine aggregate are given; to find the amount of cement required with different fine aggregates to produce a given density and strength; and to find the water content for a concrete when the water-content curve for the mortar is known. The applicability of the characteristic mortar-voids curve to these problems and the use of the analysis in the design of concrete mixtures are illustrated in the latter part of the bulletin.

It may be helpful to a quicker comprehension of the significance of the equations to know something of the range of values of the variables. For mortars of medium richness at about normal consistency v_m ranges from 0.27 when made with a coarse, well-graded sand to 0.40 with a very fine sand. For concretes the density d may range from 0.70 to 0.90. The ratio $\frac{a}{c}$ is somewhat greater than the ratio of sand to cement by loose volumes; it ranges from 1 for very rich mixtures to 5 for very lean ones. The absolute volume of the cement c ranges from 0.05 for a very lean concrete mixture to 0.15 for a rich one. The absolute volume of the coarse aggregate b will be 0 in a mortar and 0.5 or more in a coarse mixture. For a 1:2:4 mixture of fairly good materials, the values may be as follows: $c = 0.10$; $a = 0.28$; $b = 0.45$, these being the volumes of the solids in a unit of volume of concrete (cubic foot, cubic yard, etc.). For this example, it will be seen that $d = 0.83$.

II. Materials, Methods of Testing, and Tests

7. *Materials.*—Universal Portland Cement was used. It passed the standard tests* for specific gravity, fineness, soundness, and time of setting. The specific gravity of the cement was 3.10. The tensile strength of standard mortar briquettes composed of one part cement and three parts standard Ottawa sand was 232 and 320 lb. per sq. in. at 7 and 28 days, respectively, in Series 2G; and 175 and 285 lb. per sq. in. at corresponding ages in Series 211.

The fine aggregate used in Series 2G was a sand obtained from pits near the Wabash River at Attica, Indiana; it is the sand regularly used in recent years by the Engineering Experiment Station in tests of concrete and reinforced concrete. This sand was screened into several sizes and recombined in various artificial gradations. It was clean, hard, and well graded. The grains were smooth and somewhat irregular, and were mainly of calcareous material.

* Standard Specifications and Tests for Portland Cement (C9-21), 1921 Book of A. S. T. M. Standards.

For Series 211 a number of fine aggregates were used, including natural and screened sands, crushed granite, limestone, trap, slag, and chats. Special mention of the physical characteristics of these aggregates will be made in the description of these tests.

All the fine aggregates used in Series 2G and 211 were subjected to the Abrams-Harder colorimetric test* for organic impurities. All passed the test satisfactorily.

The coarse aggregate used in all concrete specimens was washed gravel from Attica, Indiana. It was composed of rounded, irregularly shaped pebbles mainly of calcareous material. All coarse aggregates were screened and separated into their several sizes and then recombined according to some artificial gradation, in a manner similar to that described for fine aggregates.

Properties of the aggregates are given in Article 20. The methods of determining these quantities follow present laboratory practice; the details will be mentioned briefly here.

8. *Determination of Specific Gravity.*—The specific gravity of fine aggregates was determined in two ways. The first method is similar to the standard specific gravity test for cement,† except that water was used in place of kerosene and the weight of sand used was smaller than the weight specified for cement. The LeChatelier flask was filled with water to just above the zero graduation, then 54 grams of sand were carefully added. Entrained air was removed by rolling the flask in an inclined position. The difference between the water level in the flask, before the sand was introduced and immediately after it was introduced, gave the absolute volume of the 54 grams of sand; this was used in the calculation of the specific gravity.

The second method of determining the specific gravity of fine aggregate was similar to a method outlined by Professor D. A. Abrams.‡ Duplicate samples of the aggregate were obtained by use of a sample splitter. Each sample was placed in a 500-cc. volumetric flask and one of them was set aside for the absorption test to be described later. The flask containing the other sample was filled with water from a burette to the 500-cc. mark, shaken well, and allowed to stand a short time; afterward sufficient water was added from the burette to bring the level to the 500-cc. mark. The specific gravity was calculated by dividing the dry weight of

* Abrams-Harder Field Test for Organic Impurities in Sand. Appendix to Report of Committee C-9. Proc. A. S. T. M Vol. XIX, Part I, p. 321.

† Standard Specifications and Tests for Portland Cement (C9-21), 1921 Book of A S. T. M. Standards.

‡ Report of Committee C-9, Proc. A. S. T. M. Vol. XX, Part I, p. 301.

the sample by the difference between 500 cc. and the total volume of water used in the flask. A second determination of the volume displaced by the sand was made after the flask had stood for 15 minutes; this value was used in determining the absorption of the fine aggregate.

The determination of the specific gravity of the coarse aggregate was made by still another method. The absolute volume, or displacement, of a given weight of gravel was found by noting its loss in weight when immersed in water. About 1000 grams of gravel were placed in a wire basket and suspended in water from one of the arms of an accurate balance. The weight of the dry gravel divided by its displacement gave the specific gravity of the material.

It should be noted that in all cases the apparent specific gravity was found. That is, the volume of a particle was taken as the space bounded by the surfaces, thus including the interior pores of the material.

It may be seen from Table 5 that the specific gravities of the various aggregates used range from 2.50 to 2.90. The determinations were made in duplicate by each method, the values obtained varying not more than 0.4 per cent from the mean.

9. *Absorptive Properties of Aggregates.*—Tests were made on all aggregates to determine the amount of water absorbed while the material was immersed for 15 minutes. This was considered a reasonable approximation to the absorption of water by the aggregate during the mixing of the concrete. The amount so determined was added to the quantity of water that was taken as effective mixing water in making the concrete specimens. For the aggregates of Series 2G longer periods of immersion were also used.

The tests on fine aggregates were made in connection with the specific gravity tests. The second of the two duplicate samples was placed in a 500-cc. volumetric flask and mixed with about 20 cc. of kerosene. The flask was then filled nearly to the 500-cc. mark with a normal NaCl solution from a burette and shaken to remove air bubbles. Due to the difference in specific gravity of the two liquids, all kerosene except that held in the pores of the aggregate by capillary attraction rose and floated on the salt solution. The line of demarcation between the kerosene and salt solution was brought just to the 500-cc. mark on the flask. This procedure determined the absolute volume of the sand, including the pore spaces. The difference between this volume and the volume displaced by the aggregate after 15 minutes immersion, as determined in connection with the specific gravity test, gave the volume of water absorbed by the aggregate.

The absorption by the coarse aggregate was determined by immersing a 1000-gram sample in water for 15 minutes; after this it was removed, surface-dried by rolling it in a burlap sack, and reweighed. The increase in weight over the original dry weight gave the weight of water absorbed by the aggregate.

In all cases the absorption has been calculated as a percentage of the original dry weight of the aggregate. All determinations were made in duplicate.

10. *Size and Gradation of Particles and Their Measurement.*—The size of particles of aggregate was found in all cases by screening the material through a series of sieves. The dimensions of the square openings of the sieves used are given in Table 1.

TABLE 1

SIZE OF SIEVE OPENINGS, TYLER STANDARD SIEVES

Sieve No.	Opening, Inches	Sieve No.	Opening, Inches
100	.0058	⅜-in.	0.371
48	.0116	¾-in.	0.742
28	.0232	1-in.	1.050
14	.0460	1¼-in.	1.250
8	.0930	1½-in.	1.500
4	.1860	2-in.	2.000

Two sieving operations were performed: (1) the sieve analysis of representative samples of a material; and (2) the screening of large quantities of material to be recombined according to artificial gradings.

The sieve analysis of aggregates was made by use of a "Rotap" mechanical sieve shaker, which held ten or more 8-in. circular sieves, nested with the coarser sizes above. A 500-gram sample of dry aggregate was put in the topmost sieve, and the motor-driven machine furnished a combination of a horizontal oscillating motion and a vertical tapping or jarring. A time switch enabled the apparatus to be run for any definite length of time. In general the practice was followed of sifting until not more than one per cent of the original sample passed through any one of the sieves in one minute. All sieve analyses were made on duplicate samples.

The screening of both fine and coarse aggregate to be recombined for use in making test pieces was done by the use of 20-in. riddles shaken by a motor-driven gyratory shaker. Screening of each size of material was continued until there was no appreciable amount of material passing through the sieve. All fine aggregate was dried before screening and was stored in covered containers after screening.

From the sieve analysis of an aggregate, certain quantities may be computed which give a measure of both the size and the gradation of the particles. Two such quantities, the surface modulus and the fineness modulus,* have been used in this investigation. The surface modulus is a number which is roughly proportional to the surface area of the particles of an aggregate. The fineness modulus is a number derived in an empirical way, which varies with the size and gradation of the particles.

For the calculation of the surface modulus, let p_1 be the percentage by weight of the sample finer than the No. 100 sieve as obtained by the sieve analysis, p_2 the percentage between the No.100 and No. 48 sieves, p_3 the percentage between the No. 48 and No. 28 sieves, and so on, using the intervals between the No. 28, No. 14, No. 8, No. 4, 3/8-, 3/4-, 1½- and 3-in. sieves. Each percentage is multiplied by a coefficient of the form $(1/2)^{m-1}$, where m indicates the number of the term in the series, so that the coefficient of p_1 is 1, that of p_2 is ½, and that of p_3 is ¼. The surface modulus is then the sum of these products

$$s.m. = p_1 + \frac{p_2}{2} + \frac{p_3}{4} + \frac{p_4}{8} + \ldots\ldots \frac{p_{10}}{512} \ldots\ldots\ldots\ldots\ldots (4)$$

The fineness modulus may be computed in a similar way by use of coefficients of the form $\frac{m-1}{100}$. The fineness modulus is thus equal to

$$f.m. = 0p_1 + 0.01p_2 + 0.02p_3 + 0.03p_4 + \ldots\ldots 0.09p_{10} \, . (5)$$

It is seen that the surface modulus is large for fine materials and smaller for coarser ones; it is most affected by a variation in the finer particles of an aggregate. The fineness modulus is large for coarse materials and small for fine ones, but does not give so much importance to the proportion of the very fine particles.

11. *Weight and Density of Aggregates.*—The unit weight of aggregates was determined† by the use of a cylindrical steel mold about 8 in. in diameter and 7 in. high and having a capacity of 0.2 cu. ft. The measure was filled in three layers, each layer being compacted in place by 25 to 30 strokes of a tamping rod. When the mold had been filled, the top was struck off evenly and the

* For detailed discussion of surface and fineness moduli, see discussion on Proportioning of Concrete, by D. A. Abrams and A. N. Talbot, Proc. A. S. T. M. Vol XIX, Part II, pp. 477-484. See also "Design of Concrete Mixtures," by D. A. Abrams, Bulletin 1, Structural Materials Research Laboratory, Lewis Institute, Chicago.

† Except for size of measure used, the method follows the "Standard Method of Test for Unit Weight of Aggregate for Concrete," 1921 Book of A. S. T. M. Standards, p. 639.

contents were weighed. The average of three determinations was used in all cases.

The density or solidity ratio of the aggregate was computed by dividing the weight per cubic foot of the aggregate by the specific gravity multiplied by 62.4. The density was also found for some of the coarse aggregates by the water-displacement method, as a verification of the accuracy of the first method. The voids, or complement of the density, are recorded in Table 5.

12. *Technique of Making Concrete Specimens.*—Since the method of planning the test mixtures differed, the method of making up the test pieces varied somewhat in the two series.

In Series 2G the principal variable was the gradation of the aggregate, as outlined in Article 15. The concrete was designed to have 1 part of cement, by volume, to 5 parts of mixed aggregate. The amount of mixing water was proportioned to produce a common consistency of the concrete, ordinarily termed normal consistency. This was done by trial, the consistency being judged by means of the slump test, and in the coarsest mixtures by the appearance of the concrete.

In Series 211 the test pieces were designed to have a certain amount of cement, coarse aggregate, and water per cubic foot of concrete. These quantities were calculated by the use of the characteristic mortar curves for the fine aggregate to be used. Since the density of the concrete could be predicted quite accurately, the quantities of materials, including the water, needed to produce a given volume of concrete were readily predetermined. In computing the amount of water to be used, an allowance for the absorption by the aggregates during a 15-minute period was added to the amount considered as effective mixing water. For use in such calculations under other circumstances this would mean also that when the unmixed aggregates are so wet that their surfaces are covered with water, this surface water should be considered a part of the effective mixing water.

Tests with the flow table were made for all concretes of this series; the results were fairly uniform for mixes of the same relative water content.

The concrete in all specimens was mixed by hand with trowels in a large sheet iron pan. Materials were weighed out and placed in a dry container and mixed dry. They were then transferred to the wet mixing pan, water was introduced from a large burette, and the batch was thoroughly mixed. The flow test or slump test was then made on a portion of the batch, and after this the specimen was molded.

In molding the cylinders, the concrete was deposited in three layers, and each layer was tamped into place with a tamping rod. The tamping rod used in Series 2G was a blunt-pointed 5/8-in. round steel bar about 20 in. long; that used in Series 2H was a wooden rod 1½ in. square and 15 in. long, which was found to be more effective, especially with the very dry and lean mixtures.

The test pieces were made in 6 by 12-in. steel molds, resting on machined cast-iron base plates. The inner surfaces of molds and plates were coated with a heavy oil. A few hours after making, the tops of the test pieces were capped with a thin layer of stiff neat cement mortar. The cap was formed by pressing down upon it a sheet of plate glass, separated from the cap by a sheet of oiled paper. The lower ends of the cylinders were not capped, unless they had been damaged in handling.

13. *Slump Test and Flow-Table Test.*—The standard tests were used in a study of the workability, or mobility, of the concrete. These tests, known as the slump test and the flow-table test, are the most common workability tests now in use, and have been tried out and developed by a number of investigators.*

The slump test was used in Series 2G in determining a standard or normal workability of the different concrete mixtures. The apparatus used consisted of a 6 by 12-in. steel cylindrical mold,† resting on a machined cast-iron base plate and fitted with guides so that it could be raised vertically. The concrete was deposited in the mold just as in making test pieces. When the mold had been filled, the top of the cylinder was struck off evenly and the mold was carefully raised clear of the cylinder. The amount of settlement of the cylinder from its original height of 12 in. has been termed the "slump" of the cylinder. The standard workability in Series 2G was that which produced a slump of ½ to ¾ in. except in the case of the coarse gradations of the aggregate, where the slump test was not applicable and the desired workability was determined by inspection.

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* "Method and Apparatus for Determining Consistency," by C. M. Chapman, Proc. A. S. T. M., Vol. XIII, Part II, 1913, p. 1045.

"Some Determinations of the Stress-Deformation Relations for Concretes under Repeated and Continuous Loadings," by G. M. Williams, Proc. A. S. T. M., Vol. XX, Part II, 1920, p. 233.

"A Comparison of the Results of the Slump Test and the Flow Table in the Measurement of the Consistency of Concrete," by W. L. Schwalbe, Proc. A. S. T. M., Vol. 21, 1921.

"Effect of Fineness of Cement," by D. A. Abrams, Proc. A. S. T. M., Vol. XIX, Part II, 1919, p. 328.

† In later work the size and shape of the molds used in the slump and flow tests have been changed to those recommended in 1921 by the American Society for Testing Materials, —Proc. A. S. T. M., Vol. 21, p. 579.

The flow-table test was used in Series 211 for a study of the workability of concretes in which the proper water content had been determined by mortar tests. The apparatus consists of a table which can be rapidly and repeatedly raised and dropped ½ in. by means of a cam. In operation a conical mold 6 in. in diameter at the top and 11 in. at the bottom and 6 in. high was placed on the table and filled with concrete, tamped in the same way as in the making of the test pieces; the mold was then removed and the table was given 15 drops at intervals of about one second. The spread of the base of the mass of concrete was taken as a measure of the workability of the concrete, the relative increase in the average diameter of the base being recorded as "per cent flow." Loose pieces of gravel were not considered in measuring the increase in diameter, but measurements were taken at several places to the edges of the coherent mass of concrete.

14. *Storage and Testing.*—All test specimens were left in the molds for 24 hours after pouring. The test specimens were then placed in the moist storage room. In Series 2G the specimens were stored in damp sand; in Series 211 they were stored in moist air which was kept saturated by means of a number of humidifying sprays operated with compressed air and water. The temperature in both the molding and storage rooms was from 70 to 75 deg. F., throughout the tests. When 27 days old, the test pieces were removed from the moist storage and allowed to dry out for one day in the air of the laboratory. They were then measured and weighed preparatory to testing.

The compression tests were made in an Olsen testing machine of 300 000-lb. capacity at a nominal speed of 0.05 in. per minute. The load was applied through a spherical bearing block carefully centered. Deformations were measured by use of a compressometer, having a gage length of 8 in., multiplication ratio of 2, and using a micrometer dial graduated to 0.001 in. Stress-strain curves were obtained in each test by use of a semi-autographic device* attached to the testing machine. The stress-strain curves found are not included in this report.

* "Rapid Semi-Autographic Tests for Determining the Proportional Limit," by H. F. Moore, Proc. A. S. T. M., Vol. XVII, Part II, 1917, p. 589.

III. Concrete with Regularly Graded Aggregate

15. *Outline of Series 2G.*—The principal tests of Series 2G followed the outline made by a subcommittee of Committee C-9 (Concrete and Concrete Aggregates) of the American Society for Testing Materials. The purpose of the tests was to obtain information "on the laws of mechanical mixtures" and on the relation between the gradation of aggregates (fine and coarse) and the strength of the resulting mortars and concretes. In the consideration of the tests it should be borne in mind that all the aggregates used in Series 2G are regularly and continuously graded from fine to coarse, except a few irregularly graded mixtures; the results, therefore, may not apply in some respects to such variations of gradation as are to be found in the aggregates used in practice where there may be a preponderance of some one size (fine, coarse, or intermediate) over the amount in other parts of the scale (or a deficiency) in such a way as to modify the properties of the aggregate considerably.

Including what would be termed mortar mixtures, ten groups of aggregates were used. The range in size of particle for the several groups by sieve sizes was 0—48, 0—28, 0—14, etc. to 0—¾ in., 0—1 in., 0—1½ in. and 0—2 in. In all cases the gradation from fine to coarse was made according to an adopted equation, and each group included eight or ten different gradings. At one end of a group the finer particles predominated and at the other end the excess of coarse particles gave a coarse mixture. The series included in all 88 of these regularly graded aggregates. The gradation of the aggregates is indicated by the sieve-analysis curves of Figs. 2 and 3. These gradations were found from the general equation

$$p = 100 \left(\frac{d}{D} \right)^n \quad \text{.................................(6)}$$

where p is the percentage of material which passes a given sieve having openings of width d, D is the maximum size of particle of the given aggregate, and n is a variable exponent. From the equation it is seen that with a certain maximum size of aggregate variation in gradation may be obtained by varying the value of the exponent n; with $n = 1.0$ the curve becomes a straight line, and with $n = 0.5$ the curve becomes a parabola. In the preparation of the aggregates according to these predetermined sieve analyses, the sand and gravel were first screened to the size indicated by the following sieves: 0—100, 100—48, 48—28, 28—14, 14—8, 8—4, 4—⅜ in., ⅜ in.—¾ in., ¾ in.—1 in., 1 in. —1½ in., 1½ in.—2 in. These mixtures were then recombined by weight to give the desired grading. In addition to the regularly graded aggre-

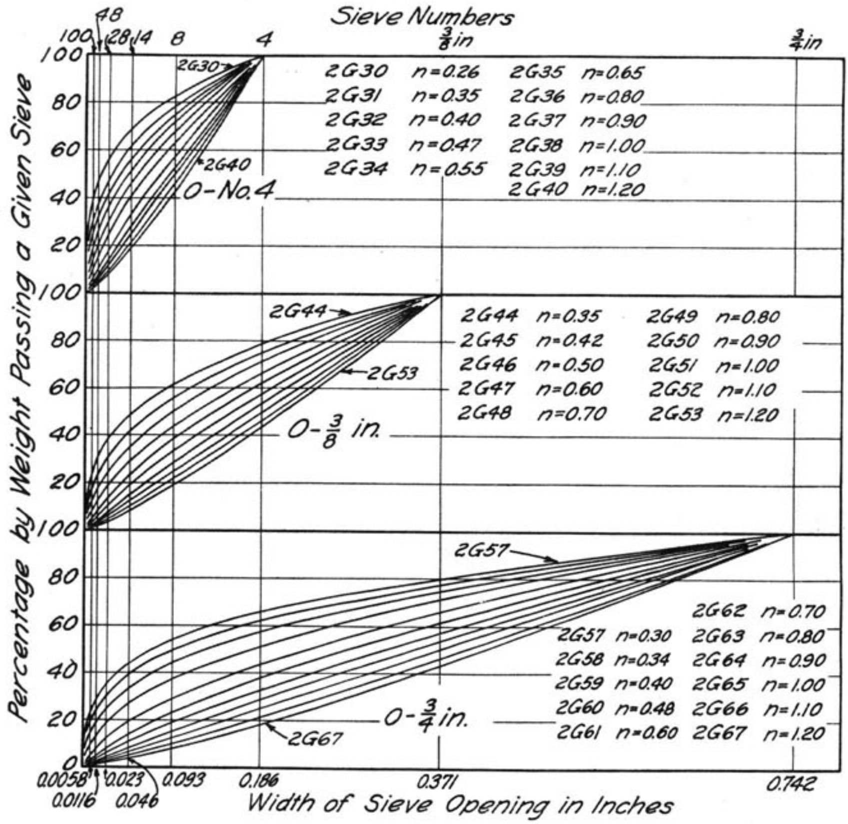

FIG. 2. SIEVE-ANALYSIS CURVES FOR MIXED AGGREGATES, SERIES 2G

gates, twelve irregularly graded mixtures of fine and coarse aggregates were used. A single mixture of 0—100 sand was also included.

All the specimens of Series 2G were made with the same proportion of cement to aggregate by loose volume: 1 part cement to 5 parts mixed aggregate, using 94 lb. of cement as a cubic foot. Due to differences in the bulking of the mixture the amount of the cement in the resulting concrete varies somewhat (a result which is always present in concretes proportioned by loose volume), ranging from say 0.17 cu. ft. of cement by bulk per cubic foot of concrete for the finer mixtures of each group having a given maximum size of aggregate to 0.21 cu. ft. per cubic foot for the coarse mixtures of each group, or in terms of the absolute volume of the cement from 0.08 to 0.10 cu. ft. of cement per cubic foot of concrete. The concrete was all of what has been termed a normal consistency, giving 0 to

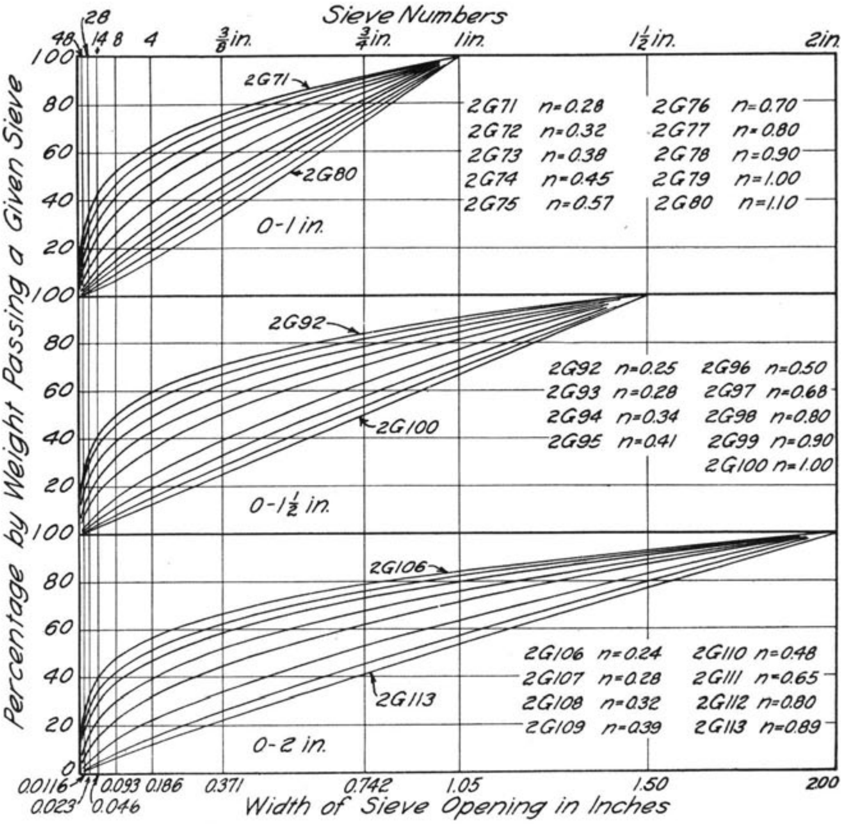

FIG. 3. SIEVE-ANALYSIS CURVES FOR MIXED AGGREGATES, SERIES 2G

¾ in. of slump for all the mixtures for which the slump test was found applicable, and the same apparent degree of wetness for the very coarse mixtures. The concrete specimens were stored under moist conditions and the test was made at 28 days, the specimens being taken from the moist room one day in advance of testing.

16. *Voids and Space in Aggregates, Mortars, and Concretes.*— It is evident that the magnitude of the voids in an aggregate or a concrete is affected by the gradation of the particles. The finer particles may be expected to fit in between the larger particles wholly or partly. If the coarser particles predominate over the somewhat finer ones to a sufficient degree, the bulk of the finer particles will not fill the space between the coarser ones. If the finer particles are too numerous, the full extent of the dilution of space by the larger particles will not be attained. It is clear that to reach a

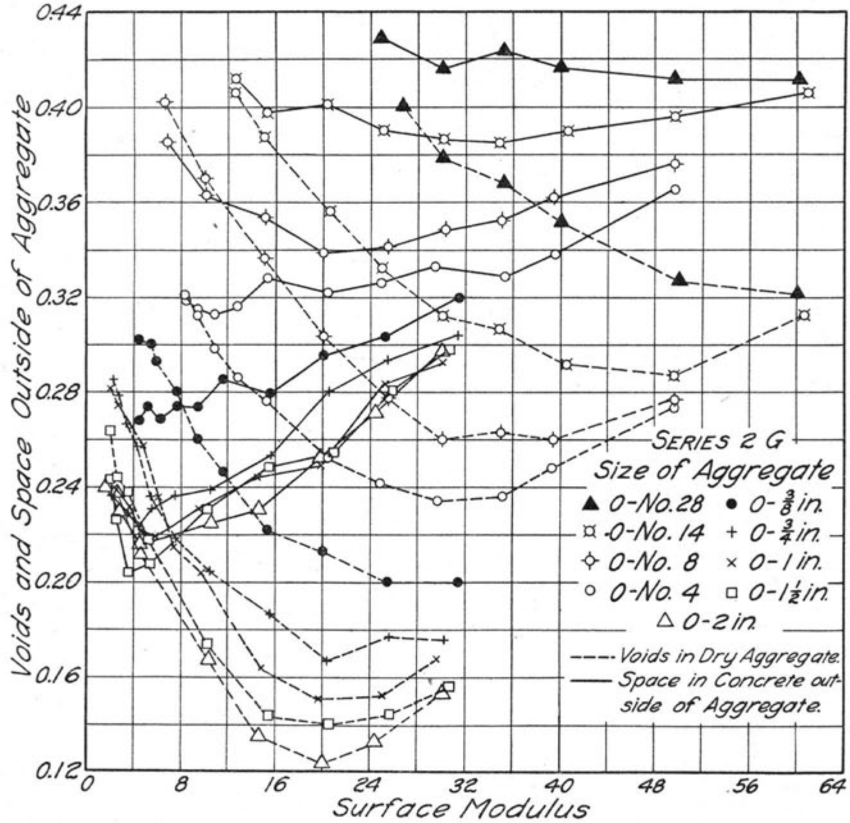

FIG. 4. VOIDS AND SPACE OUTSIDE OF AGGREGATES, SERIES 2G

minimum volume, the gradation of particles must be of a particular kind. It is also evident that the magnitude of the voids in the concrete will be affected by the amount of the cement particles as related to the amount of the finer particles of the aggregate and the available space between these particles.

The bulking of the finer particles also enters into the problem. When water in small quantities is added to dry sand, the mass of sand bulks or swells, the amount of the bulking depending upon the fineness and other qualities of the sand; an addition of 5 per cent of water will increase the volume of some sands 20 per cent. This feature of the action of fine particles must be taken into account in some way in considering voids, for it is the magnitude of the voids in the concrete in place which is found to affect strength.

In Fig. 4 the full lines give the voids in the dry aggregate of

Series 2G, that is, in the mixed aggregate free from cement and water; the dotted lines give the space outside the aggregate in the mortars and concretes, including cement, water, and air. If the absolute volume of the cement (which in Series 2G ranges from 0.08 to 0.10) be subtracted from the space outside the aggregate, the difference will give the voids in the mortars and concretes. The nature of the gradation of the aggregate—whether coarsely graded or finely graded —is here denoted by the surface modulus of the material, which is used as the abscissa of the figure. A coarsely graded material is at the left, a finely graded one at the right. The point at the left on any curve corresponds to the lower curve of the groups of Figs. 2 and 3, and the point at the right to the upper curve. If fineness modulus, or another device for indicating relative coarseness of particles, had been used, the form of the curves would be much the same.

It will be noted that for the dry aggregate the point of minimum voids in any group of aggregates, those having the same maximum size of pebble, is away from the coarse gradations and much nearer the finest gradation. With the addition of cement and water the point of minimum space outside of the aggregate, which is also the point of minimum voids in the concrete, corresponds to a much coarser gradation; for the concretes, it is at the third or fourth point from the left end of the curve and has a value of n in the gradation equation of 0.90 for the ¾-in. aggregate, 0.80 for the 1-in. and 1½-in., and 0.65 for the 2-in. size. It should be noted that at the left of the point of minimum voids the voids in the concrete increase rapidly.

For those aggregates that give the minimum voids in the concrete, the magnitude of the voids in the concrete is extremely small, perhaps at the limit of what may be obtained with a 1:5 mixture made under the most favorable conditions of gradation of particles. It should be noted also that the resulting concrete is what is termed a harsh mixture and that it would be very difficult to place so as to make homogeneous concrete free from air spaces unless it were most carefully mixed, placed, and tamped; in fact, these minimum-voids mixtures and the coarser mixtures and even the next finer ones can not be considered usable aggregates for ordinary concreting operations, and the really workable mixtures are finer than these. The value of b (see Article 6 for notation), for these very dense mixtures, ranging from 0.56 for the ¾-in. aggregate to 0.65 for the 1½-in. size, may be considered extreme examples and only possible with the best graded and rounded pebbles and the best graded fine aggregate, and for laboratory specimens. In field work the ordinary methods of placing and tamping such mixtures would result in uneven concrete, with extra voids in many places, and consequent loss of homogeneity and strength.

While the sands of any group of Series 2G have a considerable difference in voids, as is shown by the various curves of Fig. 4, the corresponding mortars have relatively little difference in voids and also little difference in the amount of space outside the sand. Thus, for the 0—No. 14 sand the voids in the dry sand range from 0.29 to 0.40, the space outside the sand in all the corresponding mortars is about 0.39 or 0.40, and the voids in the mortars 0.31 or 0.32. It should be kept in mind that these are all 1 : 5 mixtures.

The diagrams of the figure give an idea of the changes which occur when cement and water are added for this mixture and normal consistency. It is evident that in most of these mixtures the particles of sand have been pushed apart and the space outside the aggregate greatly augmented when the cement and water were added.

Tests with richer and leaner mixtures and with wetter and dryer consistencies indicate similar changes. For the aggregates used, the 1 : 5 mixture gives slightly lower voids in the resulting concrete than a 1 : 3 or a 1 : 9 mixture. Generally, the normal consistency gives less voids in the concrete than a dryer or a wetter consistency.

17. *Strength Relations.*—The principal data of Series 2G are given in Table 2. In Fig. 5 the compressive strengths of the concrete at 28 days are plotted as ordinates and the voids in the concrete as abscissas. Twelve irregular gradings are also plotted and one

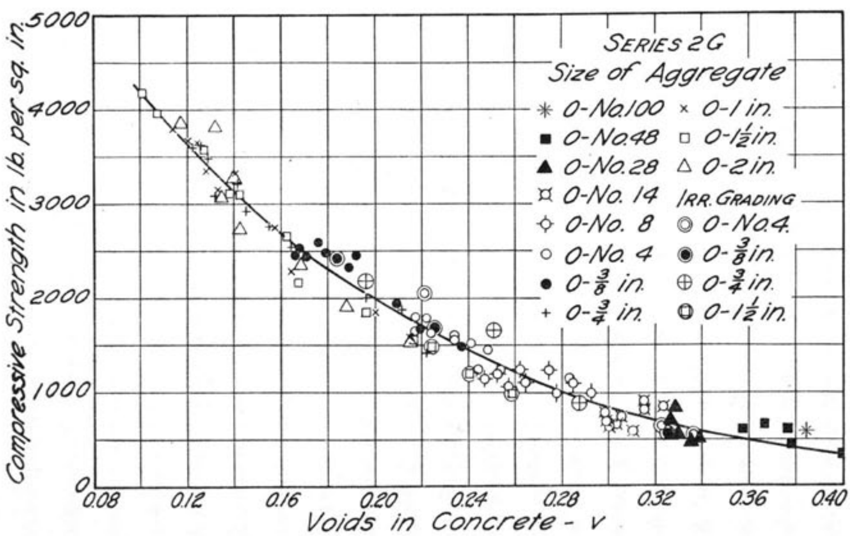

FIG. 5. VOIDS AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 2G

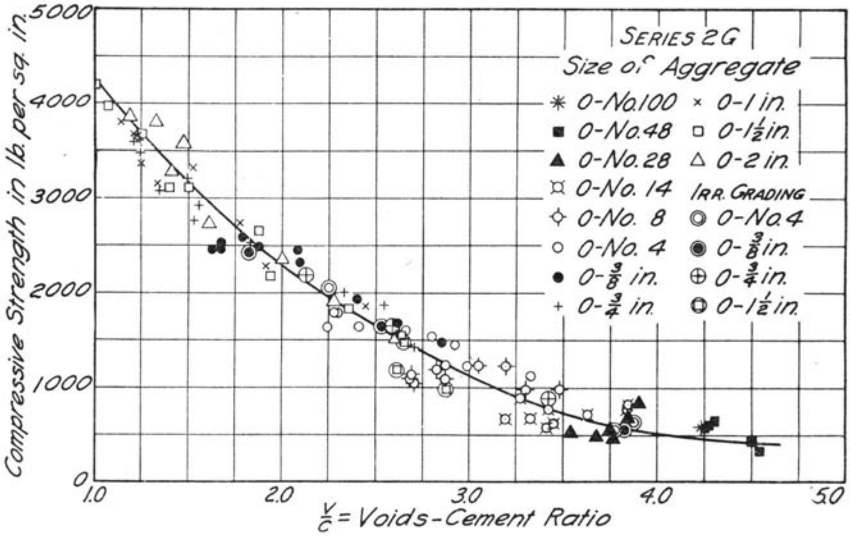

FIG. 6. VOIDS-CEMENT RATIO AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 2G

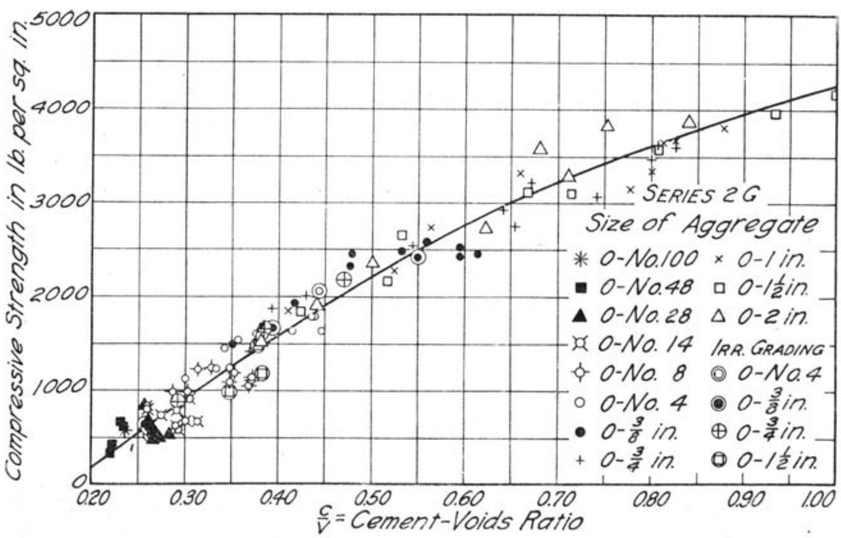

FIG. 7. CEMENT-VOIDS RATIO AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 2G

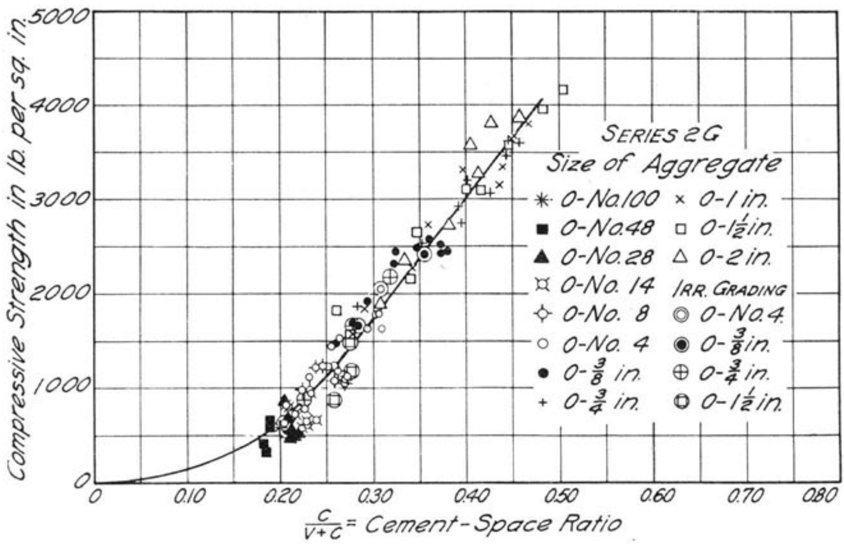

FIG. 8. CEMENT-SPACE RATIO AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 2G

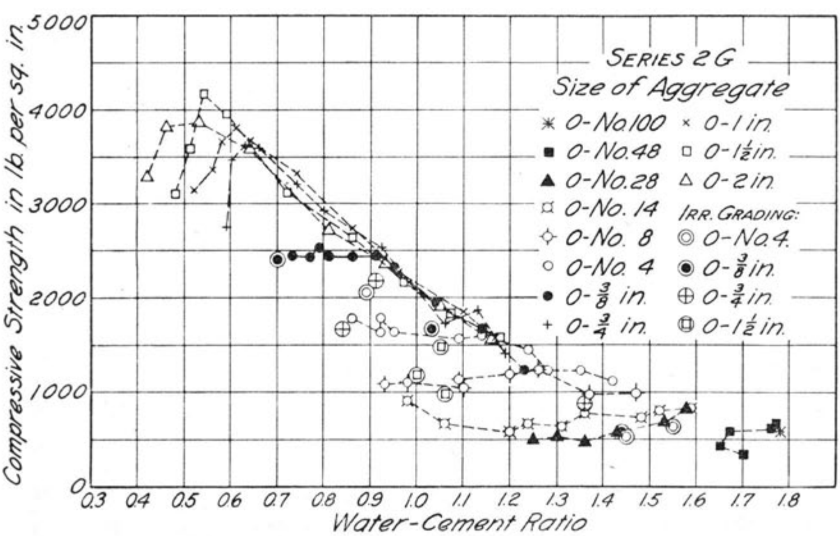

FIG. 9. WATER-CEMENT RATIO AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 2G

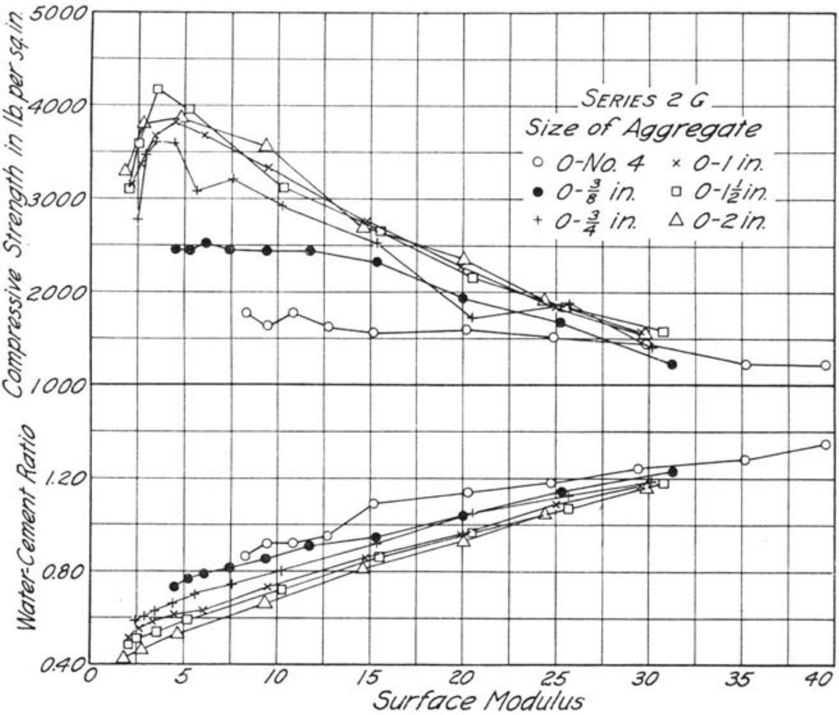

FIG. 10. SURFACE MODULUS AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 2G

mixture with 0—No. 100 sand. When the great range in size of particles and the variety of gradation are considered the close relation between the magnitude of the voids and the compressive strength of the concrete is striking. It will be noted that the voids range between 0.10 (an exceptionally dense concrete) and 0.40 (a very porous mixture). In Fig. 6 the same tests are plotted with the ratio of voids to the absolute volume of cement as abscissas; and in Fig. 7 with the ratio of cement to voids as abscissas. In Fig. 8 the tests are plotted in terms of the ratio of the absolute volume of the cement per unit volume of concrete, c, to the space around and about the particles of the aggregate (sum of the voids and the absolute volume of cement, $v + c$). This ratio gives the proportion of the space around and about the particles of the aggregate that is occupied by the cement and through which it is expected to act or be effective, and represents in a way the dilution of the cementing material.

In Fig. 9 the tests are plotted in terms of the ratio of the mixing water (exclusive of that considered to be absorbed by the aggregate)

TABLE 2

DATA OF CONCRETE TEST SPECIMENS—SERIES 2G

Artificially graded aggregates. Normal consistency; 1: 5 mix, by volume.
Tests at age of 28 days.
Each value represents the average of 3 or 4 specimens.

Cyl. No.	Size of Aggregate	$\frac{a}{c}$	Absolute Volumes			Voids v	$\frac{v}{c}$	$\frac{c}{v+c}$	w_c	Surface Modulus	Fineness Modulus	Water Cement Ratio	Comp. Strength lb. per sq. in.
			Sand a	Gravel b	Cement c								
0	0–No. 100	5.77	.525	0	.091	.384	4.24	.198	.332	100.0	0	1.78	590
1	0–No. 48	6.09	.536	0	.088	.376	4.27	.189	.320	90.0	0.20	1.76	600
2		6.46	.549	0	.085	.366	4.31	.188	.308	80.5	0.39	1.77	660
3		6.65	.559	0	.084	.357	4.25	.195	.287	70.3	0.60	1.67	590
4		6.42	.538	0	.084	.378	4.50	.181	.286	60.0	0.80	1.65	420
5		5.82	.512	0	.088	.400	4.54	.185	.309	50.0	1.00	1.70	330
6	0–No. 28	7.00	.588	0	.084	.328	3.90	.204	.269	59.6	0.99	1.55	830
7		6.93	.589	0	.085	.326	3.84	.206	.268	49.6	1.25	1.53	680
8		6.63	.583	0	.088	.329	3.74	.211	.259	40.0	1.52	1.43	560
9		6.48	.576	0	.089	.335	3.76	.210	.248	35.3	1.66	1.36	470
10		6.33	.582	0	.092	.326	3.54	.220	.245	30.1	1.82	1.30	520
11		6.19	.570	0	.092	.338	3.67	.214	.236	25.0	2.00	1.25	490
12	0–No. 14	7.06	.593	0	.084	.323	3.85	.206	.273	60.6	1.16	1.59	830
13		7.36	.603	0	.082	.315	3.84	.207	.256	49.9	1.49	1.52	810
14		7.28	.611	0	.084	.305	3.63	.216	.257	40.4	1.80	1.48	730
15		7.07	.615	0	.087	.298	3.42	.226	.242	34.8	2.00	1.36	780
16		7.05	.613	0	.087	.300	3.45	.225	.235	30.0	2.17	1.31	620
17		6.79	.611	0	.090	.299	3.32	.231	.228	25.0	2.36	1.24	670
18		6.24	.599	0	.091	.310	3.41	.227	.224	20.3	2.56	1.20	580
19		6.34	.602	0	.095	.303	3.19	.239	.209	15.0	2.83	1.06	660
20		6.13	.589	0	.096	.315	3.28	.234	.193	12.5	3.00	0.98	900
21	0–No. 8	7.42	.624	0	.084	.292	3.48	.223	.254	49.5	1.76	1.47	980
22		7.61	.639	0	.084	.277	3.30	.232	.237	39.5	2.13	1.37	980
23		7.48	.640	0	.086	.274	3.19	.239	.225	34.9	2.34	1.27	1220
24		7.61	.652	0	.086	.262	3.05	.247	.222	30.3	2.53	1.26	1240
25		7.43	.659	0	.089	.252	2.83	.261	.220	25.3	2.75	1.20	1190
26		7.15	.661	0	.092	.247	2.69	.271	.208	20.0	3.01	1.09	1140
27		6.82	.648	0	.095	.257	2.71	.270	.215	15.0	3.27	1.10	1050
28		6.48	.638	0	.098	.264	2.69	.270	.199	10.0	3.16	0.98	1100
29		6.21	.617	0	.099	.284	2.87	.258	.192	6.7	3.93	0.93	1090
30	0–No. 4	7.44	.632	0	.085	.283	3.33	.231	.249	49.6	2.01	1.42	1120
31		7.78	.661	0	.085	.254	2.99	.251	.235	39.5	2.46	1.35	1230
32		7.90	.671	0	.085	.244	2.87	.258	.225	35.1	2.67	1.28	1240
33		7.85	.667	0	.085	.248	2.92	.255	.216	29.7	2.95	1.24	1450
34		7.83	.673	0	.086	.241	2.80	.263	.210	24.8	3.19	1.18	1530
35		7.70	.678	0	.088	.234	2.66	.273	.207	20.2	3.44	1.14	1600
36		7.47	.672	0	.090	.234	2.60	.278	.202	15.2	3.75	1.09	1570
37		7.34	.685	0	.093	.224	2.41	.293	.183	12.7	3.90	0.95	1640
38		7.24	.688	0	.095	.217	2.28	.305	.180	10.8	4.05	0.92	1790
39		7.07	.686	0	.097	.217	2.24	.309	.184	9.5	4.14	0.92	1640
40		7.03	.681	0	.097	.222	2.29	.304	.173	8.3	4.25	0.86	1780
41	0–No. 4	6.95	.681	0	.098	.221	2.25	.308	.180	10.5	4.31	0.89	2050
42	Irregular	6.46	.575	0	.089	.336	3.77	.210	.265	19.1	2.40	1.45	530
43	Grading	7.17	.595	0	.083	.322	3.88	.205	.265	28.9	2.46	1.55	640
44	0–⅜ in.	6.35	.527	.153	.083	.237	2.85	.260	.209	31.3	3.23	1.23	1490
45		6.18	.519	.178	.084	.219	2.61	.277	.198	25.2	3.59	1.14	1680
46		5.48	.477	.228	.087	.209	2.40	.294	.187	20.0	3.92	1.04	1940
47		5.17	.465	.256	.090	.189	2.10	.322	.176	15.3	4.23	0.95	2320
48		4.63	.426	.290	.092	.192	2.09	.324	.172	11.8	4.52	0.91	2450
49		4.23	.402	.324	.095	.179	1.88	.347	.168	9.4	4.71	0.86	2490
50		3.89	.381	.345	.098	.176	1.79	.360	.164	7.4	4.88	0.81	2590
51		3.55	.355	.377	.100	.168	1.68	.373	.164	6.1	5.03	0.79	2520
52		3.36	.339	.389	.101	.171	1.68	.373	.160	5.3	5.13	0.77	2440
53		3.04	.310	.422	.102	.166	1.63	.380	.153	4.5	5.24	0.73	2450

TABLE 2 (CONTINUED)
DATA OF CONCRETE TEST SPECIMENS—SERIES 2G
Artificially graded aggregates. Normal consistency; 1: 5 mix, by volume.
Tests at age of 28 days.
Each value represents the average of 3 or 4 specimens.

Cyl. No.	Size of Aggregate	$\frac{a}{c}$	Absolute Volumes			Voids v	$\frac{v}{c}$	$\frac{c}{v+c}$	uc	Surface Modulus	Fineness Modulus	Water Cement Ratio	Comp. Strength lb. per sq. in.
			Sand a	Gravel b	Cement c								
54	0–⅜ in.	6.28	.533	.061	.085	.325	3.82	.207	.253	24.5	2.70	1.44	560
55	Irregular	6.14	.546	.140	.089	.225	2.53	.283	.187	15.0	3.95	1.03	1660
56	Grading	2.79	.282	.433	.101	.184	1.82	.355	.146	5.3	5.18	0.70	2410
57	0–¾ in.	5.48	.449	.247	.082	.222	2.71	.270	.201	20.1	3.70	1.19	1410
58		5.23	.434	.272	.083	.211	2.54	.282	.195	25.7	3.98	1.13	1870
59		4.84	.406	.314	.084	.196	2.33	.300	.183	20.6	4.34	1.06	2000
60		4.22	.376	.371	.089	.164	1.84	.352	.170	15.4	4.74	0.92	2530
61		3.48	.324	.438	.093	.145	1.56	.391	.153	10.3	5.19	0.80	2930
62		2.94	.279	.485	.095	.141	1.49	.402	.145	7.5	5.48	0.74	3210
63		2.55	.250	.520	.098	.132	1.35	.426	.143	5.7	5.68	0.70	3070
64		2.19	.221	.556	.101	.122	1.21	.458	.137	4.4	5.86	0.66	3600
65		1.86	.190	.582	.102	.126	1.24	.447	.134	3.4	6.03	0.63	3620
66		1.61	.166	.602	.103	.129	1.25	.444	.128	2.8	6.14	0.60	3470
67		1.36	.138	.606	.101	.155	1.53	.395	.122	2.4	6.24	0.59	2750
68	0–¾ in.	6.04	.586	.066	.097	.251	2.59	.278	.167	5.4	4.50	0.83	1660
69	Irregular	6.18	.568	.144	.092	.196	2.13	.319	.172	12.3	4.35	0.91	2180
70	Grading	6.72	.564	.065	.084	.287	3.42	.226	.233	29.4	2.70	1.35	880
71	0–1 in.	5.27	.432	.271	.082	.215	2.62	.276	.197	29.5	3.93	1.17	1570
72		4.94	.405	.313	.082	.200	2.44	.290	.186	25.0	4.26	1.09	1850
73		4.48	.385	.365	.086	.164	1.91	.344	.170	19.8	4.67	0.96	2270
74		3.89	.342	.412	.088	.157	1.78	.359	.155	14.8	5.08	0.86	2740
75		3.04	.280	.488	.092	.140	1.52	.396	.140	9.5	5.60	0.74	3320
76		2.25	.223	.558	.099	.120	1.21	.452	.129	6.0	6.02	0.64	3670
77		1.93	.193	.593	.100	.114	1.14	.467	.126	4.5	6.26	0.61	3800
78		1.58	.160	.615	.101	.124	1.23	.449	.121	3.3	6.44	0.58	3650
79		1.33	.136	.634	.102	.128	1.25	.440	.117	2.6	6.60	0.55	3350
80		1.09	.112	.652	.103	.133	1.29	.437	.109	2.1	6.72	0.51	3150
92	0–1½ in.	5.00	.410	.292	.082	.216	2.63	.275	.199	30.8	4.05	1.18	1570
93		4.73	.393	.328	.083	.196	2.36	.260	.181	25.6	4.42	1.07	1840
94		4.21	.362	.385	.086	.167	1.94	.340	.170	20.5	4.82	0.97	2160
95		3.63	.312	.440	.086	.162	1.88	.347	.153	15.5	5.27	0.86	2650
96		2.88	.265	.505	.092	.138	1.50	.400	.136	10.3	5.77	0.72	3110
97		1.87	.187	.606	.100	.107	1.07	.483	.121	5.2	6.39	0.59	3960
98		1.44	.148	.648	.103	.101	0.98	.505	.114	3.5	6.67	0.54	4170
99		1.03	.114	.660	.101	.125	1.24	.447	.108	2.5	6.88	0.51	3580
100		0.88	.089	.668	.101	.142	1.40	.416	.100	2.0	7.00	0.48	3100
103	0–1½ in.	6.38	.587	.081	.092	.240	2.61	.277	.190	10.0	4.10	1.00	1190
104	Irregular	5.77	.519	.133	.090	.258	2.87	.258	.196	15.1	3.70	1.05	980
105	Grading	5.65	.480	.211	.085	.224	2.64	.275	.184	22.3	4.10	1.05	1480
106	0–2 in.	4.76	.390	.313	.082	.215	2.62	.276	.195	29.8	4.32	1.16	1550
107		4.42	.367	.362	.083	.188	2.27	.306	.178	24.4	4.77	1.05	1900
108		4.05	.340	.408	.084	.168	2.00	.333	.161	20.0	5.14	0.93	2350
109		3.35	.295	.475	.088	.142	1.61	.382	.138	14.7	5.63	0.81	2720
110		2.61	.237	.538	.091	.134	1.47	.404	.120	9.4	6.20	0.64	3570
111		1.57	.154	.631	.098	.117	1.19	.456	.107	4.7	6.88	0.53	3860
112		1.06	.105	.664	.099	.132	1.33	.427	.094	2.7	7.26	0.46	3810
113		0.76	.075	.686	.099	.140	1.41	.413	.086	1.8	7.45	0.42	3280

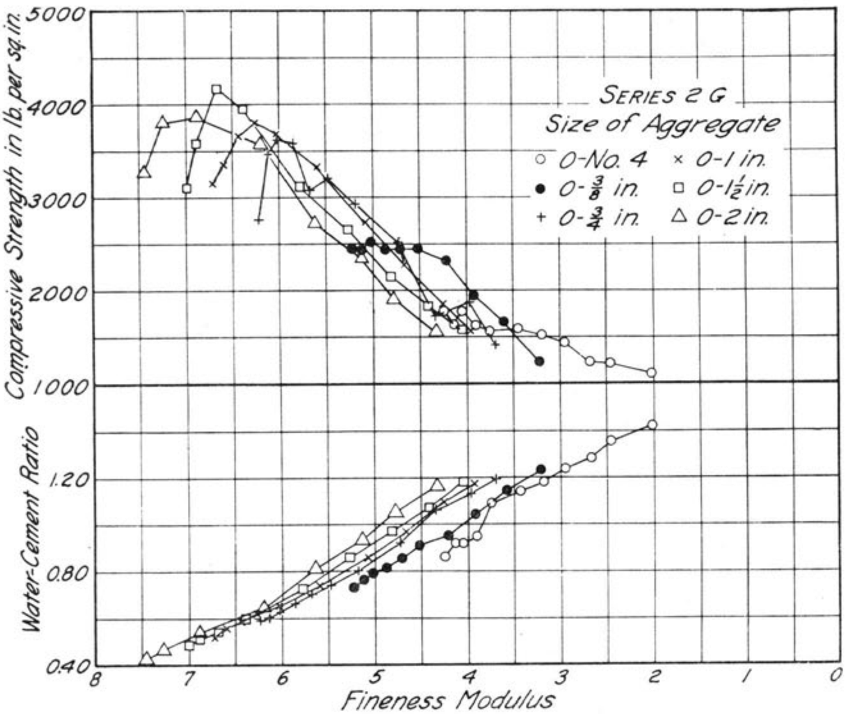

FIG. 11. FINENESS MODULUS AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 2G

to the volume of cement (bulk). The relation is close for what may be called the workable mixtures of concrete; the coarser mixtures, however, fall away. For the mortars the relation is not close. The irregular gradings are not in agreement with the regularly graded mixtures.

In Fig. 10 the tests are plotted with the surface modulus of the aggregate as abscissas and in Fig. 11 with the fineness modulus of the aggregate as abscissas, both the surface modulus and fineness modulus being used as devices to measure the gradation of fineness or coarseness of the material. The water-cement ratio (ratio of the volume of mixing water corrected for absorption by aggregate to the bulk of the cement) is also plotted in these figures. It will be noted that the maximum strength in the concretes is found at the third or fourth point and that beyond this point the strengths decrease quite regularly. In the mixtures coarser than the one giving maximum strength it was found that the mortars did not fill the voids in the coarse aggregate; all these points, however, when plotted

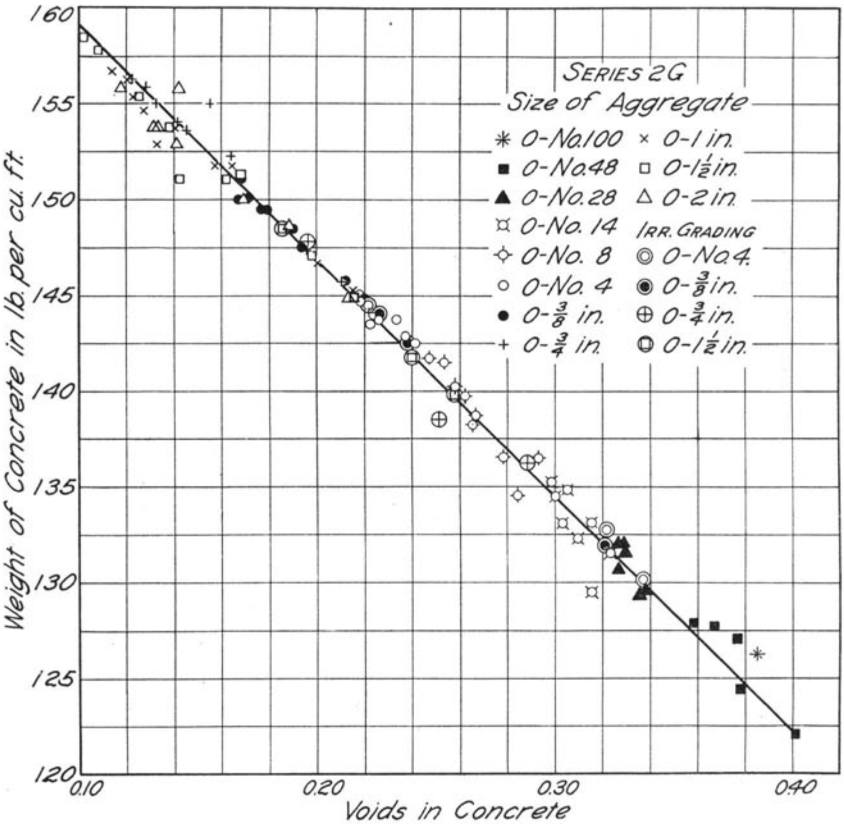

FIG. 12. VOIDS AND WEIGHT OF CONCRETE, SERIES 2G

according to voids in Fig. 5 agree closely with the general results of the series.

When the voids in concrete and weight of concrete per cubic foot were used as coördinates (see Fig. 12), a straight line relation was found, as may have been expected from the statements which have preceded. The following equation fits the weight closely:

$$W = 167\ (1 - v - c) + 193c + 55v \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where W is the weight of concrete per cu. ft., v the magnitude of the voids, and c the absolute volume of the cement. The equation assumes that 90 per cent of the voids are filled with water. It follows from this straight-line relation that for the same cement content and the same kind of aggregate the strength of the concrete is a fairly close function of its weight.

IV. Mortar Voids and Water Content

18. *Mortar Voids and Water Content.*—A distinctive feature of the method of proportioning concrete by means of the voids-cement-strength relation, as used in this bulletin, is the making of mortar-voids tests on the given fine aggregate. Tests are first made to determine the amount or magnitude of the voids for a mortar made up with a given proportion of cement. The operation is then repeated for a variety of proportions.

For each mixture the point of minimum volume of the mortar when made up with varying quantities of mixing water is determined. Tests of mortar show that, for any given proportion of cement to fine aggregate, the voids in the mortar vary with changes in the volume of mixing water used. The magnitude of the voids and its change with additions of water will depend upon the gradation of the particles of fine aggregate, each aggregate having its own individuality. Fig. 13 shows the voids in mortars made with four typical sands—very fine, fine, medium, and very coarse—for a ratio of fine aggregate to cement by absolute volumes, $\frac{a}{c}$, equal to 2. The

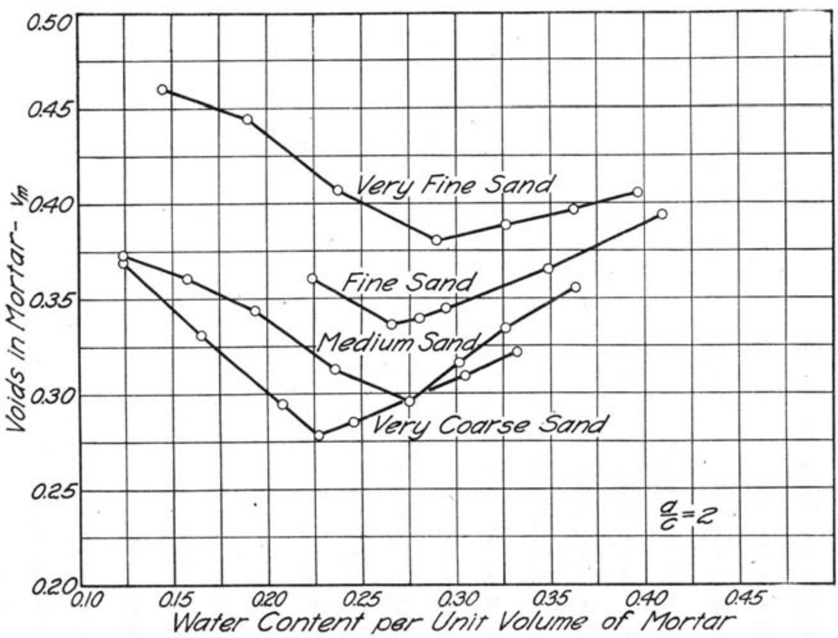

Fig. 13. Mortar Voids and Water Content for $\frac{a}{c} = 2$

water content is given in terms of the volume of the mortar; for example, 0.25 cu. ft. of water per cubic foot of mortar. For each of these sands, with additions of water the voids decrease to a minimum and then increase. The point of minimum voids (minimum volume of mortar) is fairly definite and readily fixed; it is here termed the point of basic mortar voids, and the amount of water per unit of volume of mortar that gives minimum voids is termed the basic water content. The water content in wetter mixtures may be expressed in terms of the basic water content; a 1.30 relative water content would mean that the amount of water used per unit of volume of the resulting mortar is 1.30 times that water content per unit of volume of mortar which gives the minimum volume of mortar, the swelling of the mortar through the addition of water thus being taken account of. It will be noted in Fig. 13 that the fine sand gives higher mortar voids than the coarse and that the basic water content also differs. The coarse sand used was well graded from fine to coarse. Mortars at basic water content are fairly near to what is usually considered normal consistency, or a little dryer.

By making the mortar-voids tests for several mixtures (several values of $\frac{a}{c}$) enough points may be determined to give the relation between voids in mortar and values of $\frac{a}{c}$ for the given fine aggregate. The value of the voids at minimum mortar volume (basic water content) was taken from Fig. 13 and from similar diagrams for other mixtures, and several such points were used in Fig. 14 for each of the four sands, making what is termed the characteristic mortar-voids curve at basic water content. In Fig. 15 characteristic mortar-voids curves for the coarse and the fine sand are given at three water contents, the data having been taken from Fig. 13 and others like it.

Several characteristics of such curves may be noted. For a given water content the curves for different sands converge to a single point at the left, which represents the voids in neat cement at the given consistency. Two sands may not be expected to have the point of least voids at the same mixture. The characteristic curve of a coarse well-graded sand generally shows a rapid increase in voids as the richness of the mixture is increased from $\frac{a}{c} = 2$; this property of a coarse sand will be found to tend to limit the advantages of the coarse sand in rich concrete mixtures or where a well-graded coarse aggregate is available. The relative increase

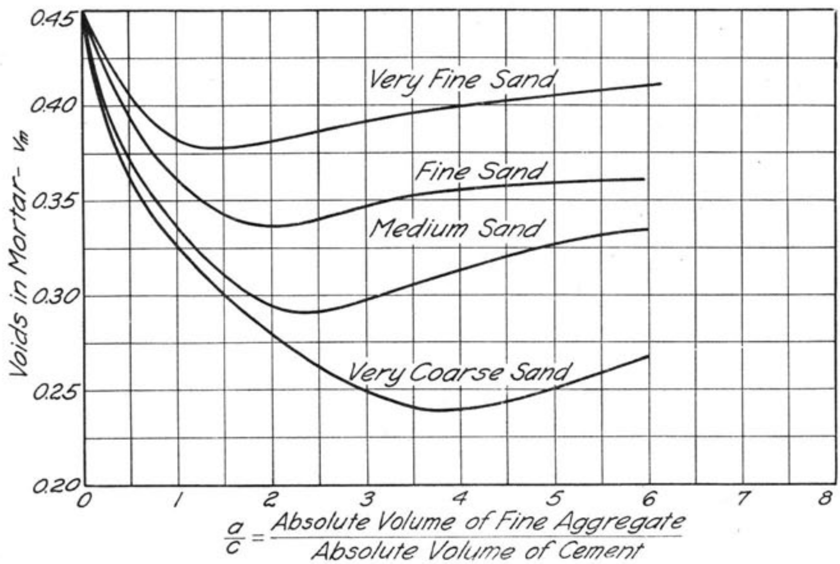

FIG. 14. CHARACTERISTIC MORTAR-VOIDS CURVES AT BASIC WATER CONTENT

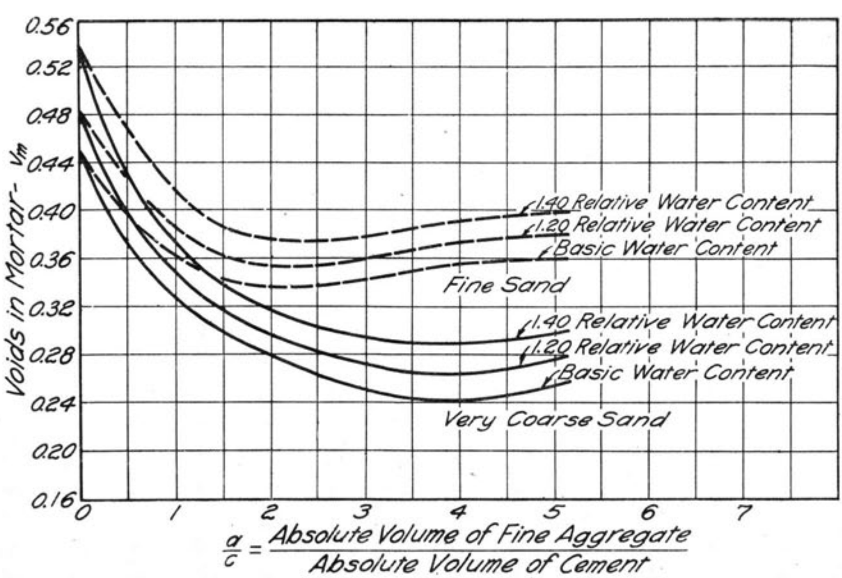

FIG. 15. CHARACTERISTIC MORTAR-VOIDS CURVES AT THREE WATER CONTENTS

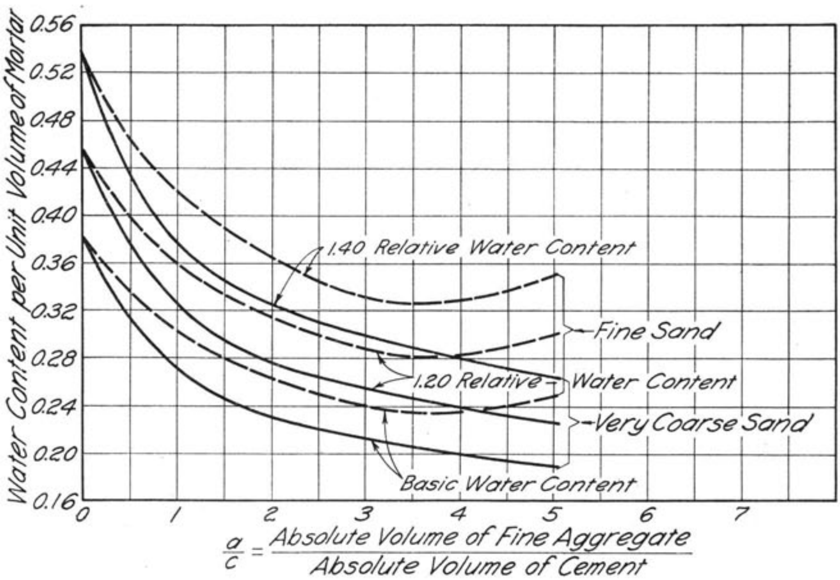

FIG. 16. WATER CONTENT OF MORTARS

in voids with the addition of more water is less for a fine sand than for a coarse sand.

For a given fine aggregate, the water content required for a unit of volume of mortar may be taken for a number of mixtures from curves such as those shown in Fig. 13 and then plotted to make a diagram such as that in Fig. 16. The three curves for each sand indicate the water content required for a unit of volume of mortar for the three conditions, 1.00 or basic water content, 1.20 relative water content, and 1.40 relative water content. This water-content curve may be constructed at the same time as the characteristic mortar curve of Fig. 15. The values take into account the bulking of the mortar with the addition of water. The curves in Fig. 16 bring out the variation in water content for different mixtures and different sands.

It is evident that the discrepancy between the value of the water content per unit volume of mortar for a given mixture and the value of the voids in the mortar is accounted for by the presence of air. The proportion of the voids in mortar that is filled with water will vary with the fineness and gradation of the sand, the richness of the mixture and the relative water content.

19. *Technique of the Mortar-Voids Test.*—The technique of the mortar-voids test used in the investigation and recommended

for future use until developments show in what way modifications should be made is as follows:

A representative sample of the fine aggregate in amount sufficient for ten or twelve batches of mortar and for the specific gravity and absorption test should be provided. For the purposes of the mortar-voids test, values of $\frac{a}{c}$ (the ratio of aggregate to cement by absolute volume) of 0, 1, 2, $3\frac{1}{2}$, and 5 will usually furnish adequate information. As a rule fifteen pounds of the aggregate will be sufficient. It should preferably have the same known moisture content that it will have when used for making concrete, or if dry, allowance may be made for the moisture present when the concrete test specimen is made. A batch of sand and cement of a given proportion will be mixed with an amount of water that will give a dry mixture and the voids determined by careful weighing and measuring of the materials. A small amount of water is added and the operation repeated. Further amounts of water are added, the mixtures passing from dry to very wet. From these data, curves such as those of Fig. 13 may be plotted.

The apparatus needed for making the mortar-voids test consists of a mold or container, accurate balance or scales, burette for measuring water, mixing pan, small trowel, and tamping rod. A 2 by 4-in. cylindrical mold, made from seamless drawn tubing and provided with a solid bottom, has proved satisfactory for mortar tests. Comparative tests have shown no appreciable difference in accuracy between results with this mold and with a larger one. The 2 by 4-in. mold can well be used with the light laboratory scales and other apparatus, and the following remarks refer to the use of this size of measuring unit. When another size of mold is used proportional quantities may be substituted that will produce a similar degree of accuracy in the results. The balance or scales should give weights accurate to the nearest gram. The burette for measuring water should have a capacity of 200 cc. and be graduated in 1-cc. divisions. The volume of the mold (approximately 200 cc.) may be found more accurately by use of the burette than by measuring with calipers and scale. The mold should be weighed carefully, or its weight may be balanced by use of a counterpoise.

The absolute volumes of aggregates and cement in a batch may be denoted by a' and c', respectively. In measuring out the materials for a batch of mortar for a given value of $\frac{a}{c}$ it is convenient to use an even value of $a' + c'$ which is about equal to the volume of the mold. A value of 200 cc. provides some excess mortar for the 2 by 4-in. mold, and also simplifies calculations. The values of a' and c' being known, the weights for use are found by multiplying a' and c' by the respective specific gravities of the two materials, and by 62.4 if English units of weight and measure are used.

After the sand and cement are weighed out, they should be mixed dry and the water added. The amount of water should be slightly less than the amount required for minimum volume of mortar, which may vary from 9 per cent of the weight of the dry materials for coarse sands and lean mixtures to 24 per cent for fine sands and rich mixtures. The mortar should be mixed thoroughly with the trowel and placed in the mold in layers of about 1 in., each layer being tamped firmly but without too much pressure. A flat-ended wooden tamping rod, about ¾ in. square, is most satisfactory, since a pointed rod leaves holes in dry mortar instead of compacting it. When the mold has been filled, the top is struck off neatly with the trowel, the outside of the mold wiped clean, and the known volume of mortar is weighed. A check reading is taken by emptying the material back into the pan, filling the mold and weighing again.

With the same batch of mortar, another increment of water is added, mixed thoroughly, and the weight of mortar in the mold is determined for this water content. The process is repeated with further increments of water, taking several values near the point of minimum voids and increasing the size of increment after this point is passed.

From the weight and volume of the mortar in the mold and the weights of the ingredients, the volume of the batch of mortar is calculated by simple proportion, it being assumed that the mortar in the mold is representative of the batch. The ratio of the absolute volume of sand and cement to the volume of the batch is equal to the density of the mortar, or to the complement of the voids. A typical form of data sheet from a mortar-voids test is given in Table 3. The computations for the first line of the table are indicated in the following illustration.

From the conditions that $a' + c' = 200$ cc. and $\frac{a}{c} = 3.5$, it follows that $c' = \frac{200}{4.5}$ and $a' = 3.5c'$. The volume of the batch is found as follows: $\frac{627}{412} \times 204 = 310$ cc. The density of the mortar is $\frac{200}{310} = 0.644$, and $v_m = 0.356$.

To allow for the loss of water due to evaporation and absorption during the repetition of void determinations, which may

TABLE 3

TYPICAL MORTAR-VOIDS TEST

Sand No. 6A
$\frac{a}{c} = 3.5$
$a' + c' = 200$ cc.

Specific gravity of sand = 2.76
Specific gravity of cement = 3.10
Volume of mold = 204 cc.

Sand		Cement		Weight of Water	Total Weight of Materials	Weight of Mortar in Mold	Volume of Batch	Density of Mortar	v_m
Weight g.	a' cc.	Weight g.	c' cc.	g.	g.	g.	cc.		
429.0	155.5	138.0	44.5	60	627	412	310	.644	.356
				70	637	442	294	.680	.320
				75	642	455	288	.695	.305
				80	647	458	289	.693	.307
				90	657	456	294	.681	.319
				105	672	448	306	.653	.347
				125	692	436	324	.617	.383
				150	717	414	353	.567	.433

last twenty to thirty minutes, two batches may be used, the first carried just beyond the point of minimum voids and the second started a little back of this point. The overlapping of the two curves plotted from these determinations in Fig. 17 shows the effect of the lost water upon the voids near the point of minimum volume. The second determination also gives a more accurate value of the water content at minimum volume. The amount of water lost will depend upon the time of mixing, the temperature, and the humidity; a correction could be found for this loss, but the method given above is simpler. It may be noted that an accurate determination of the basic water content is more difficult than the voids determination at this point of minimum voids.

The mortar-voids test may be performed by ordinary concrete workers without preliminary experience; however, a little experience in handling the mortar will generally secure increased uniformity and reliability of results. Complete data on one fine aggregate may usually be obtained and computed in 3 or 4 hours by one operator and one recorder. Skill is required principally in tamping the mortar into the mold, as the usefulness of the mortar-voids test depends upon producing about the same degree of compactness in the mortar as may be found in the concrete test specimens containing the same proportions of sand, cement, and water.

20. *Description of Fine Aggregates Used in Series 211.*—Twenty-two of the fine aggregates used in Series 211 were received from

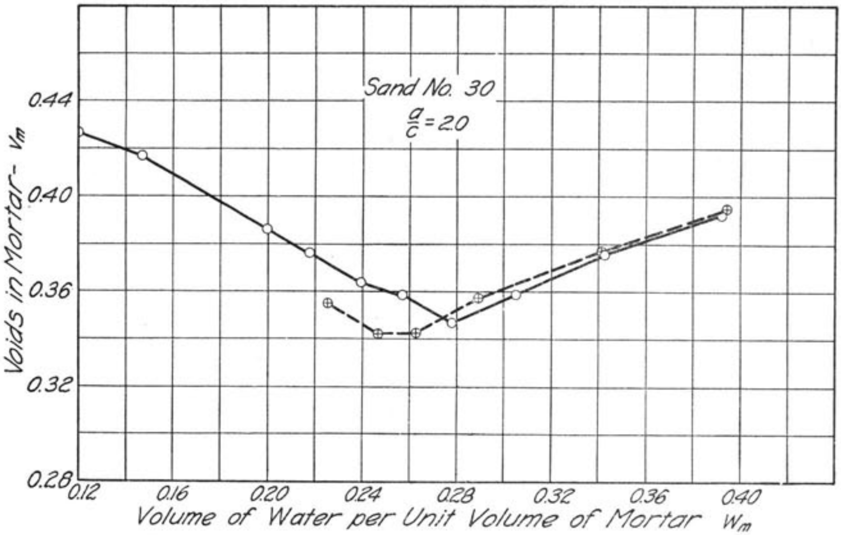

FIG. 17. DIAGRAM SHOWING LOSS OF WATER IN MORTAR-VOIDS TEST

the Structural Materials Research Laboratory of Lewis Institute and were taken from materials collected there for the investigation of concrete. A fine bank sand from Greenup, Illinois, was received from the Illinois Division of Highways. The other fine aggregates were Attica sands. The various fine aggregates may be described as follows:

No. 0.—Standard Ottawa sand, specified by American Society for Testing Materials for making comparative tests of cement. Screened to sizes between the No. 20 and No. 30 sieves. Grains nearly spherical, composed of a dull white translucent quartz. Obtained from Ottawa, Illinois.

No. 1.—Fine sand from Joliet, Illinois, obtained by screening through a No. 28 sieve. Light yellow in color. Principally irregular grains of limestone, with the balance mainly quartz. Slight amount of clay and traces of mica.

No. 2.—Trap screenings, with coarser particles predominating. From Trap Rock, Pennsylvania. Very dark gray with porphyritic structure. Particles flat, irregular, and elongated. Had highest specific gravity of the aggregates used.

No. 3.—Bank sand from Algonquin, Illinois. Light yellow in color. Consisted largely of quartz, with considerable calcareous material and some conglomerate particles. Had a predominance of fine particles.

No. 4.—Kaw River sand from Kansas City, Missouri. Composed mostly of quartz with some particles of granite. Bright yellow in color, with many translucent grains which were quite uniformly round.

No. 5.—Very fine lake sand, light yellow in color, with white quartz grains predominant. Particles passing the No. 100 sieve were black. Obtained from Chicago, Illinois.

No. 6.—Limestone screenings from Milwaukee, Wisconsin. Had a large proportion of finely powdered particles. Larger particles were elongated and irregular. Dull white in color.

No. 7.—Chats from Joplin, Missouri. Particles had sharp edges, were irregular, elongated, and smooth. Color, gray for larger particles, white for finer ones. Specific gravity, 2.60, is among the lowest for the aggregates used.

No. 8.—Banding sand, apparently a fine pure white quartz sand. Probably from Ottawa, Illinois. Grains spherical in shape. Apparently no absorption of water in absorption test.

No. 9.—Limestone screenings from Chicago, Illinois. Nearly white in color. Particles angular and of uniform size.

No. 10.—Lake Michigan dune sand, consisting mainly of quartz, with some particles of limestone, granite and a fine black substance. Light yellow in color.

No. 11.—Crushed slag from Chicago, Illinois. Gray in color. Particles porous in texture, irregular in shape and generally flat.

No. 12.—Crushed granite from Berlin, Wisconsin. Particles dark red in color, angular, and rough. The finely powdered particles appeared nearly white.

No. 13.—Platte River "gravel" from Fremont, Nebraska. Larger particles were angular pebbles of granite; finer ones well rounded grains of quartz.

No. 14.—"Glass" sand from Mapleton, Pennsylvania. Fine particles of white and light yellow quartz or similar material. Particles round, but apparently rough on the surface.

No. 15.—Crushed slag, from Lorain, Ohio. Light gray in color. Particles angular and porous.

No. 16.—Granulated slag from Buffington, Indiana. Glassy brown in color. Grains somewhat rounded and porous in texture. Specific gravity, 2.50, was the lowest among the aggregates used.

No. 17.—Fine sea sand from Atlantic City, New Jersey. Grayish white in color. Consisted largely of quartz grains, together with a fine black powder passing No. 100 sieve.

No. 18.—Conglomerate sand from San Diego, California. Grains slightly shell-like in appearance, irregular in shape, and having small surface cavities. Finer particles predominant, composed of red, black and green material probably quartz and basalt. Apparently a disintegration product from granite. Also some mica present.

No. 19.—Bank sand from Janesville, Wisconsin. Composed of basalt and impure quartz. Coarser particles smooth and irregular. Finer particles more uniform in size and shape.

No. 20.—Washed sand from Elgin, Illinois. Dull yellow in color, with well rounded grains.

No. 21.—Sand from Medicine Hat, Canada. Principally silica sand, with some mica and basaltic material. Particles generally irregular and sharp. Light yellow in color, with many translucent grains.

No. 22.—Same as No. 20, except that it had not been washed, and contained a considerable amount of clay.

No. 23.—Fine quartz sand from Greenup, Illinois. Grains slightly angular and light yellow in color.

No. 24.—Sand from Attica, Indiana. Particles smooth and somewhat irregular. Mainly of calcareous material, dull yellow in color, with some quartz and mineral fragments. This sand, artificially graded, was used in Series 2G.

Nos. 31, 32 and *36.*—Artificial gradations of Sand No. 24.

All the fine aggregates were used with one coarse aggregate, a washed gravel from Attica, Indiana. This gravel was also used in Series 2G. The artificial gradation of the gravel used in all the concretes in Series 211 was 60 per cent between the ⅜- and ¾-in. screens, and 40 per cent between the ¾- and 1-in. screens.

The nature of the fine and coarse aggregates, the location or source of the supply, and the sieve analysis of each are given in Table 4. Table 5 gives the specific gravity, weight, voids, absorption, fineness modulus, and surface modulus of these fine aggregates.

21. *Data and Discussion.*—The characteristic mortar-voids curves given in Figs. 18 and 19 are representative of the data of the tests. Fine aggregate No. 23 is a fine sand, finer than would generally be considered acceptable for ordinary concrete work. No. 24 is a coarse well-graded sand and is considered an excellent sand for making concrete. At the top of the figure are given the mortar voids for several values of $\frac{a}{c}$ with water content ranging from dry to wet. From these diagrams values are taken for the mortar-voids curves shown on the figures—at basic water content, 1.20 relative water content, and 1.40 relative water content. The water content of the mortar, w_m, and the absolute volume of cement per unit of volume of mortar, c_m, are also indicated.

TABLE 4

SIEVE ANALYSES OF FINE AGGREGATES, SERIES 211

No.	Origin	Description	Per Cent by Weight Passing Given Sieve						
			100	48	28	14	8	4	⅜
0	Ottawa, Ill.	Standard Ottawa Sand	..	0	1	100	....	....	....
1	Joliet, Ill.	Sand	34	75	100	....	....	....	....
2	Trap Rock, Pa.	Trap Screenings	11	16	23	33	52	91	100
3	Algonquin, Ill.	Sand	5	19	46	67	84	100	....
4	Kansas City, Mo.	Kaw River Sand	0	3	40	75	91	98	100
5	Chicago, Ill.	Lake Sand	1	60	99	100	....	....	....
6	Milwaukee, Wis.	Limestone Screenings	23	37	52	67	84	97	100
7	Joplin, Mo.	Chats	3	9	20	32	57	100	....
8	Ottawa, Ill.	Banding Sand	43	99	100	....	....	....	....
9	Chicago, Ill.	Limestone Screenings	3	3	3	3	6	100	....
10	Gary, Ind.	Dune Sand	6	95	100	....	....	....	....
11	Chicago, Ill.	Slag	5	9	16	25	45	89	100
12	Berlin, Wis.	Crushed Granite	15	21	28	39	61	96	100
13	Fremont, Neb.	Platte River "Gravel"	3	19	42	57	71	88	98
14	Mapleton, Pa.	Glass Sand	3	32	93	99	100	....	....
15	Lorain, Ohio	Crushed Slag	9	15	24	34	55	93	100
16	Buffington, Ind.	Granulated Slag	7	15	37	76	98	99	100
17	Atlantic City, N. J.	Sea Sand	9	95	100	....	....	....	....
18	San Diego, Cal.	Sand	1	7	24	48	75	94	100
19	Janesville, Wis.	Sand	1	14	57	73	82	90	99
20	Elgin, Ill.	Washed Sand	2	12	51	71	86	98	100
21	Medicine Hat, Can.	Sand	1	5	32	61	79	92	99
22	Elgin, Ill.	No. 20 Unwashed	1	7	37	57	72	95	100
23	Greenup, Ill.	Sand	6	48	99	100	....	....	....
24	Attica, Ind.	Natural Sand	4	10	25	51	78	97	100
31	Attica, Ind.	No. 24, Artificial Grading	3	14	29	55	78	99	100
32	Attica, Ind.	No. 24, Artificial Grading	7	43	78	100	....	...	....
36	Attica, Ind.	No. 24, Artificial Grading	0	0	3	63	88	100	....
	Size of Sieve Opening—Inches		.0058	.0116	.0232	.046	.093	.186	.372

The characteristic mortar-voids curves at basic water content for the fine aggregates of Series 211 are given in Fig. 20. It will be noted that the various fine aggregates show marked individuality. The point of least voids is found at different values of $\frac{a}{c}$, ranging from 1 to 5. Some curves like No. 24 show small voids between $\frac{a}{c} = 2$ and $\frac{a}{c} = 5$. In others, like No. 15, the voids increase rapidly on each side of the minimum. Sand No. 10 (a very fine sand) has its minimum well to the left. Sands No. 0 and No. 36, which are free from fine particles, have their minimum at $\frac{a}{c} = 2$. Sand No. 13, which has a considerable amount of coarse particles, has the minimum well to the right. As already pointed out, all the curves converge

TABLE 5

PHYSICAL PROPERTIES OF AGGREGATES, SERIES 211

No.	Specific Gravity	Weight lb. per cu. ft.	Voids	Fineness Modulus	Surface Modulus	Per Cent Absorption by Weight
			Fine Aggregate			
0	2.65	105.4	.363	2.99	12.6	0.05
1	2.76	107.0	.380	0.91	60.7	0.16
2	2.90	121.5	.329	3.74	19.1	0.10
3	2.71	116.6	.309	2.79	22.6	0.36
4	2.63	109.5	.333	2.91	16.5	0.16
5	2.64	103.0	.375	1.44	39.2	0.20
6	2.75	123.4	.280	2.40	37.0	0.40
7	2.60	101.6	.373	3.79	13.2	0.10
8	2.65	106.1	.358	0.58	71.0	0.00
9	2.77	96.9	.440	4.82	5.9	0.16
10	2.67	101.1	.393	0.99	51.6	0.20
11	2.68	103.7	.379	4.11	12.8	0.56
12	2.61	114.1	.300	3.40	23.5	0.20
13	2.63	121.0	.263	3.23	19.9	0.10
14	2.62	95.1	.419	1.73	33.3	0.40
15	2.59	91.3	.435	3.70	17.9	0.94
16	2.50	80.5	.483	2.69	22.6	0.34
17	2.73	107.4	.370	0.95	53.4	0.25
18	2.65	105.6	.361	3.50	13.7	0.52
19	2.65	116.1	.298	2.84	21.4	0.12
20	2.69	114.0	.321	2.81	20.2	0.52
21	2.64	112.5	.317	3.30	15.0	0.32
22	2.69	116.0	.309	3.30	15.7	0.36
23	2.63	101.5	.382	1.47	39.8	0.65
24	2.67	112.1	.332	3.33	16.6	0.63
31	2.67	116.7	.301	3.22	17.6	0.63
32	2.67	108.5	.349	1.73	36.4	0.63
36	2.67	103.7	.378	3.46	10.2	0.63
			Coarse Aggregate			
	2.69	102.5	.394	7.40	0.62	1.00

to 0.45 at $\frac{a}{c} = 0$, this being the magnitude of the voids in the neat cement paste for the cement used.

In Figs. 21 and 22 are shown the characteristic mortar-voids curves for the same fine aggregates at 1.20 and 1.40 relative water content; that is, for the water content 1.20 and 1.40 times those used in the mixtures denoted as being at basic water content, which was taken as that amount of mixing water that produced the minimum volume of the mortar. The curves differ somewhat in shape from those at basic water content, though retaining in general the same representative characteristics.

In Fig. 23 are given curves from which may be obtained the basic water content for any value of $\frac{a}{c}$; that is, the amount of

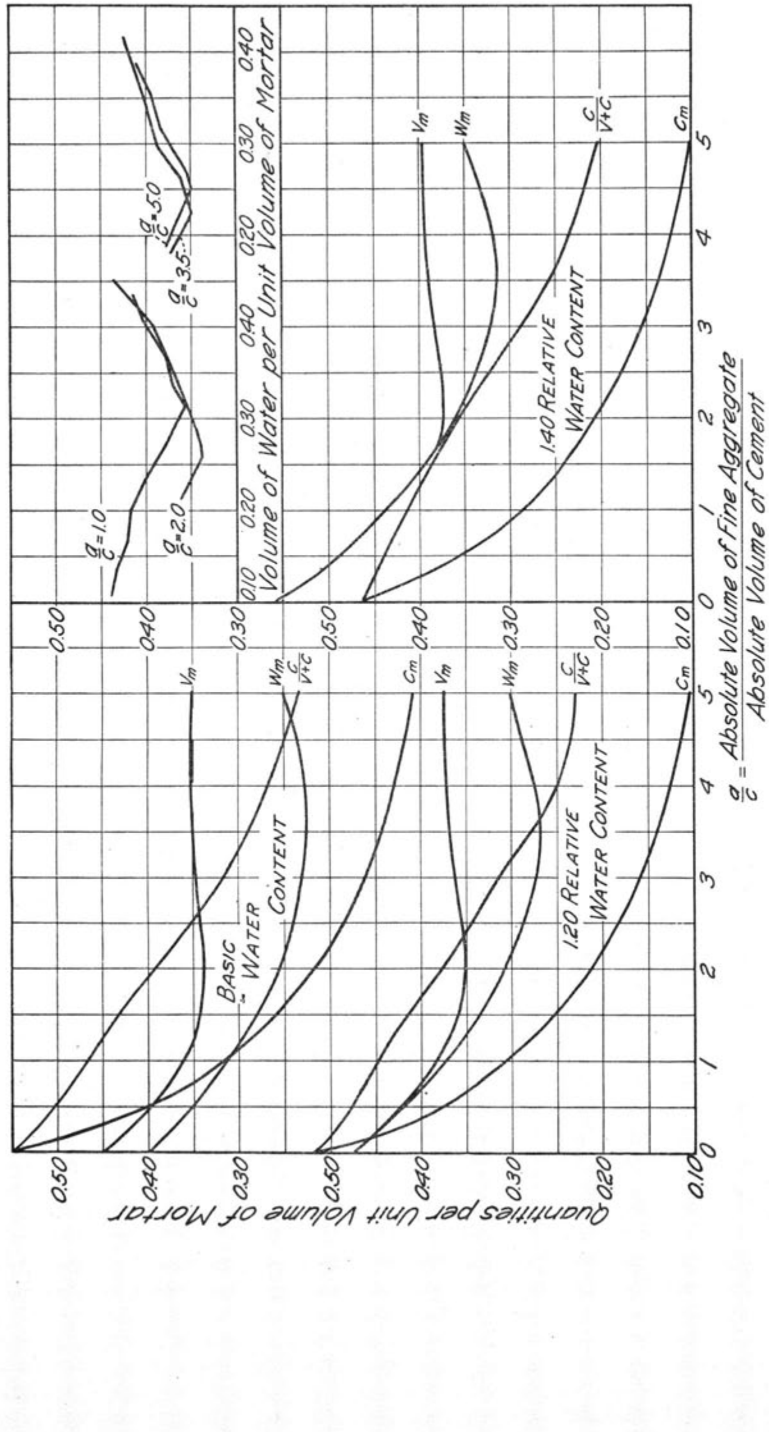

FIG. 18. CHARACTERISTIC MORTAR-VOIDS CURVES FOR SAND NO. 23

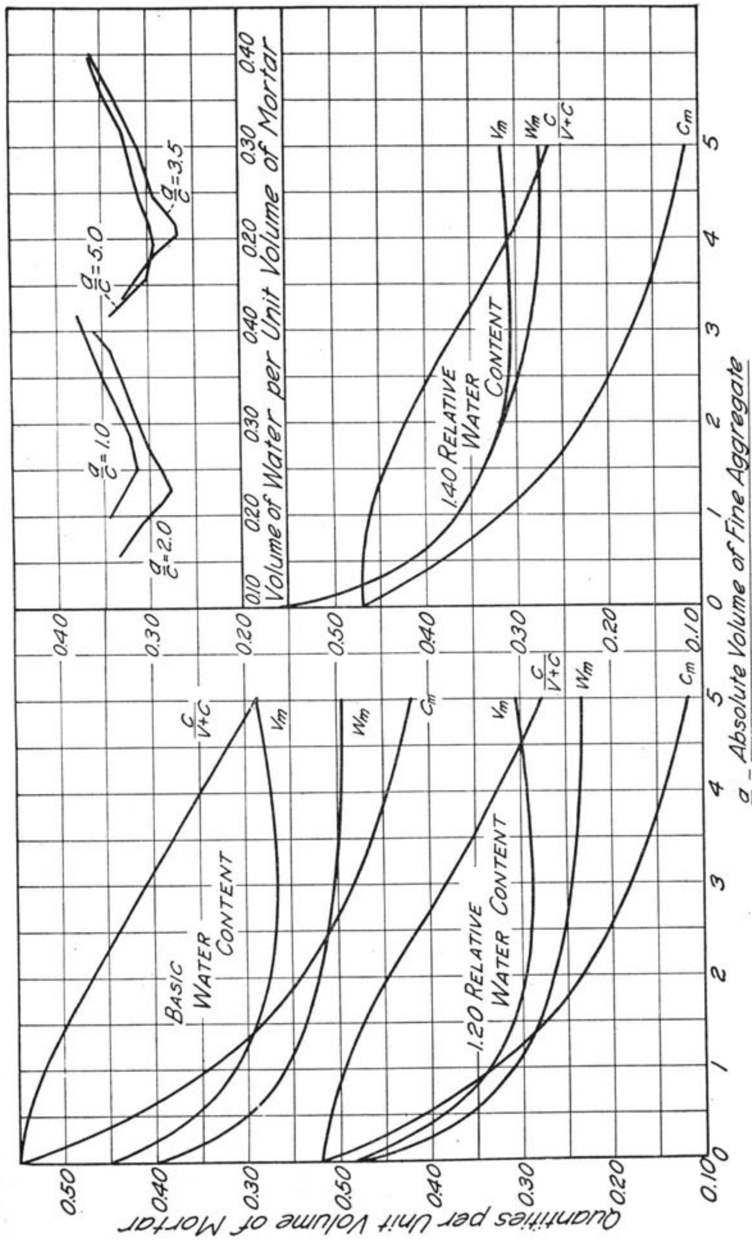

FIG. 19. CHARACTERISTIC MORTAR-VOIDS CURVES FOR SAND NO. 24

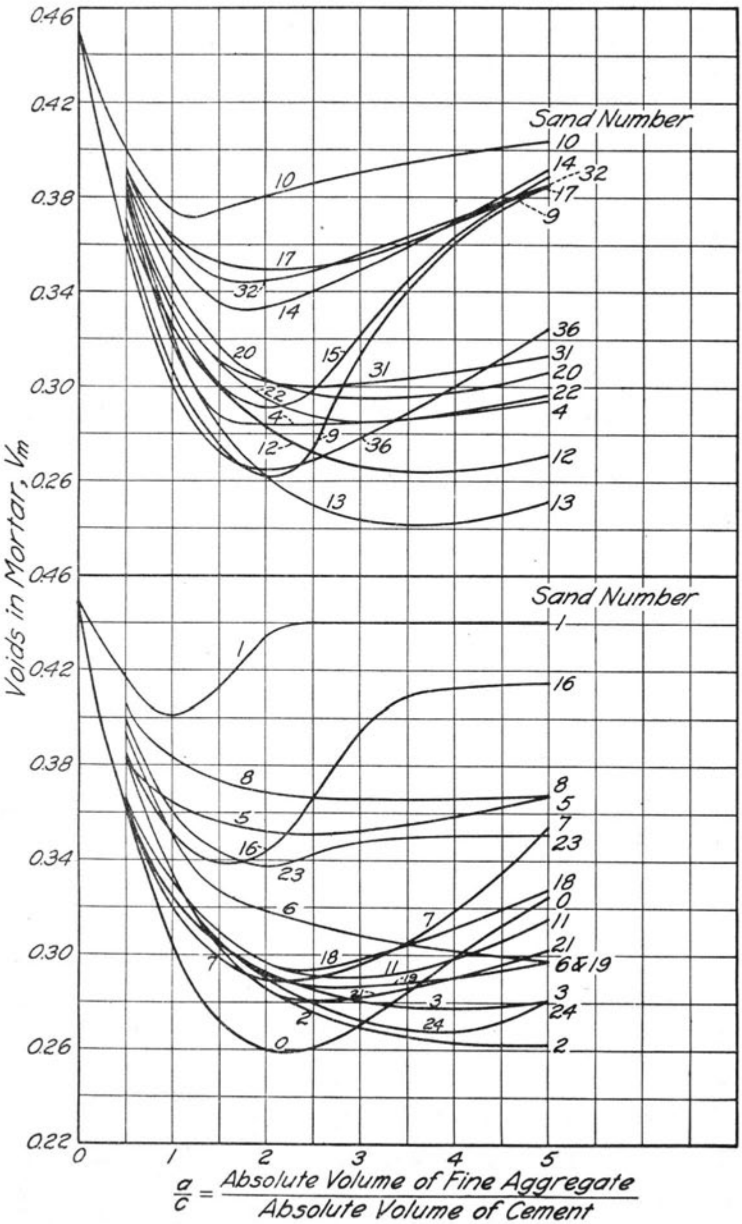

FIG. 20. CHARACTERISTIC MORTAR-VOIDS CURVES, BASIC WATER CONTENT

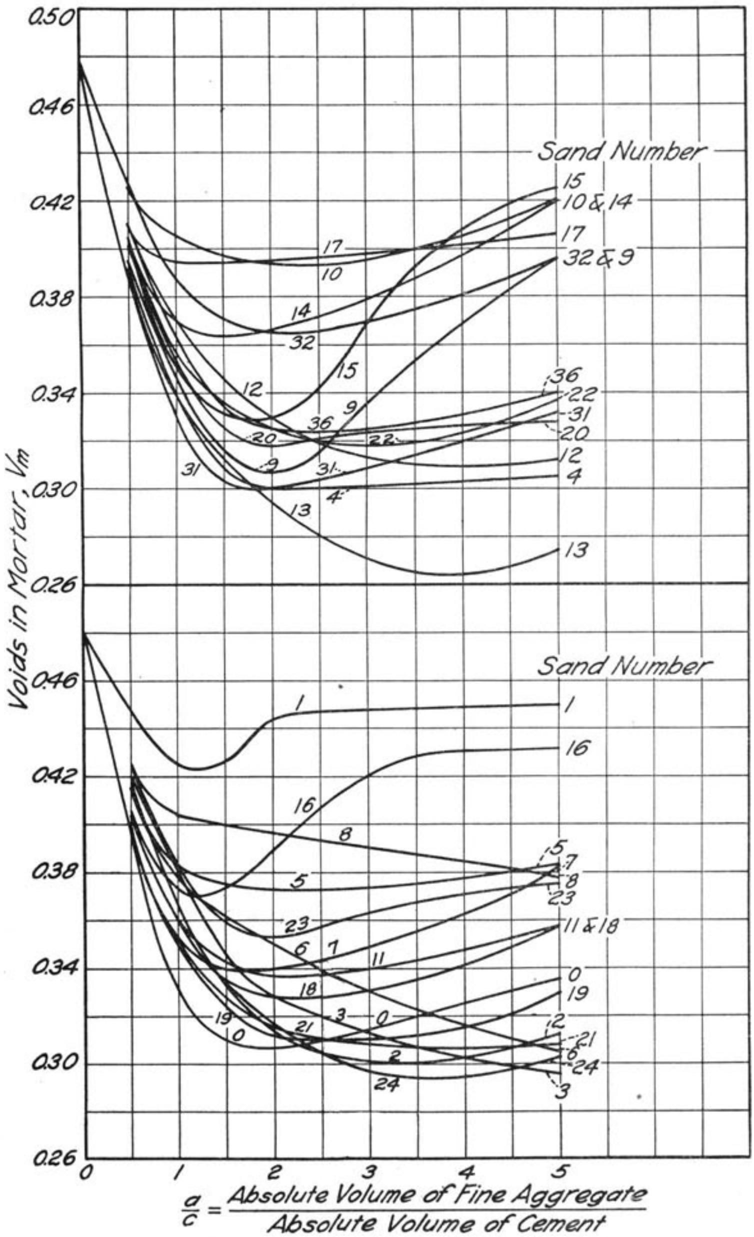

FIG. 21. CHARACTERISTIC MORTAR-VOIDS CURVES, 1.20 RELATIVE WATER CONTENT

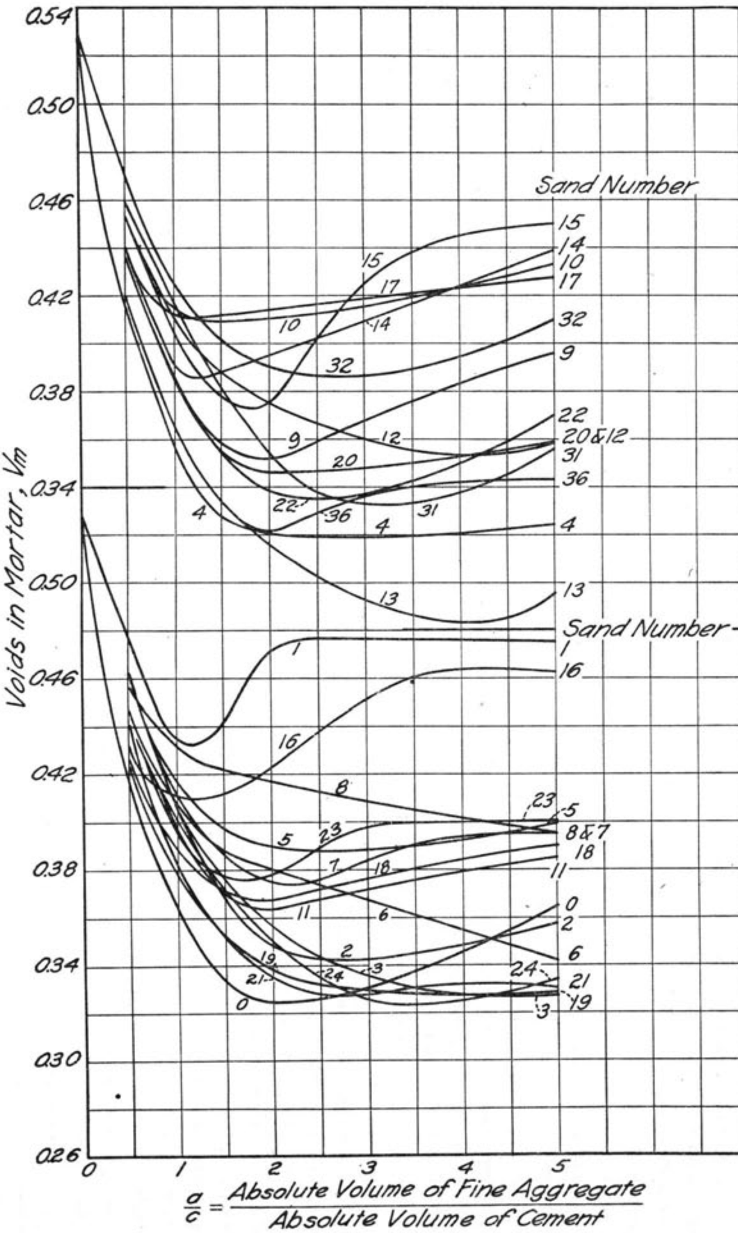

FIG. 22. CHARACTERISTIC MORTAR-VOIDS CURVES, 1.40 RELATIVE WATER CONTENT

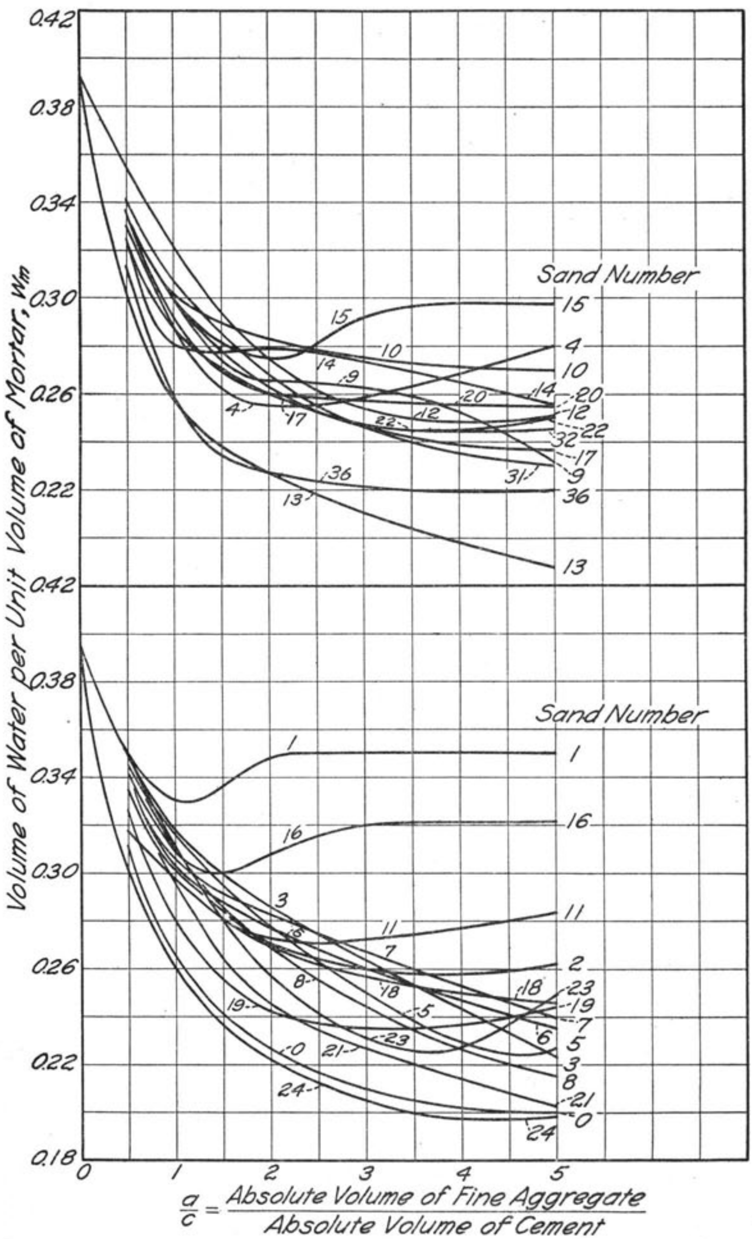

FIG. 23. WATER CONTENT OF MORTARS AT BASIC WATER CONTENT

mixing water per unit of volume of mortar required to produce a mortar with minimum voids. The value given does not include the water absorbed by the aggregate or lost through evaporation or otherwise; the proper allowance for this was made according to the nature of the aggregate used. For other relative water contents, values taken from the diagram may be multiplied by the relative water content, 1.20, 1.30, etc.

Fig. 24 gives both the voids in the mortar and the water content at minimum volume (basic water content) for a number of the fine aggregates. It is apparent that in general the voids of the mortar are not entirely filled with water, the proportion filled varying with the nature or gradation of the fine aggregate and the richness of the mixture and ranging from 60 to nearly 100 per cent; 75 to 90 per cent are common values. It will be noted that, for Nos. 2, 4, and 12 (all well-graded, rather coarse materials—trap screenings, sand, and granite screenings), the voids are quite well filled with water through the whole range of $\frac{a}{c}$, and the air voids are correspondingly small. For Nos. 13, 20, and 24 (all well-graded sands) the voids are fairly well filled with water, say 75 to 85 per cent. For Nos. 10, 17, and 23 (all fine sands) the voids are poorly filled, say 60 to 70 per cent, except for rich mixtures. It will also be noted that Nos. 0, 9, and 15, in which the particles are uniform in size, are peculiar in that the proportions of unfilled voids are small for the rich mixes and increase materially throughout the range of leaner mixtures.

It appears that the leaner mixtures in general have more air voids than mixtures near the one that gives the lowest percentage of voids; richer mixtures than the latter tend toward the value for neat cement paste, about 15 per cent of the total voids. It seems that the nature of the surface of the particles, as well as the gradation, affects the proportion of air voids. It should be noted that these amounts of air voids were found in mortars that were thoroughly mixed.

When a larger amount of mixing water than that giving minimum mortar volume is used, the proportion of air voids is found to be smaller. The change is dependent upon the richness of the mixture as well as the nature and gradation of the fine aggregate. For example, for No. 31 in the lean mixture ($c = 0.06$) at basic water content, the air voids were about 25 per cent of the total voids, about 15 per cent at a relative water content of 1.20, and almost nothing at 1.50. For a medium mixture ($c = 0.10$), the air voids at basic water content were about 20 per cent of the total voids,

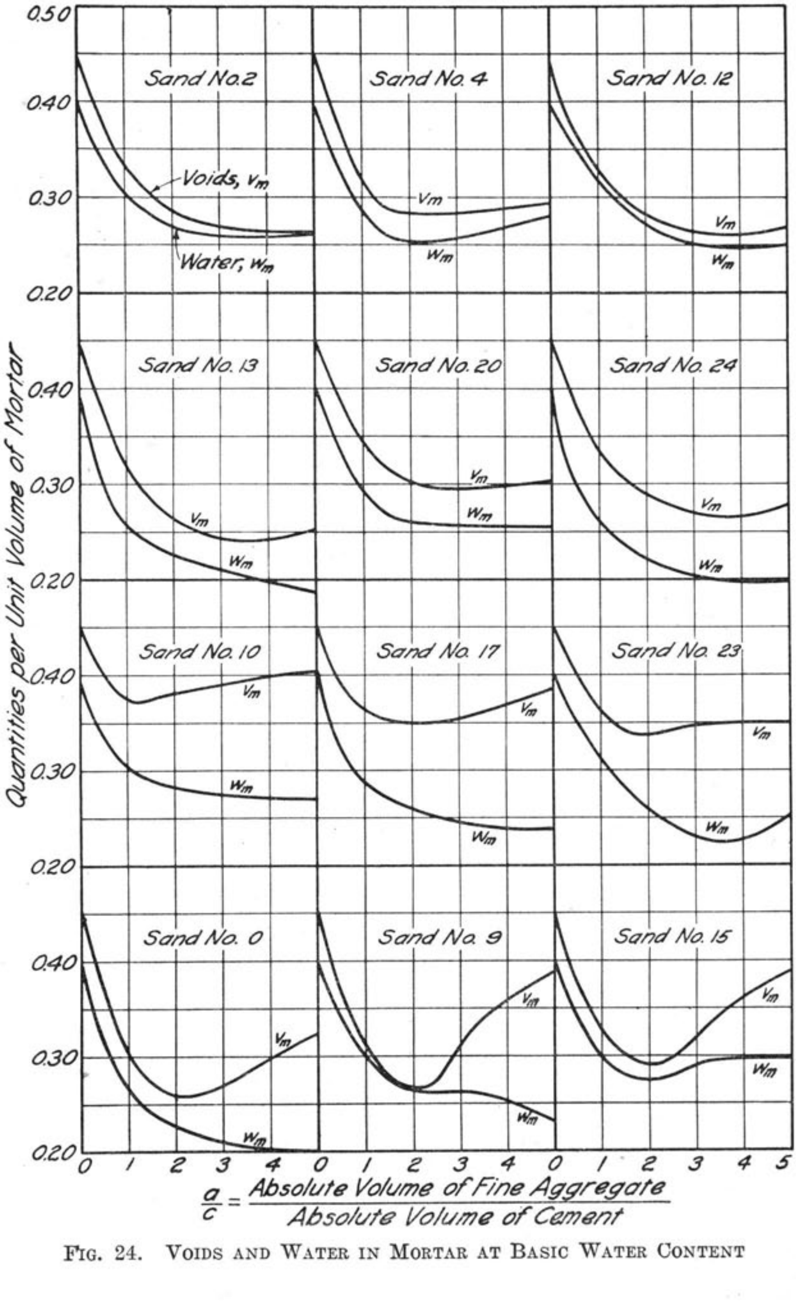

FIG. 24. VOIDS AND WATER IN MORTAR AT BASIC WATER CONTENT

10 per cent at a relative water content of 1.20, and almost nothing at 1.50. For a rich mixture ($c = 0.15$), the air voids were about 15 per cent at basic water content and perhaps 5 per cent at a relative water content of 1.25. For No. 32, a finer sand, but well graded, the proportion of air voids at basic water content to total voids was about 5 per cent more for all three mixtures than was the proportion for No. 31; the air voids vanished at relative water contents somewhat higher than was the case with the coarser sand. For still finer sands the air voids were even greater and required the addition of more water before vanishing. On the other hand such materials as Nos. 2, 4, and 12 were practically filled with water at basic water content. For the medium and finer materials the addition of 30 or 40 per cent water to the basic mixture may be accompanied by an increase in voids in the mortar of only 10 to 20 per cent of their value, though some of the sands gave a larger increase than this.

For the wetter mixtures, then, the water content is a fairly good measure of the voids in the mortar.

V. Concretes at Basic Water Content—Series 211

22. *Layout of Tests.*—The concrete test specimens of Series 211 were made up with three proportions of cement, the values of the absolute volume of cement per cubic foot of concrete being about 0.06, 0.10, and 0.15 cu. ft., corresponding approximately to 1 part of cement to 8 parts mixed aggregate by bulk, 1 part of cement to 5, and 1 part to 2½ or 3, respectively. The twenty-five fine aggregates already described were used. All concrete was made with basic water content. Due to the smallness of the amounts of the fine aggregates available and also to the limit in the number of test pieces which could be handled in the time available, in general only two of these mixes were made with each of the fine aggregates. For one mixture with each fine aggregate $c = 0.10$ was used; for half of the fine aggregates the rich mix ($c = 0.15$) was used as the second mixture and for the other half the lean mix ($c = 0.06$). With each of the fine aggregates and for each value of c used, the coarse aggregate was taken in two proportions, one with $b = 0.45$, a value which was considered to be near the approximate limit that would surely give a fairly workable mixture at ordinary consistencies, and the other with $b = 0.30$, chosen as a relatively low proportion of coarse materials. This plan resulted in giving a fairly even distribution of the values of strength and voids-cement ratio over the whole field

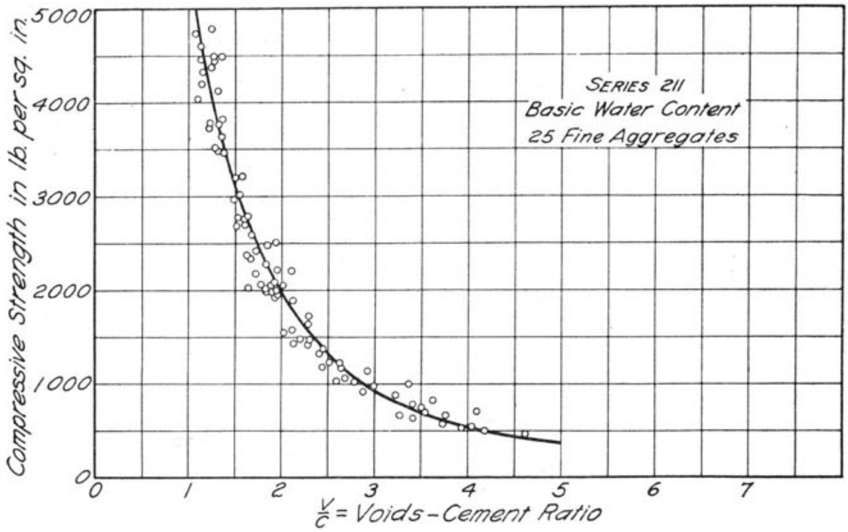

FIG. 25. VOIDS-CEMENT RATIO AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 211

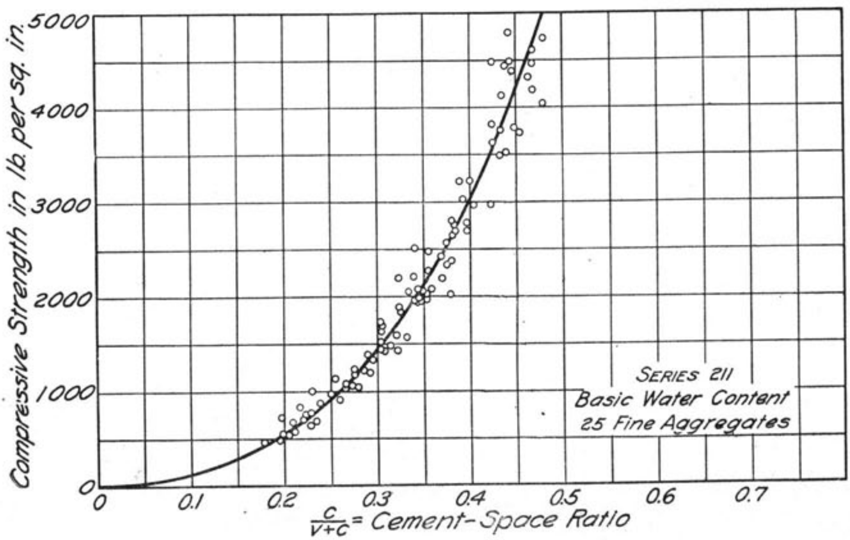

FIG. 26. CEMENT-SPACE RATIO AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 211

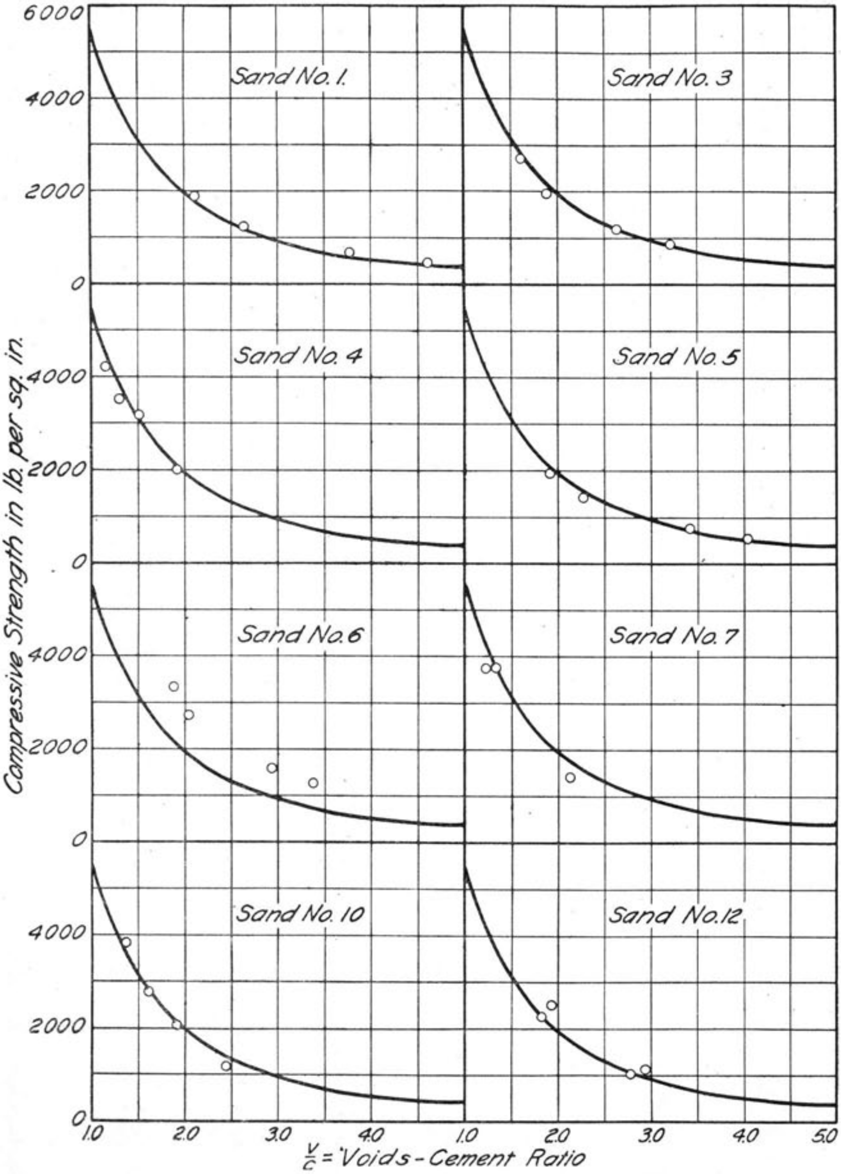

FIG. 27. VOIDS-CEMENT RATIO AND COMPRESSIVE STRENGTH OF CONCRETE MADE WITH VARIOUS FINE AGGREGATES, SERIES 211

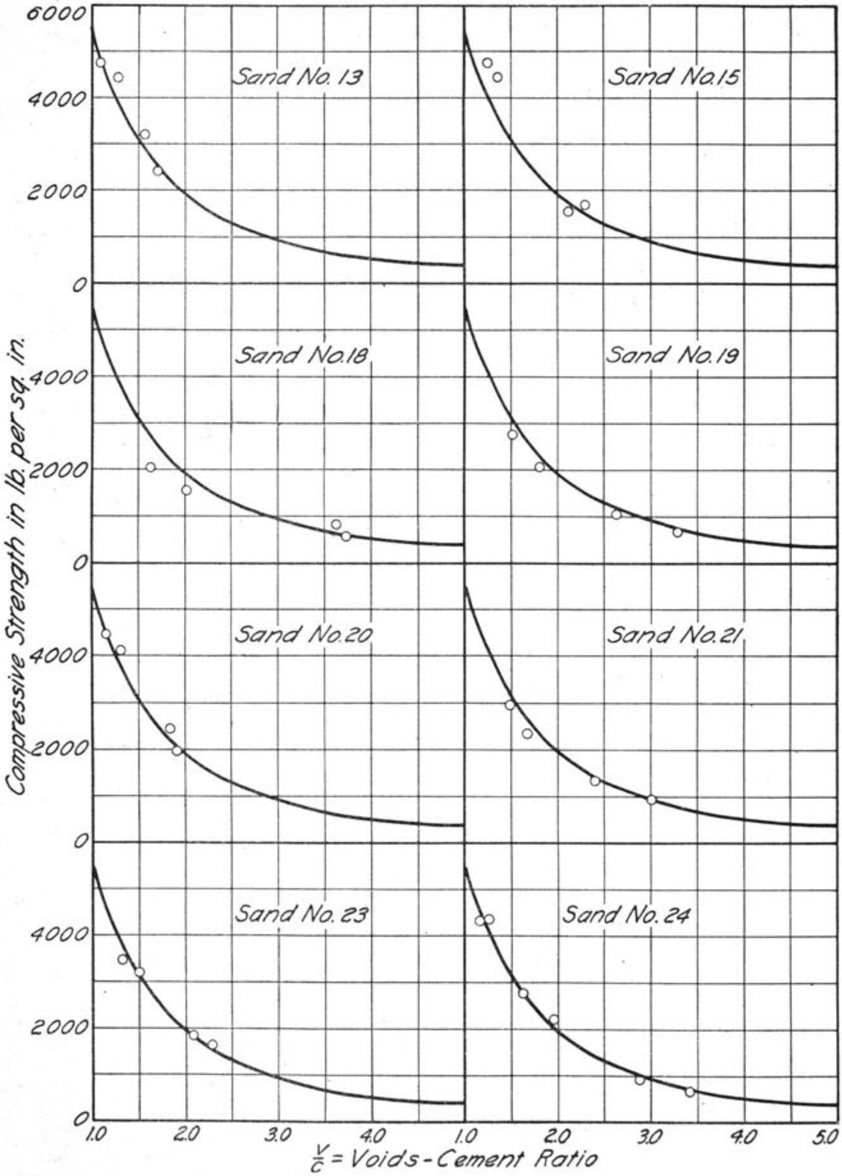

FIG. 28. VOIDS-CEMENT RATIO AND COMPRESSIVE STRENGTH OF CONCRETE MADE WITH VARIOUS FINE AGGREGATES, SERIES 211

TABLE 6

DATA OF CONCRETE TEST SPECIMENS—SERIES 211, BASIC WATER CONTENT
Various fine aggregates; ⅜- to 1-in. gravel. Tests at age of 28 days.
Each value represents the average of 3 or 4 specimens

Cyl. No.	$\frac{a}{c}$	Absolute Volume			Voids v	$\frac{v}{c}$	$\frac{c}{v+c}$	w_c	Water-Cement Ratio	Comp. Strength lb. per sq. in.
		Sand a	Gravel b	Cement c						
1–50	5.12	.309	.353	.060	.278	4.61	.179	.216	1.74	460
51	1.84	.185	.503	.100	.212	2.11	.322	.177	0.86	1890
52	4.09	.248	.463	.061	.228	3.76	.210	.181	1.44	680
53	2.70	.274	.356	.102	.268	2.64	.275	.226	1.07	1230
2–70	2.11	.313	.349	.150	.184	1.23	.448	.177	0.57	3780
71	1.49	.222	.465	.149	.163	1.10	.479	.158	0.51	4040
73	3.75	.377	.351	.100	.171	1.71	.370	.178	0.86	2180
3– 1	2.82	.278	.464	.099	.159	1.61	.384	.137	0.67	2700
2	3.68	.365	.348	.099	.188	1.90	.345	.157	0.78	1980
3	5.44	.321	.462	.059	.158	2.64	.275	.117	0.96	1180
4	6.88	.405	.346	.059	.190	3.22	.237	.137	1.13	880
4– 5	1.51	.224	.460	.148	.169	1.15	.467	.138	0.45	4200
6	3.62	.364	.352	.100	.183	1.83	.354	.153	0.74	2000
7	2.10	.313	.347	.149	.191	1.29	.437	.152	0.49	3510
8	2.80	.281	.468	.100	.151	1.51	.398	.116	0.56	2690
5–38	4.71	.281	.456	.060	.203	3.41	.227	.141	1.13	780
39	5.73	.346	.352	.060	.242	4.04	.199	.165	1.33	550
40	2.22	.219	.493	.099	.190	1.92	.342	.146	0.72	1920
41	3.20	.322	.352	.100	.227	2.27	.307	.169	0.82	1410
6–54	2.64	.259	.459	.098	.184	1.88	.348	.149	0.74	3330
55	2.83	.349	.350	.099	.202	2.04	.329	.173	0.85	2720
56	6.66	.393	.349	.059	.199	3.37	.229	.164	1.35	1280
57	5.30	.313	.455	.059	.173	2.93	.255	.145	1.19	1600
7–58	3.41	.348	.342	.098	.209	2.12	.321	.172	0.85	1410
59	2.69	.243	.456	.091	.210	2.31	.302	.115	0.62	2180
60	2.09	.310	.346	.148	.196	1.32	.432	.174	0.57	3760
61	1.40	.202	.480	.144	.174	1.21	.453	.148	0.50	3730
8–74	1.68	.251	.349	.150	.251	1.68	.374	.195	0.63	2590
75	2.12	.207	.489	.098	.206	2.11	.322	.144	0.71	2200
76	1.05	.156	.493	.148	.203	1.37	.422	.159	0.52	3460
77	3.07	.307	.350	.100	.244	2.45	.290	.175	0.85	1390
9–96	2.85	.268	.422	.094	.216	2.31	.302	.148	0.77	1510
97	3.35	.330	.345	.099	.226	2.29	.303	.173	0.85	1440
10–46	1.69	.254	.354	.150	.242	1.61	.383	.192	0.62	2760
47	3.01	.301	.355	.100	.244	2.44	.291	.194	0.94	1180
48	2.31	.233	.473	.101	.193	1.91	.344	.148	0.71	2070
49	1.21	.181	.467	.149	.203	1.36	.424	.163	0.53	3820
11–92	6.16	.357	.343	.058	.242	4.10	.196	.165	1.38	710
93	2.89	.280	.439	.097	.184	1.90	.345	.167	0.83	2050
94	3.59	.355	.347	.099	.199	2.01	.332	.181	0.89	2050
95	5.14	.298	.449	.058	.195	3.36	.229	.153	1.28	1000
12–82	6.65	.406	.354	.061	.179	2.94	.254	.153	1.21	1130
83	3.77	.369	.343	.098	.190	1.94	.341	.161	0.80	2500
84	5.50	.325	.453	.059	.164	2.78	.265	.147	1.21	1020
85	3.01	.292	.436	.096	.176	1.83	.354	.156	0.79	2280
13–26	2.14	.317	.345	.148	.189	1.28	.439	.149	0.49	4440
27	3.84	.386	.345	.099	.170	1.72	.368	.129	0.63	2420

TABLE 6 (CONTINUED)

DATA OF CONCRETE TEST SPECIMENS—SERIES 211, BASIC WATER CONTENT
Various fine aggregates; ⅜ to 1-in. gravel. Tests at age of 28 days.
Each value represents the average of 3 or 4 specimens

Cyl. No.	$\frac{a}{c}$	Absolute Volume			Voids v	$\frac{v}{c}$	$\frac{c}{v+c}$	w_c	Water-Cement Ratio	Comp. Strength lb. per sq. in.
		Sand a	Gravel b	Cement c						
13–28	2.69	.261	.488	.097	.154	1.58	.388	.110	0.55	3210
29	1.30	.193	.498	.149	.163	1.09	.480	.140	0.45	4740
14–42	4.69	.277	.456	.059	.208	3.53	.220	.149	1.22	700
43	5.60	.336	.354	.060	.250	4.17	.194	.182	1.47	490
44	3.29	.316	.340	.096	.248	2.58	.279	.178	0.90	1030
45	2.54	.249	.463	.098	.190	1.94	.340	.146	0.72	1940
15–78	2.13	.311	.345	.146	.198	1.36	.425	.185	0.61	4480
79	3.33	.333	.355	.100	.212	2.12	.320	.184	0.89	1570
80	2.82	.265	.425	.094	.216	2.30	.303	.158	0.82	1720
81	1.61	.234	.439	.145	.182	1.25	.443	.164	0.55	4780
16–62	1.30	.189	.467	.146	.198	1.35	.425	.164	0.54	3640
63	1.86	.271	.345	.146	.238	1.63	.380	.205	0.68	2370
64	2.39	.232	.458	.097	.213	2.20	.313	.166	0.83	1480
65	2.98	.289	.344	.097	.270	2.78	.265	.208	1.03	1040
17–66	3.16	.310	.346	.098	.246	2.51	.285	.181	0.90	1220
67	2.23	.221	.498	.099	.182	1.84	.352	.138	0.68	1990
68	4.68	.276	.459	.059	.206	3.50	.222	.145	1.19	750
69	5.59	.341	.359	.061	.239	3.92	.203	.177	1.41	540
18– 9	2.70	.267	.472	.099	.162	1.64	.379	.136	0.66	2010
10	5.09	.290	.446	.057	.207	3.63	.216	.133	1.13	840
11	6.32	.373	.348	.059	.220	3.73	.211	.157	1.29	560
12	3.58	.354	.348	.099	.199	2.01	.332	.163	0.81	1550
19–30	5.30	.318	.461	.060	.161	2.68	.271	.133	1.08	1060
31	6.52	.391	.352	.060	.197	3.28	.233	.164	1.33	660
32	2.55	.253	.497	.099	.151	1.52	.396	.130	0.64	2780
33	3.58	.362	.356	.101	.181	1.79	.358	.158	0.76	2060
20–17	3.65	.361	.351	.099	.189	1.91	.344	.166	0.81	1950
18	2.08	.309	.349	.148	.194	1.31	.433	.165	0.54	4120
19	2.78	.270	.455	.097	.178	1.84	.353	.133	0.67	2470
20	1.49	.220	.463	.148	.169	1.14	.467	.146	0.48	4450
21–34	5.30	.323	.470	.061	.146	2.40	.295	.109	0.87	1340
35	6.49	.396	.360	.061	.183	3.00	.250	.126	0.99	970
36	2.83	.283	.469	.100	.148	1.48	.403	.123	0.60	2970
37	3.68	.372	.359	.101	.168	1.66	.375	.145	0.69	2300
22–22	2.13	.315	.348	.148	.189	1.28	.439	.165	0.54	4460
23	2.80	.280	.465	.100	.155	1.55	.392	.130	0.64	3010
24	1.42	.211	.473	.148	.168	1.13	.468	.151	0.49	4600
23–13	3.19	.319	.352	.100	.229	2.29	.304	.142	0.69	1640
14	1.86	.277	.350	.149	.224	1.50	.400	.170	0.56	3210
15	2.50	.243	.458	.097	.202	2.08	.325	.132	0.66	1840
16	1.30	.193	.464	.148	.195	1.32	.432	.155	0.50	3490
24–86	3.80	.369	.345	.097	.189	1.95	.339	.159	0.80	2210
87	2.11	.315	.350	.149	.186	1.25	.445	.170	0.55	4370
88	6.61	.390	.350	.059	.201	3.41	.227	.160	1.31	640
89	5.54	.327	.445	.059	.169	2.87	.259	.135	1.11	900
90	3.14	.301	.442	.098	.159	1.62	.381	.137	0.68	2780
91	1.59	.235	.446	.148	.171	1.15	.464	.161	0.53	4310

and also gave opportunities for comparisons of the aggregates in different ways.

The values of a, b, c, and the water content for the specimens were predetermined in each case, using the data of the mortar-voids tests. Enough additional water was added to allow for the absorption of water by the fine and coarse aggregates during the time of mixing and placing in the mold. The amount of this is not included in the water contents reported.

23. *Data and Discussion.*—The data of the tests are given in Table 6. In this table each cylinder number listed indicates the reference number of the sand used and the serial number of the cylinder. Thus, Sand No. 1 was used in Cylinder No. 1—50. The results of the compressive strengths have been plotted in Fig. 25, with strength as ordinates, and ratio of voids to absolute volume of cement $(\frac{v}{c})$ as abscissas. In Fig. 26 strength is given as ordinates and cement-space ratio $(\frac{c}{v+c})$ as abscissas. The curves are given by the equation

$$S = \frac{32000}{\left(1+\frac{v}{c}\right)^{2.5}} = 32000\left(\frac{c}{v+c}\right)^{2.5} \qquad (8)$$

where S is the strength in pounds per square inch at 28 days, c the absolute volume of cement per unit of volume of concrete, and v the voids in a unit of volume of concrete. In Figs. 27 and 28 the results from concrete made with a number of the fine aggregates are shown separately. The curve on these diagrams is that of the foregoing formula.

In general the values of the test results fall near the curves represented by the equation. A study of the results for the twenty-five fine aggregates shows that generally all the values fall so close to the curve as to indicate that a curve of this nature may be expected to be representative of the strength relation. Only two of the aggregates give values markedly higher than the curves and none fall far below. No. 6, limestone screenings, is noticeably higher, the results being as much as 50 per cent higher than the curve. It seems probable that the chemical action of the fine particles of the limestone has contributed to the compressive strength of the concrete. The rich mixtures of No. 15 have values 25 per cent above the curve. This cementing action with puzzolanic materials has been noted by various writers. Except for these two fine aggregates the agreement between the curve and the test values was generally very good. It should be noted that there is no special divergence for any of the values of b and c used; that is, the rich, medium, and lean mixtures, and the mixtures with large and with small amounts of the coarse materials, all conform closely to one curve.

VI. Concretes with Varied Water Content—Series 211

24. *Layout of Tests.*—Included in Series 211 were tests to find the effect of varying the water content—changing the amount of mixing water. Specimens were made up at basic water content and at water contents having nominal values of 1.20 and 1.40 times basic water content, and also of 1.60 and even more. In order to insure uniformity of materials throughout the tests, the three fine aggregates used were artificially graded, the combination being made up from the Attica sand of the laboratory. Sand No. 31 corresponds closely to fine aggregate No. 24, a well-graded sand of excellent quality. No 32 was graded from 0 to the No. 14 sieve and represents a well-graded fine sand. Sand No. 36 ranges from the No. 28 to the No. 4 sieve—a coarse sand free from fine particles. The sieve analysis of these sands is given in Table 4 and the fineness modulus and surface modulus in Table 5. For No. 31 and No. 32 three cement contents were used, $c = 0.06$, $c = 0.10$, and $c = 0.15$. For No. 36, $c = 0.10$ only was used in the wet mixtures. The variations made in

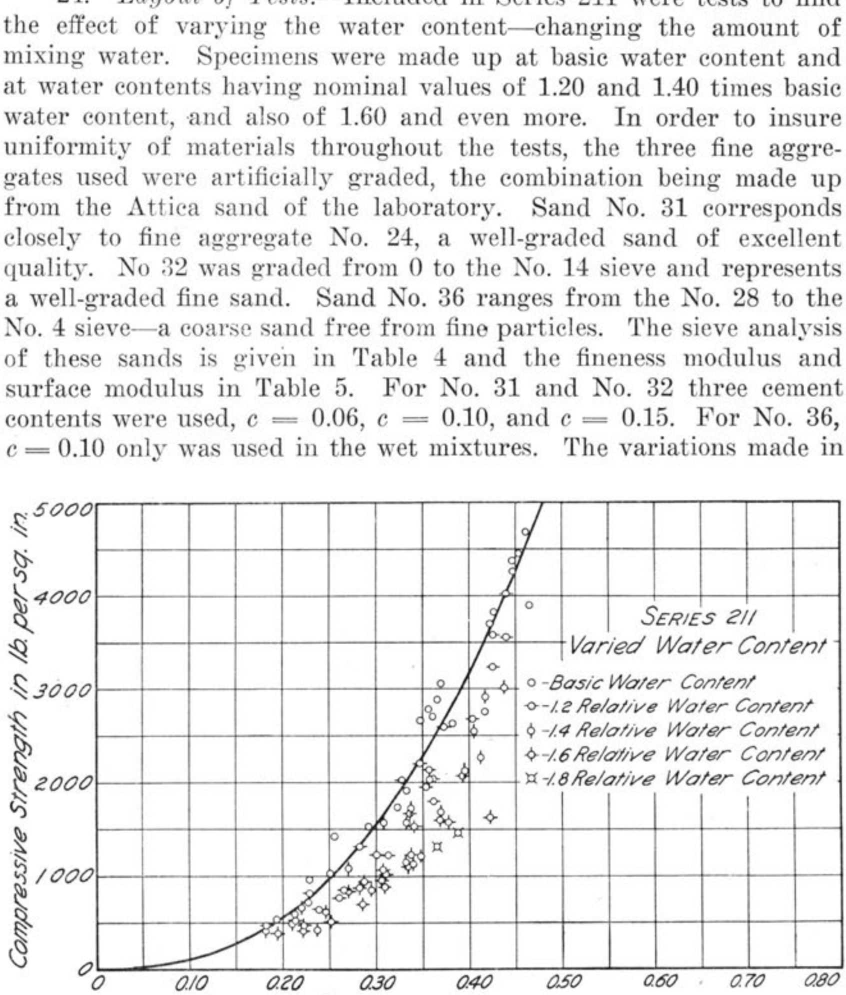

FIG. 29. CEMENT-SPACE RATIO AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 211, VARIED WATER CONTENT

TABLE 7

DATA OF CONCRETE TEST SPECIMENS—SERIES 211, VARIED WATER CONTENT

Artificial gradations of sand; ⅜- to 1-in. gravel. Tests at age of 28 days. Each value represents the average of 3 or 4 specimens

Cyl. No.	Relative Water Content	$\frac{a}{c}$	Absolute Volume			Voids v	$\frac{v}{c}$	$\frac{c}{v+c}$	w_c	Water-Cement Ratio	Comp. Strength lb. per sq. in.
			Sand a	Gravel b	Cement c						
31–1	0.97	7.05	.427	.304	.061	.208	3.41	.227	.161	1.29	710
31–2	1.14	7.05	.419	.299	.060	.222	3.70	.213	.188	1.52	530
31–3	1.30	7.05	.410	.293	.058	.239	4.12	.195	.215	1.79	390
31–4	1.01	5.37	.323	.455	.060	.162	2.70	.270	.130	1.05	1140
31–5	1.19	5.37	.319	.450	.060	.171	2.85	.260	.154	1.24	760
31–6	1.38	5.37	.313	.442	.058	.187	3.23	.236	.176	1.47	430
31–7	1.50	5.37	.306	.437	.057	.200	3.51	.222	.199	1.69	480
31–8	1.17	4.79	.284	.493	.059	.164	2.78	.265	.142	1.17	850
31–9	1.33	4.79	.277	.486	.058	.179	3.09	.245	.163	1.36	610
31–10	1.54	4.79	.279	.490	.058	.173	2.98	.251	.186	1.55	510
31–11	1.04	3.90	.392	.305	.101	.202	2.00	.333	.177	0.85	1920
31–12	1.22	3.90	.384	.298	.099	.219	2.21	.312	.207	1.02	1220
31–13	1.38	3.90	.377	.294	.097	.232	2.39	.295	.236	1.18	860
31–14	1.56	3.90	.367	.285	.094	.254	2.70	.270	.268	1.38	840
31–15	1.00	2.88	.283	.447	.099	.171	1.73	.361	.141	0.69	2710
31–16	1.18	2.88	.282	.444	.098	.176	1.80	.357	.168	0.83	1950
31–17	1.33	2.88	.276	.435	.096	.193	2.01	.332	.192	0.98	1150
31–18	1.47	2.88	.271	.426	.094	.209	2.23	.310	.215	1.11	1020
31–19	1.06	2.52	.254	.506	.100	.140	1.40	.417	.137	0.66	2750
31–20	1.18	2.52	.246	.489	.097	.168	1.73	.361	.158	0.79	1800
31–21	1.33	2.52	.241	.479	.095	.185	1.95	.339	.180	0.92	1120
31–22	1.47	2.52	.236	.465	.092	.207	2.25	.305	.200	1.05	960
31–23	.99	2.29	.343	.302	.150	.205	1.37	.422	.186	0.60	3700
31–24	1.17	2.29	.338	.297	.147	.218	1.48	.403	.220	0.73	2680
31–25	1.31	2.29	.326	.287	.143	.244	1.71	.369	.249	0.84	1680
31–26	1.43	2.29	.316	.278	.138	.268	1.94	.340	.274	0.95	1530
31–27	.98	1.55	.231	.449	.149	.171	1.15	.465	.156	0.51	3900
31–28	1.12	1.55	.224	.436	.145	.195	1.35	.425	.182	0.61	3250
31–29	1.28	1.55	.219	.424	.141	.216	1.53	.395	.211	0.73	2130
31–30	1.37	1.55	.2[illegible]3	.412	.137	.238	1.74	.365	.230	0.81	1800
31–31	.93	1.28	.186	.489	.145	.180	1.24	.446	.144	0.48	4370
31–32	1.12	1.28	.185	.486	.145	.184	1.27	.440	.172	0.58	3560
31–33	1.25	1.28	.181	.477	.141	.201	1.42	.413	.195	0.67	2260
31–34	1.32	1.28	.174	.457	.136	.233	1.71	.369	.214	0.76	1600
32–1	1.16	4.32	.258	.503	.060	.179	2.98	.251	.144	1.16	1020
32–2	1.31	4.32	.258	.494	.059	.189	3.20	.238	.165	1.36	650
32–3	1.49	4.32	.254	.494	.059	.193	3.27	.234	.187	1.54	630
32–4	1.61	4.32	.247	.487	.057	.199	3.49	.223	.205	1.74	420
32–5	.99	4.79	.295	.465	.061	.179	2.94	.254	.134	1.06	1430
32–6	1.14	4.79	.286	.452	.060	.202	3.37	.229	.158	1.28	820
32–7	1.35	4.79	.284	.448	.059	.209	3.55	.220	.187	1.54	660
32–8	1.50	4.79	.281	.443	.058	.218	3.76	.210	.211	1.76	530
32–9	.89	6.17	.378	.308	.061	.253	4.15	.194	.162	1.29	530
32–10	1.07	6.17	.371	.303	.060	.266	4.43	.184	.194	1.57	460
32–11	1.24	6.17	.370	.302	.060	.268	4.47	.183	.226	1.83	440
32–12	1.49	6.17	.369	.323	.060	.248	4.13	.195	.257	2.08	400

TABLE 7 (CONTINUED)

DATA OF CONCRETE TEST SPECIMENS—SERIES 211, VARIED WATER CONTENT

Artificial gradations of sand; ⅜- to 1-in. gravel. Tests at age of 28 days. Each value represents the average of 3 or 4 specimens

Cyl. No.	Relative Water Content	$\frac{a}{c}$	Absolute Volume: Sand a	Gravel b	Cement c	Voids v	$\frac{v}{c}$	$\frac{c}{v+c}$	w_c	Water-Cement Ratio	Comp. Strength lb. per sq. in.
32–13	.99	3.59	.357	.301	.100	.242	2.42	.292	.173	0.79	1510
32–14	1.17	3.59	.352	.297	.099	.252	2.55	.282	.204	1.00	1310
32–15	1.33	3.59	.347	.292	.097	.264	2.72	.269	.234	1.17	1100
32–16	1.51	3.59	.344	.289	.096	.271	2.82	.282	.266	1.34	870
32–17	1.01	2.62	.262	.453	.099	.186	1.88	.347	.143	0.70	2660
32–18	1.17	2.62	.257	.441	.099	.203	2.05	.328	.167	0.82	2020
32–19	1.32	2.62	.251	.438	.096	.215	2.24	.308	.190	0.96	1560
32–20	1.47	2.62	.252	.435	.096	.217	2.26	.307	.214	1.08	1050
32–21	0.97	2.30	.226	.496	.099	.179	1.81	.356	.128	0.63	2780
32–22	1.15	2.30	.225	.493	.098	.184	1.88	.347	.153	0.76	2200
32–23	1.33	2.30	.223	.490	.097	.190	1.96	.338	.177	0.89	1730
32–24	1.48	2.30	.221	.484	.096	.189	1.97	.337	.200	1.01	1230
32–25	1.56	2.30	.214	.468	.093	.225	2.42	.293	.217	1.13	890
32–26	1.13	1.96	.191	.541	.098	.170	1.73	.366	.140	0.69	2890
32–27	1.31	1.96	.191	.540	.097	.172	1.77	.361	.162	0.81	2040
32–28	1.46	1.96	.189	.534	.096	.181	1.88	.347	.184	0.93	1210
32–29	1.53	1.96	.182	.516	.093	.209	2.24	.309	.199	1.04	870
32–30	0.99	2.07	.309	.301	.149	.241	1.62	.382	.184	0.60	2630
32–31	1.17	2.07	.306	.298	.148	.248	1.68	.373	.219	0.72	2600
32–32	1.32	2.07	.298	.300	.144	.258	1.79	.359	.248	0.84	2020
32–33	1.47	2.07	.294	.286	.142	.278	1.96	.338	.280	0.96	1680
32–34	1.00	1.37	.209	.458	.151	.182	1.21	.453	.156	0.50	4450
32–35	1.15	1.37	.204	.448	.148	.200	1.35	.426	.182	0.60	3580
32–36	1.30	1.37	.200	.440	.146	.214	1.47	.405	.209	0.69	2550
32–37	1.43	1.37	.195	.429	.142	.234	1.65	.378	.233	0.80	1570
32–38	0.96	1.14	.169	.499	.149	.183	1.23	.448	.143	0.47	4260
32–39	1.14	1.14	.168	.497	.148	.187	1.27	.440	.170	0.56	4020
32–40	1.28	1.14	.165	.489	.145	.201	1.39	.418	.195	0.65	2920
32–41	1.40	1.14	.164	.478	.142	.216	1.52	.394	.216	0.74	2070
32–42	1.50	1.14	.157	.465	.138	.240	1.74	.365	.238	0.84	1310
32–43	1.27	0.89	.128	.540	.145	.187	1.29	.437	.182	0.61	3010
32–44	1.42	0.89	.127	.534	.143	.196	1.37	.422	.205	0.70	1620
32–45	1.50	0.89	.123	.518	.139	.220	1.58	.388	.223	0.78	1460
36–1	0.95	3.93	.392	.301	.099	.208	2.10	.322	.151	0.74	1740
36–2	1.11	3.93	.382	.294	.097	.227	2.34	.300	.176	0.88	1230
36–3	1.28	3.93	.377	.290	.096	.237	2.47	.288	.202	1.02	930
36–4	1.45	3.93	.376	.288	.096	.240	2.50	.286	.230	1.16	700
36–5	0.96	2.99	.292	.443	.098	.167	1.70	.370	.119	0.59	3060
36–6	1.13	2.99	.289	.438	.097	.176	1.81	.356	.141	0.71	2130
36–7	1.28	2.99	.284	.430	.095	.191	2.01	.332	.161	0.82	1560
36–8	1.47	2.99	.285	.431	.095	.189	1.99	.334	.185	0.94	1120
36–9	0.89	6.67	.406	.307	.061	.226	3.71	.212	.149	1.18	600
36–10	0.87	5.20	.305	.442	.058	.195	3.37	.229	.110	0.92	970
36–11	1.06	2.42	.357	.297	.147	.199	1.35	.426	.156	0.51	3830
36–12	1.02	1.65	.226	.451	.149	.174	1.17	.461	.132	0.43	4680

the amount of coarse aggregate included the following values of b: for Sand No. 31, 0.30, 0.45, and 0.50; for Sand No. 32, 0.30, 0.45, 0.50, 0.55; for Sand No. 36, 0.30, and 0.45.

25. *Data and Discussion.*—Table 7 gives general data of the tests of concretes with varied water content. Each cylinder number listed indicates the reference number of the sand used and the serial number of the cylinder. The compressive strengths are plotted in Fig. 29. It is apparent from the data that the strength of the specimens for the wetter mixtures is considerably less than that which would correspond to the strength for the voids found in these mixtures as judged from the strength of concrete at basic water content. The discrepancy is small at relative water contents close to basic, but it increases as water is added. It may be considered that there are two sources of the lower strength of such concretes: (a) the increase in the voids which comes with an increase in the water, and (b) an additional effect of the presence of excess water. It would appear that the decrease in strength due to the presence of the excess water is about the same as or somewhat more than that due to the increase in voids. A study of the results was made by calculating the relative strength of the concrete at a given water content in terms of the value which would correspond to the voids of this mixture, and plotting the results. From such a study the diagram in Fig. 30 was devised as a tentative reduction method; it is applicable to the results of these tests and may be expected to apply to materials other than Sands Nos. 31, 32, and 36. This reduction curve means that at a relative water content of say 1.30, the

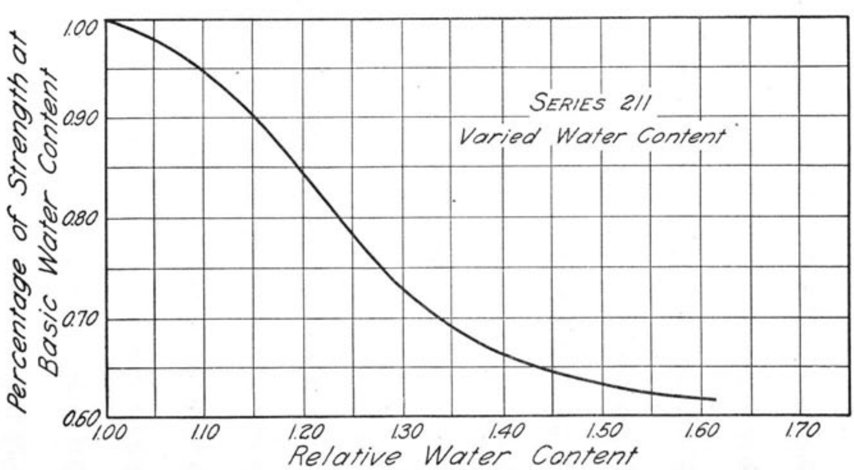

FIG. 30. REDUCTION CURVE FOR CONCRETES OF VARIED WATER CONTENT

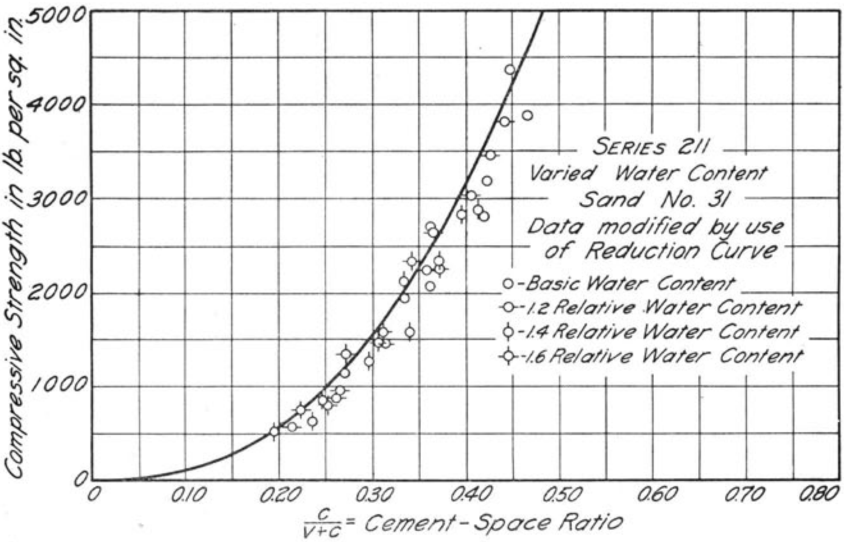

FIG. 31. CEMENT-SPACE RATIO AND COMPRESSIVE STRENGTH, MODIFIED BY REDUCTION CURVE, SAND No. 31

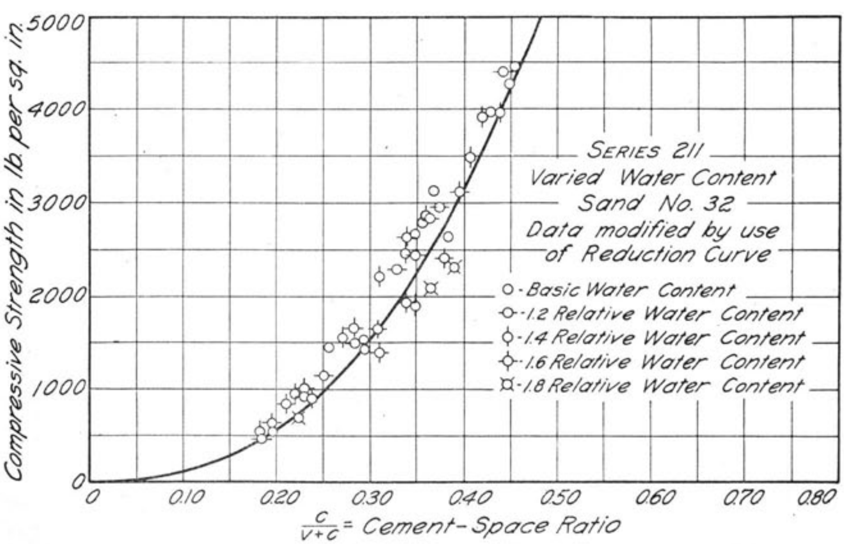

FIG. 32. CEMENT-SPACE RATIO AND COMPRESSIVE STRENGTH, MODIFIED BY REDUCTION CURVE, SAND No. 32

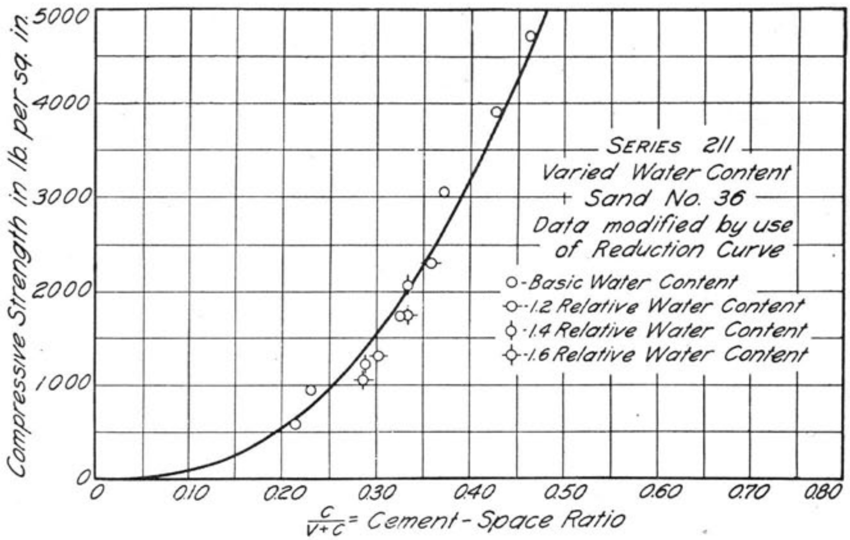

FIG. 33. CEMENT-SPACE RATIO AND COMPRESSIVE STRENGTH, MODIFIED BY REDUCTION CURVE, SAND NO. 36

strength of the concrete is 0.73 of that which would correspond to the same voids at basic water content. Conversely, the strength at basic water content for a given value of the voids may be obtained by dividing the strength for the given water content by a ratio taken from the reduction curve.

The reduction curve given in Fig. 30 was applied to the results of the tests of the series. The strength of a specimen was divided by the factor of the reduction curve corresponding to its relative water content. Figs. 31, 32, and 33 give the values found in this way. It will be seen that the diagrams made up in this way give a very good general agreement with the curve for basic water content and that there is no more variation in the wetter mixtures than is found in the specimens made up with basic water content.

The reverse use of the reduction diagram would be as follows: Divide the required strength of the desired concrete by the factor of the reduction curve corresponding to the relative water content to be used, as found from Fig. 30. The mixture of the concrete may then be designed by the ordinary consideration of voids, using the values which would be found at the relative water content to be used.

It should be repeated that the use of such a reduction diagram is offered tentatively. Investigations with other materials and other mixtures are needed to determine whether the accuracy and usefulness of this method may be established for general purposes.

The direct use of the ratio of the volume of cement used to the volume of the mixing water, as an indication of the strength of the concrete, is discussed in Article 27.

It would be interesting to know the cause of the decreased strength with the wetter mixes.

VII. General Discussion

26. *Qualities of the Aggregates.*—It will be recognized that the fine aggregates used in Series 211, twenty-five in number, comprised a variety of materials, materials that differ considerably in mineralogical composition, in nature of surface, in size and gradation of grain, and in general shape of grain. In Series 2G differences in the gradation of size of grain constituted the principal variation. It will be interesting to consider the effects of the several kinds of variation in the fine aggregate upon the strength of the concretes.

Specifications usually provide that the fine aggregate shall consist of inert material having clean, hard, strong, uncoated grains and free from injurious amounts of dust and soft or flaky particles. The fine aggregates used included sands, both siliceous and partly calcareous, crushed granite and trap and limestone, and blast furnace slag. The grains of these aggregates were hard, strong, and structurally sound, as is common with such materials. The strength of the rock and slag in which these fine aggregates had their source is, of course, well above the strength of ordinary concrete. It is only in rich mixtures at advanced ages that the strength of the concrete may be affected by the hardness and strength of fine aggregate, unless the particles should be structurally weak.

Such weak material as mica was present in such small quantities as not to affect the strength of the concrete. The dust and other fine particles found in the fine aggregates were generally fairly well distributed through the mass. A study of the results of the tests will show that the differences in mineralogical composition have no noticeable effects on the strength of the concretes, except as the limestone screenings having a considerable amount of fine particles gave indications of cementing activity and consequent higher strength in the 28-day tests, as is brought out elsewhere. It has already been noted that the aggregates did not contain injurious organic material so far as is shown by the colorimetric test.

Nothing was brought out in the tests to show marked differences in strengths because of differences in the nature of the surface of the grains and the effect of smoothness or of differences in the porosity of the grain on the adhesion of the cement to the grain. There was some indication that the smooth and non-porous grain of the Ottawa sand gave slightly lower strength results.

The effect of the size and the gradation of particles of the fine aggregate upon the strength of the mortar and concrete follows very closely the indications of the mortar-voids tests, as well as that of the voids-cement ratio relations, as is shown in the discussions elsewhere.

The shape of the particles in the fine aggregate, whether round or sharp and angular, affects the strength of the mortar and concrete principally through the difference in opportunity for the particles to fit in among each other, which results in a change in the density of the mortars and concretes. The fine aggregates having rounded particles and also the same gradation of sizes give the denser mortars and the higher strengths, as is shown by the tests. There is nothing in the tests to indicate any advantage in sharpness or angularity of grain.

The gravel used as the coarse aggregate in the tests was all of the same general nature, of good quality, and of fairly well rounded shape. The effect of the differences in gradation is discussed in the next article.

It should be noted that no investigation was made of properties of the aggregate that would affect the durability of the concrete.

27. *Water-Cement Ratio and Strength.*—When the compressive strength of concrete for a series of tests is plotted as ordinates and the ratio between the volume of mixing water and the volume of cement in a given volume of concrete as abscissas, a relation between the points is found which may serve a useful purpose in the design of concrete mixtures.* It would be well to learn something of the conditions under which variations from a single relation occur and whether a single law is generally applicable. In the use of the water-cement ratio, the volume of the cement is usually taken by bulk, counting 94 lb. of cement as a cubic foot. This volume of the cement used in the specimens in the series of tests under consideration may be obtained by multiplying the absolute volume of the cement c by the ratio $\frac{193}{94}$. This bulk volume will be denoted by C where used in connection with the water-cement ratio. In the calculation of the water-cement ratio the amount of water that may be expected to be

* See article "Revolutionary Results Obtained in Proportioning Water in Concrete Tests," Engineering News-Record, May 2, 1918, Vol. 80, p. 873, and Bulletin 1 of Structural Materials Research Laboratory, "Design of Concrete Mixtures," both by Duff A. Abrams. See also, Johnson, "Materials of Construction," Fifth Edition, Appendix B, p. 815.

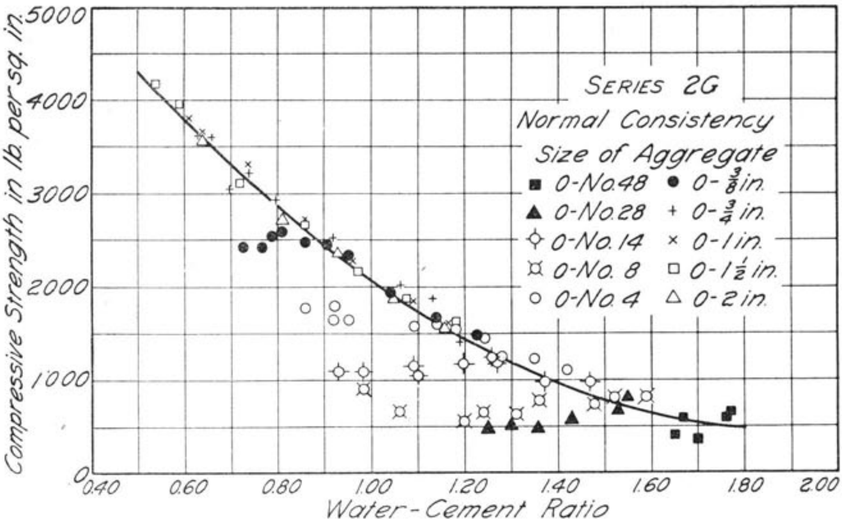

FIG. 34. WATER-CEMENT RATIO AND COMPRESSIVE STRENGTH, SERIES 2G

absorbed by the aggregate before the concrete is finally in place, or perhaps before it takes its initial set, is deducted from the amount of mixing water used.

The compressive strengths of Series 2G have been plotted against water-cement ratios in Fig. 34. Specimens containing so much coarse material that their strength is thereby impaired are not included in this diagram. In these very coarse mixtures the voids of the coarse aggregate are not filled with mortar; they are so coarse that they should not be used for concrete. The strengths of the concretes given in the diagram follow a very definite line. The mixtures made up with sand and cement only do not follow the general line; for a given maximum size of particle there is relatively little difference in the strength of these mortar specimens with any of the gradations. It is apparent that in the coarser gradations of these sands the voids in the mortars hold a relatively small proportion of water and hence the water content is not representative of the voids. It should be kept in mind that in Series 2G the aggregates had a regular gradation from fine to coarse and that the same consistency was used in all specimens.

Fig. 35 contains the results of Series 211 on concretes made with twenty-five fine aggregates at basic water content. Here again the agreement is fair, though there is a scattering of the points, and the position of the curve differs from that of Fig. 34, the strength at a given ratio being smaller. Fig. 36 represents the results of Series

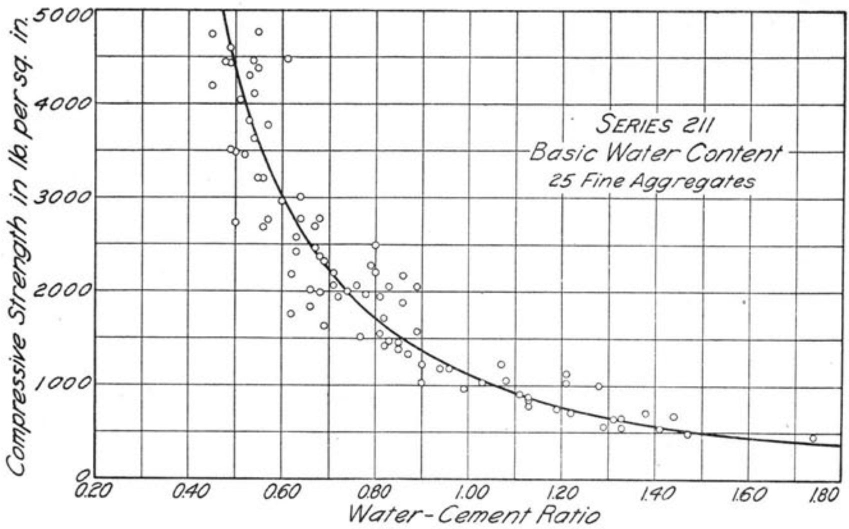

FIG. 35. WATER-CEMENT RATIO AND COMPRESSIVE STRENGTH, SERIES 211, BASIC WATER CONTENT

211 having varied water contents, one sand with three different gradations being used. An identification and study of the data from which Figs. 35 and 36 were plotted show that for the fine aggregates that have the voids well filled with water at basic water content, as Nos. 2 and 12, the points fall to the right of the curve, while for those with poorly filled voids at basic water content, as Nos. 10 and 23, the points fall to the left of the curve. It would appear then, that variations in the proportion of water to voids affect the water-cement strength relation.

In Fig. 37 are shown the curves for strength and water-cement ratio taken from Series 2G and 211, the curve for Series 83 given by Professor D. A. Abrams,* and the curves for Series 201 derived from the results of tests made at other laboratories.† It appears from this diagram that while a curve may be found to fit a given set of tests, the different sets of tests give different positions of the water-cement ratio curve. It would appear that the different sets of tests have cements of different strengths, or that different proportions of the voids are filled with water, or that different standards or methods are used in the testing work, or that there are other variations in the relation between the strength and the water-cement ratio. Variations from

* Bulletin 1, Structural Materials Research Laboratory, Lewis Institute, Chicago. The curve is given here in a revised position due to allowance for absorption by aggregate.

† Report of Committee C-9, Proc. A. S. T. M., Vol. 22, p. 329.

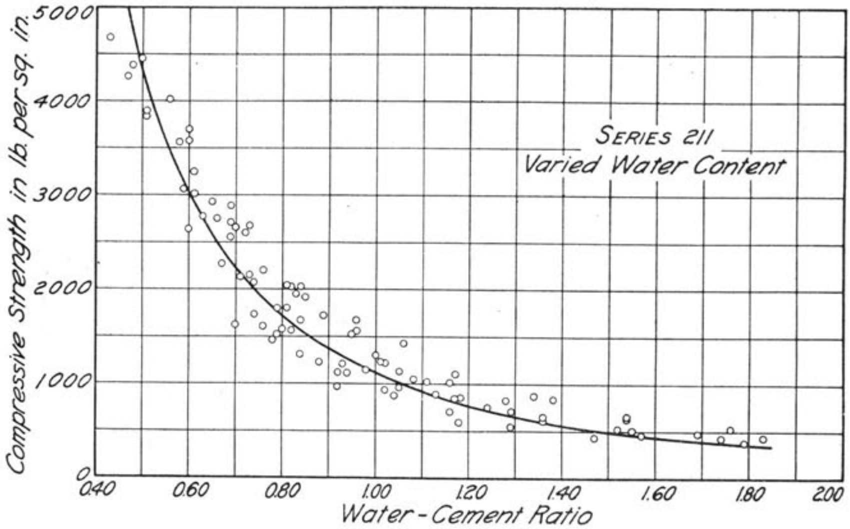

FIG. 36. WATER-CEMENT RATIO AND COMPRESSIVE STRENGTH, SERIES 211, VARIED WATER CONTENT

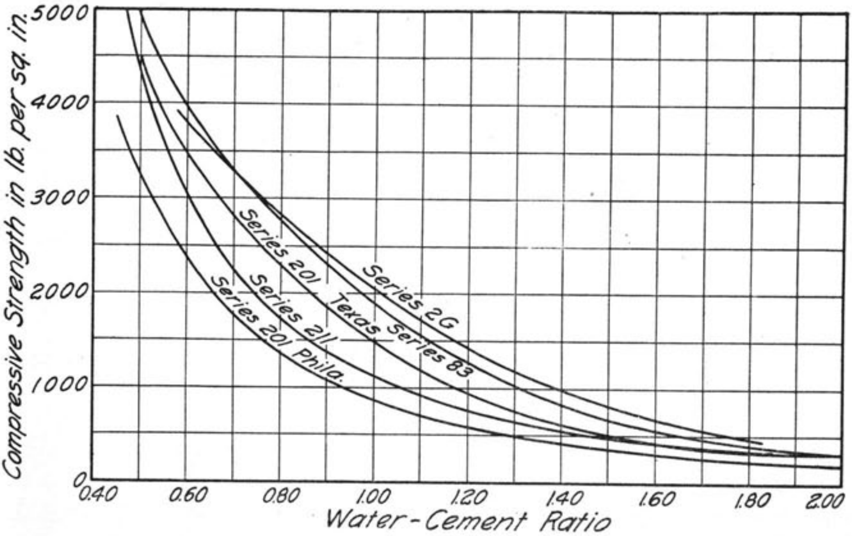

FIG. 37. WATER-CEMENT RATIO AND COMPRESSIVE STRENGTH, SEVERAL SERIES OF TESTS

a single curve were found by Roderick B. Young, when changes were made in consistency, in gradations of particles, and in richness of mixture.*

The curve drawn to represent the average of a large number of tests must be considered to have only a general significance; when there is much variation from this curve, the effect of the variables needs to be taken into account. More study is needed and more definite information on such matters as the effect of changes in relative water content or consistency, the effect of differences in richness and the effect of the nature of the aggregate.

It may be added that regardless of the variations found in different groups of tests the water-cement ratio relation has useful applications in a variety of ways in the consideration of concrete mixtures, and in problems relating to the design of concrete. One illustration is in finding the effect of increasing the water content in a given mixture; if the strength of a concrete at basic water content is known, the strength at a relative water content of 1.40 may be determined with a fair degree of accuracy by multiplying the basic strength by a coefficient taken from a curve made up from data with various water contents.

28. *Voids-Cement Ratio and Strength.*—In Article 17 on "Strength Relations" and Article 23 on "Data and Discussion" it has been shown that for Series 2G and Series 211 at and near basic water content when the compressive strength is plotted as ordinates and the ratio between the voids and the absolute volume of the cement as abscissas, the points have a fairly definite relation which is well represented by a curve. When the cement-space ratio is used, a similar relation is found to exist. The results of the two series of tests are plotted in Fig. 6, page 31, and Fig. 25, page 59. Considering that the fine aggregates used cover a very wide range of gradations, and also a very wide range of materials, twenty-five fine aggregates of quite different characteristics being included in Series 211, and also a considerable range in richness of mixture and amount of coarse aggregate used, the marked agreement of the test results with a single voids-cement curve appears to have significance. The average variation of the compressive strength of the test specimens from the curve shown in Fig. 25 is about 9 per cent of the compressive strength itself; there are very few variations greater than 15 per cent.

It is evident that if the voids in the concrete were entirely filled with water the voids-cement ratio and the water-cement ratio would be identical provided the quantity of cement was expressed the same

* Journal of the Boston Society of Civil Engineers, March, 1921.

way in both cases. Differences in the two curves may be expected to be caused mainly by differences in the degree to which the voids are filled with water in mixtures made up with a variety of richness, nature and gradation of aggregate, and consistency. If the proportion of the voids filled with water were constant, there would be a constant similarity between the voids-cement relation and the water-cement relation. It may be noted that for a number of sands the proportion of voids that was filled with water remained nearly constant throughout a considerable range of richness (see Fig. 24, page 57), although the proportion was not the same for the different sands. Sands of different nature and gradation may then not be expected to show agreement in both the voids-cement and the water-cement relations.

For concretes made up with water contents higher than the basic (wetter than normal consistency), the application of a reduction curve, such as that shown in Fig. 30, page 68, gives results which conform fairly well to the same voids-cement relation, as is shown by the points plotted in Figs. 31, 32, and 33. The three sands used in the tests plotted in these three figures varied considerably in gradation, though they were of the same nature.

The results of Series 211 at basic water content have been plotted in two other ways in Figs. 38 and 39.

Fig. 38 shows the relation of voids and strength at basic water content for the twenty-five fine aggregates and for three amounts of cement, nominally c of 0.06, 0.10, and 0.15—corresponding to lean, medium, and rich mixtures. The results include the two proportions of coarse aggregate, represented by $b = 0.30$ and 0.45. All the fine aggregates are represented in the medium mixtures, half of them in the lean mixtures, and the other half in the rich mixtures. It is seen that for a given amount of cement the strength is fairly close to an average curve, even with the great variety of fine aggregates used. The curve for $c = 0.10$ is quite similar to that in Fig. 5, page 30. From these curves it may be said that, when the amount of cement is kept constant, an increase in the voids of the concrete of 25 per cent of themselves results in a reduction of strength of 25 to 30 per cent.

To enable a comparison of the relations between strength and amount of cement to be made, the curves of Fig. 38 were used to reduce or correct the strength values of the test results to correspond to the nearest of three voids values, $v = 0.16$, $v = 0.20$, and $v = 0.24$; and these strength values so reduced are plotted in Fig. 39. The increase of strength with increased amount of cement, the voids in the concrete remaining constant, is very apparent. The

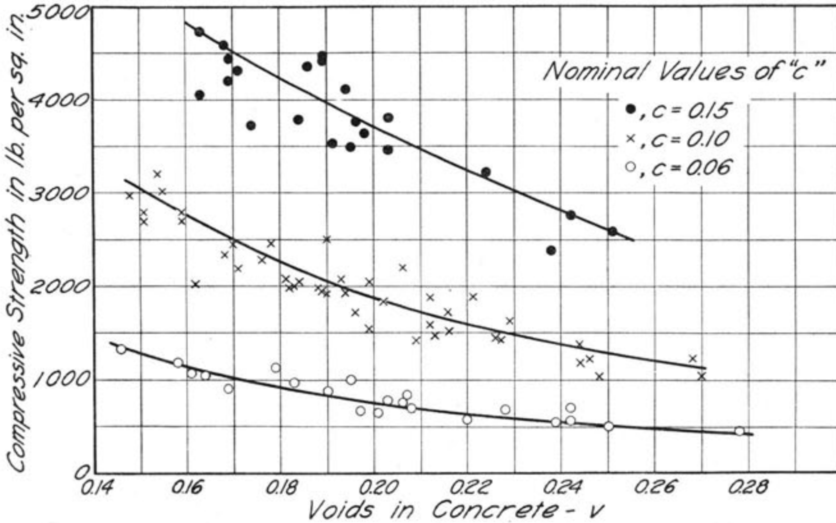

FIG. 38. VOIDS AND COMPRESSIVE STRENGTH OF CONCRETE, SERIES 211, BASIC WATER CONTENT

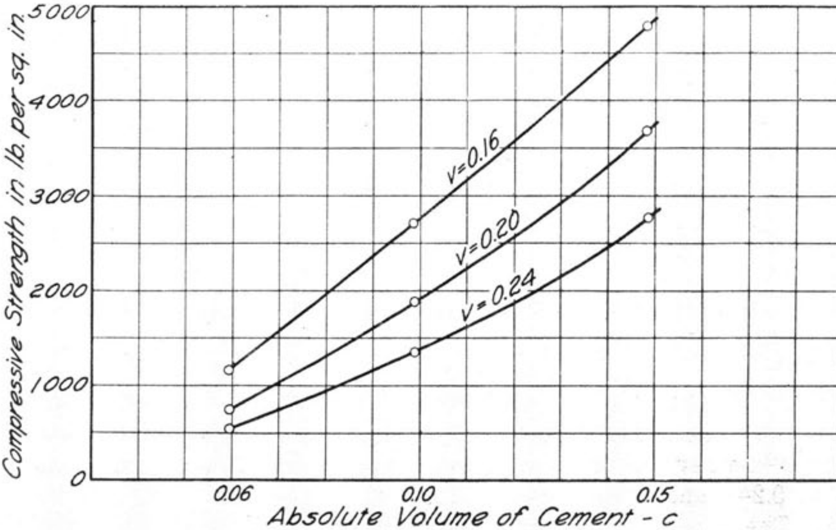

FIG. 39. CEMENT CONTENT AND COMPRESSIVE STRENGTH OF CONCRETE

ratio of increased strength is nearly the same in the three curves and in the different parts of each. By taking values from the three curves at various places and averaging the resulting ratios it will be found that if the voids in the concrete were to remain the same the addition of 50 per cent to the cement content of a concrete would almost double its strength. As has been seen from the characteristic mortar-voids curves, however, an increase in cement for values of $\frac{a}{c}$ less than 2 is generally accompanied by an increase in voids, and hence the effect of the addition of cement is generally less than this. Part of the increase is probably due to the more rapid rate of hardening of the richer mixtures, but the ratio of increase of strength at greater ages may be expected to be nearly as much as at 28 days.

It should be borne in mind that the relation between strength, voids, and amount of cement shown in Figs. 38 and 39 represents the results obtained with a great variety of fine aggregates and that large and small amounts of the coarse aggregate were used in the tests. It is evident that the dependence of strength of concrete upon amount of cement and amount of voids in the concrete must be true for a wide range of materials and conditions.

The results of tests made in a number of laboratories have been studied with reference to the relation between strength of concrete and the voids-cement ratio. Diagrams similar to Figs. 6 and 25 have been made from the data of tests conducted by the Structural Materials Research Laboratory, the Hydro-Electric Power Commission of Ontario, the University of Texas, and the City of Philadelphia, already referred to in Article 27, "Water-Cement Ratio and Strength," page 72, but are not reproduced here. The curve of the relation between strength and voids-cement ratio for Series 83 of the Structural Materials Research Laboratory and that for the University of Texas tests approach closely the curves for the Illinois tests. The curves for the tests of the Hydro-Electric Power Commission lie somewhat above, about 400 lb. per sq. in at $\frac{v}{c} = 2.0$, and that for the tests of the City of Philadelphia lies below, about 600 lb. per sq. in. at $\frac{v}{c} = 2.0$. All the curves so made up are for mixtures at or near normal consistency, but the use of the reduction diagram gives results for the wetter mixtures which fall fairly close to the curve for normal consistency.

29. *Relation between Strength of Mortar and Strength of Concrete.*—An auxiliary set of tests on mortars may be used to

compare the strength properties of mortars with those of concretes. These tests with mortars were made for the purpose of comparing the strength properties of mortars made of the various fine aggregates with mortars of Attica sand made up with the same gradation of particles, and to learn whether differences in adhesion might be present. The mortars were 1 part cement and 2.5 parts fine aggregate by absolute volume, with basic water content in all cases. Compression tests of 2 by 4-in. mortar cylinders were made at 28 days. The average of the strengths of the specimens made up with twenty-five fine aggregates was almost exactly the same as the average of the strengths of the specimens with Attica sand made up in the same gradings, the average of the voids-cement ratios of the specimens also being the same in the two cases. It is evident that, in general, adhesion to the grains was not a governing consideration in these tests.

To make a comparison of the strength properties of these mortars with those of concretes, the strength of mortar specimens was plotted as ordinates and the voids-cement ratio as abscissas, and the curve for the concretes of Series 211, given by equation (8), was drawn on the diagram. The points representing the mortars were somewhat more scattered than the points in diagrams representing the tests of concrete, and they generally fell below the curve of the concrete tests. Two other sets of mortar specimens, 6 by 12-in. cylinders, made in connection with later series of tests of concretes, gave compressive strengths at or above the curve for the relation between voids-cement ratio and strength of concrete. Several series of tests made elsewhere indicate that the voids-cement-strength relation holds for both mortars and concretes. Differences between the strength of mortars and concretes as indicated by the voids-cement relation may be expected to be due to differences in relative water content, in manipulation in making the specimen (such as those affecting compactness), and in conditions attendant on differences in size of specimen.

Other things being the same, it may be asserted that below a reasonable limit for the amount of the coarse aggregate, the strength of the concrete is identical with the strength of the mortar composing it. "Other things" refer to relative water content, manipulation in placing, and compactness of mortar and concrete, absorptive properties of coarse aggregate, manner of curing and testing, etc. In making a comparison of the relative strengths of mortars and concretes, special care should be taken to secure identical conditions. It is further assumed that the strength of the particles of coarse aggregate is at least as great as the strength of the mortar and that the adhesion of cement to the coarse aggregate is as great as to the fine aggregate. Under this view, the coarse aggregate becomes only a filler, concentrating the mortar and making it richer for a given amount of cement in a fixed volume of concrete. This conception of the function of the mortar simplifies the process of designing the mixture, for the voids-cement ratio of the concrete in good workable mixtures will be essentially the same as that of the contained mortar. This conception also leads to a better comprehension of the source of the strength and the effect of variations in mixtures.

30. *Sieve Analysis and Concrete-Making Properties of Aggregates.*—Sieve analyses of fine aggregates and coarse aggregates and combinations of the two give important information on the concrete-making properties of materials. Specifications not uncommonly set limits for the proportions passing certain sieves or retained thereon. Characteristics of the aggregate may be judged from the sieve analysis. To what extent the sieve analysis or functions derived therefrom may be used in judging and comparing the concrete-making properties of materials may well be considered.

The coarser sands included in the variety of fine aggregates used in Series 211 give sieve analysis having the same general characteristics. Differences in strengths are found, however, in concretes made with fine aggregates of similar sieve analysis. It will be found that the crushed stone and crushed slags give properties which differ from the sands, particularly for concretes having medium or lean mortars. As an example, No. 15, crushed slag, which resembles No. 24 in gradation, gives a low strength for the medium mixture ($c = 0.10$), though it has the highest strength in the rich mixture ($c = 0.15$). This result could be anticipated from the mortar-voids curve of the material given in Fig. 20, page 52.

To learn whether there is a clearly marked relation between the compressive strength of the concrete and a function of the sieve analysis, Fig. 40 has been plotted from the results of Series 211. The fineness modulus of the fine aggregate has been used for the abscissas of the diagram, since this function of the sieve analysis is frequently used and is well understood; the use of surface area or surface modulus would give a quite similar diagram. The medium richness of the tests ($c = 0.10$) has been used in making the diagram. The amount of the coarse aggregate in all tests plotted is so nearly the same (approximately $b = 0.45$), and this value is sufficiently below the limiting value for coarse aggregate, that the compressive strengths may be said to be fairly comparable for all the materials. It is evident that for the coarser fine aggregates, including the ones generally giving the best results and of the kind commonly specified

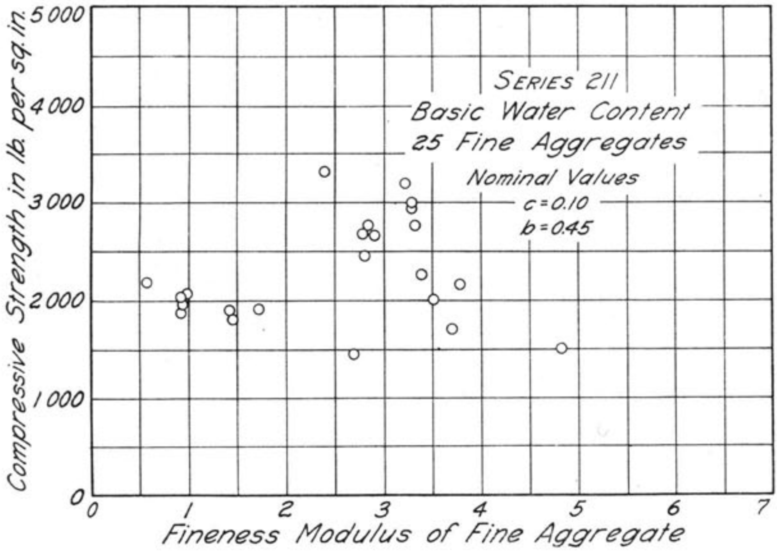

FIG. 40. STRENGTH OF CONCRETE AND FINENESS MODULUS OF FINE AGGREGATE, SERIES 211

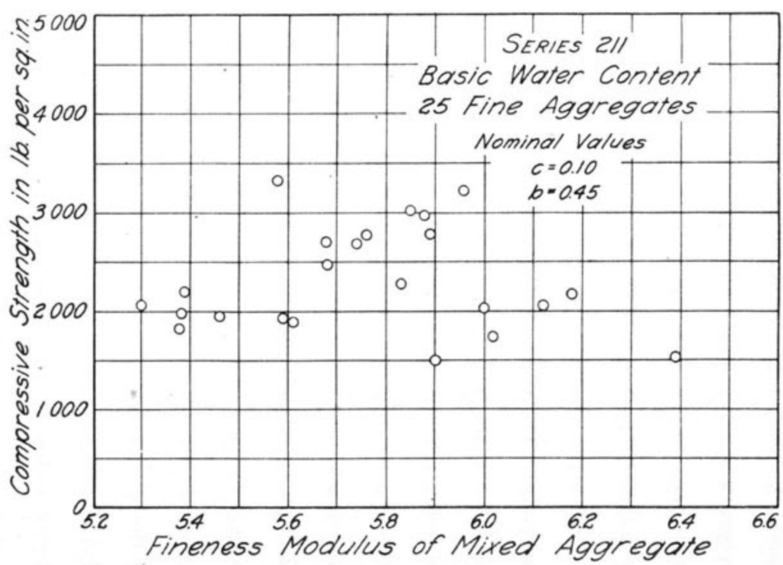

FIG. 41. STRENGTH OF CONCRETE AND FINENESS MODULUS OF MIXED AGGREGATE, SERIES 211

for the best work, considerable variations in strength exist for the same value of the fineness modulus. Fig. 41 indicates a similar diversity of results when the strengths are plotted against the fineness modulus of the mixed fine and coarse aggregate. Similar variations will be found for the lean mixture ($c = 0.06$) and the rich mixture ($c = 0.15$), though it may be expected that with rich mortars the differences will be relatively smaller, and in the lean mortars relatively larger.

The voids in the mortars made with the same fine aggregates at a value of $\frac{a}{c} = 2.5$ and basic water-content plotted in Fig. 42 show a similar variation. Fig. 43 giving mortar voids at 1.40 relative water content shows an even greater variation. The values of the voids were taken from Figs. 20 and 22, pages 52 and 54. On the other hand the strength of the concretes corresponds closely with the curve for the voids-cement ratio, as a reference to Figs. 27 and 28, pages 60 and 61 will show, these results being also in accord with the voids properties of the materials shown in Figs. 20 and 22. If the aggregates were uniformly graded according to a regular scale, as in Series 2G, a function like the fineness modulus might have a definite relation to the strength, as seen in Fig. 11, page 36, which shows a clearly defined relation for each series of aggregates having the same maximum size, when approximately the same amount of cement is used.

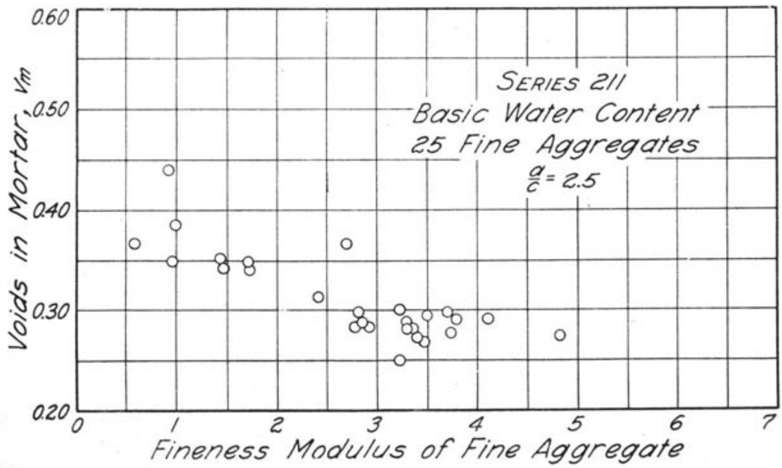

FIG. 42. MORTAR VOIDS AND FINENESS MODULUS, BASIC WATER CONTENT, SERIES 211

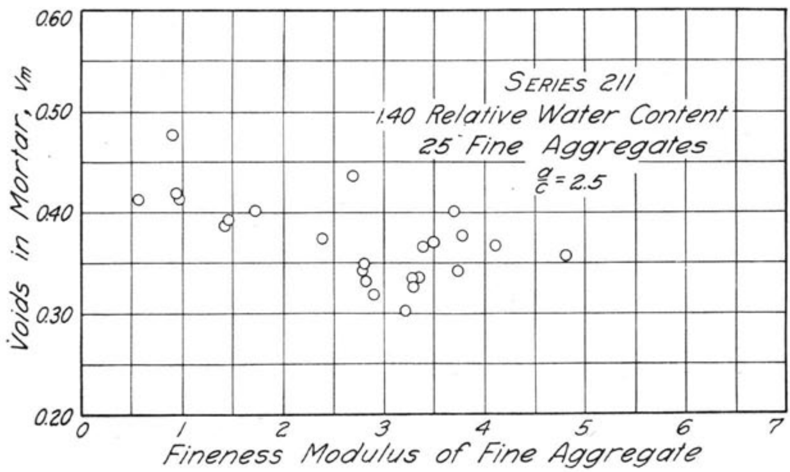

FIG. 43. MORTAR VOIDS AND FINENESS MODULUS, 1.40 RELATIVE WATER CONTENT, SERIES 211

It appears also that the quantity of mixing water necessary to give a normal consistency or a minimum volume of the mortar and concrete can not be told accurately for any material from the sieve analysis or a function like the fineness modulus. This may be seen by plotting the water content of the mortars of Series 211 in terms of the fineness modulus in a manner similar to that used in plotting the mortar voids in Figs. 42 and 43; a diagram so made will show the points widely scattered. Values at the higher relative water contents give similar diversity in the relation between the water content and the fineness modulus. Even when the aggregates have a uniformly varied gradation, as in Series 2G, no simple equation applicable to all the aggregates may be found for calculating the mixing water required to make a given consistency. In Fig. 44 the water-cement ratios of the mixtures used in Series 2G have been plotted in terms of the fineness modulus of the mixed aggregate. It will be recalled that these mixtures were made up of 1 part cement to 5 parts of mixed aggregate (the amount of cement in a cubic foot of the concrete or mortar necessarily varying somewhat according to the size and gradation of the aggregate, c thus varying from 0.082 to 0.103), and that the consistency of all the mixtures as determined generally by the slump test, or by judgment in the case of the coarsest mixtures, was the same, being that commonly called normal consistency. The curve representing the formula for mixing water given by Prof. D. A. Abrams in Bulletin 1 of the Structural Materials

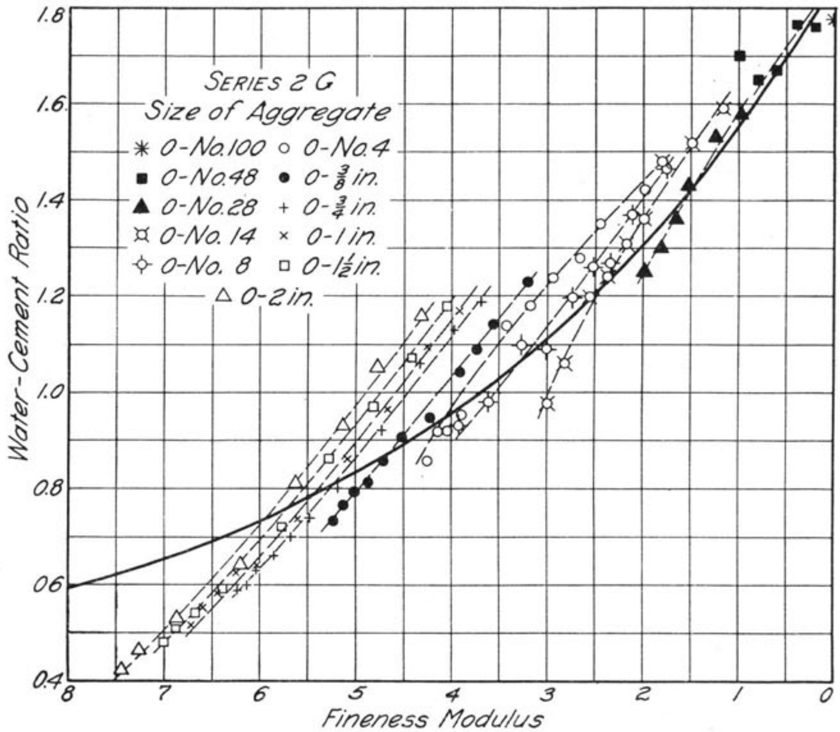

FIG. 44. WATER-CEMENT RATIO AND FINENESS MODULUS, SERIES 2G

Research Laboratory is also plotted on the diagram. A study of sieve-analysis curves and devices such as fineness modulus and surface modulus makes it seem probable that a formula for water content dependent upon them can not be formulated which will be applicable to the variety of aggregates and the variety of gradations found in the materials used in making concrete. Instead, an experimental determination, such as the mortar-voids test, may be found useful for the purpose.

It is evident that the sieve analyses do not bring out all the differences in the characteristics of the fine aggregates; they, of course, show general differences in gradation. From physical considerations it seems evident that a sieve analysis, or a function derived therefrom, can not be expected generally to measure or determine the voids in the concrete, because (1) a group of particles of about the same size near the upper or lower size, or at some intermediate size, may be sufficiently in excess of its proportionate amount of volume to interfere with the law of voids that may govern with regularly graded particles; (2) the sieve analysis does not take into account the bulking or swelling of the mass when wet, or the difference in the amount of bulking for particles of different size or different nature of surface; and (3) the sieve analysis for the reason given in (1) can not take into account the changes in bulk, due to a change in the amount of cement used in the mixture.

It follows then that while such devices as fineness modulus, surface modulus, and surface area may give a fairly satisfactory way of representing the voids function in material having certain regular gradations of size used with a certain proportion of cement and having certain degrees of bulking, they can not represent at all generally the variety of differences in grading and bulking and the variety in the cement proportions that are found in the general run of aggregates and concrete mixtures. At best with materials of the same general nature and same general gradations they must be used only as rough guides. Since it has been shown that the voids in mortars and concretes are an indication of their compressive strengths, it follows that such functions as fineness modulus can not be expected to be definitely indicative of the compressive strength which will be attained.

31. *Limiting Values of the Amount of Coarse Aggregate.*—Assuming the use of a constant amount of cement, in a general way it may be said that the greater the value of b (the absolute volume of the coarse aggregate in a unit volume of concrete; in other words, the amount of the solids in the coarse aggregate put into this volume of concrete), the greater the density of the concrete, up to the point where the voids in the coarse aggregate are not filled by the mortar. This increased density with increased values of b is evident from equation (3) for all cases where the values of v_m (the voids in the mortar used) do not increase so much with the decreased value of a that goes with the increased value of b as to offset the effect of the increase in b—a condition which may exist with very rich mixtures. Concrete in which the mortar does not fill the voids in the coarse aggregate should, of course, not be used when strength, homogeneity, and properties other than space-filling and weight-producing ones are desired.

It will be well, then, to find what it is that limits the value of b which may be used in concrete, and also what the limiting values of b may be expected to be, both the extreme limit and the usable limit for different conditions of ordinary work. It will also be well to learn what happens to the strength and other properties of concrete when a value of b smaller than a limiting value is used.

For use in the discussion call b_0 the density or absolute volume in a unit of volume (bulk) of a coarse aggregate. For the coarse aggregates ordinarily used this may range from 0.55 to 0.67, when tamped into the measuring receptable according to the standard method of the American Society for Testing Materials. For gravel the values usually range between 0.60 and 0.64; for broken stone, between 0.56 and 0.62. The difference between these values and 1.0 are the voids in the mass.

It is apparent that the ratio $\frac{b}{b_0}$ can not reach 1.0; particles of the mortar will be found between the particles of the coarse aggregate, holding them apart, producing the so-called wedging action; also the particles of the coarse aggregate may not be arranged among themselves as well as when measured alone, thus leaving spaces not filled with mortar. Concrete made up in proportions attempting to secure a ratio of 1.0 for $\frac{b}{b_0}$ will result in a volume greater than the volume or bulk of the coarse aggregate, and generally considerably greater. It is apparent also that the limit of the ratio $\frac{b}{b_0}$ that may be obtained in a laboratory specimen will depend upon the care exercised and the amount of tamping used in making the specimen. It is also apparent that on construction work the limit will depend upon the manner of placing and tamping; a tamping machine on a concrete pavement will permit a higher ratio than would be obtainable where little tamping is given. Besides, the ordinary methods of placing may be expected to bring about a lower limit than would be obtainable in laboratory specimens. The higher values of $\frac{b}{b_0}$ and of b give a very harsh coarse mixture, not easily worked and not applicable in concrete work under usual conditions of tamping and placing.

Tests to determine just what the working limit of b and $\frac{b}{b_0}$ should be for any given aggregate and any working condition may readily be made. Well-rounded pebbles will give a higher allowable value than elongated ones. The limit of b for broken stone may be expected to be lower than for gravel, the difference depending upon the angularity of the pieces.

Tests indicate that for usual working conditions in the field the limiting value of the bulk of the coarse aggregate that may properly be put into a unit of volume of concrete may be said to range from 65 to 75 per cent of the volume of the resulting concrete when made

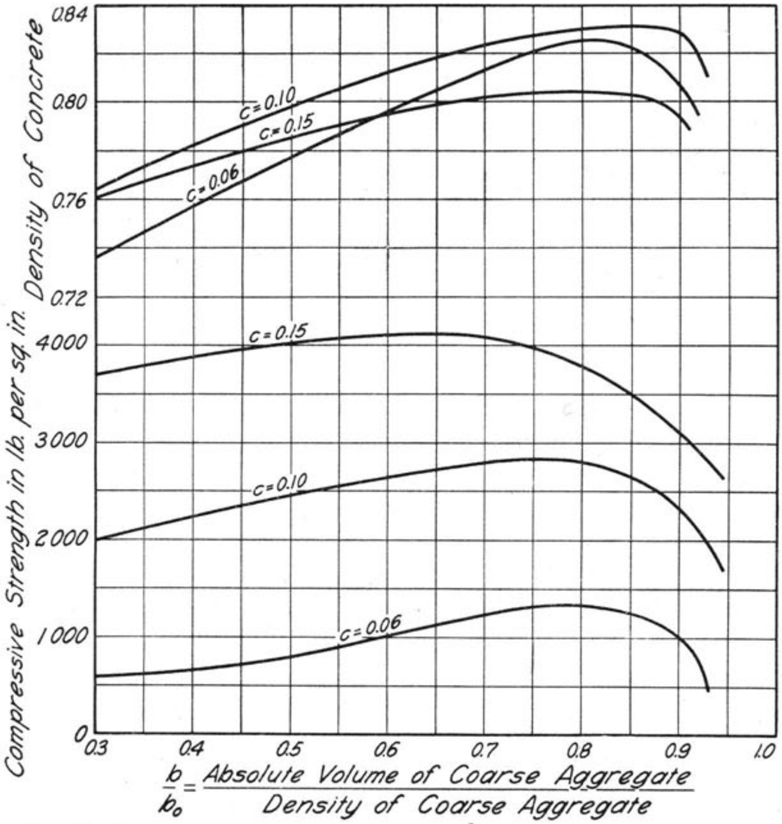

FIG. 45. DENSITY, STRENGTH, AND RATIO $\frac{b}{b_0}$ FOR CONCRETE MADE WITH COARSE SAND

with the usual coarse sands, (that is, $\frac{b}{b_0}$ will range from 0.65 to 0.75), and somewhat greater values when the concrete is made with the finer sands. The three curves in the upper part of Fig. 45, derived from test data, represent the density in concretes made with different values of $\frac{b}{b_0}$, using gravel and a coarse sand, in very rich, medium, and lean mixtures; the point of downturn represents the limiting value of the ratio for carefully made laboratory specimens. The three curves in the lower part of Fig. 45 give the corresponding compressive strengths at 28 days for the three mixtures. It is evident that there is little change in strength for concretes with $c = 0.15$ (very rich mixture) between $\frac{b}{b_0} = 0.45$ and $\frac{b}{b_0} = 0.80$. This may be

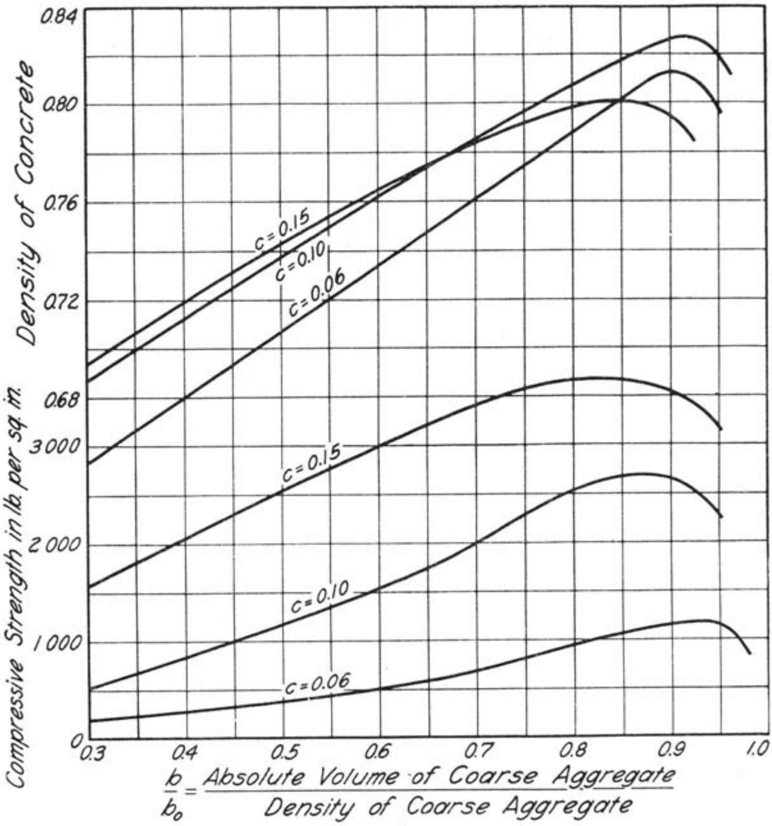

FIG. 46. DENSITY, STRENGTH, AND RATIO $\frac{b}{b_0}$ FOR CONCRETE MADE WITH FINE SAND

expected from the fact that for coarse sand the voids in the mortar increase rapidly as $\frac{a}{c}$ is decreased from 2.0 to 1.0 or less, as is seen from Fig. 20, page 52. It is true also that for $c = 0.10$ (medium mixture) there is no great change in strength between $\frac{b}{b_0} = 0.60$ and $\frac{b}{b_0} = 0.80$. It is evident then that when fine aggregates of this character are used, the strength is not greatly affected by changes in b within the ranges indicated, though it must be understood that the upper limit named will not give concrete that is easily workable. The range in the values of $\frac{b}{b_0}$ named above correspond to a range in the fineness modulus of the mixed aggregate of say 5.0 to 6.0.

Fig. 46 is a similar representation of density and strength for concretes made with a very fine sand. It is apparent that the lines representing the density for a given amount of cement have a greater slope and break off more sharply at the upper limit of density, and that this point of maximum density is reached at a higher value of $\frac{b}{b_0}$ than is the case with coarser sand. It has not been established that a much higher limiting value of $\frac{b}{b_0}$ may be used with the very fine sand under workable conditions than with the coarser sand. It will be seen from the diagram for strength that the range of $\frac{b}{b_0}$ giving approximately the same strength is smaller than for the coarser sand, and that the strength of the concrete drops off rapidly with smaller values of $\frac{b}{b_0}$. It is plain then that to obtain the best strength when fine sand is used the largest possible value of b or $\frac{b}{b_0}$ consistent with workable conditions should be used.

It is well to call attention to the necessity for choosing limiting values of b for use in construction work that are considerably smaller than the values obtainable in laboratory practice. The limiting values for $\frac{b}{b_0}$ of 0.65 to 0.75 already mentioned are as great as will be generally satisfactory. These correspond to values of b of about 0.40 to 0.46 for gravel and about 0.38 to 0.43 for broken stone. It is also well to note again that with the coarser of the fine aggregates the strength of the concrete does not fall off materially even with a considerable decrease in the value of b below the limiting value. In reinforced concrete work, it may be expected that the limiting value of b that will result in good construction will be smaller than that usable in mass concrete. It is also found that the reduction and variability of strength of concrete attendant upon careless or indifferent placing is less for the smaller values of b.

The values of $\frac{b}{b_0}$ given in the foregoing discussion imply a measurement of b_0 (the density of the coarse aggregate) determined by the method specified by the American Society for Testing Materials and described in Article 11. If the coarse aggregate is thrown loosely into the measure, as is quite commonly done on construction work, the amount of solids put into the measure will be much smaller than that obtained by the standard laboratory method. A broken stone giving a density of 0.58 by the standard method may give only 0.51 by loose measure in the field, and a gravel giving 0.64 in the laboratory, only 0.59 in the field. This means that if the coarse aggregate is to be measured by loose volume, the amount of solids in a measure may be only about nine-tenths as much as that obtained by the laboratory method. Variations in moisture content and in method of filling will, of course, vary this ratio. As it is the amount of solids deposited in the concrete which counts, the limiting value of b used with the common methods of measuring the aggregate will, of course, be the same as the limiting value of b given for field work in the discussion when laboratory methods of measuring the density of the aggregate were used.

Tests are in progress to determine the limiting values of b which may be used in concrete made with a variety of coarse aggregates and with different methods of placing and tamping and also to fit different conditions of size of member and arrangement of reinforcement. It may be said, however, that for many conditions, covering much of ordinary concrete construction, the value of 0.45 for gravel and 0.40 for broken stone will be a satisfactory limit for b. This limit means that, for medium richness and coarse sand with measurement by loose volumes in the field, the volume of the coarse aggregate will be about twice that of the fine aggregate when gravel is used, and somewhat less than twice when broken stone is used. For work where more than ordinary care is taken in placing, spading, and tamping, a higher limit may be used. The limits found in Series 2G, 0.59 to 0.65, were obtained under unusual conditions. The aggregates, both fine and coarse, were artificially graded from fine to coarse and the concrete was worked in a manner which would not be feasible on construction work. The values found, however, are interesting examples of what may be done. The highest density of concrete obtained, 0.90, is probably not far from the limit of what is possible even with artificially graded materials.

The coarse aggregate will frequently contain fine material which should properly be classed as fine aggregate. This may be taken into account in the mortar-voids test by adding to the fine aggregate the fine material screened from the coarse aggregate in an amount proportional to the ratio of the two aggregates as they are to be used. Whether the overlap will greatly affect the voids in the mortar and concrete will depend upon the gradation of the fine aggregate and the preponderance of sizes just above and below the separation size. Tests for the voids in mortar and concrete may easily be made which will determine whether the overlap is injurious.

Nothing has been said of the need of gradation in the coarse aggregate. With coarse aggregate of the larger sizes, the limiting value of b may be expected to be somewhat greater when the aggregate is well graded from fine to coarse than when it is poorly graded. For aggregate of ordinary size there seems to be little difference in the limiting values regardless of the grading. It should be noted that when the value of b used is less than the limiting value, the differences in the size and grading of the coarse aggregate will not result in any change in the density of the concrete, provided the absolute volume of the coarse aggregate remains the same; a coarse aggregate of uniform size will give the same density as one that is well graded, and it follows that the same strength will be attained.

The effect upon the mobility or workability of the concrete produced by using a value of b smaller than the limiting value is discussed in the succeeding article.

32. *Elements Affecting Workability.*—An important property of a concrete mixture lies in the ease of placing or pouring. It is important that the concrete fill every part of the form and come in complete contact with the reinforcing bars. It is also of advantage to have the concrete homogeneous throughout; that is, to have the large and small particles uniformly distributed through the mass and to have freedom from pockets or even very small spaces not occupied by mortar. It is desirable that this condition be obtained with little effort and expense. This property of the concrete is now known as workability or mobility. The means of measuring the degree of workability of concrete so far devised are not fully satisfactory. The slump test is at best applicable to comparisons with the same gradations of materials and the same richness of mixture; for comparisons with widely different materials and richness, it is crude. The flow table is a better instrument for general comparative purposes, since the jar produced by the drop tends to overcome the viscosity of the mixture; it does not, however, take into account all conditions that enter into the workability of concrete. In considering the workability of concrete it is well to keep in mind that the cement, fine aggregate, coarse aggregate, and water—in amount or nature—each has a bearing on the workability of the mixture.

At the drier mixtures, a mixture rich in cement has sufficient viscosity to give low workability to the concrete. With the addition of water beyond the basic water content the effect of the presence of the larger amount of cement is to increase the workability.

A fine sand acts somewhat as does the cement. With a fine sand the voids in the mortar are not filled until a high relative water content is reached, and a noticeable effect of the increase in mixing water upon mobility is not found until considerable water has been added. With a coarse sand this condition is not apparent.

Water aids in lubrication. An increase in the mixing water beyond the point where the air voids become relatively small increases the workability, but a point is finally reached where the addition of water has little effect.

The shape of the particles of the coarse aggregate affects the mobility of the mixture, especially in a coarse mixture. Round particles make a most easily worked concrete. As may be expected, broken stone with rough surfaces and marked angularity gives difficulty in placing, especially when its amount is close to the upper limit of what may be put into concrete. A very well-graded coarse aggregate is less easily worked than one of more nearly uniform size, at and near the upper limit of b. When the amount of coarse aggregate used is decreased to values well below the upper limit, the workability of the concrete is improved, and it is possible by such a change to effect a decreased relative water content and yet secure increased mobility with no loss in strength.

The usual method of trying to secure increased workability is to increase the amount of the mixing water. This method is fairly effective through a certain range, say perhaps from a relative water content of 1.10 to 1.50, the limits depending upon the richness of the mixture, the nature of the fine aggregate and the coarse aggregate and other matters. Beyond this range, the excess of water has little effect on workability and tends to produce segregation in the mass, and the water will not remain uniformly distributed throughout the concrete.

In using added water to improve workability, it should always be kept in mind that the presence of additional water beyond what is termed the basic water content results in decreased strength in the concrete. In many cases the addition of 50 per cent of water to the amount which will give what is ordinarily termed normal consistency will produce a concrete having only about one-half the strength of that at basic water content. In many cases it will be possible to obtain the requisite mobility in other ways, such as a decrease in the value of b, and to maintain much of the strength lost through adding water. Frequently, excess water is used through lack of knowledge of the great loss of strength resulting.

The elements affecting workability and the means that may be used to improve it are worthy of much more study and investigation than have yet been given to the problem.

VIII. Applications to Proportioning of Concrete Mixtures

33. *Control of Properties of Concrete.*—Under the conditions more generally existing, concrete may not be as uniform and certain in qualities as some other materials of construction. The aggregate used in a given piece of work will vary considerably in gradation of particles, in moisture content, in void-producing properties, and in other qualities, and thus different batches of concrete may be expected to have different strength properties. The control of the mixing water and of the manner of mixing and placing is imperfect, and the operations may be conducted by workmen who do not have much knowledge of the effect of the manner of doing the work. Variations in curing conditions add to the uncertainties. What is wanted is to reduce the variations and uncertainties to a reasonable minimum. It is in this direction that the effort should be made to improve the state of the art.

The conditions in laboratory investigations are under better control. Aggregates of uniform quality and of a known condition of moisture may be provided. Proportioning, mixing, placing, and curing may be done with reasonable uniformity. The cement may be of uniform quality. With such control there need not be much divergence in the results. Figs. 25 and 36, pages 59 and 75, may be taken as illustrations of variations which may be expected under careful laboratory control for fine aggregates of widely different character. Each point on these diagrams represents the average of the strengths of three companion specimens; if individual tests were plotted, the points would be somewhat more scattered than those shown in the diagrams. It is seen that the extreme variation from the mean curve generally does not exceed 500 lb. per sq. in. (less than this at the lower strengths), or say 20 per cent variation from the mean curve. The average variation is much less, about 9 per cent in Fig. 25 and 15 per cent in Fig. 36. For fine aggregate having more nearly the same general characteristics, it may be expected that the variation will be less.

It is to be expected that the proportioning of the materials in the field will be subject to much greater variations than laboratory operations, and hence that a high degree of accuracy is difficult to attain. Variations in the measurement of the aggregates put into a batch of concrete will occur; different amounts of solids will get into the measuring receptacle, depending upon the moisture content and variations in gradation; especially will the method of depositing the coarse aggregate in the measuring receptacle by the workmen affect the absolute volume put in. The control of the proper amount of mixing water is rendered more difficult by the varying condition of the aggregates with respect to amounts of water already absorbed or carried on the surface of the particles.

It is believed that the use of the methods outlined in this bulletin will give useful information for aiding in the control of the uniformity of concrete in the field. It will be possible to determine the effect of a change in the grading of the aggregate. The methods may be applied to finding the differences in amount of mixing water required when different aggregates and different proportioning are used. The effect of excess of water is readily seen. The need for knowing just what amount of aggregate goes into the mixture (in real volume as a concrete-making material, not bulk) and its control is shown. In the following sections solutions of various practical problems are outlined. In some applications the relative strengths of different mixtures are as important as their absolute strengths; for such cases the principles here developed are of particular value.

34. *Bulk and Weight Measurement of Materials.*—The absolute volumes of the ingredients were used in the analysis and in the tests. For field use these may readily be converted into either bulk measurement or weight measurement. The methods and the formulas for use in the conversion will now be given.

Let W_a, W_b, and W_c represent the weight per unit of volume (bulk measurement) of the fine aggregate, coarse aggregate, and cement, respectively, in the condition and with the compactness they will have as measured on the work. W_c is conventionally taken as 94 lb. per cu. ft. The unit weights of the aggregates may be easily determined. It is important that these weights be obtained for exactly the conditions that will be used on the work, as the weights will vary considerably by different methods of depositing.

Let v_a, v_b, and v_c represent the voids per unit of volume in the fine aggregate, coarse aggregate, and cement, respectively, in the condition in which they will be measured on the work. The conventional weight, 94 lb. per cu. ft., makes v_c equal to 0.513. If the specific gravity of the aggregate is known or the weight per cubic foot of the solid material, v_a and v_b may be found from W_a and W_b. The weight of a cubic foot of solid particles of cement may generally be taken as 193 lb., and 167 lb. per cu. ft. is quite a common value for the solids of gravel and sand.

Let V_a, V_b, and V_c represent the bulk of the fine aggregate, coarse aggregate, and cement, respectively, in a cubic foot of concrete, and let V'_a and V'_b represent the bulk of the fine and coarse aggregates in the mixture corresponding to one cubic foot of cement (one bag). Then

$$V_c = \frac{c}{1 - v_c} = 2.05c, \quad V_a = \frac{a}{1 - v_a}, \quad V_b = \frac{b}{1 - v_b}, \ldots\ldots\ldots (9)$$

$$V'_c = 1,\ V'_a = \frac{0.487a}{(1 - v_a)\ c},\ \text{and}\ V'_b = \frac{0.487\,b}{(1 - v_b)\ c} \quad \ldots\ldots\ldots (10)$$

For aggregate with solid matter weighing 167 lb. per cubic foot the last two equations become

$$V'_a = \frac{81.3\,a}{W_a c},\ \text{and}\ V'_b = \frac{81.3\,b}{W_b c}. \quad \ldots\ldots\ldots (11)$$

If the aggregate is to be measured by weight, then the weights of the fine aggregate W'_a and coarse aggregate W'_b corresponding to one cubic foot of cement in the mixture will be, for aggregate with solid matter weighing 167 lb. per cu. ft.,

$$W'_a = \frac{81.3\,a}{c},\ \text{and}\ W'_b = \frac{81.3\,b}{c}. \quad \ldots\ldots\ldots (12)$$

Measurement of the aggregates by bulk is the method usually employed on work. It is known that bulk measurement is subject to considerable variation, due to variation in the gradation of the material, the amount of moisture contained, the manner of filling the measure, etc. Some of the variations found in concrete are caused by such variations in the measurement of the aggregate. It is felt by many that a method of measurement by weight would give much closer uniformity in the quality of the concrete, and especially that it would help in overcoming the variable effects of the bulking or swelling of fine aggregate when small amounts of moisture get into it. It would also ensure that the same amount of coarse aggregate would be used in each batch. For aggregates having about the same specific gravity this would be similar to the use of absolute volumes in proportioning.

35. *Calculation of Quantities of Materials.*—An important use of the data obtained from the mortar-voids test of a fine aggregate, together with the analytical relations outlined, is found in the calculation of quantities of materials required to produce a given volume of concrete. Tables of quantities of cement, sand, and gravel required for a cubic yard of concrete are given in various textbooks; but these tables can not be expected to apply to aggregates of widely different qualities or moisture contents, or to unusual proportions, or to different relative water contents. However, given the mortar-voids curves for a sand, the proportions of the concrete, and the relative water content, the quantities of materials required are easily found. As examples of the method of calculation of the quantities and of the difference in the results obtained by two ways of measuring the materials, Tables 8 and 9 are given. The quantities required for concretes of several proportions are included.

Table 8 is based on the use of dry materials, with the bulk of the aggregates measured by the standard laboratory method described in

Article 11. The aggregates used are Sand No. 24, having 33.2 per cent voids when measured under the conditions named, and a 3/8- to 1-in. gravel having 42 per cent voids when measured in a similar condition. Mortar-voids curves for Sand No. 24 are given in Fig. 19, page 51.

Table 9 is based upon loose measurement of aggregates containing moisture. It has been noted elsewhere that aggregates exposed to the weather on construction work take up moisture and increase in bulk materially over that obtaining in a dry state; also that materials measured by shoveling loosely into a measure take up more space than when tamped into the measure in a systematic way. It seems desirable, then, in computing a table of quantities to consider the moisture conditions and methods of measurement of materials that will exist on field work. Table 9 has been calculated on such a basis, the voids used, 0.47 in the sand and 0.46 in the gravel, being the values found in the materials as measured on a construction job.

The procedure in computing the quantities for the two tables may best be shown by a numerical example, such as that of the 1:1:2 mixture at basic water content in Table 8. In the first column the proportions by volume (bulk) are given in bold-faced type for each mixture. The proportions by absolute volume, given in the second line, are found by multiplying the values in the first line by the density of the individual material. These absolute volumes may be considered as the absolute volume in cubic feet in a one-bag batch. For the ratio of the absolute volumes of the sand and cement given for the first mixture, $\frac{a}{c} = 1.38$. It is found from Fig. 19 that $v_m = 0.294$; the density of the mortar in the mixture is 0.706. The volume of the mortar then is $\frac{c' + a'}{1 - v_m} = \frac{0.487 + 0.668}{0.706} = 1.63$ cu. ft. It is plain that for such mixtures the sum of the volume of the mortar and the absolute volume of the coarse aggregate is equal to the volume of the resulting concrete, if there are no voids outside the mortar. Adding 1.63 to the absolute volume of gravel (1.16) gives the volume of concrete in the batch as 2.79 cu. ft. The quantities for a cubic foot of concrete then are: sand, $\frac{1}{2.79} = 0.36$ cu. ft.; gravel, $\frac{2}{2.79} = 0.72$ cu. ft.; and cement, $\frac{1}{2.79} = 0.36$ cu. ft., or bags. Similarly, the quantities for a cubic yard of concrete will be: sand,

TABLE 8

QUANTITIES OF MATERIALS BY LABORATORY MEASUREMENT

Materials measured dry by standard laboratory methods
Sand No. 24 (Voids = 0.332), ⅜- to 1-in. gravel (Voids = 0.420)
No allowance has been made in these quantities to cover waste of materials

Proportions by Volume and by Absolute Volume	Relative Water Content	v_m from Fig. 19	w_m from Fig. 19	Volume in 1-bag batch cu. ft.			Quantities for one cubic yard of Concrete							
				$a'+c'$	Mortar	Concrete	Cement bbl.	Sand cu. yd.	Gravel cu. yd	Water lb.	Absolute Vol.—cu. yd.			
											a	b	c	d
1:1:2	1.00	.294	.240	1.15	1.63	2.79	2.41	.36	.72	265	.239	.415	.174	.828
0.487:0.668:1.160	1.20	.315	.288	1.15	1.70	2.86	2.37	.35	.70	318	.234	.407	.171	.812
$(a/c = 1.38)$	1.40	.340	.336	1.15	1.75	2.91	2.33	.34	.69	370	.230	.399	.168	.797
1:1½:3	1.00	.275	.223	1.49	2.05	3.79	1.78	.40	.79	236	.264	.459	.129	.852
0.487:1.002:1.740	1.20	.294	.268	1.49	2.11	3.85	1.75	.39	.78	279	.260	.452	.127	.839
$(a/c = 2.07)$	1.40	.315	.312	1.49	2.17	3.91	1.73	.38	.77	324	.256	.445	.125	.826
1:2:3	1.00	.266	.207	1.82	2.48	4.22	1.60	.47	.71	237	.316	.412	.115	.843
0.487:1.336:1.740	1.20	.288	.248	1.82	2.56	4.30	1.57	.47	.70	281	.311	.405	.113	.829
$(a/c = 2.76)$	1.40	.307	.290	1.82	2.63	4.37	1.54	.46	.69	325	.306	.398	.111	.815
1:2:3½	1.00	.266	.207	1.82	2.48	4.51	1.49	.44	.77	225	.296	.450	.107	.853
0.487:1.336:2.030	1.20	.288	.248	1.82	2.56	4.59	1.47	.44	.76	266	.291	.443	.106	.839
$(a/c = 2.76)$	1.40	.307	.290	1.82	2.63	4.66	1.45	.43	.75	308	.286	.436	.104	.826
1:2:4	1.00	.266	.207	1.82	2.48	4.80	1.41	.42	.83	214	.278	.483	.101	.862
0.487:1.336:2.320	1.20	.288	.248	1.82	2.56	4.88	1.38	.41	.82	253	.274	.476	.099	.849
$(a/c = 2.76)$	1.40	.307	.290	1.82	2.63	4.95	1.36	.40	.81	292	.270	.469	.098	.837
1:3:5	1.00	.275	.195	2.49	3.43	6.33	1.07	.47	.79	212	.316	.458	.077	.851
0.487:2.004:2.900	1.20	.294	.234	2.49	3.53	6.43	1.06	.47	.78	250	.312	.451	.076	.839
$(a/c = 4.14)$	1.40	.310	.273	2.49	3.61	6.51	1.04	.46	.77	289	.308	.445	.075	.828
1:3:6	1.00	.275	.195	2.49	3.43	6.91	0.98	.44	.87	199	.290	.503	.070	.863
0.487:2.004:3 480	1.20	.294	.234	2.49	3.53	7.01	0.96	.43	.86	234	.286	.497	.069	.852
$(a/c = 4.14)$	1.40	.310	.273	2.49	3.61	7.09	0.95	.42	.85	269	.282	.491	.068	.841

TABLE 9

QUANTITIES OF MATERIALS BY LOOSE MEASUREMENT

Sand No. 24, measured loose and containing 5 per cent moisture by weight (Voids = 0.47)
⅜- to 1-in. gravel, measured loose and containing 3 per cent moisture by weight (Voids = 0.46)
No allowance has been made in these quantities to cover waste of materials

Proportions by Volume and by Absolute Volume	Relative Water Content	v_m from Fig. 19	w_m from Fig. 19	Volume in 1-bag batch cu. ft.			Quantities for one cubic yard of Concrete							
				$a'+c'$	Mortar	Concrete	Cement bbl.	Sand cu. yd.	Gravel cu. yd.	Additional Water lb.	Absolute Vol.—cu. yd. a	b	c	d
1:1:2	1.00	0.308	0.256	1.02	1.47	2.55	2.65	.39	.78	177	.208	.423	.191	.822
0.487:0.530:1.080	1.20	0.330	0.308	1.02	1.52	2.60	2.59	.38	.77	233	.204	.416	.187	.807
($a/c=1.09$)	1.40	0.360	0.360	1.02	1.59	2.67	2.54	.37	.75	293	.198	.405	.183	.786
1:1½:3	1.00	0.285	0.233	1.28	1.79	3.41	1.98	.44	.88	124	.233	.475	.143	.851
0.487:0.795:1.620	1.20	0.308	0.280	1.28	1.85	3.47	1.94	.43	.87	170	.229	.467	.140	.836
($a/c=1.63$)	1.40	0.330	0.327	1.28	1.92	3.54	1.91	.42	.85	221	.225	.458	.138	.821
1:2:3	1.00	0.272	0.217	1.55	2.13	3.75	1.80	.52	.80	122	.283	.432	.130	.845
0.487:1.060:1.620	1.20	0.292	0.261	1.55	2.19	3.81	1.77	.52	.79	168	.278	.425	.128	.831
($a/c=2.18$)	1.40	0.312	0.305	1.55	2.25	3.87	1.74	.52	.78	216	.274	.418	.126	.818
1:2:3½	1.00	0.272	0.217	1.55	2.13	4.02	1.68	.50	.87	108	.264	.470	.121	.855
0.487:1.060:1.890	1.20	0.292	0.261	1.55	2.19	4.08	1.65	.49	.86	151	.260	.463	.119	.842
($a/c=2.18$)	1.40	0.312	0.305	1.55	2.25	4.14	1.63	.48	.85	196	.256	.456	.118	.830
1:2:4	1.00	0.272	0.217	1.55	2.13	4.29	1.58	.47	.93	95	.248	.504	.114	.866
0.487:1.060:2.160	1.20	0.292	0.261	1.55	2.19	4.35	1.55	.46	.92	137	.244	.497	.112	.853
($a/c=2.18$)	1.40	0.312	0.305	1.55	2.25	4.41	1.52	.45	.91	179	.240	.490	.110	.840
1:3:5	1.00	0.268	0.201	2.08	2.84	5.54	1.22	.54	.90	82	.287	.488	.088	.863
0.487:1.590:2.700	1.20	0.288	0.241	2.08	2.92	5.62	1.21	.53	.89	121	.283	.480	.087	.850
($a/c=3.27$)	1.40	0.307	0.281	2.08	3.00	5.70	1.20	.53	.88	161	.279	.473	.086	.838
1:3:6	1.00	0.268	0.201	2.08	2.84	6.08	1.11	.50	.99	68	.262	.533	.080	.875
0.487:1.590:3.240	1.20	0.288	0.241	2.08	2.92	6.16	1.10	.49	.98	103	.258	.526	.079	.863
($a/c=3.27$)	1.40	0.307	0.281	2.08	3.00	6.24	1.08	.48	.96	140	.255	.519	.078	.852

0.36 cu. yd.; gravel, 0.72 cu. yd.; and cement, $\frac{0.36 \times 27}{4} = 2.41$ bbl.

The water required per cubic foot of concrete at basic water content, working from w_m, is $0.240 \times \frac{1.63}{2.79} = 0.140$ cu. ft. To this must be added an allowance for absorption, which for these materials may be taken as 3 per cent of the absolute volume of the gravel (0.013 cu. ft.) and 2 per cent of the absolute volume of the sand (0.004 cu. ft.) making a total of 0.157 cu. ft. of water, equal to 9.8 lb. For 1 cu. yd. of concrete, the water required will then be 264 lb., equal to 15.7 per cent of the volume of the concrete.

The absolute volumes of the materials, *a*, *b*, and *c*, which are frequently desired, may be calculated as follows: $a = \frac{0.668}{2.79} = 0.208$, $b = \frac{1.16}{2.79} = 0.423$, and $c = \frac{0.485}{2.79} = 0.191$. The density of the concrete, *d*, is the sum of *a*, *b*, and *c*.

The quantities for the other mixtures given are calculated in a similar way. The computations for Table 9 were made similarly to those for Table 8, the only material difference being in regard to the computation of the amount of mixing water. It was considered that the assumed moisture content of the aggregate consisted of absorbed water and surface water; the absorbed water was taken as $0.02a + 0.03b$, as in Table 8; the remaining water, constituting the larger part, was considered to be available as mixing water. The additional amount of water required to provide the desired water content was then calculated and is given in the table of quantities.

Comparison of the quantities of the two tables shows that, with the same nominal proportions, the mixtures of Table 9 use from 10 to 13 per cent more cement than those of Table 8, and also that the bulk of the sand and the gravel is increased in the same proportion. The absolute volumes (and weights) of sand used, however, are 10 to 13 per cent less in Table 9, while the absolute volumes (and weights) of gravel are 3 to 5 per cent greater than those in Table 8. It is evident that the corresponding mixtures of Table 9 are richer, and therefore stronger and more expensive, than those of Table 8.

It should be noted that the increase in voids in the sand, by the method of measurement used in Table 9, over the voids in the same sand, by the method of measurement in Table 8, is due principally to the bulking produced by the moisture, while the increase of voids in the coarse aggregate is due to the lack of tamping and not to differences in moisture. It may be added that the moisture content assumed in the aggregates of Table 9 is perhaps greater than will be encountered in outdoor work with these materials.

Attention is called to the fact that in the calculations and in the tables no allowance has been made in the quantities to cover waste of materials.

The specific gravity and voids in the various aggregates used in this bulletin are given in Table 5. As a further guide to the probable limits of these quantities in concrete aggregates of the ordinary types, as found in the various parts of the United States, the following list showing the common range of variation and the average values is presented.* There may be occasional extreme values outside the ranges given. The voids noted were determined by the standard laboratory method, on dry materials; with loose measurement of materials the voids would be considerably greater.

Material	Specific Gravity		Voids	
	Common Range	Average	Common Range	Average
Sand	2.35–2.85	2.60	0.30–0.42	0.36
Stone Screenings	2.50–2.90	2.70	0.35–0.45	0.39
Gravel	2.45–2.80	2.60	0.35–0.41	0.37
Trap Rock	2.60–2.90	2.75	0.43–0.48	0.45
Limestone and Granite	2.45–2.70	2.60	0.42–0.46	0.43
Slag	1.90–2.40	2.25	0.37–0.45	0.42

36. *Design of Concrete Mixtures.*—The proportioning of concrete to attain a specified strength with the materials available is an important problem, especially for reinforced concrete structures in which the concrete is to be highly stressed. The problem of designing concrete mixtures may present itself in many ways, some of which are illustrated in the following examples:

(a) Given certain materials, to determine proportions that will produce a desired strength under certain conditions affecting workability.

In this case the mortar-voids curves for the fine aggregate used, such as those of Fig. 18, page 50, should be constructed and curves for the quantity $\frac{c}{v+c}$ which may be derived from the mortar-voids curves will also be found useful. Curves for $\frac{c}{v}$ or $\frac{v}{c}$ may, of course, be used instead of $\frac{c}{v+c}$.

* For more complete data on specific gravity, voids, and other properties of aggregates, see the following references:

1. Technologic Paper No. 58, U. S. Bureau of Standards.
2. Johnson, "Materials of Construction."
3. Baker, "Masonry Construction."

To take a numerical example, consider that concrete is to be made from Sand No. 23 and a ¼- to 1-in. gravel to produce a strength of 2000 lb. per sq. in. at 28 days at a relative water content of 1.40, which will be assumed to give the desired workability. Fig. 30, page 68, indicates that the desired strength should be divided by a reduction factor of 0.67, an operation which gives an expected strength of 3000 lb. per sq. in. at basic water content. From Fig. 26, the value of $\frac{c}{v + c}$ required is 0.39. From Fig. 18 at 1.40 relative water content, the value of $\frac{a}{c}$ to be used is 1.4. The limiting value of this coarse aggregate which may well be used is probably given by $b = 0.45$; that is, the limit of the bulk of the coarse aggregate may be expected to be about three-fourths of the volume of the concrete in place for the usual gradations of the gravel. Using $b = 0.45$ and noting from Fig. 18 that $v_m = 0.40$, $v = v_m\ (1{-}b) = 0.22$ (equation [3]), and the density is 0.78. $a + c = d - b = 0.78 - 0.45 = 0.33$. Since $\frac{a}{c} = 1.4$, $c = \frac{0.33}{2.4} = 0.137$ and $a = 0.193$.

If the proportioning is to be by volume, the quantities for a cubic foot of concrete are obtained by dividing c, a, and b by the respective densities of the cement, sand, and gravel. Thus, if the densities of the dry cement, sand, and gravel are 0.487, 0.62, and 0.58, respectively, as determined by the method of measurement used, the bulk quantities required for a cubic foot of mixed concrete are: cement, $\frac{0.137}{0.487} = 0.28$ cu. ft.; sand, $\frac{0.193}{0.62} = 0.31$ cu. ft.; gravel, $\frac{0.45}{0.58} = 0.78$ cu. ft. The approximate proportions, by bulk, then are 1 : 1.1 : 2.8.

From Fig. 18 the water, w_m, required for a cubic foot of mortar of the foregoing mixture is 0.40 cu. ft. Hence, in the concrete, which is 55 per cent mortar, the water required is $0.55 \times 0.40 = 0.22$ cu. ft. In addition, an allowance must be made for absorption of water by the aggregate during mixing and placing; for materials of the kind specified an addition of $0.03\ b + 0.02\ a$ or 0.017 cu. ft. has been found sufficient, thus making a total of 0.237 cu. ft. or 14.8 lb. of water per cu. ft. of fresh concrete. If the aggregates are in such condition that their surfaces carry water, this surface water should be considered as mixing water and a proper allowance made for the amount of water to be used. If the water-cement ratio is desired, since the bulk of the cement per cu. ft., C, is 0.28 cu. ft. and the water exclusive of absorption is 0.22 cu. ft., $\frac{w}{C} = \frac{0.22}{0.28} = 0.79$.

If the materials are to be proportioned by weight, the weights for a cubic foot of mixed concrete are obtained by multiplying c, a, and b by 62.4 times the specific gravities of the materials. Using the quantities in the example of the preceding paragraph, the weight of cement is $62.4 \times 3.10c = 25.6$ lb.; of sand, $62.4 \times 2.67a = 32.2$ lb.; and of gravel, $62.4 \times 2.69b = 75.7$ lb. Thus a cubic foot of concrete requires 134.4 lb. of solids and 14.8 lb. of water. By proportion, the quantities for a one-bag batch are: cement, 94 lb.; sand, 114 lb.; gravel, 269 lb.; and water 54 lb.

(b) Given a definite amount of cement per unit volume of concrete and a value of $\frac{a}{c}$, $\frac{b}{a}$, or b to be used, to produce concrete of the most favorable strength for a given condition of workability.

Let it be assumed that a cement content of 6 bags per cubic yard is specified, and that the proportion of cement to sand is to be 1 : 2 by bulk. Then the absolute volume of cement per cubic foot of concrete is $c = \frac{6 \times 0.487}{27} = 0.108$ cu. ft. With a sand such as the Kaw River Sand (No. 4, Table 4) having voids of 33.3 per cent, as used, $\frac{a}{c} = \frac{2 \times 0.667}{0.487} = 2.74$, and $a = 2.74 \times 0.108 = 0.296$. If the desired workability is such that a relative water content of 1.20 can be used, from Fig. 21, v_m is 0.305, and from Fig. 23, $w_m = 0.254 \times 1.20 = 0.305$. From equation (2), $\frac{0.108 + 0.296}{0.695} + b = 1.0$; and $b = 0.419$. The density of the concrete is 0.823, and the ratio $\frac{c}{v + c}$ is equal to $\frac{0.108}{0.177 + 0.108} = 0.379$. From Figs. 29 and 30 the strength of this concrete should be 2200 lb. per sq. in.

The bulk quantities and the amount of mixing water may be found as explained in detail in Case (a).

As a second problem, let c and the ratio $\frac{b}{a}$ of the absolute volumes of coarse and fine aggregate be specified. Again using $c = 0.108$ and a relative water content of 1.20, let the ratio $\frac{b}{a}$ be specified as 1.50. In this problem a solution is found by trial, one of the unknowns being assumed, and the resulting values being then corrected. For example, suppose Sand No. 24 (Fig. 19) is to be used. With $c =$

0.108, and $\frac{b}{a} = 1.50$, try $v_m = 0.30$. Then by equation (2), $\frac{0.108 + a}{0.70} + 1.50a = 1.0$; $a = 0.289$ and $\frac{a}{c} = 2.67$. From the curve of Fig. 19, for this value of $\frac{a}{c}$, $v_m = 0.285$. Hence, rewriting the equation, $\frac{0.108 + a}{0.715} + 1.50a = 1.0$, the result is $a = 0.293$, $b = 1.50a = 0.440$, and $\frac{a}{c} = 2.71$. It is evident that v_m is not changed appreciably by a small change in $\frac{a}{c}$, so that the result of the second trial is usually sufficiently accurate. With these proportions the density is $d = 0.108 + 0.293 + 0.440 = 0.839$; $\frac{c}{v + c} = \frac{0.108}{0.269} = 0.402$, and the strength from Figs. 29 and 30, is 2700 lb. per sq. in. The volumes of the materials (bulk) per cu. ft. of concrete are seen to be as follows: cement, $\frac{0.108}{0.487} = 0.221$; sand, $\frac{0.293}{0.668} = 0.439$; and coarse aggregate (say 40 per cent voids), $\frac{0.440}{0.60} = 0.733$. The proportions by bulk then are 1 : 2 : 3 . 3.

As a third form of problem, assume that values of c and b are specified. Thus, c may again be taken at 0.108 and b at 0.42. Again using a 1.20 relative water content and Sand No. 24, the solution is effected by trial. From equation (2), $\frac{0.108 + a}{1 - v_m} + 0.42 = 1$. Assuming $v_m = 0.285$, a is found to be 0.307, and $\frac{a}{c}$, 2.84. As the value of v_m from Fig. 19 with $\frac{a}{c} = 2.84$ is 0.285, this solution is satisfactory. The density is found from the relation, $d = 0.108 + 0.307 + 0.420 = 0.835$. The bulk quantities and strength may now be found as in the preceding examples.

(c) Given the ratio of the cement to total aggregate and the conditions which will produce the desired workability, to produce concrete of the best strength.

Assume that a 1:5 mixture (1 part of cement to 5 parts of mixed fine and coarse aggregates) is to be used at 1.10 relative water content, and that the materials are Attica sand (voids = 33.2 per cent when dry) and ⅜- to 1½-in. gravel (voids = 40 per cent). Any method of proportioning by the use of the volume of the mixed aggregates requires additional data, such as the shrinkage in the volumes of fine and coarse aggregates when mixed together, and the yield (the amount of concrete resulting from the use of a unit volume of mixed aggregate). The shrinkage may range from 0 to 30 per cent, with common values ranging from 12 to 20 per cent. With a 1:5 mixture it was found in Series 2G that the yield ranged from 0.96 to 1.18, with a value of about 1.00 for the mixtures giving the highest strengths, and with the larger values in mixtures containing a large amount of the fine material producing the bulking or swelling effect. With wetter mixtures the yield increases slightly; an increase in the richness of mixture produces a large increase in the yield. For the mixture specified above a yield of say 1.02 may be assumed, from which it is seen that 1 volume of cement and 5 volumes of aggregate would produce 5.10 volumes of concrete; hence $c = \frac{1.0 \times 0.487}{5.10} = 0.095$. The next step is to choose the value of b, which should be made as large as possible without sacrificing workability; consider in this case that $b = 0.45$. Then the cement content for the mortar contained in the mixture must be $\frac{0.095}{1-b} = 0.173$, whence from Fig. 19, (interpolating for 1.10 relative water content), $\frac{a}{c} = 3.20$ and $a = 0.304$. The density is $0.095 + 0.304 + 0.450 = 0.849$, $\frac{c}{v + c} = \frac{0.095}{0.246} = 0.385$ and the strength from Figs. 29 and 30 is about 2700 lb. per sq. in.

If the amounts of separately measured fine and coarse aggregates are desired, their shrinkage when mixed must be determined by trial. For the aggregates and proportions used here the shrinkage has been estimated to be about 20 per cent, so that a volume of about $\frac{5.0}{0.80}$ or about 6.25 cu. ft. of separately measured aggregates, 2.35 cu. ft. of fine and 3.90 cu. ft. of coarse aggregate, would be required for a one-bag batch. With this information and the value of v_m from the mortar-voids curves of Fig. 19, the value of the yield used may be checked up. For a one-bag batch the volume of concrete may be computed as $\frac{0.487 + (2.35 \times 0.668)}{0.722} + (3.90 \times 0.60) = 5.19$ cu. ft., and the yield is 1.04.

As in the preceding examples, the mixing water required by the mortar may be computed from values of w_m from Fig. 19 and the value of b; to this is added the necessary allowance for absorption by the aggregates. Thus, since w_m is found by interpolation to be equal to 0.220 and b is 0.45, the water required for the mortar in one cubic foot of concrete is $0.220\ (1-b) = 0.121$ cu. ft. To this may be added the allowance for absorption by the aggregate of perhaps 1 per cent by weight for these materials, or 2.7 per cent by absolute volume, amounting to 0.021 cu. ft. The total water required then is 0.142 cu. ft., or 8.9 lb. per cubic foot of concrete, or $8.9 \times 5.19 = 46$ lb. for a one-bag batch.

It should be noted that the use of separate proportions for the fine aggregate and coarse aggregate obviates the need of determining the shrinkage and yield of the mixed aggregate found necessary in the foregoing examples of proportioning by ratio of cement to mixed aggregate.

(d) Given a required strength of mortar, to compare two fine aggregates.

Assume that a mortar having a strength of 2000 lb. per sq. in. at 1.30 relative water content is to be made. From Figs. 29 and 30, this requires a value of $\frac{c}{v + c} = 0.38$. It may also be assumed that Sands Nos. 23 and 24 are available. From Fig. 18 for Sand No. 23, it is found by interpolation that for the above conditions $\frac{a}{c} = 1.80$, $v_m = 0.365$, whence $c = \frac{1-v_m}{\frac{a}{c} + 1} = 0.227$ and $a = 0.408$. Similarly, from Fig. 19 for Sand No. 24, for this case $\frac{a}{c} = 2.85$, $v_m = 0.295$, $c = 0.183$, and $a = 0.522$. This comparison of quantities in a given volume of mortar shows that the use of Sand No. 23 requires 24 per cent more cement than Sand No. 24, but on the other hand Sand No. 24 requires 28 per cent more absolute volume of sand, or 14 per cent more bulk than the former.

A comparison of concretes made from these two sands will show a variation in quantities of sand and cement similar to that found with mortars. Thus, if a value $b = 0.40$ were used with both sands, the above values of a and c for the mortar should be multiplied by 0.60 to find the absolute volumes of sand and cement per unit volume of concrete. If a larger value of b could be used with the fine sand (No. 23) than with the coarser sand, the high cement requirement of the fine sand could be reduced somewhat. A comparison of costs of various mixtures will be given in Article 39.

(e) Given arbitrary proportions of the materials, to determine the probable strength of the concrete.

The strength naturally depends upon the quality of the materials and the relative water content, as well as upon the proportions; it can be estimated quite closely from information obtained through the mortar-voids test.

Assume the concrete to be of the proportions 1 : 2 : 3½ by volume and also that a relative water content of 1.40 is needed to give the desired workability. Consider that Sand No. 23, having 38 per cent voids, as measured and used on the work, is to be used and that the coarse aggregate is a ½- to 1½-in. gravel having 42 per cent voids. The voids in the dry cement will be taken at 51.3 per cent. The absolute volumes in a one-bag batch then are: cement, 0.487 cu. ft.; fine aggregate, $2 \times 0.62 = 1.24$ cu. ft.; and coarse aggregate, $3\frac{1}{2} \times 0.58 = 2.03$ cu. ft. The value of $\frac{a}{c}$ is $\frac{1.24}{0.487} = 2.54$. From Fig. 18 the voids in the mortar, v_m, $= 0.375$. The cement content of the mortar, c_m, $= 0.177$; and the cement-space ratio, $\frac{c}{v + c}$, $= 0.320$. By reference to Figs. 29 and 30 it is seen that the probable strength of concrete containing these proportions of sand and cement is 1300 lb. per sq. in. at the age of 28 days.

The volume of the one-bag batch of concrete $= \frac{0.487 + 1.24}{0.625} + 2.03 = 4.79$ cu. ft. of concrete. The absolute volumes of the materials in a unit volume of concrete are as follows: $c = \frac{0.487}{4.79} = 0.102$; $a = \frac{1.24}{4.79} = 0.259$; $b = \frac{2.03}{4.79} = 0.424$; and $d = a + b + c = 0.785$. From these values, $\frac{c}{v + c} = \frac{0.102}{0.317}$ or 0.32 as was found from the mortar-voids data. It will be seen that the value of b comes within the limiting value for a uniform workable mixture.

Another method of estimating the strength is by the use of the water-cement ratio curve of Fig. 35. For the 1.40 relative water content used in the preceding example it is seen from Fig. 18 that $w_m = 0.335$, $c_m = 0.177$ and the ratio of volume of water to bulk of cement is $\frac{w}{C} = 0.487 \times \frac{w_m}{c_m} = 0.92$. From Fig. 35 the strength is found to be 1300 lb. per sq. in. at 28 days. The water-cement relation will be found useful in studying the effect of varying the water. Thus, if the water in the batch is decreased 10 per cent the value of $\frac{w}{C}$ becomes $0.92 \times 0.90 = 0.83$ and the indicated strength is 1600 lb. per sq. in.; an increase of 10 per cent over the original amount of water makes $\frac{w}{C}$ equal 1.01 and the corresponding strength is 1100 lb. per sq. in.

37. *Comparison of Quality of Fine Aggregates.*—It is usually specified that fine aggregates shall be clean, hard, strong, and free from organic and other deleterious substances. With these requirements fulfilled, the gradation of the particles becomes an important element in fixing the concrete-making properties of a material. It is difficult, however, to compare the quality of sands or other fine aggregates by the use of the sieve analysis. An ideal sieve-analysis curve may be of little value in comparing the quality of several sands of quite different granular composition. Even in making specifications there are great differences in opinion as to what percentage may be permitted to pass certain sieves and as to whether certain minimum percentages should be specified. One reason for such differences in views is that the concretes used include marked differences in the richness of the mortar; for a common richness of mortar a closer agreement would be found. A similar trouble is found with the tension or compression test comparing mortar made with the given fine aggregate and mortar made with a standard sand; the comparison should be made for the given richness, and a different relative value may be needed for different degrees of richness. A further objection to such tests lies in the necessity for awaiting a period of hardening, such as 28 days, before information for making comparison is available.

The mortar-voids test furnishes another means of comparing the concrete-making quality of fine aggregates, which has several excellent features. The characteristic mortar-voids curve of a fine aggregate shows its properties throughout the range of richness. The probable strength of mortar and concrete made up with the material in different proportions may be estimated with a fair degree of accuracy. Unlike strength determinations, the test may be made at once.

For the purpose of making such comparisons the mortar-voids curves, such as those shown in Fig. 20, should be plotted from the data of the mortar-voids test for the fine aggregates to be compared. If rather wet concrete is to be used, it will be well also to plot the mortar-voids curves for the relative water content that will probably be used. As an example, consider fine aggregate No. 3 (Algonquin sand), No. 12 (crushed granite), No. 15 (crushed slag), No. 20 (Elgin sand, washed), and No. 0 (standard Ottawa sand). Nos. 3, 12, and 20 have quite similar sieve analyses. No. 15 is peculiar in having a large proportion of particles between sieves No. 8 and No. 4. Take values of the mortar-voids from Fig. 20 for $\frac{a}{c}$ of $1\frac{1}{2}$ (rich mortar mixture), $2\frac{1}{2}$ (medium mortar mixture), and 4 (lean mortar mixture). It will be seen that at $\frac{a}{c} = 1\frac{1}{2}$ the mortar voids for Nos. 3, 12, and 15 are almost the same, about 0.30. No. 0 gives less voids and No. 20 greater voids. At $\frac{a}{c} = 2\frac{1}{2}$, the voids in all mortars are somewhat smaller than at $\frac{a}{c} = 1\frac{1}{2}$, the order from least to greatest being, Nos. 0, 12, 3, 20 and 15. At $\frac{a}{c} = 4$, in the order from least to greatest, the voids are: No. 12. 0.26; No. 3, 0.28; No. 20 and No. 0, 0.30; and No. 15, 0.36. If a relative water content of 1.40 were used (see Fig. 22), the values would give a similar comparison, except that No. 12 will have a greater relative increase of voids with the added water than the others, due to the voids at basic water content being relatively free from air. It is evident that for rich mixtures and basic water content, the Ottawa standard sand will give less voids than the other fine aggregates, and that for lean mixtures the crushed granite will give the least voids and the crushed slag the most. For wetter concrete, with 1.40 relative water content, the comparisons are about the same for the rich mixture; for the lean, the voids for No. 15 are quite high, and there is little difference among the others. A comparison of fine aggregates having dissimilar granular composition, such as Nos. 1, 23, and 2, would give more striking differences.

A comparison of the relative strengths of mortars and concretes made with the fine aggregates may be made by the use of the relation between strength and the voids-cement ratio—equation (8) for the materials of Series 211,—or the cement-space ratio—also given by equation (8),—which are applicable to both mortars and concretes. Figs. 25 and 26 give curves representing these equations for the constants derived from Series 211. The details of the procedure to be used have been given in Article 36.

It may be added that, when strength comparisons are made with mortars of standard sand, it would be well to have the ratio of cement and sand (both the Ottawa sand and the others) made the same as that to be used in the concrete, and that for application to the most common concrete mixtures a mortar of 1 part cement and 2 parts fine aggregate by weight, corresponding approximately to $\frac{a}{c} = 2\frac{1}{2}$, would give more representative results than the usual ratio of 1:3.

38. *A Method of Specifying Quality of Fine Aggregate.*—The ease of making the mortar-voids test and the definiteness in the location of the point of minimum volume and in the increase of voids with the addition of water, together with the relation of the voids in the mortar to the strength of the mortar and concrete, suggests the value of this method as a means of rating fine aggregates so far as the effect of the size of particles is concerned. There is such a variety of fine aggregates available in different parts of the country, very divergent in character, and the needs in concrete construction are so divergent that difficulty has been found in specifying a gradation or a size of particles that is generally acceptable. The test of a sand to determine the mortar voids, with characteristic mortar curves at several water contents, such as are shown in Fig. 18, will enable a study to be made of the concrete-making properties of the sand, and it may be rated or judged from these curves. In the making up of specifications for fine aggregate, generally only the voids at minimum mortar volume for a definite richness of mixture need be considered, though perhaps the voids at the relative water content to be used in the concrete may be a better criterion. The requirement may take one of several forms:

(1) A maximum limit of voids for a given mixture may be specified, say for a ratio $\frac{a}{c}$ of $2\frac{1}{2}$, or for the ratio to be used in the concrete mixture. The value of this specified maximum may then be fixed for a given locality or a given purpose, according to the materials available.

(2) A maximum limit of voids for a given mixture may be specified for use with the specified proportion of cement to aggregate; a fine aggregate with mortar voids exceeding this, up to a given limit, may be used, provided the amount of cement per unit of volume of concrete is increased sufficiently to give a concrete coming within a cement-voids ratio which would be specified.

(3) The limit of voids may be set quite high or omitted and the requirement made that for the fine aggregate used the amount of cement and the proportions of the cement, fine and coarse aggregate, and water shall be such that the cement-voids ratio of the resulting concrete shall come within a specified limit at a given relative water content.

It would appear that all three forms would be useful; one might be preferable under some circumstances and another under others.

TABLE 10

COSTS OF MATERIALS FOR VARIOUS CONCRETE MIXTURES

All mixtures have a value of $\frac{a}{c}$ equal to 2.75 and 1.40 relative water content.

Prices assumed: cement, $2.50 per barrel, Attica sand and gravel, $2.50 per cu. yd.

Mixture	Concrete in One Bag Batch cu. ft.	b	Cost of Materials in Dollars	
			per batch	per cu. yd.
1:2:0	2.64	.00	.810	8.28
1:2:1	3.24	.19	.903	7.52
1:2:2	3.84	.31	.996	7.00
1-2:3	4.44	.41	1.088	6.61
1·2:4	5.04	.48	1.181	6.33

If the general quality of the fine aggregate in other respects than size and gradation were not known, as in a sand which may be weak in structure, or which may have a poor surface for adhesion, or in a new and untried light aggregate, a strength test should be made in such a way as to give a comparison with mortar or concrete made with a fine aggregate of known quality. For such a test the two fine aggregates should have the same gradation of particles.

39. *Methods of Comparing Cost of. Concrete Mixtures.*—The amount of cement in a mixture is often used as a criterion of the cost of the concrete, but it must be borne in mind that less aggregate is used in the richer mixtures and that this will partly compensate for the extra cost of the cement, especially in localities where aggregates must be transported a long distance and therefore are expensive. The relative costs of materials vary greatly in different localities, but it will be assumed for the purpose of illustration and comparison in this article that the cost of a barrel of cement is equal to the cost of a cubic yard of sand or gravel, and this price will be taken at $2.50. The economic aspect of several problems may be studied on this assumption.

(a) The effect of varying the proportion of the coarse aggregate, with strength kept constant.

According to the principles outlined in this bulletin, the ratio $\frac{a}{c}$ of the mortar content of the concrete is of primary importance in determining the voids and the cement content, and hence, also, the strength of the concrete mixture for a given consistency and grade of materials. It appears, therefore, that with a constant proportion of cement and sand, the strength will remain constant or nearly so, even when a large variation is made in the proportion of the

TABLE 11

QUANTITIES AND COST OF MATERIALS IN CONCRETE MIXTURES FOR DIFFERENT FINE AGGREGATES

Estimated compressive strength, 1500 lb. per sq. in. 1.40 relative water content. Prices assumed: cement, \$2.50 per barrel, fine and coarse aggregate, \$2.50 per cu. yd.

Fine Aggregate No.	Description	c	a	b	$\frac{a}{c}$	Bulk Proportions	Quantities per cubic yard			Cost of Materials, dollars per cu. yd.
							Cement bbl.	Fine Aggregate cu. yd.	Gravel cu. yd.	
2	Trap screenings	.106	.255	.45	2.40	1:1.74:3.39	1.48	.380	.75	6.52
10	Dune sand	.127	.196	.45	1.54	1:1.23:2.84	1.77	.323	.75	7.11
15	Crushed slag	.116	.228	.45	1.97	1:1.69:3.10	1.61	.403	.75	6.91
18	Medium sand	.114	.233	.45	2.05	1:1.56:3.16	1.59	.365	.75	6.76
23	Fine sand	.116	.227	.45	1.95	1:1.53:3.10	1.61	.368	.75	6.82
24	Well-graded sand	.101	.265	.45	2.54	1:1.85:3.56	1.41	.397	.75	6.39

coarse aggregate. The value of b, of course, must not exceed the limit for good workability, say usually 0.40 to 0.45 for ordinary materials. The quantities of materials involved in the cost of the various mixtures given in Table 10 were calculated by the methods already outlined. The costs of the various mixtures, presumably having about the same strengths, show that the cost decreases as b increases. There is no great advantage in using a value of b greater than 0.45. Coarser mixtures would be difficult to place and are likely to result in uneven quality of concrete.

(b) The effect of quality of sand upon the cost of concrete.

Consider that a concrete with an estimated strength of 1500 lb. per sq. in. at 1.40 relative water content is desired, and that gravel is used. For the comparison of fine aggregates, use fine aggregates Nos. 2, 10, 15, 18, 23, and 24. From Figs. 29 and 30 it is found that for a strength of 1500 lb. per sq. in. at 28 days and with 1.40 relative water content, $\frac{c}{v + c} = 0.36$, and $\frac{v}{c} = 1.77$. Although a somewhat higher value of b may be used with a fine sand than with a coarse one, the difference is not great and for this comparison a value of $b = 0.45$ will be taken for all mixtures. By using the mortar-voids curves of Fig. 22 and choosing values of a and c which make $\frac{c}{v + c} = 0.36$, the proportions and quantities given in Table 11 have been calculated by the principles already outlined. As before, \$2.50 is used as the price of a barrel of cement, and $2.50 as that of a cubic yard of each aggregate. If it were the case that the price of fine sand, for example, was considerably less than that of the better sands, the comparison in cost of concretes may readily take this into account. If it is known how much greater the value of *b* may be taken for fine sands than for coarse, this condition also may be considered.

(c) Relative water content and its effect upon the cost of concrete of given strength.

Suppose that a concrete is required to have an estimated strength of 2500 lb. per sq. in. at 28 days. To compare costs for different consistencies, consider concrete mixtures made with Sands Nos. 23 and 24 at relative water contents of 1.0, 1.2, and 1.4. By using the methods of Article 35 and considering that in all cases a gravel having 40 per cent voids will be used, the quantities given in Table 12 have been determined. The cost of the different mixtures is based upon the same prices of materials as those used in Tables 10 and 11.

Table 12 shows a considerable increase in cost in using a wet concrete mixture which has enough additional cement to produce the same strength as the drier mixtures. The cost of the concrete in which Sand No. 23 is used at 1.40 relative water content seems abnormally high, but the strength of 2500 lb. per sq. in. is nearly at the limit of the possibilities of this sand at this water content, and a mortar that approaches a neat cement mixture is required to produce the required strength. This condition will often obtain when a rather high strength is required from a poor sand at a high water content. The results with Sand No. 24 are representative of costs of concrete

TABLE 12

COST OF MATERIALS FOR CONCRETES OF DIFFERENT WATER CONTENT
Estimated compressive strength, 2500 lb. per sq. in.
Prices assumed: cement, $2.50 per barrel, sand and gravel, $2.50 per cu. yd.

Sand No.	Relative Water Content	*c*	*a*	*b*	Quantities per cu. yd. of Concrete			Cost of Materials, dollars per cu. yd.
					Cement bbl.	Sand cu. yd.	Gravel cu. yd.	
23	1.0	.110	.253	.45	1.53	.42	.75	6.75
	1.2	.128	.226	.45	1.78	.37	.75	7.25
	1.4	.217	.051	.45	3.02	.09	.75	9.65
24	1.0	.087	.317	.45	1.21	.47	.75	6.08
	1.2	.103	.289	.45	1.44	.43	.75	6.55
	1.4	.132	.242	.45	1.84	.38	.75	7.43

in which a well-graded sand is used, since very rich mortar mixtures are not required here to develop the specified concrete strength.

The foregoing examples of comparisons of the cost of concretes are illustrative of the applicability of the method.

IX. Conclusion

40. *Comments.*—The combination of the mortar-voids tests, the analytical relations of cement, aggregates, voids, and water, and the experimental determination of strengths, by the methods outlined in this bulletin, offers a useful method for the study of materials, the design of concretes, the calculation of quantities, the comparison of mixtures, and the estimation of strength.

The mortar-voids test takes account of the factors which give uncertainty to the sieve-analysis function. If the requirements for structural strength and surface adhesibility be otherwise guarded, it will enable new or untried materials to be judged. It will permit easy comparison of aggregates. It is of interest in showing what range of mortar richness will produce the best results with a given fine aggregate and thus in aiding to find conditions under which the use of what may have been thought to be a poor aggregate can be permitted.

The design and estimation of strength of mixtures may be made for a given amount of cement, for a given limit of quantity of coarse aggregate or any smaller value, or for a given ratio of cement and sand. A reduction curve may be obtained for finding the relative strengths for different relative water contents, based on the basic water content, that giving the minimum volume of the mortar and concrete.

The use of the absolute volume of the materials gives simple and convenient methods of analysis and calculation, and in many ways is advantageous. To think in terms of the absolute volume of the materials in a unit of volume of mortar or concrete itself gives an advantage in making comparisons and judging effects. When the density of the material as it is found by the method of measurement to be used on the work is known, the volumes by bulk measurement may easily be obtained. The advantage of measuring aggregates by weight, in producing concrete of uniform quality, is apparent. Since the specific gravity of sands in a given locality varies but little, the use of an average specific gravity is sufficiently accurate for most purposes.

The method of treatment brings out the old principle that other things being similar the strength of the concrete is equal to the strength of the mortar of the concrete. "Other things" refers to the water content, to the degree of compactness, to the conditions of storage, etc. Limitations in such matters as amount of coarse aggregate may be expected to come within ordinary practice in well-made concrete.

It should be noted again that the equation for strength of concrete in terms of the voids-cement ratio or cement-space ratio will be dependent upon the quality of the cement, and that some variation in the constants of the equations may be expected. The tests were made at an age of 28 days, but equations of the same general form for strength at other ages may be determined from experimental data.

The matter of allowance for the water absorbed by the coarse aggregate during the period of placing and setting will need careful attention in so far as this affects the amount of water left to be counted as mixing water. Any water carried into the mixture on the surface of the particles of the coarse aggregate and not absorbed by the aggregate during the period of mixing and placing will, by the technique described for the tests, be included in the amount considered as mixing water.

Further study and experience will doubtless develop limitations and suggest modifications and standardization of methods. To what extent and in what way account should be taken of the possibility that the sum of the volume of the mortar and the coarse aggregate may not reach unity, is one question. What limit to the volume of the coarse aggregate may best be used for different aggregates, different water contents, and different kinds of construction, may well be studied. The shrinkage of wet mixtures and rich mixtures needs consideration. The effect of air voids upon the strength of concrete, and also the effect of the absorption of water by the aggregates during and after the setting of the cement, should be studied. The effect upon strength of the addition of water beyond basic water content for a variety of fine aggregates is worthy of experimentation and study, as well as methods of allowing for the reduction in strength accompanying an increase in water content. These and other topics deserve to be taken up by a number of research laboratories and given careful study, regardless of whether the methods outlined in this bulletin are considered the most satisfactory for the study of the designing, comparison, and determination of quantities for concretes.

41. *Résumé of Method.*—In review of the steps to be taken in the application of the methods outlined in this bulletin, the following is presented:

(1) The mortar-voids curve is determined experimentally. For certain purposes the use of a general mortar-voids curve of

materials having the same characteristics may be sufficiently accurate. Mortar-voids curves may well be made up for two or more water contents.

(2) Curves giving the water content, cement content, and ratio of voids to cement for varying ratios of fine aggregate to cement may be made up from the data of the mortar-voids test.

(3) An equation or a diagram giving the relation between the strength of mortar and concrete at a given age for the common run of cements should be available, as should a reduction curve for the effect of relative water content.

(4) The limiting value should be known or judged for the absolute volume of the coarse aggregate that may be used under the conditions of placing and tamping obtaining on the work.

(5) For use in making measurements by bulk, the density of the fine aggregate and the coarse aggregate for the method of measurement to be used on the work should be known.

(6) With such information the use of the analytical relations given by the equations will permit designs, comparisons, and estimates of strength and quantities of materials to be made.

It is felt that the method of attack outlined in this bulletin will be helpful in the work of analyzing, comprehending, and judging the effects of the various elements entering into the strength properties of concrete, as well as in many applications to the problems of practice.

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