UNIVERSITY OF ILLINOIS BULLETIN

ISSUED WEEKLY

Vol. XIV APRIL 30, 1917 No. 35

[Entered as second-class matter Dec. 11, 1912, at the Post Office at Urbana, Ill., under the Act of Aug. 24, 1912.]

THE EFFECT OF MOUTHPIECES ON THE FLOW OF WATER THROUGH A SUBMERGED SHORT PIPE

BY

FRED B. SEELY

BULLETIN No. 96

ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

PRICE: TWENTY-FIVE CENTS

EUROPEAN AGENT

CHAPMAN & HALL, LTD., LONDON

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THE EFFECT OF MOUTHPIECES ON THE FLOW OF WATER THROUGH A SUBMERGED SHORT PIPE

BY
FRED B. SEELY
ASSOCIATE IN THEORETICAL AND APPLIED MECHANICS

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THE EFFECT OF MOUTHPIECES ON THE FLOW OF WATER THROUGH A SUBMERGED SHORT PIPE

I. Introduction

1. *Preliminary.*—This bulletin presents the results of experiments on the flow of water through a submerged short pipe with and without entrance and discharge mouthpieces of a variety of angles and lengths. It treats of the loss of head which occurs when a stream contracts or expands under differing conditions of flow and emphasizes the marked effect that turbulence of flow may have upon the amount of head lost. The discussions have a direct bearing upon various problems in hydraulic practice which involve the contraction and expansion of a stream in flowing through passages.

Comparatively little experimental work has been done to determine the value of conical mouthpieces of various angles and lengths in reducing the lost head at the entrance to and discharge from a submerged pipe, particularly for mouthpieces of the sizes and proportions comparable with those met in engineering practice. The need for such experiments is, therefore, apparent. The minimizing of the lost head due to the contraction and expansion of a stream may be of considerable importance in a variety of hydraulic problems; for example, the intake to a pipe particularly when the pipe is of short length and of large diameter, the suction and discharge pipes of a low head pump, the reduction or expansion from one pipe to another of different diameter or of different shape, the passages through a large valve, the passages through locomotive water columns, the draft tube to a turbine, the connection from a centrifugal pump to a main, the sluice ways through dams, the slat screens at head gates, culverts and short tunnels, jet pumps, the Boyden diffuser as formerly used for the outward flow turbine, the Venturi meter, the suction and discharge pipes of dredges, and the guide vanes and runner of a turbine.

Losses due to this cause are difficult to estimate and easy to overlook. Even where such losses are in themselves of little consequence as compared with other quantities involved, they may have a considerable influence upon subsequent losses on account of the turbulent motion started by the contraction or expansion. The efficiency of a drainage pump or other low head pump, for example, may be increased by an entrance mouthpiece on the suction pipe because it allows the pump to receive the water in a smoother condition of flow. It is well known that a turbine must receive the water from the guide vanes without shock if, in the subsequent flow through the runner, the energy of the water is to be absorbed efficiently by the turbine. The loss of head through a Venturi meter may be considerably increased if the meter is placed too short a distance downstream from a valve, elbow, or other obstruction or cause of disturbance in the pipe. The friction factor for a pipe following an obstruction or bend may be changed by the disturbance thus caused; the lost head at the entrance to a pipe, particularly when inward-projecting, may be more than that ordinarily assumed for a tube three diameters long. There is but little definite knowledge on the whole subject of the effect of abnormal conditions, and it offers a large scope for investigation. The fact that a comparatively small change in the form of the blades of a turbine runner may result in a large effect on the efficiency of the turbine should prove suggestive when estimating the probable effect of turbulent flow in less severe or critical cases. It is also worth mentioning in this connection that the recent advances in turbine design have been due largely to the attention given to the approach channels to the guide vanes and to the design of the draft tube.

The flow of water usual in engineering practice is more or less turbulent. The general equation of energy, or Bernoulli's theorem, so generally used in hydraulics, applies only when the particles of water move with uniform velocity in parallel stream-lines. Although this condition of flow seldom occurs, satisfactory analyses may often be made by using an average velocity and introducing empirical constants. However, a very slight change in the conditions under which flow takes place may cause, in some cases, a large difference in the action or behavior of the water. There is, therefore, always danger in extending the use of experimental data or empirical constants to apply to conditions of flow quite different from those under which the data were obtained.

2. *Acknowledgment.*—All the experimenting was done in the hydraulic laboratory of the University of Illinois under the general direction of Professor A. N. Talbot. A part of the problem and some

of the methods had been developed by Professor Talbot through experimental work which had been carried on in the hydraulic laboratory for a number of years. The types and proportions of the mouthpieces and the general features of the apparatus had been planned by him. Thesis work of the following students has been utilized as preliminary material in helping to determine the methods used in the investigation: W. P. Ireland, "Entrance Head in Pipes and Conduits," 1903; C. C. Wiley, "Entrance Head and Discharge Head in Pipes," 1904; W. R. Robinson, "Entrance Head and Discharge Head in Pipes," 1906; W. R. Robinson, "An Investigation of the Flow of Water through Submerged Orifices and Pipes," 1909. Although but few of the data given in these theses were incorporated in the final results as reported in this bulletin, they were of considerable value in determining the influence of certain factors involved in the method of experimenting.

The major part of the experimenting was carried out by the writer during 1914 and 1915, with the help of Mr. L. J. Larsen and Mr. R. L. Templin, research fellows in the Engineering Experiment Station, whose careful work is gratefully acknowledged.

II. Apparatus and Method of Experimenting

3. *Short Pipe.*—The cast-iron short pipe to which the mouthpieces were attached was 22½ in. long, bored to a smooth surface and to a 6-in. diameter; an average of thirty micrometer readings taken across three diameters at each of ten successive sections along the pipe gave 0.5995 in. It was threaded at each end so that a mouthpiece could be screwed on either end or on both ends. Fig. 1 shows the 6-in. short pipe used in the experiments. Fig. 5 is from a photograph and shows the tank used in the experiments and also the 6-in. short pipe and some of the mouthpieces. A flange near the middle of the pipe was used to attach it to a partition separating the two compartments of the tank, allowing the pipe to project into each compartment. Fig. 2 shows the short pipe in place in the tank with mouthpieces attached.

Experiments were also made on a steel tube 3.11 in. in diameter by 12 in. long used as an inward-projecting short pipe only with no mouthpiece attached.

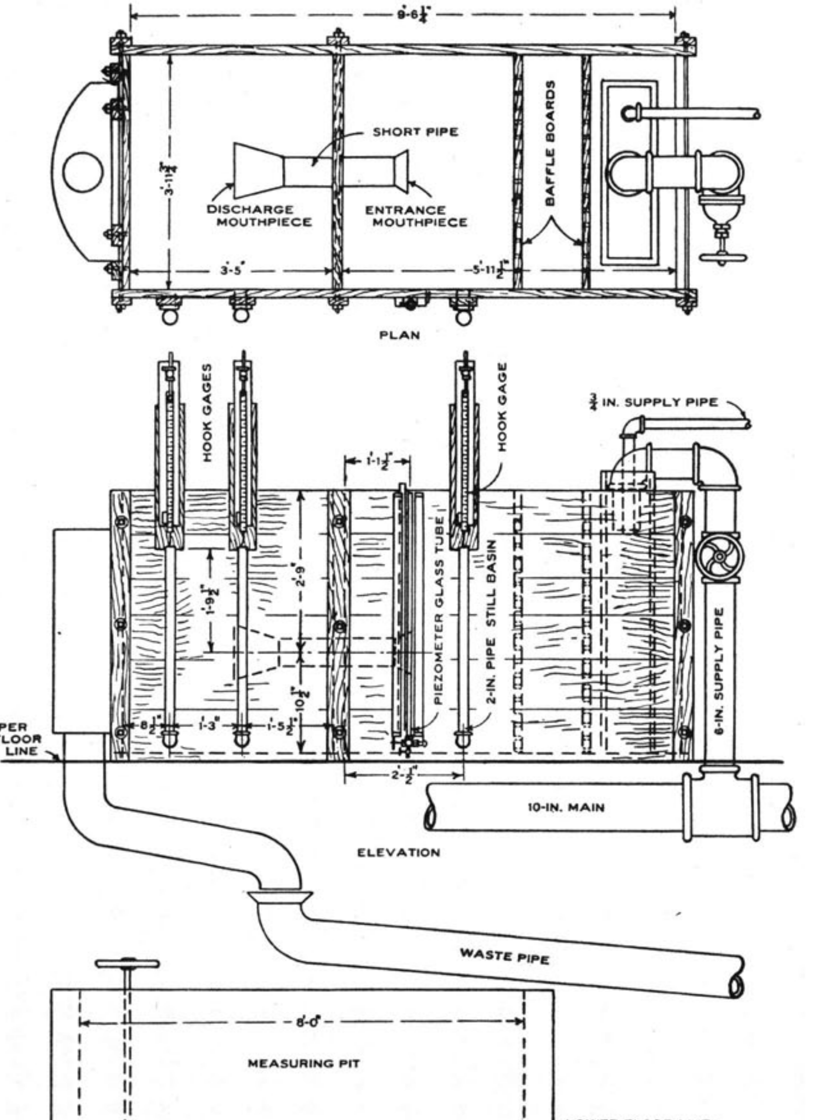

FIG. 2. TANK AND ARRANGEMENT OF APPARATUS

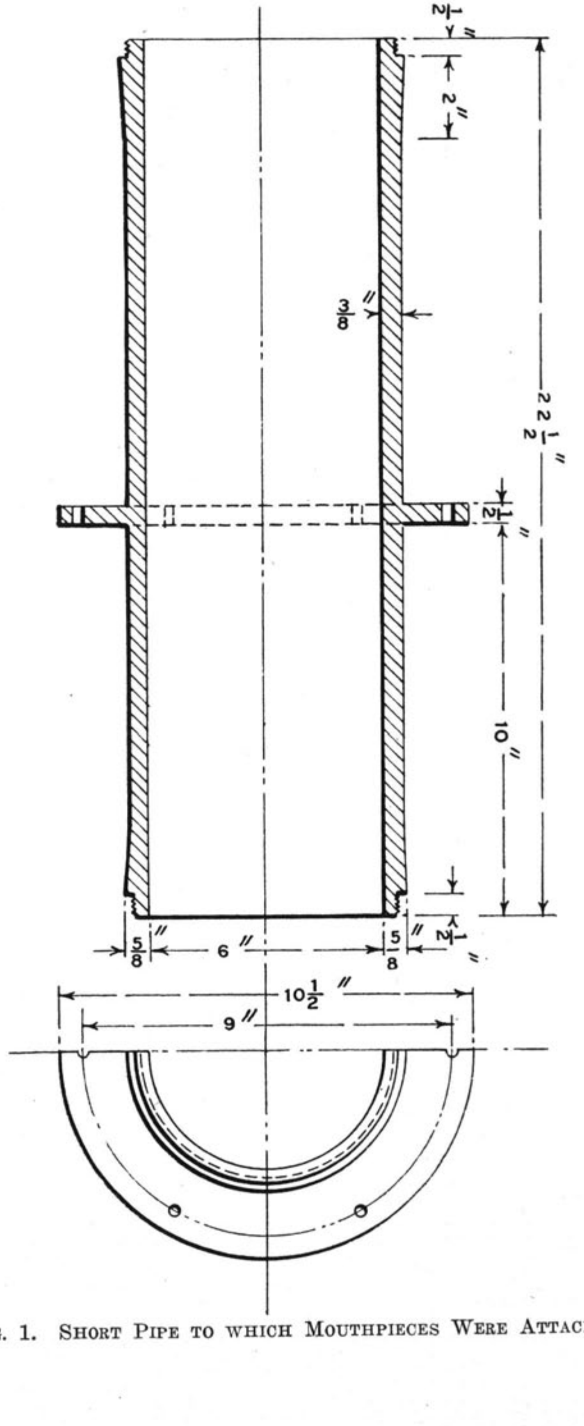

FIG. 1. SHORT PIPE TO WHICH MOUTHPIECES WERE ATTACHED

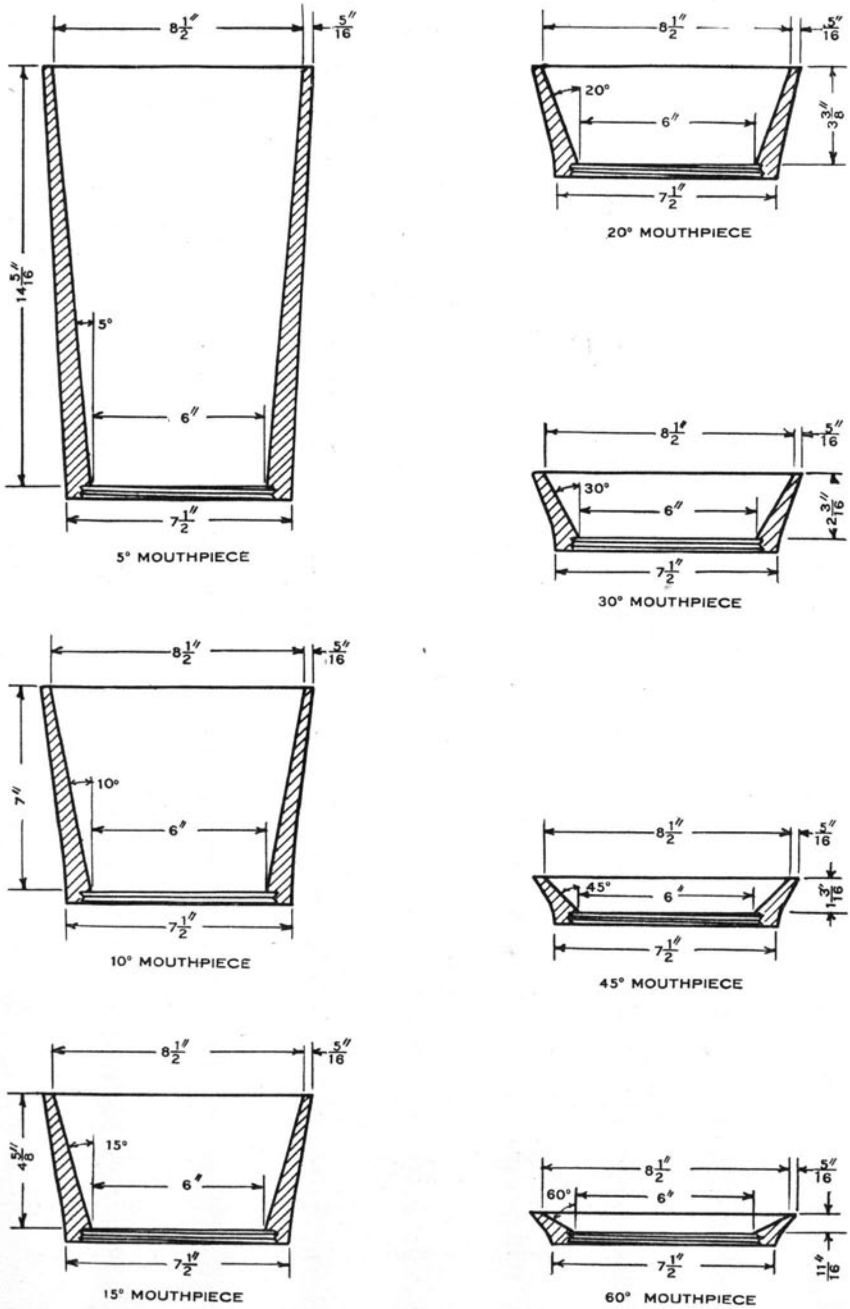

Fig. 3. Conical Mouthpieces having an Area Ratio of 1 to 2

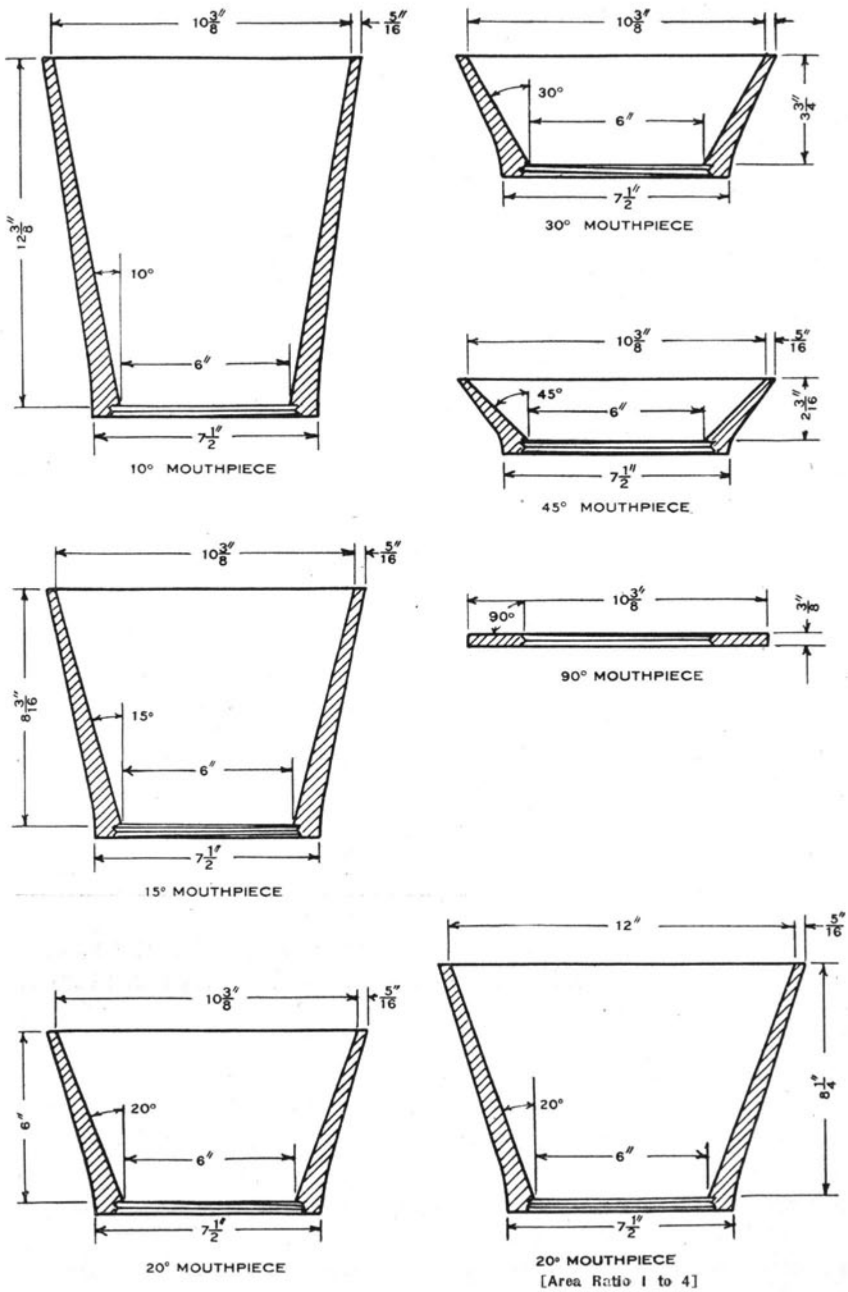

FIG. 4. CONICAL MOUTHPIECES HAVING AN AREA RATIO OF 1 TO 3

4. *Mouthpieces.*—The cast-iron conical mouthpieces which were screwed on the ends of the 6-in. pipe are shown in Figs. 3 and 4. The smallest cross-sectional area of each mouthpiece was the same as the area of the pipe with which the mouthpiece made a smooth connection. The largest or outer area of one series of mouthpieces was twice the area of the pipe, that is, the ratio of the area of the pipe to the largest area of the mouthpiece was 1 to 2. Another series had an area ratio of 1 to 3, and one mouthpiece (20 degree angle) had an area ratio of 1 to 4. The length of a mouthpiece for any area ratio depends, of course, upon the angle of the mouthpiece. Table 1 gives

TABLE 1

LIST OF MOUTHPIECES USED ON THE SHORT PIPE

Each Mouthpiece Tested Singly on Entrance End and Discharge End				Combinations			
				Entrance End		Discharge End	
Area Ratio	Angle of Mouthpiece Degrees	Area Ratio	Angle of Mouthpiece Degrees	Degrees	Area Ratio	Degrees	Area Ratio
1 to 2	5			20	1 to 2	5	1 to 2
	10	1 to 3	10	20	1 to 3	5	1 to 2
	15		15	20	1 to 2	10	1 to 3
	20		20	20	1 to 2	10	1 to 2
	30		30	30	1 to 2	10	1 to 2
	45		45	20	1 to 2	15	1 to 3
	60			15	1 to 2	15	1 to 2
			90	20	1 to 2	20	1 to 4
				20	1 to 2	20	1 to 3
		1 to 4	20	20	1 to 3	20	1 to 2
				30	1 to 2	30	1 to 2

a list of the mouthpieces used, together with the particular combinations of an entrance and a discharge (exit or diverging) mouthpiece employed. In no case was a mouthpiece used alone, that is, without being attached to the short pipe. It will be noted that any mouthpiece could be attached to the entrance end of the short pipe only, or to the discharge end only, or two could be attached, one on each end of the pipe. In the case of a few of the mouthpieces, duplicates were made. The angle of the mouthpiece as given in Table 1 and in the various figures, and as used in these pages, means the angle between the axis line of the pipe and one element of the cone, not the total angle of convergence or divergence. Hence, a 90-degree entrance mouthpiece is a flat disc giving a square or flush entrance to the pipe.

5. *Tank and Method of Experimenting.*—The same tank was used in all the experiments. The dimensions of the tank are shown in Fig. 2 and a photograph gives other details in Fig. 5. The tank was divided into two compartments by a vertical partition to which the short pipe was attached in a horizontal position.

Water from the laboratory standpipe was supplied to the tank through a 6-in. supply pipe and also through a ¾-in. pipe, the latter making possible a finer adjustment in maintaining a constant head. After passing through baffle boards the water flowed through the short pipe and finally left the downstream compartment by passing out of the small openings in the end of the tank, the flow through which was regulated by placing stoppers in some of the holes. These holes were arranged in two long vertical rows in the end of the tank, one near each side, and the stoppers were arranged so as to give nearly the same distribution of flow from each row. This, it was found, helped to maintain steady conditions.

The quantity of water discharged was measured in a pit about 6 ft. deep, with a diameter of 7.995 ft. as obtained from readings of a micrometer attached to a rigid stick. The rise in the pit was determined by a vertical graduated rod which was read directly to 0.02 ft. and to 0.004 ft. by estimating. A float was attached to the bottom of the rod and a still basin was provided. The water was wasted into another pit through a movable spout until the surface of the water in the measuring pit became fairly still so that an accurate reading of the rod could be taken. A hook gauge was used to test the accuracy of the float and rod. At the end of the experiment the water was again wasted in the same manner. A calibrated stop watch gave the time corresponding to the rise in the pit.

The head causing flow through the short pipe is the difference in the levels of the water surfaces in the two compartments of the tank. The head was measured in nearly all the experiments by means of hook gauges. These gauges were read directly to 0.001 ft. and to 0.0005 ft. by estimating. Vertical 2-in. pipes attached toward the bottom of the tank served as still basins for the hook gauges (see Figs. 2 and 5). The level of the water in the upstream compartment was determined by the use of one hook gauge only, but two gauges were used on the downstream compartment in the earlier experiments. It was found, however, that for the lower heads the two gauges gave practically the same result and for the higher heads the gauge nearer the partition gave less fluctuation. For these reasons, and because of less difficulty in getting simultaneous readings of only two gauges, it was decided to take readings with one gauge only on each compartment.

Zero readings of the hook gauge were obtained by reading the gauges when the tank was nearly full and when no water was allowed to escape, the levels of the water surfaces in the two compartments then being the same. Zero readings were taken frequently during the experiments.

For most of the heads above 0.3 ft. the head was measured by a differential gauge which was connected to each compartment of the tank by means of rubber hose. A mixture of carbon tetrachloride and gasoline having a specific gravity of 1.25 was used; thus making the gauge reading four times the head. The gauge was provided with a scale graduated to 0.005 ft. In a few cases two vertical piezometer glasses were used, one attached near the bottom of each compartment, the difference in readings of which (corrected for zero reading) gave the head to 0.001 ft. These three methods overlapped somewhat so that certain heads were measured by all three methods.

Leakage from the tank and from the measuring pit was determined several times during the progress of the experiments and was found to be negligible.

The following procedure comprised an experiment: Stoppers were removed from the end of the tank in sufficient number to give the desired discharge, and the inflow through the 6-in. and ¾-in. pipes was then adjusted until the difference in levels of the water surfaces in the two compartments of the tank became constant. The ¾-in. supply pipe was used to make the final adjustment of the head and to hold the head constant throughout the experiment. After obtaining a constant head, the waste pipe shown in Fig. 2 was pulled from beneath the discharge pipe, allowing the water to discharge into the measuring pit until the rise in the pit was sufficient to allow of its measurement without appreciable error, and to allow an accurate measurement of the head to be made. The head was taken as an average of from two to ten readings of the hook gauges, the larger number being necessary with the higher velocities on account of the greater fluctuations of the water levels due to the more turbulent conditions of the water, especially in the downstream compartment. Each experiment was repeated three times, as a rule, although in the case of some of the small-angle discharge mouthpieces and at the higher velocities, as many as six or eight runs were made.

III. Experimental Results and Discussion

6. *Brief Analysis of Flow.*—The term short pipe or tube as technically used in hydraulics applies to a tube merely long enough to allow the stream to expand after contraction at entrance and flow full at the discharge end. A length equal to three diameters is usually considered sufficient although the pipe used in these experiments was 3.75 diameters long.

The head, h, causing flow through a tube is divided into two items, (1) the velocity head in the tube, which represents energy per pound of water transformed from potential or pressure energy into kinetic energy, and (2) lost head,* h', representing energy per pound of water dissipated (transferred into heat) chiefly by impact and friction of the water particles. The velocity head is usually represented by $\frac{v^2}{2g}$, where v is the mean axial velocity in the pipe. It is clear that if the water has a mean axial velocity, v, and at the same time is in a turbulent state of motion, probably also with some rotation as a whole, the water will possess more kinetic energy than if parallel stream-line flow occurred with the same axial velocity. Hence with turbulent flow, $\frac{v^2}{2g}$ does not represent the total kinetic energy created. Furthermore, since dissipation of energy (lost head) accompanies the transfer of energy, it is evident that more head will be lost when turbulent flow is produced and also wherever the velocity head is transferred back into pressure head as in reducing the velocity by means of a diverging mouthpiece. It is plain that the flow of water through a short pipe with or without mouthpieces does not yield to a simple detailed analysis.

In equation form we have, then,

$$h = h' + \frac{2g}{v^2} \qquad (1)$$

$$\text{But } v = c\sqrt{2gh}. \qquad (2)$$

*The term "lost head" is sometimes used to denote the head which causes the flow, but as used in these pages it will always mean dissipated energy.

$$\text{Therefore, } h' = \left(\frac{1}{c^2} - 1\right)\frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which c is the coefficient of discharge since the pipe flows full at the discharge end, and is the ratio of the measured rate of discharge, q, to the theoretical rate of discharge, or, in equation form,

$$c = \frac{q}{a\sqrt{2\ gh}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which a is the area of the cross-section of the pipe. The expression $\left(\frac{1}{c^2} - 1\right)$ is called the coefficient of loss and is denoted by m.

If a converging or entrance mouthpiece is attached to the short pipe, the contraction of the stream will be somewhat suppressed. The discharge, therefore, will be increased, with a corresponding reduction in the lost head. How much of this decrease in energy loss occurs at the entrance and how much during the subsequent flow in the pipe for any given case is difficult to say. The loss of head which would occur in a 6-in. pipe 22½ in. long, if it were a part of a longer pipe and preceded by a considerable length of straight pipe of the same diameter, would be that due to pipe friction,

$$f\frac{1}{d}\,\frac{v^2}{2g} \ or\ 0.086\frac{v^2}{2g}.$$

This loss is not considered in these experiments for, although the pipe may flow full for the greater part of its length when the contraction at entrance is largely suppressed by an entrance mouthpiece, the state of flow is no doubt determined chiefly by the entrance conditions and the loss of head should all be considered as entrance loss.

If a diverging mouthpiece is attached to the discharge end of the short pipe, a part of the velocity head in the pipe may be regained. Theoretically, the amount possible of recovery is the difference between the velocity head in the pipe, $\frac{v^2}{2g}$, and the velocity head at the outer or discharge area of the mouthpiece, $\frac{v_0^2}{2g}$. This may be

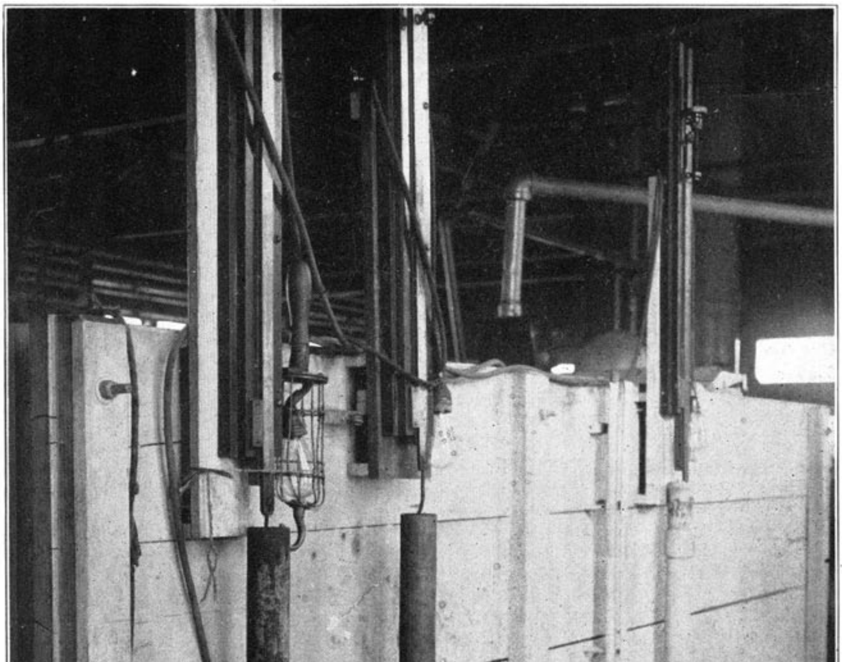

FIG. 5. VIEW OF TANK, OF SHORT PIPE, AND OF SOME OF THE MOUTHPIECES

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expressed as

$$\left[1-\left(\frac{a}{A}\right)^2\right]\frac{v^2}{2g},$$

in which $\frac{a}{A}$ is the ratio of the area of the pipe to the outer area of the discharge mouthpiece. In other words, if the water approaches the mouthpiece with parallel stream-lines (smooth flow) and if the mouthpiece could reduce the velocity without eddying—ideal conditions—a discharge mouthpiece with an area ratio of 1 to 2 would regain 75 per cent of the velocity head in the pipe, while a mouthpiece having an area ratio of 1 to 3 would regain 88 per cent of the velocity head and a 1 to 4 mouthpiece would recover 94 per cent.

The flow in a short pipe, particularly when no mouthpiece is attached to the entrance end, is far from smooth. And while this turbulence of flow would be expected to change materially the action or effect of the discharge mouthpiece from that occurring under ideal conditions, there is no rational method of taking this into account. It seems probable, however, that the discharge mouthpiece influences the flow for some distance back in the pipe so that the loss of head in the tubes is less than when no discharge mouthpiece is used. This action in turn allows the mouthpiece to act with greater efficiency. This will be discussed later.

It should be noted that in the expression for the coefficient of loss,

$$m = \left(\frac{1}{c^2} - 1\right). \qquad (5)$$

c is the coefficient of discharge based on the area of exit from the system. Hence in the case of the short pipe with a discharge mouthpiece attached, the lost head is expressed in terms of the velocity head at exit from the mouthpiece. But since $av = Av_0$ we have the lost head expressed in terms of the velocity head in the pipe as

$$h' = \left(\frac{1}{c_0^2} - 1\right)\left(\frac{a}{A}\right)\frac{v^2}{2g} \qquad (6)$$

7. *Tables and Figures.*—Table 2 gives the condensed experi-

TABLE 2

CONDENSED EXPERIMENTAL DATA FOR 6-INCH SUBMERGED PIPE HAVING CONICAL MOUTHPIECES

(Depth of Submergence, about 3½ Diameters)

Mouthpiece Attached to Pipe: Entrance	Mouthpiece Attached to Pipe: Discharge	Head Causing Flow Feet h	Measured Discharge Cubic Feet per Second q	Mean Velocity in Pipe Feet per Second v	Coefficient of Discharge Based on Area of Pipe c
None (Inward-Projecting)	None	.0074	.102	.520	.756
		.0128	.135	.689	.757
		.0153	.153	.778	.779
		.0190	.170	.868	.785
		.0292	.210	1.07	.785
		.0327	.223	1.14	.784
		.0563	.294	1.50	.785
		.0745	.336	1.71	.783
		.1112	.405	2.07	.784
		.1976	.551	2.81	.787
		.296	.668	3.41	.780
		.4420	.819	4.17	.783
5° Area Ratio 1 to 2	None	.0093	.130	.665	.854
		.0157	.176	.899	.886
		.0357	.265	1.35	.895
		.0503	.319	1.63	.903
		.0750	.432	2.20	.908
		.0880	.425	2.16	.904
		.0886	.425	2.16	.906
		.1700	.577	2.94	.895
		.2234	.671	3.42	.903
		.3520	.838	4.27	.899
10° Area Ratio 1 to 2	None	.0089	.130	.661	.875
		.0087	.132	.673	.899
		.0116	.146	.743	.869
		.0170	.183	.934	.893
		.0309	.250	1.28	.903
		.0378	.276	1.41	.902
		.0545	.333	1.70	.906
		.0582	.340	1.73	.893
		.0600	.347	1.77	.900
		.0656	.369	1.88	.915
		.0798	.400	2.04	.899
		.1005	.453	2.31	.908
		.1057	.465	2.37	.910
		.1880	.621	3.16	.910
		.2170	.672	3.42	.913
		.2540	.730	3.72	.920
		.2557	.722	3.68	.908
		.3090	.797	4.06	.910

Mouthpiece Attached to Pipe: Entrance	Mouthpiece Attached to Pipe: Discharge	Head Causing Flow Feet h	Measured Discharge Cubic Feet per Second q	Mean Velocity in Pipe Feet per Second v	Coefficient of Discharge Based on Area of Pipe c
90° Area Ratio 1 to 3 (Flush Entrance)	None	.0075	.104	.530	.766
		.0100	.123	.625	.778
		.0114	.131	.665	.775
		.0135	.144	.735	.783
		.0148	.151	.770	.793
		.0215	.185	.940	.799
		.0294	.224	1.14	.807
		.0310	.215	1.10	.795
		.0428	.257	1.32	.795
		.0535	.290	1.48	.798
		.0942	.389	1.98	.802
		.1727	.520	2.65	.798
		.1840	.540	2.76	.802
		.2636	.645	3.28	.802
		.3410	.739	3.76	.803
None	5° Area Ratio 1 to 2	.0172	.198	1.01	.961
		.0310	.271	1.38	.975
		.0313	.273	1.39	.976
		.0408	.311	1.58	.977
		.0489	.344	1.75	.986
		.0589	.378	1.92	.985
		.0700	.411	2.10	.987
		.0930	.473	2.41	.988
		.1060	.502	2.56	.980
		.1135	.524	2.67	.986
		.1155	.525	2.67	.978
		.1400	.594	3.03	1.01
		.1633	.623	3.17	.980
		.2560	.792	4.04	.994
		.3670	.948	4.83	.993
None	10° Area Ratio 1 to 2	.0182	.193	.985	.910
		.0361	.278	1.42	.930
		.0600	.362	1.85	.939
		.0807	.418	2.13	.932
		.1335	.543	2.76	.944
		.1560	.589	3.00	.945
		.1680	.606	3.08	.938
		.2930	.790	4.02	.934
		.4120	.960	4.89	.943

TABLE 2 (Continued)

CONDENSED EXPERIMENTAL DATA FOR 6-INCH SUBMERGED PIPE HAVING CONICAL MOUTHPIECES

(Depth of Submergence, about 3½ Diameters)

Mouthpiece Attached to Pipe		Head Causing Flow Feet	Measured Discharge Cubic Feet per Second	Mean Velocity in Pipe Feet per Second	Coefficient of Discharge Based on Area of Pipe	Mouthpiece Attached to Pipe		Head Causing Flow Feet	Measured Discharge Cubic Feet per Second	Mean Velocity in Pipe Feet per Second	Coefficient of Discharge Based on Area of Pipe
Entrance	Discharge	h	q	v	c	Entrance	Discharge	h	q	v	c
15° Area Ratio 1 to 2	None	.0161	.175	.893	.878	None	15° Area Ratio 1 to 2	.0120	.146	.743	.845
		.0288	.241	1.23	.902			.0185	.190	.968	.888
		.0450	.303	1.54	.906			.0230	.200	1.02	.861
		.0582	.350	1.78	.921			.0406	.281	1.43	.884
		.0633	.365	1.86	.909			.0444	.294	1.50	.885
		.0662	.405	2.07	.911			.0595	.344	1.75	.895
		.0852	.417	2.12	.906			.0610	.349	1.78	.900
		.1030	.463	2.36	.917			.0660	.367	1.87	.902
		.1827	.622	3.17	.926			.0670	.365	1.86	.904
		.2540	.728	3.71	.915			.0898	.428	2.18	.905
		.3520	.864	4.40	.925			.0891	.425	2.16	.901
		.4230	.952	4.85	.925			.1242	.500	2.55	.899
		.7650	1.25	6.40	.915			.1692	.578	2.95	.902
								.2637	.729	3.72	.893
								.2820	.746	3.80	.907
								.3400	.833	4.24	
20° Area Ratio 1 to 2	None	.0080	.123	.628	.877	None	20° Area Ratio 1 to 2	.0090	.123	.625	.821
		.0105	.147	.752	.914			.0148	.161	.821	.844
		.0191	.196	1.00	.905			.0153	.162	.827	.834
		.0306	.252	1.28	.912			.0160	.167	.849	.837
		.0355	.269	1.37	.908			.0244	.211	1.07	.859
		.0550	.338	1.72	.917			.0383	.269	1.37	.870
		.0695	.381	1.94	.917			.0404	.271	1.38	.856
		.0754	.394	2.01	.912			.0404	.272	1.38	.853
		.0846	.416	2.12	.908			.0437	.283	1.44	.860
		.1380	.540	2.75	.920			.0505	.305	1.55	.864
		.1736	.606	3.09	.924			.0747	.372	1.89	.863
		.2320	.707	3.60	.927			.0782	.385	1.96	.876
		.3075	.804	4.10	.921			.0953	.423	2.15	.870
		.4270	.964	4.91	.927			.1385	.514	2.62	.877
								.1867	.598	3.05	.879
								.2025	.626	3.19	.884
								.330	.795	4.05	.880
30° Area Ratio 1 to 2	None	.0080	.121	.619	.862	None	30° Area Ratio 1 to 2	.0096	.115	.586	.746
		.0083	.124	.633	.866			.0152	.146	.744	.751
		.0185	.187	.955	.874			.0152	.153	.779	.789
		.0221	.212	1.08	.907			.0176	.165	.841	.791
		.0367	.272	1.38	.900			.0291	.211	1.08	.786
		.0410	.287	1.46	.900			.0506	.281	1.43	.792
		.0470	.309	1.58	.906			.0519	.283	1.44	.789
		.0594	.347	1.77	.902			.0804	.359	1.83	.792
		.0804	.406	2.07	.911			.0852	.367	1.87	.798
		.0818	.414	2.11	.919			.1690	.513	2.61	.793
		.0967	.448	2.28	.915			.2331	.606	3.08	.797
		.1524	.563	2.87	.916			.2715	.657	3.35	.802
		.2404	.710	3.62	.921			.3990	.794	4.04	.798
		.3341	.838	4.27	.921						
		.390	.906	4.61	.923						
45° Area Ratio 1 to 2	None	.0189	.192	.975	.884	None	45° Area Ratio 1 to 2	.0078	.102	.521	.733
		.0278	.231	1.18	.885			.0163	.154	.787	.773
		.0404	.281	1.43	.888			.0340	.227	1.16	.785
		.0462	.304	1.55	.902			.0521	.282	1.44	.790
		.0721	.381	1.94	.902			.0546	.288	1.46	.782
		.0762	.386	1.97	.889			.0774	.343	1.75	.784
		.0965	.438	2.23	.896			.1185	.426	2.17	.785
		.1535	.557	2.84	.902			.2480	.617	3.14	.787
		.1857	.615	3.14	.907			.3970	.782	3.98	.789
		.325	.803	4.09	.890						
		.399	.902	4.59	.907						
		.451	.961	4.89	.910						

TABLE 2 (Continued)

CONDENSED EXPERIMENTAL DATA FOR 6-INCH SUBMERGED PIPE HAVING CONICAL MOUTHPIECES

(Depth of Submergence, about 3½ Diameters)

Mouthpiece Attached to Pipe		Head Causing Flow Feet	Measured Discharge Cubic Feet per Second	Mean Velocity in Pipe Feet per Second	Coefficient of Discharge Based on Area of Pipe	Mouthpiece Attached to Pipe		Head Causing Flow Feet	Measured Discharge Cubic Feet per Second	Mean Velocity in Pipe Feet per Second	Coefficient of Discharge Based on Area of Pipe
Entrance	Discharge	*h*	*q*	*v*	*c*	Entrance	Discharge	*h*	*q*	*v*	*c*
60° Area Ratio 1 to 2	None	.0079	.119	.606	.852	None	60° Area Ratio 1 to 2	.0116	.126	.644	.743
		.0083	.120	.614	.845			.0154	.149	.758	.762
		.0159	.174	.886	.876			.0396	.243	1.24	.776
		.0255	.219	1.11	.870			.0400	.246	1.25	.780
		.0383	.272	1.38	.883			.0577	.296	1.51	.780
		.0526	.316	1.61	.874			.0600	.304	1.55	.786
		.0553	.326	1.66	.877			.1035	.399	2.03	.787
		.0798	.394	2.01	.889			.1786	.526	2.68	.790
		.0800	.391	1.99	.880			.1940	.547	2.79	.792
		.1088	.465	2.37	.895			.2605	.642	3.27	.796
		.1437	.526	2.68	.882			.3101	.696	3.55	.795
		.2657	.716	3.65	.883			.4777	.859	4.38	.790
		.4129	.896	4.56	.886						
10° Area Ratio 1 to 3	None	.0080	.128	.654	.907	None	10° Area Ratio 1 to 3	.0103	.147	.750	.920
		.0150	.177	.900	.900			.0135	.168	.858	.924
		.0160	.179	.911	.899			.0185	.201	1.03	.941
		.0170	.185	.942	.901			.0304	.255	1.30	.931
		.0181	.190	.968	.897			.0319	.264	1.35	.940
		.0187	.196	1.00	.913			.0461	.317	1.61	.938
		.0198	.200	1.02	.902			.0530	.345	1.76	.956
		.0210	.210	1.07	.922			.0600	.360	1.83	.929
		.0500	.325	1 66	.922			.0749	.403	2.06	.934
		.0505	.325	1.66	.918			.0750	.404	2.06	.938
		.0516	.331	1.69	.924			.1000	.469	2.39	.951
		.0770	.398	2.03	.914			.1028	.479	2.44	.948
		.0960	.452	2.30	.928			.1378	.548	2.80	.938
		.1176	.497	2.53	.921			.1450	.572	2.92	.954
		.1570	.574	2.92	.921			.1740	.628	3.20	.956
		.1737	.609	3.11	.929			.1880	.662	3.37	.967
		.2375	.708	3.61	.925			.2175	.699	3.56	.954
		.2585	.745	3.80	.930			.2751	.794	4.04	.961
15° Area Ratio 1 to 3	None	.0080	.129	.660	.916	None	15° Area Ratio 1 to 3	.0090	.133	.675	.885
		.0165	.187	.955	.925			.0145	.170	.866	.896
		.0332	.257	1.31	.930			.0149	.169	.862	.880
		.0497	.324	1.65	.921			.0152	.172	.874	.880
		.0513	.330	1.68	.926			.0293	.242	1.24	.897
		.0778	.400	2.04	.912			.0588	.340	1.74	.893
		.1058	.476	2.34	.930			.0589	.345	1.76	.901
		.1200	.505	2.57	.927			.0812	.401	2.04	.896
		.1924	.638	3.25	.924			.1060	.466	2.38	.910
		.3634	.880	4.49	.928			.1238	.501	2.55	.905
								.1855	.610	3.11	.901
								.2043	.648	3.30	.914
								.2380	.705	3.59	.918
								.2685	.742	3.78	.910
								.2895	.777	3.96	.918
20° Area Ratio 1 to 3	None	.0080	.126	.644	.895	None	20° Area Ratio 1 to 3	.0080	.119	.607	.845
		.0140	.172	.874	.921			.0141	.160	.818	.867
		.0260	.234	1.19	.919			.0185	.182	.925	.848
		.0630	.366	1.87	.925			.0224	.204	1.04	.864
		.1010	.463	2.36	.924			.0225	.215	1.10	.855
		.1095	.483	2.46	.927			.0414	.278	1.41	.867
		.2101	.687	3.50	.930			.0440	.284	1.45	.872
		.3614	.880	4.49	.929			.0503	.310	1.58	.879
								.0855	.411	2.10	.892
								.1199	.486	2.48	.890
								.2029	.633	3.23	.892
								.2422	.694	3.54	.892
								.3071	.782	3.99	.897

TABLE 2 (Continued)

CONDENSED EXPERIMENTAL DATA FOR 6-INCH SUBMERGED PIPE HAVING CONICAL MOUTHPIECES

(Depth of Submergence, about 3½ Diameters)

Mouthpiece Attached to Pipe		Head Causing Flow Feet	Measured Discharge Cubic Feet per Second	Mean Velocity in Pipe Feet per Second	Coefficient of Discharge Based on Area of Pipe	Mouthpiece Attached to Pipe		Head Causing Flow Feet	Measured Discharge Cubic Feet per Second	Mean Velocity in Pipe Feet per Second	Coefficient of Discharge Based on Area of Pipe
Entrance	Discharge	h	q	v	c	Entrance	Discharge	h	q	v	c
30° Area Ratio 1 to 4	None	.0097	.144	.736	.932	None	30° Area Ratio 1 to 4	.0085	.122	.623	.841
		.0106	.149	.761	.920			.0206	.192	.980	.851
		.0306	.254	1.29	.922			.0241	.214	1.09	.874
		.0440	.306	1.56	.925			.0475	.297	1.51	.865
		.0599	.353	1.80	.918			.0584	.335	1.71	.880
		.0755	.397	2.02	.920			.0803	.394	2.03	.884
		.1079	.475	2.42	.920			.1112	.463	2.36	.882
		.1525	.570	2.91	.928			.1276	.500	2.55	.889
		.1870	.632	3.22	.930			.1600	.566	2.88	.899
		.1906	.638	3.25	.927			.2203	.666	3.39	.902
		.2801	.780	3.97	.936			.2789	.756	3.85	.910
								.2975	.777	3.98	.905
30° Area Ratio 1 to 3	None	.0085	.124	.630	.853	None	30° Area Ratio 1 to 3	.0076	.107	.546	.780
		.0165	.178	.909	.881			.0082	.106	.543	.770
		.0235	.220	1.12	.909			.0147	.150	.764	.785
		.0301	.247	1.26	.906			.0195	.173	.883	.788
		.0303	.247	1.26	.901			.0300	.215	1.09	.787
		.0643	.369	1.88	.925			.0310	.221	1.12	.796
		.0663	.374	1.91	.923			.0320	.217	1.11	.794
		.0950	.450	2.29	.928			.0520	.281	1.43	.795
		.1730	.611	3.11	.933			.0831	.360	1.83	.794
		.3053	.810	4.13	.932			.1454	.480	2.44	.800
								.2901	.682	3.47	.803
								.4600	.855	4.36	.801
45° Area Ratio 1 to 3	None	.0080	.119	.607	.845	None	45° Area Ratio 1 to 3	.0115	.131	.666	.773
		.0129	.153	.780	.853			.0178	.162	.825	.771
		.0232	.214	1.09	.893			.0305	.215	1.09	.779
		.0262	.231	1.17	.905			.0580	.297	1.52	.784
		.0412	.285	1.45	.890			.0993	.390	1.98	.785
		.0602	.346	1.76	.895			.1493	.475	2.42	.781
		.0783	.397	2.02	.901			.1970	.546	2.79	.782
		.1164	.490	2.50	.912			.2896	.669	3.41	.789
		.1256	.502	2.56	.900			.350	.737	3.76	.795
		.1801	.600	3.06	.898			.452	.836	4.26	.792
		.2233	.668	3.40	.896			.507	.917	4.67	.796
		.3117	.788	4.02	.897						
		.3223	.813	4.14	.906						
20° Area Ratio 1 to 2	5° Area Ratio 1 to 2	.0130	.232	1.18	1.29	20° Area Ratio 1 to 3	5° Area Ratio 1 to 2	.0091	.190	.968	1.27
		.0209	.301	1.54	1.32			.0221	.302	1.54	1.29
		.0280	.348	1.77	1.32			.0413	.416	2.12	1.30
		.0504	.464	2.37	1.32			.0626	.519	2.65	1.32
		.0520	.484	2.46	1.34			.1900	.686	3.49	1.35
		.0651	.539	2.75	1.34						
		.1083	.700	3.56	1.35						
		.1548	.836	4.26	1.35						
20° Area Ratio 1 to 2	10° Area Ratio 1 to 2	.0076	.158	.802	1.15	20° Area Ratio 1 to 2	10° Area Ratio 1 to 3	.0194	.261	1.33	1.19
		.0204	.263	1.34	1.17			.0274	.317	1.61	1.22
		.0397	.371	1.89	1.18			.0480	.345	2.16	1.23
		.0628	.470	2.35	1.18			.0734	.519	2.64	1.21
		.0915	.476	2.89	1.19			.1110	.639	3.26	1.22
		.1449	.708	3.60	1.18						
		.496	1.33	6.80	1.18						

Table 2 (Concluded)

Condensed Experimental Data for 6-inch Submerged Pipe Having Conical Mouthpieces

(Depth of Submergence, about 3½ Diameters)

Mouthpiece Attached to Pipe: Entrance	Mouthpiece Attached to Pipe: Discharge	Head Causing Flow Feet h	Measured Discharge Cubic Feet per Second q	Mean Velocity in Pipe Feet per Second v	Coefficient of Discharge Based on Area of Pipe c	Mouthpiece Attached to Pipe: Entrance	Mouthpiece Attached to Pipe: Discharge	Head Causing Flow Feet h	Measured Discharge Cubic Feet per Second q	Mean Velocity in Pipe Feet per Second v	Coefficient of Discharge Based on Area of Pipe c
15° Area Ratio 1 to 2	15° Area Ratio 1 to 2	.0062	.128	.650	1.03	20° Area Ratio 1 to 2	15° Area Ratio 1 to 3	.0089	.160	.817	1.08
		.0192	.239	1.22	1.09			.0098	.170	.866	1.10
		.0433	.363	1.85	1.11			.0118	.196	1.00	1.14
		.0643	.443	2.25	1.11			.0224	.261	1.33	1.11
		.0690	.452	2.30	1.09			.0260	.292	1.49	1.15
		.3810	.975	4.97	1.09			.0456	.377	1.92	1.12
								.0479	.393	2.00	1.13
								.0624	.445	2.27	1.13
								.0890	.524	2.67	1.12
								.2022	.818	4.17	1.15
20° Area Ratio 1 to 3	20° Area Ratio 1 to 2	.0152	.191	.971	.984	20° Area Ratio 1 to 2	20° Area Ratio 1 to 3	.0388	.312	1.59	1.00
		.0333	.283	1.44	.985			.0651	.410	2.09	1.02
		.0621	.407	2.48	1.04			.0907	.494	2.52	1.04
		.0898	.486	2.07	1.03			.1434	.621	3.16	1.04
		.1512	.643	3.27	1.06			.1791	.695	3.55	1.04
		.233	.804	4.09							
20° Area Ratio 1 to 2	20° Area Ratio 1 to 4	.0343	.319	1.63	1.09	30° Area Ratio 1 to 2	10° Area Ratio 1 to 2	.0077	.160	.817	1.16
		.0559	.400	2.04	1.08			.0237	.282	1.44	1.17
		.0848	.481	2.46	1.05			.0340	.334	1.70	1.13
		.1233	.605	3.08	1.10			.0437	.381	1.94	1.16
		.1750	.726	3.70	1.10			.0630	.464	2.36	1.17
								.0650	.464	2.36	1.16
								.1860	.817	4.16	1.21
								.3030	1.04	5.30	1.20
								.500	1.37	6.97	1.23
30° Area Ratio 1 to 2	30° Area Ratio 1 to 2	.0227	.219	1.11	.921	45° Area Ratio 1 to 2	45° Area Ratio 1 to 2	.0108	.141	.721	.867
		.0625	.363	1.85	.922			.0301	.246	1.25	.901
		.0671	.381	1.94	.932			.0525	.330	1.68	.912
		.0719	.396	2.02	.940			.0633	.357	1.82	.900
		.1009	.469	2.37	.932			.0966	.443	2.25	.904
		.1560	.585	2.98	.944			.1411	.535	2.73	.905
		.1693	.608	3.11	.940			.2254	.681	3.47	.911
		.293	.796	4.06	.934			.466	.985	5.02	.912
		.373	.903	4.60	.932			.828	1.32	6.71	.915
		.465	1.07	5.49	.935						
		.742	1.27	6.48	.935						

TABLE 3

RESULTS OF EARLIER EXPERIMENTS ON CONVERGING MOUTHPIECES[1]

Source	Coefficient of Discharge Based on Smallest Area	Mean Velocity at Smallest Area Feet per Second		Diameter or Side of Smallest Area	Ratio of Length of Straight Pipe Attached to d	Form of Mouthpiece
	c	v		d	$\frac{l}{d}$	
Balch	.73 to .92	4 to 15		4 in diam.	0.25 to 1.5	Conical, 45 degrees. Area Ratio 1 to 1.9
Davis and Balch	.829 to .955	4.5 to 13		1.51 ft. sq.	0.65	Approach to each side of square was circular. Area Ratio 1 to 3. Discharge vertical
Stewart	.928 to .894	1 to 4		4 ft. sq.	.077 to 3.5	Approach to each side of square was a quarter ellipse, the outer area being 8 feet square, the plane of which was 3 feet from entrance to straight pipe. Area Ratio 1 to 4
Ellis	.951 to .944	12 to 32		1 ft. sq.	0	Approach to each side of square was a quarter ellipse with semi-diameters of 0.5 feet (vertical) and 0.33 feet (horizontal). Discharge vertical
Francis	.927 to .944	5.5 to 9.2		1.22 in diam.	.082	Cycloidal. Area Ratio 1 to 1.95
Brownlee	.941 to .966	7.5 to 27.2		0.1982 in. diam.	0	Cycloidal. Area Ratio 1 to 4
Weisbach	.959 to .994	?		0.396 in diam.	0	Same as form of jet issuing from sharp edged orifice. Discharge into air
		Angles				
		Degrees	Minutes		Area Ratio	
Castel	.924	5	26		1 to 1.5	Conical. c varied but little with v. "Angle" means one-half total angle of convergence. l/d = zero. There was contraction at exit. Discharge into air
	.946	13	24			
	.924	19	24	0.6 in diam.	to	
	.895	30	00			
	.870	40	20		1 to 1.88	
	.930	5	26		1 to 1.7	
	.956	13	40	0.782 in diam.	to	
	.920	35	52		1 to 10	

[1] For reference to source see Appendix.

mental data in which each set of values represents, as a rule, the average of three experiments or runs although in some cases as many as six or eight experiments were made under practically the same head as explained above. In some cases, also, experiments were repeated at times differing by several days or even weeks. This was done mainly to check the work of different experimenters and to determine the effect of certain factors which caused trouble during the experimenting.

In Figs. 6, 7, 8, and 9 the experimental values of the coefficients of discharge for the short pipe with the various mouthpieces attached

TABLE 4

RESULTS OF EARLIER EXPERIMENTS IN WHICH DISCHARGE MOUTHPIECES WERE USED[1]

Source	Form and Angle of Entrance Mouthpiece or Approach		Area Ratio	Angle of Discharge Mouthpiece (One-Half of Total Angle)		Area Ratio	Coefficient of Discharge Based on Smallest Area	Diameter or Side of Smallest Section Inches	Connection
		Degrees[2]	$\frac{a}{A}$	Degrees	Minutes	$\frac{a}{A}$	c		
Weisbach	Sharp edge (no mouthpiece)		....	2	27	$\frac{1}{1.72}$	0.946	0.972	
Eytelwein	Circular	45	$\frac{1}{2}$	2	35	$\frac{1}{3}$	1.55	1.06	Connected without a straight pipe or throat
Francis	Cycloidal	45	$\frac{1}{1.95}$	2	30	$\frac{1}{2}$	1.55	1.22	Connected by a straight pipe or throat 0.1 inch long
Francis	Cycloidal	45	$\frac{1}{1.95}$	2	30	$\frac{1}{5.3}$	2.14	1.22	
Francis	Cycloidal	45	$\frac{1}{1.95}$	2	30	$\frac{1}{10}$	2.35	1.22	
Francis	Cycloidal	45	$\frac{1}{1.95}$	2	30	$\frac{1}{16}$	2.35	1.22	
Brownlee	Cycloidal	30	$\frac{1}{4}$	3 (slightly	33 curved)	$\frac{1}{22}$	2.20	0.1982	Connected without a straight pipe
Venturi	Rounded corners		?	2	43	$\frac{1}{2.7}$	1.46	1.33	Connected without a straight pipe
Davis and Balch	Circular	45	$\frac{1}{3}$	14+		$\frac{1}{1.78}$	1.03	12.51 (sq.)	Connected by a straight pipe 6 inches long. Vertical discharge
	Circular	45	$\frac{1}{3}$	curved		$\frac{1}{1.78}$	1.00	12.51 (sq.)	

[1] For references to source see Appendix. Velocities range about as in Table 3.
[2] Approximate.

have been plotted as ordinates, and the mean velocities in the pipe as abscissas. It will be noted that the mean velocity of flow through the pipe varies from about 0.5 ft. to 6 ft. per sec., although in many cases the upper limit was between 4 and 5 ft. per sec. These curves show that in all cases the coefficient of discharge is practically constant for velocities above 2 ft. per sec. and in some cases above 1 ft. per sec. However, for discharge mouthpieces having small angles of divergence the curves show a tendency for the coefficient of discharge to increase slightly with the velocity. This, it is thought, is due chiefly to the more turbulent condition of the water in the downstream compartment caused by these longer mouthpieces, which in turn made the measurement of the water level slightly too large. For the same reason the results for the longer discharge mouthpieces show the greater fluctuations. In all

TABLE 5

FINAL RESULTS FOR 6-INCH SUBMERGED SHORT PIPE HAVING MOUTHPIECES WITH AN AREA RATIO OF 1 TO 2 FOR VELOCITIES FROM 2 TO 5 FEET PER SECOND

(Depth of Submergence about 3½ Diameters)

Mouthpiece[1] Attached to Short Pipe, Area Ratio 1 to 2: Entrance Degrees	Mouthpiece[1] Attached to Short Pipe, Area Ratio 1 to 2: Discharge Degrees	Coefficient of Discharge from Experiment Based on Area of Pipe	Coefficient of Loss Based on Velocity Head in Pipe: $\left(\frac{1}{c^2}-1\right)$; Column 6 times $\left(\frac{a}{A}\right)^2$	Coefficient of Discharge Based on Outer Area of Mouthpiece	Coefficient of Loss Based on Outer Area of Mouthpiece $\left(\frac{1}{c_o^2}-1\right)$	Head Required on Pipe with No Mouthpiece Attached to Give Same Discharge[2] $H=\left(\frac{c}{.785}\right)^2 h$		Gain in Equivalent Head for Discharge Mouthpiece Due to Entrance Mouthpiece (See col. 8, table 7)	Coefficient of Discharge for Discharge Mouthpiece when Entrance Mouthpiece is Attached $c_d = .785 \times \sqrt{\text{Col. 7} + \text{Col. 8}}$	Per Cent Increase in c for Discharge Mouthpiece Due to Entrance Mouthpiece	Gain in Velocity Head Recovered by Discharge Mouthpiece Due to Entrance Mouthpiece, Expressed in Per Cent of Maximum Theoretical Amount Possible of Recovery. (See cols. 11 and 12, table 7)
		c	m	c_o	m_o	H_E	H_D		c_d	Per Cent	Per Cent
1	2	3	4	5	6	7		8	9	10	11
0	0	.785	.620								
5	..	.900	.235			1.32h					
..	5	.990	.770	.495	3.08		1.59h	.51h	1.137	14.9	33.0
10	..	.910	.205			1.34h					
..	10	.940	.872	.470	3.50		1.44h	.19h	1.000	6.39	19.0
15	..	.920	.180			1.38h					
..	15	.900	.975	.450	3.90		1.32h	.11h	.938	4.22	7.9
20	..	.925	.165			1.39h					
..	20	.880	.104	.440	4.15		1.26h	.01h	.884	0.45	0
30	..	.920	.180			1.38h					
..	30	.800	1.31	.400	5.25		1.04h	0	.800	0	0
45	..	.900	.235			1.32h					
..	45	.790	1.37	.395	5.40		1.01h	0	.790	0	0
60	..	.885	.275			1.27h					
..	60	.790	1.37	.395	5.40		1.01h	0	.790	0	0

[1] Angle given is one-half total angle of convergence or divergence.

[2] This is called "equivalent head."

TABLE 6

FINAL RESULTS FOR A 6-INCH SUBMERGED SHORT PIPE HAVING MOUTHPIECES WITH AN AREA RATIO OF 1 TO 3 FOR VELOCITIES FROM 2 TO 5 FEET PER SECOND

(Depth of Submergence about 3½ Diameters)

Mouthpiece[1] Attached to Short Pipe, Area Ratio 1 to 3: Entrance Degrees	Discharge Degrees	Coefficient of Discharge from Experiment Based on Area of Pipe	Coefficient of Loss Based on Velocity Head in Pipe: $\left(\frac{1}{c^2}-1\right)$ / Column 6 times $\left(\frac{a}{A}\right)^2$	Coefficient of Discharge Based on Outer Area of Mouthpiece	Coefficient of Loss Based on Velocity Head at Outer Area of Mouthpiece $\left(\frac{1}{c_o{}^2}-1\right)$	Head Required on Pipe with No Mouthpiece Attached to Give Same Discharge $H=\left(\frac{c}{.785}\right)^2 h$		Gain in Equivalent Head for Discharge Mouthpiece Due to Entrance Mouthpiece (See col. 8, table 7)	Coefficient of Discharge for Discharge Mouthpiece when Entrance Mouthpiece is Attached $c_d = .785 \times \sqrt{\text{Col. 7 + Col. 8}}$	Per Cent Increase in c for Discharge Mouthpiece Due to Entrance Mouthpiece	Increase in Velocity Head Recovered by Discharge Mouthpiece Due to Entrance Mouthpiece, Expressed in Per Cent of Maximum Theoretical Amount Possible of Recovery. (See cols. 11 and 12, table 7)
		c	m	c_o	m_o	H_E	H_D		c_d	Per Cent	Per Cent
1	2	3	4	5	6	7		8	9	10	11
0	0	.785	.620								
10	..	.925	.165			1.39h					
..	10	.950	1.00	.317	9.00		1.47h	0.27h	1.035	8.95	14.7
15	..	.925	.165			1.39h					
..	15	.910	1.095	.303	9.86		1.32h	0.17h	.958	5.28	11.5
20	..	.925	.165			1.39h					
..	20	.890	1.14	.297	10.30		1.28h	0	.890	0	0
30	..	.925	.165			1.39h					
..	30	.800	1.44	.267	13.00		1.04h	0	.800	0	0
45	..	.900	.235			1.31h					
..	45	.790	1.49	.263	13.45		1.01h	0	.790	0	0
90	..	.800	.560			1.04h					
20 (1 to 4)	..	.925	.165			1.39h					
..	20 (1 to 4)	.900	1.23	.225	18.70		1.31h	0.04h	.912	1.33	9.0

[1] Angle given is one-half total angle of convergence or divergence.

TABLE 7

FINAL RESULTS FOR 6-INCH SUBMERGED SHORT PIPE HAVING A MOUTHPIECE ATTACHED TO EACH END FOR VELOCITIES FROM 2 TO 5 FEET PER SECOND

(Depth of Submergence about 3½ Diameters)

Mouthpiece[1] Attached to Short Pipe		Coefficient of Discharge from Experiment Based on Area of Pipe	Same as Col. 3 Except Based on Outer Area of Mouthpiece	Coefficient of Loss Based on Velocity Head in Pipe	"Equivalent Head" or Head on Straight Pipe to Give Same Discharge	That Part of Equivalent Head Due to Discharge Mouthpiece	Increase in Equivalent Head for Discharge Mouthpiece Due to Entrance Mouthpiece	Coefficient of Discharge Calculated from Data for Mouthpieces When Tested Separately	Coefficient of Loss Based on c' but on Velocity Head in Pipe	Ratio of Velocity Head Recovered to Maximum Possible of Recovery (in Per Cent)	
Area	Ratio										
Entrance Degrees	Discharge Degrees	c	c_o	$m_o=\left(\frac{1}{c_o^2}-1\right)\left(\frac{a}{A}\right)^2$	$H_S=\left(\frac{c}{.785}\right)^2 h$	$H'_D=\frac{H_S}{H_E}$	H'_D-H_D	$c'=.785\times\sqrt{H_E\times H_D}$	$m_o=\left(\frac{1}{c'^2_o}-1\right)\left(\frac{a}{A}\right)^2$	Contraction Suppressed	Contraction not Suppressed
1	2	3	4	5	6	7	8	9	10	11	12
20 (1–2)	5 (1–2)	1.34	.720	.232	2.92h	2.10h	0.51h	1.17	.480	91.0	58.0
20 (1–2)	10 (1–3)	1.22	.407	.558	2.42h	1.74h	0.27h	1.12	.687	55.4	40.7
20 (1–2)	10 (1–2)	1.18	.590	.468	2.26h	1.63h	0.19h	1.11	.610	59.7	40.7
15 (1–2)	15 (1–2)	1.10	.550	.572	1.97h	1.43h	0.11h	1.06	.632	47.7	39.8
20 (1–2)	20 (1–3)	1.04+	.347	.811	1.76h	1.28h	0.00h	1.04	.811	26.6	26.6
20 (1–3)	20 (1–2)	1.04	.520	.672	1.76h	1.27h	0.01h	1.04	.672	32.4	32.4
30 (1–2)	30 (1–2)	0.940	.470	.875	1.44h	1.04h	0.00h	0.940	.875	7.35	7.35
45 (1–2)	45 (1–2)	0.910	.455	.955	1.34h	1.02h	0.01h	0.906	.962	4.00	4.00
20 (1–2)	20 (1–4)	1.075	.270	.795	1.88h	1.35h	0.04h	1.065	.880	33.0	24.0
20 (1–2)	15 (1–3)	1.13	.377	.675	2.07h	1.49h	0.17h	1.06	.777	42.0	30.5

[1] Angle given is one-half total angle of convergence or divergence.

cases, the coefficient of discharge decreases more or less rapidly as the velocity decreases from 1 or 2 ft. per sec.

Tables 5, 6, and 7 give the values of the coefficients of discharge and the corresponding values of the coefficients of loss for the short pipe with the various mouthpieces for velocities of from 2 to 5 ft. per sec. as taken from the curves in Figs. 6, 7, 8, and 9, together with certain other data used in the discussion which follows. Figs. 12 and 14 show the influence upon the action of discharge mouthpieces of attaching an entrance mouthpiece to the short pipe. The entrance mouthpiece suppresses the contraction at entrance to the pipe and allows the discharge mouthpiece to receive the water in a smoother state of flow than when the entrance end of the pipe is simply inward projecting. The influence of smooth flow is shown in Figs. 12 and 14 which are obtained by plotting the gain in rate of discharge and gain in velocity head recovered (expressed in per cent) as ordinates and the angle of discharge mouthpieces as abscissas.

8. *Inward-Projecting and Flush Entrance.*—The values of the coefficients of discharge for the short pipe with inward-projecting entrance (no mouthpiece attached) were determined with special care since the effect of attaching a mouthpiece could not otherwise be found. It will be noted from Fig. 6 and Tables 5 and 6 that the value of the coefficient of discharge and the coefficient of loss are respectively $c = 0.785$ and $m = 0.62$ while the values generally given in texts for an inward-projecting pipe are $c = 0.72$ and $m = 0.93$. That is, the head lost at the entrance to an inward projecting pipe is 0.62 of the velocity head in the pipe ($0.62 \frac{v^2}{2g}$) instead of $0.93 \frac{v^2}{2g}$ It would hardly be expected that all inward-projecting pipes would give the same coefficient of discharge, for such factors as the condition of the edge at entrance to the pipe, the diameter of the pipe or perhaps the ratio of the thickness of the pipe to the diameter, the degree of wetness of the material (effect of oil, etc.), the temperature and velocity of the water, the form and size of tank together with the location and form of piezometer orifice, and the conditions of discharge (submerged or into air) might easily influence the flow.

In order to get further data on this form of entrance another short pipe, 3.11 in. in diameter by 12 in. long, was tested in the same

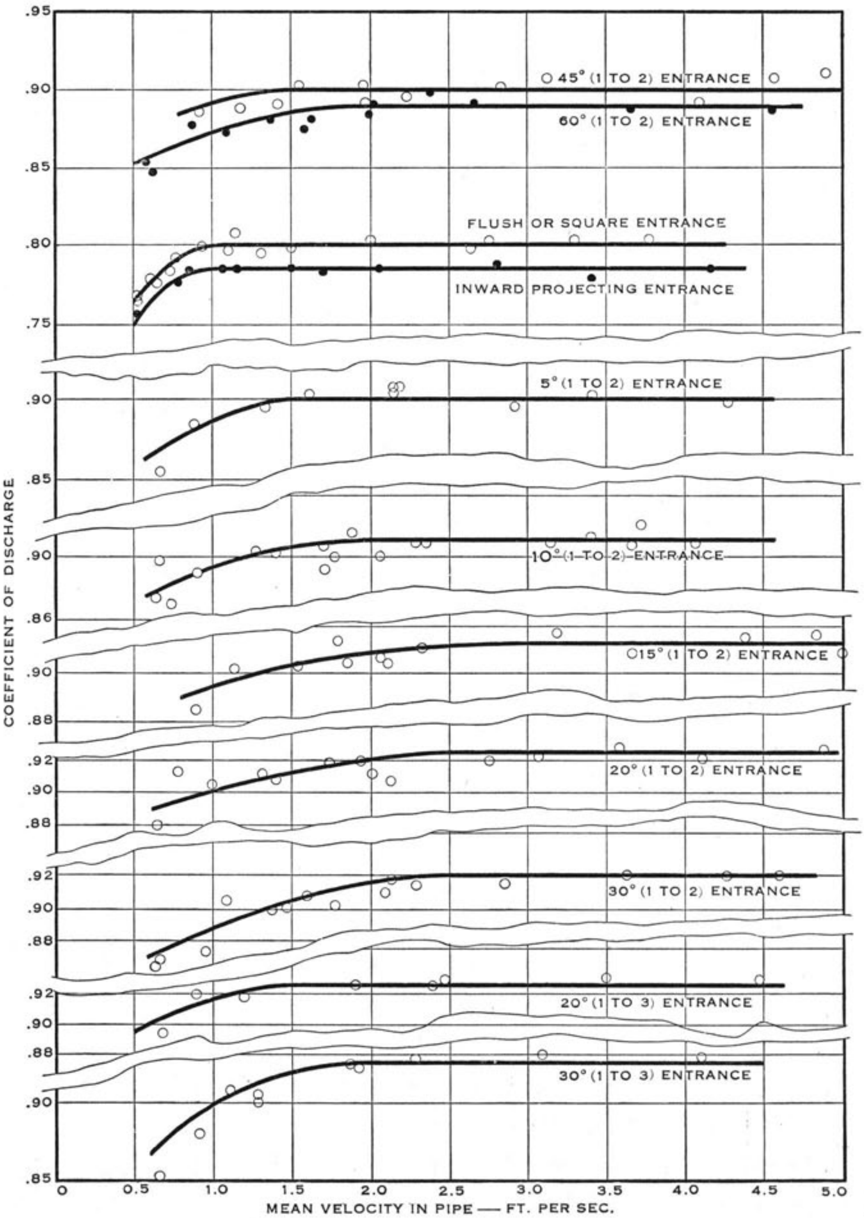

FIG. 6. RELATION BETWEEN COEFFICIENT OF DISCHARGE FOR THE SHORT PIPE WITH MOUTHPIECES AND THE MEAN VELOCITY IN PIPE

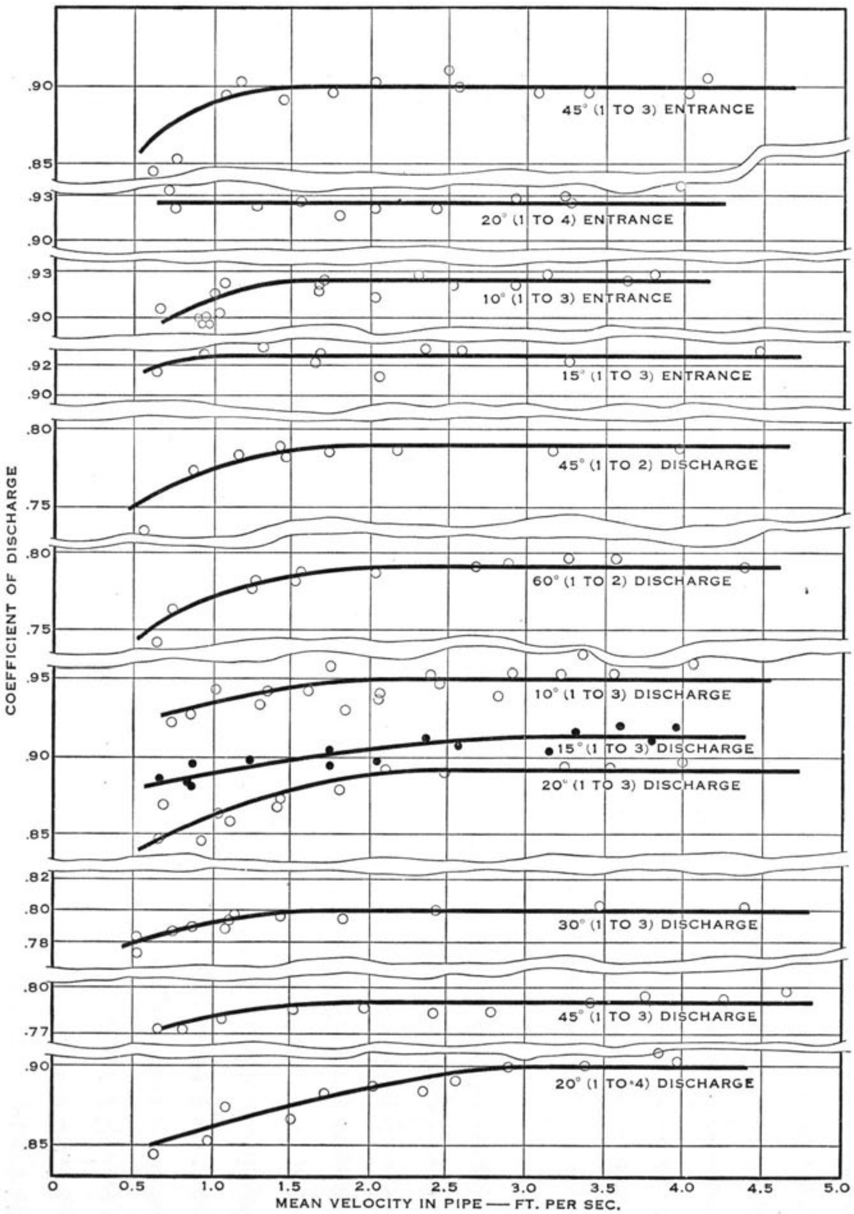

FIG. 7. RELATION BETWEEN COEFFICIENT OF DISCHARGE FOR THE SHORT PIPE WITH MOUTHPIECES AND THE MEAN VELOCITY IN PIPE (CONTINUED)

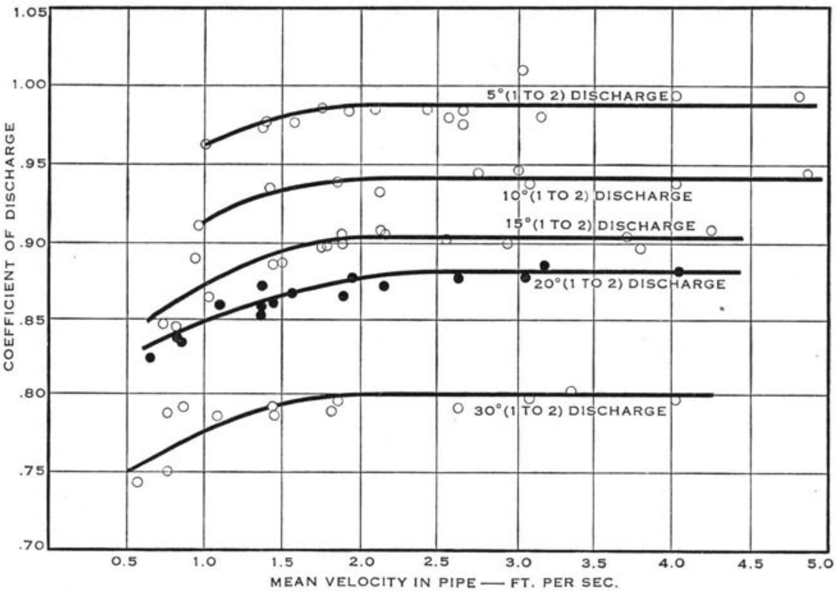

FIG. 8. RELATION BETWEEN COEFFICIENT OF DISCHARGE FOR THE SHORT PIPE WITH MOUTHPIECES AND THE MEAN VELOCITY IN PIPE (CONTINUED)

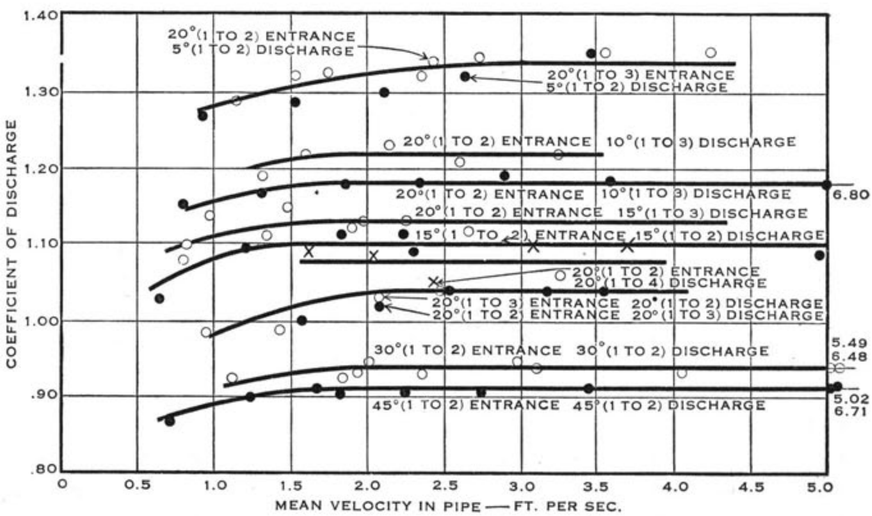

FIG. 9. RELATION BETWEEN COEFFICIENT OF DISCHARGE FOR THE SHORT PIPE WITH MOUTHPIECES AND THE MEAN VELOCITY IN PIPE (CONTINUED)

tank, the maximum velocity being slightly greater than 5 ft. per sec. The values of the coefficient of discharge varied but little, ranging from 0.783 to 0.788. The ratio of the thickness of the pipe at entrance to the diameter for the 6-in. pipe was 0.05 and for the 3-in. pipe this ratio was 0.054. The entering edge of the 3-in. pipe was somewhat sharper than that of the 6-in. pipe.

The value for the lost head at entrance to an inward-projecting pipe as usually given seems to be based on rather meager data obtained chiefly from experiments with discharge into air at rather high velocities through tubes of small diameters. The value $c = 0.72$ for the coefficient of discharge seems to have been handed down from Weisbach. Bidone* reported a value of $c = 0.767$ and Bilton* a value of $c = 0.75$ for a 2½-in. pipe, increasing to 0.79 for a 1-in. pipe and to 0.93 for a ⅛-in. pipe, the length in each case being 2½ diameters.

The values of the coefficient of discharge and the coefficient of loss for a flush entrance (corresponding in these experiments to a 90-degree entrance mouthpiece having an area ratio of 1 to 3) also fail to check closely the values so generally given in texts and so generally used, namely, $c = 0.82$ and $m = 0.49$. As shown in Fig. 6 and Table 6, the values found in these experiments are $c = 0.80$ and $m = 0.56$. That is, the lost head at entrance to a short pipe having a flush entrance is $0.56 \frac{v^2}{2g}$. A 90-degree mouthpiece with an area ratio of 1 to 3 is ample to give the same conditions of flow as a reservoir wall which is flush with end of the pipe and hence it should give the same rate of discharge.

More experimental data on short pipes having flush entrances are available, and they cover a wider range of sizes and conditions than do those for inward-projecting pipes, the value of the coefficient of discharge ranging from 0.785 to 0.84 with the majority of the values lying below 0.82.*

The lost head at the entrance to a pipe can probably be determined better from a submerged tube than from one discharging into air. It would seem, therefore, that the value of the lost head at the entrance to an inward-projecting pipe is not so different from that

*See Appendix for references.

for a pipe having a flush entrance as is usually believed, and that the common text book values need revision to apply to conditions commonly met in hydraulics.

9. *Entrance Mouthpieces.*—From Tables 5 and 6 and the curves in Figs. 7 and 8, it will be seen that entrance mouthpieces having angles of from 10 degrees to 30 degrees (20 degrees to 60 degrees total angle of convergence) give practically the same discharge, while all the entrance mouthpieces having angles of from 5 degrees to 60 degrees (10 degrees to 120 degrees total angle) give only about 5 per cent range in the rate of discharge. In other words, the lost head at the entrance to an inward-projecting short pipe may be reduced from 0.62 of the velocity head in the pipe to 0.18 of the velocity head by a conical mouthpiece having an angle ranging from 10 degrees to 30 degrees. Also, the lost head will vary but little from 0.20 of the velocity head in the pipe for all entrance mouthpieces having angles between 10 degrees and 45 degrees (20 degrees and 90 degrees total angle). It should be stated, however, that conditions surrounding the entrance to the mouthpiece may have some effect, such as the accumulation of dirt in a passage or any other obstruction. Furthermore, it is clear from a comparison of Figs. 7 and 8 that no advantage results from increasing the length of the entrance mouthpiece beyond that corresponding to an area ratio of 1 to 2. The lost head at the entrance to a mouthpiece is sometimes considered the same as would occur at the entrance to an inward projecting pipe of the same area as that of the mouthpiece; that is, the lost head is found by multiplying the velocity head at entrance to the mouthpiece by the coefficient of loss for an inward projecting pipe. There seems to be little reason to justify such a method, since an entrance mouthpiece having an area ratio of 1 to 3 gives almost the same lost head as one with an area ratio of 1 to 2 while the velocity head at entrance to the former mouthpiece would be, of course, only one ninth of that of the latter.

It is not clear just what effect a straight throat or pipe has when added to an entrance mouthpiece. The discharge through the mouthpieces alone was not determined in these experiments. It would seem that if the mouthpiece suppressed the contraction very completely, the pipe would cause added resistance only, and hence decrease the discharge, while if the suppression was rather incomplete, the pipe might recover some of the velocity head during the expansion in it.

If this was in excess of the head lost during the expansion, the net results might be an increase in the discharge. Further, the suppression of the contraction by an entrance mouthpiece not only decreases the entrance loss but also reduces the turbulence of the subsequent flow in the pipe, thereby increasing the effect of a discharge mouthpiece. This will be discussed later under the heading of Combinations of Mouthpieces and Effect of Smooth Flow.

10. *Earlier Experiments with Entrance Mouthpieces.*—The results of other experimenters on entrance mouthpieces are not entirely consistent but give data of considerable importance. Table 3 gives in condensed form the results of the more important experiments. From this table it will be seen that adding a straight pipe to the mouthpiece used by Balch increased the discharge while in the experiments by Stewart the discharge was decreased. Futhermore, the coefficient of discharge increased with the velocity in the experiments by Balch and also in those by Davis and Balch but decreased in the experiments by Ellis, while Stewart found the coefficient to decrease at first and then to increase (not shown in Table 3). In the experiments recorded in this bulletin, the coefficient remained nearly constant. It will be observed that the mouthpieces used by the last four experimenters named in Table 3 have a throat diameter less than 1¼ in.; in fact in only one case is the throat diameter above 0.6 in. These mouthpieces give somewhat higher values for c than do mouthpieces of larger throat diameters. Perhaps the higher values for the coefficients of discharge for these small mouthpieces may be due to a lesser amount of turbulence; that is, it may be that the water flowing through a small mouthpiece is affected, or controlled more by the sides than in the case of a large mouthpiece, at least when the water enters or is received by the mouthpiece in a somewhat disturbed state of flow.

11. *Discharge Mouthpieces.*—As might be expected, the angle of the discharge mouthpieces influences the flow in a very different way from that of the entrance mouthpieces. Tables 5 and 6 and the curves of Figs. 7 and 8 show that when there is no mouthpiece on the entrance end of the short pipe, the coefficient of discharge for a discharge mouthpiece (attached to the pipe) diminishes rather rapidly as the angle of the mouthpiece increases, dropping somewhat

abruptly to practically no effect for an angle slightly greater than 20 degrees (40 degrees total angle).

It will be noted also by a comparison of Figs. 10 and 11 that increasing the length of the discharge mouthpiece beyond that corresponding to an area ratio of 1 to 2 has comparatively little effect on the rate of discharge. It may be, however, that an increase in length would show a greater effect for smaller angles than were used in these experiments, that is, for angles less than 5 degrees for an area ratio of 1 to 2 or less than 10 degrees for an area ratio of 1 to 3. But, it will appear (see article 12) that the size of the pipe, or the smallest area of the mouthpiece, is a much more important factor in the flow through a discharge mouthpiece than it is for an entrance mouthpiece. In other words, a discharge mouthpiece with a throat diameter of ½ in. may give quite different results from that of a discharge mouthpiece having the same area ratio and the same angle of divergence but with a throat diameter of 6 in., at least when the water is received by the mouthpiece in a turbulent state of flow. The discharge mouthpiece with the small throat diameter (having a relatively large ratio of circumference to cross-sectional area) seems to be able to affect a greater percentage of the water flowing and thus regain more energy per pound of water discharged. Furthermore, the chances of having the water approach the discharge mouthpiece with smooth flow is greater in the case of the small pipe, hence perhaps the two causes work together, and for any given case they would be difficult to separate. For these reasons a comparison with earlier experiments on discharge mouthpieces for the purpose of extending the present experiments is apt to be misleading.

It is clear from Figs. 10 and 11 that the governing factor in the recovering of velocity head by means of a discharge mouthpiece attached to a short pipe, having a diameter of several inches or more, is the angle at which expansion begins—rate of expansion at the start—at least when the total angle of divergence is not less than 10 degrees and the area ratio not less than 1 to 2.

12. *Earlier Experiments with Discharge Mouthpieces.*—Table 4 gives, in condensed form, the results of the more important earlier experiments with discharge mouthpieces. These experiments seem to show the influence of the size of throat area as discussed above. The increase in the coefficient of discharge, c, with an increase in

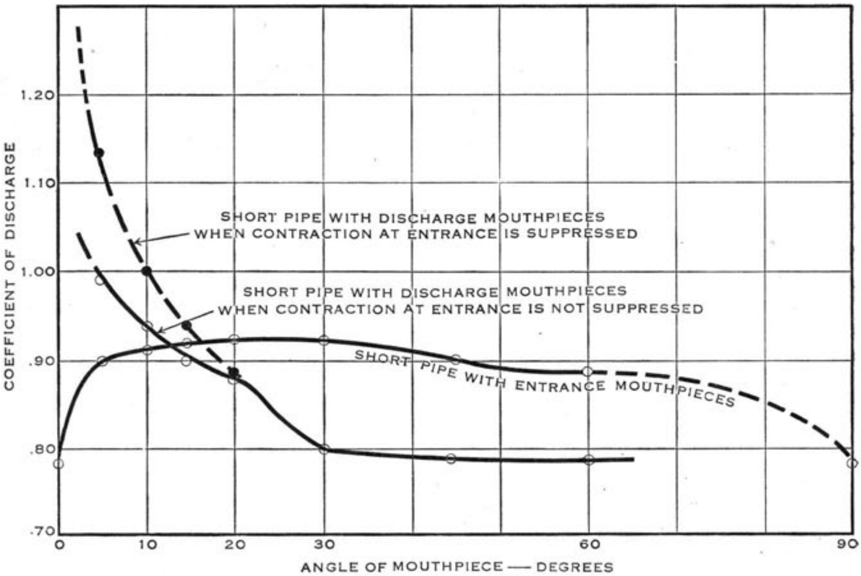

FIG. 10. RELATION BETWEEN COEFFICIENT OF DISCHARGE FOR THE SHORT PIPE WITH MOUTHPIECES AND ANGLE OF MOUTHPIECE FOR AREA RATIO OF 1 TO 2

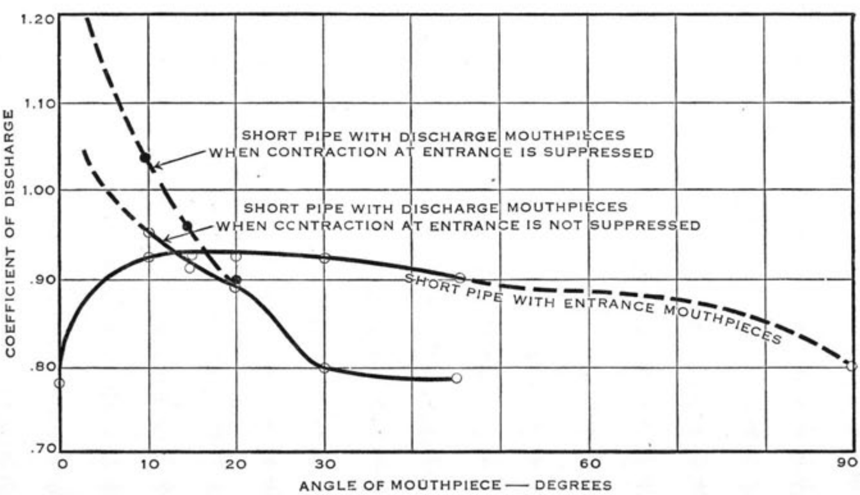

FIG. 11. RELATION BETWEEN COEFFICIENT OF DISCHARGE FOR THE SHORT PIPE WITH MOUTHPIECES AND ANGLE OF MOUTHPIECE FOR AREA RATIO OF 1 TO 3

the length of the discharge mouthpieces having small throat diameters is probably somewhat larger than would be obtained with mouthpieces having large throat diameters but with the same angles of divergence. As already noted in the experiments herein recorded the value of c increased very little for an increase in length corresponding to an increase in area ratio from 1 to 2 to 1 to 3 when the total angle of divergence was 20 degrees or more. It is clear from Table 4 that for discharge mouthpieces having small throat diameters and with the water in a state of smooth flow as it approaches the mouthpiece (entrance mouthpiece attached), an increase in length has a marked effect on the discharge. It will be noted that the greatest increase in c in the experiments reported by Francis occurred when the area ratio was increased from 1 to 2 to 1 to 5. It is probable that the increase in length would have been more noticeable in the present experiments for smaller angles of divergence, particularly for the smoother conditions of flow.

13. *Combinations of Mouthpieces and Effect of Smooth Flow.*—In order to get some measure of the influence of smooth flow upon the rate of discharge and the amount of velocity head recovered by a discharge mouthpiece, experiments were made using combinations of an entrance and a discharge mouthpiece when attached to the short pipe. The results are given in Table 7. It will be seen from this table that a discharge mouthpiece acts more effectively in recovering velocity head when a mouthpiece is attached to the entrance end. The following example will show this in the case of the 5-degree (1 to 2) discharge mouthpiece. This mouthpiece gave a coefficient of discharge of 0.99 (Table 5) when no mouthpiece was attached to the entrance end of the short pipe. From Table 7 it will be seen that the short pipe with a combination of the 5-degree (1 to 2) discharge mouthpiece and the 20-degree (1 to 2) entrance mouthpiece gave a coefficient of discharge of 1.34. This means that the 5-degree mouthpiece has a coefficient of discharge of 1.137 when used in combination with the 20-degree entrance mouthpiece. This value is obtained from the following steps: Attaching a 5-degree (1 to 2) discharge mouthpiece to the short pipe when no entrance mouthpiece is used is equivalent to raising the head on the inward projecting pipe from h to $1.59h$ as obtained by equating the rates of discharge,

$$0.99\ a\ \sqrt{2gh} = 0.785\ a\ \sqrt{2gH_D}, \text{ from which } H_D = 1.59h.$$

In like manner it is found that attaching a 20-degree (1 to 2) mouthpiece to the entrance end of the pipe when no mouthpiece is attached to the disharge end is equivalent to raising the head on the inward projecting pipe from h to $1.39h$ ($H_E = 1.39h$).

Now if both of these mouthpieces were attached to the short pipe, it might be expected that the head on the inward projecting pipe required to give the same discharge (equivalent head) would be $1.39h$ times 1.59 or $2.21h$ and that the coefficient of discharge for the combination, c_c, would be found from,

$$c_c\ a\sqrt{2gh} = 0.785\ a\sqrt{2g\ (2.21h)} \text{ or, } c_c = 1.17^* \quad . \quad . \quad (7)$$

As already noted, however, the coefficient of discharge for this combination as found from experiment is 1.34 (Table 7) which corresponds to an equivalent head of $2.92h$. If all of this increase is attributed to the more efficient action of the discharge mouthpiece due to the fact that it receives the water in more nearly parallel stream-lines (smooth flow), the result is an equivalent head for the 5-degree (1 to 2) discharge mouthpiece of $2.10h$ (Col. 7, Table 7) instead of $1.59h$, an increase of $0.51h$ due to smooth flow. Hence, the coefficient of discharge for the short pipe with the 5-degree (1 to 2) discharge mouthpiece, assuming smooth flow as the water approaches the mouthpiece, would be found from

$$c_d\ a\sqrt{2gh} = 0.785\ a\sqrt{2g\ (2.10h)}, \text{ or } c_d = 1.137^\dagger \quad . \quad . \quad (8)$$

as compared with 0.99. In other words, this particular mouthpiece gives an increase, due to smooth flow, of 14.9 per cent in the rate of discharge.

Tables 5 and 6 give the values of the coefficient of discharge for the short pipe with the various discharge mouthpieces attached when the water flowed through an entrance mouthpiece on its way to the discharge mouthpiece. These values are represented by the dash lines in Figs. 10 and 11. Even though the entrance mouthpiece used gives stream lines that are far from parallel, it is thought that a less turbulent condition of flow would seldom be found at least in a 6-in. pipe, and for velocities above 2 ft. per sec. Hence, the values of the

*See Table 7.

†See Col. 9, Table 5.

coefficient of discharge given by the dash lines are about the maximum to be expected. In each combination the entrance mouthpiece used was one giving about the minimum contraction.

Tables 7 and 8 also give the percentage gain in the coefficient of

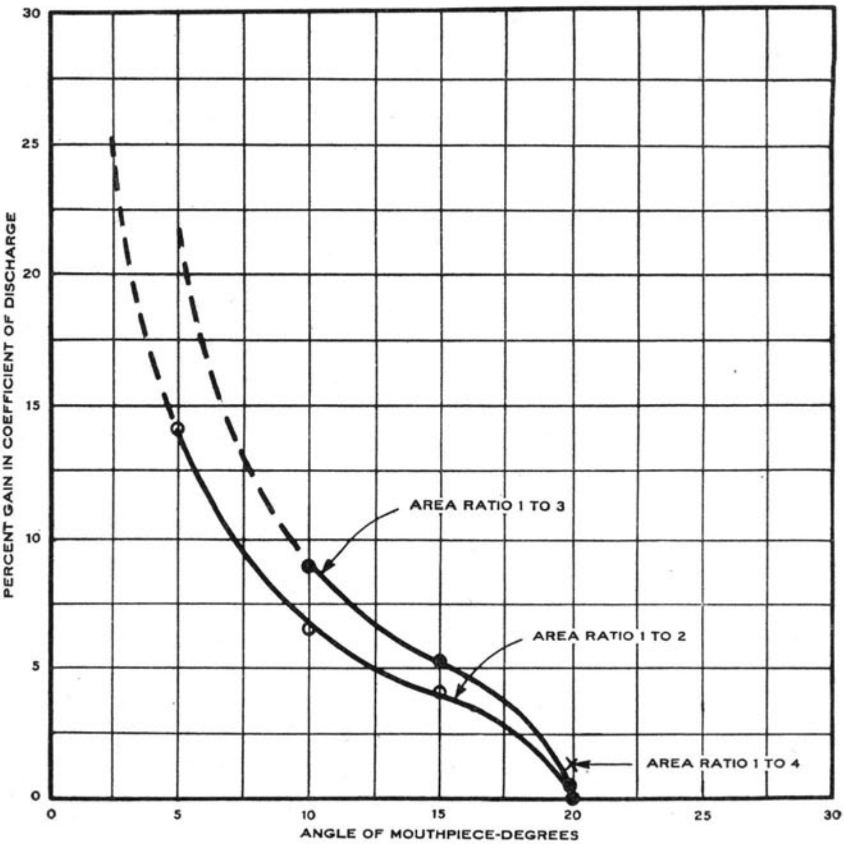

FIG. 12. RELATION BETWEEN PER CENT GAIN IN COEFFICIENT OF DISCHARGE FOR DISCHARGE MOUTHPIECES DUE TO ENTRANCE MOUTHPIECE, AND ANGLE OF DISCHARGE MOUTHPIECES

discharge for the various discharge mouthpieces due to the entrance mouthpiece. The relation of this gain to the angle of the discharge mouthpiece is shown in Fig. 12 for both series of mouthpieces. It will be noted that as the angle of the discharge mouthpiece increases from 5 degrees (10 degrees total angle) the gain in the coefficient decreases rather rapidly, reaching zero for both series of mouthpieces at an angle of about 20 degrees (40 degrees total angle of divergence). Furthermore, this is the same angle at which a discharge mouthpiece rather abruptly ceases to regain any velocity head when no mouthpiece is used on the entrance end (see Figs. 10 and 11). It would seem, therefore, that a 20-degree discharge mouthpiece (total angle of divergence of 40 degrees), or one with a greater angle, allows such a turbulent condition of the water to develop that it is unable to recover any velocity head no matter how smooth the flow may be as the water approaches the mouthpiece. It is worth noting also that a comparison of the results in Table 5, Col. 10, and Table 6, Col. 10, indicates that the length of a discharge mouthpiece has a somewhat larger influence on the discharge when smooth flow exists than when the flow is more turbulent, and it is probable that this effect would have been more noticeable for smaller angles of divergence.

Attention has been called to the fact that the theoretical amount of velocity head which is possible of recovery by discharge mouthpieces having area ratios of 1 to 2, 1 to 3, and 1 to 4 is respectively 75, 88, and 94 per cent of the velocity head in the pipe. The effect of smooth flow may also be measured in terms of the increase in the amount of velocity head recovered by the discharge mouthpieces. For example, the coefficient of loss for the particular combination discussed above is 0.232 (Col. 5, Table 7) and the coefficient of loss for the pipe with the 20-degree (1 to 2) entrance mouthpiece only, is 0.165. Hence the loss of head in the discharge mouthpiece alone is 0.067 times the velocity head in the pipe. But 0.75 of the velocity head is the maximum amount possible of recovery, and since the mouthpiece lost 0.067 velocity heads, the amount recovered is 0.683 velocity heads which is 91 per cent of the maximum amount possible of recovery (Table 7, Col. 11). By a similar analysis it is shown that these same mouthpieces would have regained 58 per cent of the theoretical amount possible of recovery if the 5-degree (1 to 2) discharge mouthpiece had given the same action when in combination as it did when it was the only mouthpiece attached. That is, smooth flow allows a 5-degree (1 to 2) discharge mouthpiece to recover 33 per cent more velocity head than when the water approaches this mouthpiece in a turbulent state of flow (Table 5, Col. 11). The relation between the increase in the velocity head recovered by the discharge mouthpieces due to an entrance mouthpiece, expressed in per cent of the maximum theoretical amount possible of recovery, and the angle of the discharge mouthpieces is shown in Fig. 11. It is probable that the two curves should have the same ordinate for a 15-degree mouthpiece. At least, the curve for the mouthpieces having an area ratio of 1 to 3 should not be above the other curve. If the same entrance mouthpiece had been used in each case, the difference would probably have been negligible. The curves, therefore, are drawn to give the same ordinate for the 15-degree mouthpiece.

In the foregoing discussion it has been assumed that adding a discharge mouthpiece to the short pipe will not change the state of flow in the pipe. This is probably true if the pipe has an entrance mouthpiece attached, suppressing the contraction. If, however, there is no mouthpiece on the entrance end, it appears that the influence of the discharge mouthpiece extends back into the pipe, producing, in effect, a mouthpiece whose smallest cross-sectional area is somewhat less than the area of the pipe and perhaps allowing the expansion of the stream in the pipe to be continuous with that in the mouthpiece. For example, the short pipe with a 10-degree (1 to 2, 20-degree total angle) discharge mouthpiece gave a coefficient of loss of 0.872 (Table 5), and since the short pipe without any mouthpiece attached gives a coefficient of loss of 0.62, the mouthpiece alone should give a loss of 0.252 times the velocity head in the pipe. But from Table 7 it will be seen that this same mouthpiece gives in combination with a 20-degree (1 to 2) entrance mouthpiece a coefficient of loss of 0.468, and after deducting 0.165, which is the loss for the short pipe with the 20-degree entrance mouthpiece, 0.303 is left as the coefficient of loss for the discharge mouthpiece alone. This is inconsistent with the value, 0.252, already established. A similar effect occurs with all the mouthpieces except the 5-degree. The explanation, as already suggested, seems to be that the coefficient of loss for the short pipe is less than 0.62 when a discharge mouthpiece is attached; that the influence of the mouthpiece is felt back into the pipe. This also helps to explain why the coefficients of discharge for mouthpieces with small throat diameters and connected by a short throat are relatively large.

Fig. 13 shows the relation between the coefficient of loss for the short pipe with entrance mouthpiece attached and the angle of mouthpiece. It also shows the relation for the discharge mouthpieces alone as obtained by deducting the loss up to the mouthpiece. The values for the discharge mouthpieces alone are the larger ones as indicated in the example given. The values are given in Table 5. The number of velocity heads in the pipe recovered by any discharge mouthpiece would be the value obtained by subtracting the ordinate to the curve in Fig. 11 from 0.75 for an area ratio of 1 to 2 and from 0.88 for an area ratio of 1 to 3.

14. *Experimental Difficulties Encountered.*—During the early part of the investigation great difficulty was experienced in getting

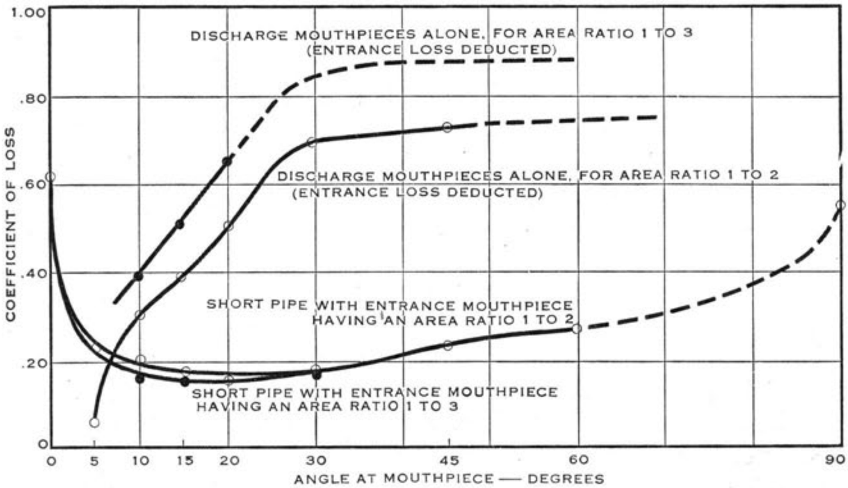

FIG. 13. RELATION BETWEEN COEFFICIENT OF LOSS AND ANGLE OF MOUTHPIECE

consistent results at the higher velocities, especially with mouthpieces having small angles of divergence. The head would fluctuate at times for apparently no good cause, and the value of the coefficient of discharge would vary considerably in successive experiments. It was noticed finally that vortices sometimes formed in the upstream compartment causing a "suck hole" as large as $1\frac{1}{2}$ in. in diameter at the water surface and tapering to a fine point some 10 or 12 in. below the surface. It was observed also that this vortex motion was more active and persistent when the unsteady conditions prevailed, but in no case did it appear to allow air to enter the pipe. In order to prevent the formation of the vortex a float was made fitting closely in the upstream compartment. Grill work on the under part of the float extended deep enough to suppress the vortex before it could fully form. Results were more consistent after the float was used, and it was kept in use for the remainder of the experiments. It was at once noticed, however, that the coefficient of discharge for the longer discharge mouthpieces was lowered by the use of the float and considerable time was spent in attempting to measure the effect

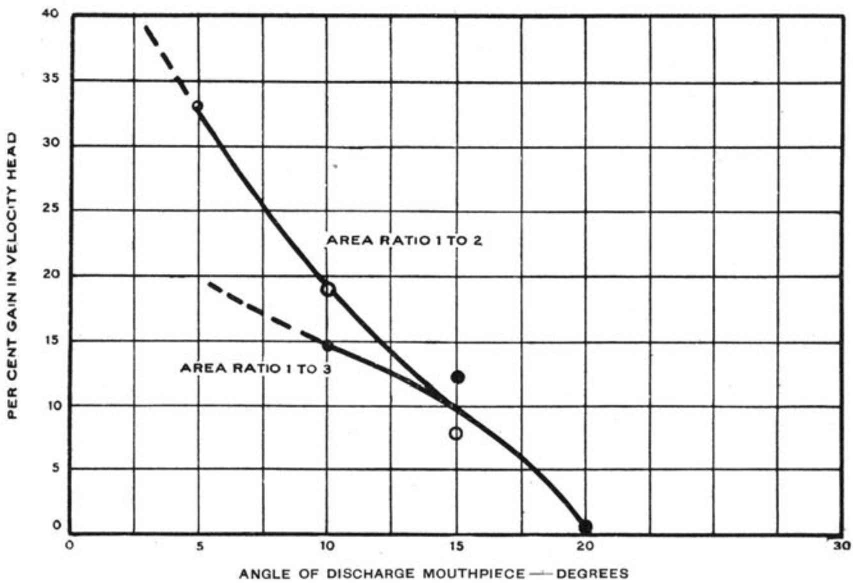

FIG. 14. RELATION BETWEEN GAIN IN VELOCITY HEAD RECOVERED BY DISCHARGE MOUTHPIECES DUE TO ENTRANCE MOUTHPIECE, EXPRESSED IN PER CENT OF THEORETICAL AMOUNT POSSIBLE OF RECOVERY, AND ANGLE OF DISCHARGE MOUTHPIECE

of the vortex. The experimental results clearly indicated that the vortex motion may increase the discharge by at least 2 per cent in the case of the discharge mouthpieces with the smaller angles. If air had entered the pipe, the discharge would, of course, have been decreased. It would appear, therefore, that the whirling motion of the water allowed it to enter the pipe with less contraction. With a 15-degree discharge mouthpiece, the vortex seemed to show no effect although the vortex was not so large nor active as for the 5 and 10-degree mouthpiece. This suggests that perhaps the increase due to the vortex is caused more by the smoother flow of the water as it approaches the discharge mouthpiece, resulting in more efficient action by the mouthpiece (as already discussed), than by reduction in the entrance loss.

Another troublesome factor met with during the experiments was that of temperature changes of the water in the 2-in. pipes used as still basins for the hook gauges. The zero readings of the gauges were taken frequently during the experiments and were found to remain constant until the summer months when results became very erratic. After considerable time it was discovered that the trouble was due to the fact that the sun shone only on the pipe attached to the downstream compartment early in the afternoon, later shifting so as to shine only on the pipe attached to the upstream compartment. After correcting for the difference in temperature thus caused in the two still-basin pipes, consistent results were obtained. It was found, however, that the whole trouble could be avoided by frequently taking water from the tank and pouring it down the pipes.

15. *Conclusions.*—The preceding discussion has shown that the losses accompanying the flow of water depend largely upon the state of its motion which in turn is influenced by many factors, the effects of which in many cases can be but roughly estimated. While the results of these experiments tend to define the range of such effects for certain conditions of flow, additional experiments would be necessary to establish all the inferences which have been suggested. The following conclusions, however, seem justified:

a. As applying to conditions likely to be met in engineering practice, the value for the head lost at the entrance to an inward-projecting pipe (i. e. without entrance mouthpiece and not flush with wall of the reservoir) is 0.62 of the velocity head in the pipe ($0.62\frac{v^2}{2g}$) instead of $0.93\frac{v^2}{2g}$, as usually assumed. To put it in another form, the coefficient of discharge for a submerged short pipe with an inward-projecting entrance is 0.785 instead of 0.72 as given in nearly all books on hydraulics. Further, the lost head at the entrance to a pipe having a flush or square entrance is 0.56 of the velocity head in the pipe ($0.56\frac{v^2}{2g}$)

instead of $0.49 \frac{v^2}{2g}$ as usually assumed. In other words, the coefficient of discharge for a submerged short pipe with a flush entrance is 0.80 instead of 0.82 as given by nearly all authorities.

b. The loss of head resulting from the flow of water through a submerged short pipe when a conical mouthpiece is attached to the entrance end, may be as low as 0.165 of the velocity head in the pipe ($0.165 \frac{v^2}{2g}$) if the mouthpiece has a total angle of convergence between 30 and 60 degrees and an area of ratio of end sections between 1 to 2 and 1 to 4 or somewhat greater. In other words, the coefficient of discharge for a submerged short pipe with an entrance mouthpiece as specified above is 0.915.

c. The loss of head which occurs when water flows through a submerged short pipe having an entrance mouthpiece varies but little with the angle of the mouthpiece if the total angle of convergence is between 20 and 90 degrees and if the area ratio is between 1 to 2 and 1 to 4 or somewhat more. The loss of head for any mouthpiece within this range would be approximately 0.20 of the velocity head in the pipe ($0.20 \frac{v^2}{2g}$). There is, therefore, little advantage to be gained by making an entrance mouthpiece longer than that corresponding to an area ratio of 1 to 2. Thus, an entrance mouthpiece with a total angle of convergence of 90 degrees and the length of which is only 0.2 of the diameter of the pipe gives approximately $0.20 \frac{v^2}{2g}$ for the loss of head.

d. The amount of velocity head recovered by a conical mouthpiece when attached to the discharge end of a submerged short pipe depends largely upon the angle of divergence of the mouthpiece, but comparatively little upon the length of the mouthpiece. This is true for lengths greater than that corresponding to an area ratio of 1 to 2 and for total angles of divergence of 10 degrees or more. The amount of velocity head recovered decreases rather rapidly as the angle of divergence increases from a total angle of 10 to 40 degrees. At or near 40 degrees the amount of velocity head recovered rather abruptly falls to approximately zero.

e. A conical discharge mouthpiece having a total angle of divergence of 10 degrees and an area ratio of 1 to 2, when attached to a submerged short pipe, will recover 0.435 of the velocity head in the pipe, which is 58 per cent of the theoretical amount possible of recovery.

f. The amount of velocity head recovered by a diverging or discharge mouthpiece when attached to a submerged short pipe is considerably more when a converging or entrance mouthpiece is also attached than it is when the entrance end of the short pipe is simply inward-projecting (no mouthpiece attached). This excess in the velocity head recovered diminishes rather rapidly as the angle of the discharge mouthpiece increases, and it becomes zero for a discharge mouthpiece having a total angle of divergence of approximately 40 degrees. This increase in the velocity head recovered is probably due to the effect of smooth flow in the pipe as the water approaches the discharge mouthpiece. The smooth flow allows the mouthpiece to recover more of the velocity head in the pipe than when a more turbulent flow exists; this increase amounts to as much as 33 per cent in the case of the discharge mouthpiece having a total angle of divergence of 10 degrees and an area ratio of 1 to 2.

While these conclusions are drawn from experiments on the flow of water through a particular short pipe having various entrance and discharge conditions, it is felt that the results of the experiments are applicable in a general way to a large variety of cases in engineering practice where the contraction and expansion of a stream of water occurs. A number of such cases are suggested in the introduction.

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*This reference was not found until after this bulletin had been written. The sizes of the mouthpieces and their angles of divergence are so small (about 0.6 in. throat diameter and 4 to 12 degrees total angle of divergence) and the velocity is so high (32 to 131 ft. per sec. throat velocity) that the results are not directly comparable with the results given in this bulletin. The article contains, however, much valuable information, emphasizing in particular the decrease in lost head when the water is caused to whirl or rotate about the axis of the tube as it enters the tube. The above article also refers to experiments of Banninger, published in "Zeitschrift für das Gesamte Turbinenwesen," 1906, page 12. The results of these experiments have not been available for examination.

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