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THE FAILURE OF PLAIN AND SPIRALLY REINFORCED CONCRETE IN COMPRESSION

BY

FRANK E. RICHART
ANTON BRANDTZAEG
REX L. BROWN

BULLETIN No. 190

ENGINEERING EXPERIMENT STATION

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THE FAILURE OF PLAIN AND SPIRALLY REINFORCED CONCRETE IN COMPRESSION

BY

FRANK E. RICHART
RESEARCH ASSOCIATE PROFESSOR OF THEORETICAL AND APPLIED MECHANICS

ANTON BRANDTZAEG
GRADUATE STUDENT IN THEORETICAL AND APPLIED MECHANICS
(PRESIDENT, A/S BETONGBYGG, TRONDHJEM, NORWAY)

AND

REX L. BROWN
RESEARCH ASSOCIATE IN THEORETICAL AND APPLIED MECHANICS

ENGINEERING EXPERIMENT STATION
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THE FAILURE OF PLAIN AND SPIRALLY REINFORCED CONCRETE IN COMPRESSION

I. Introduction

1. *Introduction.*—This bulletin deals with an investigation of the action of concrete under compressive stresses applied in one or more directions. A part of the investigation has already been reported in Bulletin 185 of the Engineering Experiment Station, which contains the results of tests of concrete cylinders subjected to combined stresses applied by means of hydraulic pressure. The present bulletin contains the results of a closely allied series of tests on short columns or cylinders of plain and spirally reinforced concrete. The tests were made with two distinct objects: to study the general behavior of concrete under compression in one direction or in three directions, and to study directly certain phenomena relating to spirally reinforced columns. The tests described in Bulletin 185, wherein the lateral pressure on a cylinder was directly controlled, gave information on a range of stresses that could not be included in any ordinary series of column tests. However, the tests described herein afforded an opportunity for a large number of deformation measurements, and in this respect are far superior to the tests employing hydraulic pressure.

The spirally reinforced column is an important practical example of the use of concrete under three-dimensional compression. Analyses and tests have furnished a fund of information which serves as a basis for the design of such members; there is still lacking, however, a knowledge of the general principles governing the action of such members by means of which the limited test results available may safely be extended and generalized. Certain principles regarding the effect of lateral restraint in raising the ultimate strength of concrete were stated many years ago by Considère.* One of these is that the strength of hooped concrete may be considered as being the sum of two essentially unlike quantities: a resistance proportional to the strength of the concrete itself, and an added strength which is a function of the lateral restraint applied by the hooping. Considère reasoned that the second contribution to the strength was similar to the bearing power of a non-cohesive granular material when laterally restrained, a property dependent upon the internal friction of the

*Considère, A. "Résistance à la compression du béton armé et du béton fretté," Génie Civil, 1903. See Translation, "Experimental Researches on Reinforced Concrete," by L. S. Moisseiff, 1906.

material. This conception agrees in many respects with the results of tests and is commonly employed in the analysis of spirally reinforced columns.

The tests described in this bulletin were intended to throw light on a number of questions, of which the following are typical, regarding the action of spirally reinforced members: What is the relation between the lateral pressure developed by the reinforcement and the added strength (above that of the plain concrete) at various stages of loading? Is the added strength dependent only upon the lateral pressure, and, as in the case of a granular material, independent of the amount of deformation produced? What is the character of the action by which the material in the column changes from an almost elastic solid to an apparently plastic material after the spiral reinforcement has become effective? What conditions determine the maximum load carried by a column in which failure of the spiral reinforcement is not reached? In view of the large deformations accompanying the use of spiral reinforcement, how much of the strength of such members can be utilized in buildings, bridges, or similar structures?

Previous tests have answered certain of these questions to some extent. Among the foremost investigators of the spirally reinforced column may be mentioned Considère, Talbot, Bach, Withey, Rudeloff, Saliger, Mörsch and others; important tests have also been carried on by various committees of technical organizations. A bibliography of selected references to research on spirally reinforced columns is given in the Appendix.

In the tests of this bulletin an especial attempt was made to secure complete and systematic observations of the behavior of plain and spirally reinforced concrete under load, and an unusually large number of deformation readings were taken as the maximum load was approached. The group of columns tested contained widely varying amounts and kinds of reinforcement.

2. *Acknowledgment.*—The tests reported in this bulletin were performed in 1925-6 as a part of the regular work of the Engineering Experiment Station under the administrative direction of DEAN M. S. KETCHUM, Director of the Station, and of PROFESSOR A. N. TALBOT, then in charge of the Department of Theoretical and Applied Mechanics. Professor Talbot, whose wide experience in investigations of reinforced concrete made his advice invaluable, directed the planning of the tests and the choice of materials, thus insuring much of the value of the results.

The investigation was carried on as a basis for a Master's thesis by Mr. Brandtzaeg, who chose the line of study, outlined the tests, and took much of the responsibility of carrying on the tests and analyzing the results. Most of the laboratory work, which required special technique, was performed by Mr. Brown and Professor Richart, who also supervised the investigation and gave special attention to the interpretation of the test results.

Acknowledgment is due the AMERICAN STEEL AND WIRE COMPANY, the ILLINOIS STEEL COMPANY and the AMERICAN SYSTEM OF REINFORCING, all of Chicago, who furnished the steel used in the column spirals. The company last mentioned also assisted in the securing of suitable grades of material and fabricated all of the test specimens.

3. *Notation.*—The following notation will be used throughout the bulletin:

d = diameter of spiral wire.
D = outside diameter of spiral.
ϵ_1 = axial unit deformation in column.
ϵ_2 = lateral unit deformation in column.
ϵ_s = unit deformation in spiral reinforcement.
$\epsilon_v = \epsilon_1 + 2\epsilon_2$ = volumetric deformation or unit change in volume of column.
E = initial modulus of elasticity of concrete.
f_1 = axial compressive unit stress.
f_2 = lateral compressive unit stress.
f_s = tensile unit stress in spiral reinforcement.
$f_v = f_1 + 2f_2$ = "volume" stress.
f_c' = compressive strength of plain concrete.
f_b = bearing unit stress between concrete and spiral.
k = rate of depression of spiral into concrete, expressed as the difference between lateral unit deformations in concrete and in spiral for an increment of spiral stress of 10 000 lb. per sq. in.
K = initial bulk modulus of concrete.
$N = \frac{f_b}{k}$ = modulus of depression.
p = percentage of spiral reinforcement, based upon outside diameter of spiral.
μ = initial value of Poisson's ratio for concrete.

II. Materials, Test Pieces, and Testing

4. *Outline of Tests.*—The investigation of the failure of concrete in compression made in 1925 included four series of tests; the part described in this bulletin was designated as Series 1, and consisted of one group of plain concrete and six groups of spirally reinforced concrete compression test pieces, all made of a 1:2.1:2.5 mixture. All test pieces were cylinders, or short columns, 10 in. in diameter and 40 in. long; the size of cross-section was limited by the capacity of the testing machine, while the length was chosen so as to avoid the bending effects to be found in long columns and still to furnish a satisfactory length of member unaffected by the restraining influence of the end bearing plates. Three test pieces of each kind were used except in the case of the plain concrete, where five were used. Because of the limited number of test pieces, the only variables introduced in the series were the amount and kind of spiral reinforcement. Table 1 gives an outline of the series.

5. *Materials.*—The concrete materials were identical with those used in the tests described in Bulletin 185.

Universal portland cement was used in all specimens. Immediately before its use a supply sufficient to make all specimens was screened into tight metal containers and thoroughly mixed. The cement complied with the Standard Specifications for Portland Cement of the American Society for Testing Materials, as shown by the physical properties listed in Section 8, Bulletin 185. The sand and gravel used were washed materials from the Wabash River at Attica, Indiana. Sieve analyses and other properties of these materials are given in Table 2.

The proportions and consistency of the concrete were carefully chosen in order to secure uniformity and workability of the mixture. The proportions of the mixture employed were 1:2.1:2.5, by loose volumes, though all measuring of ingredients was done by weight. This mixture had a corrected water-cement ratio of 0.87, an average slump of 6.9 in., and a flow, or ratio of final to initial base diameter of the standard specimen on the flow table, of 215 per cent. The relative absolute volumes of the ingredients of the concrete used, as determined from test cylinders made with the test pieces were: cement, 0.118; sand, 0.328; gravel, 0.337; water, 0.212. Detailed data regarding the concrete are given in Table 3.

The reinforcing spirals were intended to be of mild steel, with the exception of one lot of special high strength cable steel which hap-

TABLE 1

OUTLINE OF TESTS OF SERIES 1

All test pieces 10 in. in diameter, 40 in. long
Concrete, 1: 2.1: 2.5, by loose volumes

Group No.	Column No.	Number of Columns in Group	Percentage of Spiral Reinforcement	Nominal Diameter of Steel, in.	Kind of Steel
0	02–06	5	0	..	
1	11–13	3	0.50	1/8	Annealed drawn wire
2	21–23	3	1.11	3/16	Annealed drawn wire
3	31–33	3	2.07	1/4	Rolled mild steel
4	41–43	3	2.64	5/16	Rolled mild steel
5	51–53	3	4.41	3/8	Rolled mild steel
13	131–133	3	1.96	1/4	Suspension cable wire

TABLE 2

SIEVE ANALYSES AND OTHER PROPERTIES OF AGGREGATES

Item	Sieve No.*	Sand	Gravel
Percentage of Material Passing a Given Sieve	100	1	
	48	2	
	28	21	
	14	54	
	8	71	
	4	93	4
	3		14
	3/8-in.		33
	1/2-in.		65
	3/4-in.		96
Per cent moisture content, as used		0.2	0.3
Per cent moisture absorption		0.6	1.0
Specific gravity		2.67	2.69
Unit weight, lb. per cu. ft.			
(a) Measured loose, struck off		107	94
(b) Tamped, A.S.T.M. method†		113	103

*Tyler Standard Screen Scale Sieves.

†Standard Method of Test for Unit Weight of Aggregate for Concrete, A.S.T.M. Book of Standards, Part II, p. 120, 1927.

pened to be available. Five sizes of mild steel were chosen, having diameters of 1/8, 3/16, 1/4, 5/16, and 3/8 in., while the cable steel stock was of 1/4-in. diameter. It was found that the smallest two sizes—1/8-in. and 3/16-in. rounds—could not be secured in rolled bars, so that drawn wire was substituted in these sizes. This wire, although annealed, varied greatly in quality from the rolled steel used in the larger sizes.

TABLE 3

DATA OF CONCRETE MIXTURES

From tests of three 6 by 12-in. control cylinders made with each column

Column No.	Slump, in.	Flow, per cent	Cement-Space Ratio	Water-Cement Ratio	Compressive Strength* lb. per sq. in.
02.......	6.6	209			2530
03.......	7.0	216			2550
04.......	6.0	217			2290
05.......	6.8	216			2280
06.......	7.0	210			2475
Average...	6.7	214	0.353	0.87	2425
11.......	6.9	229			2675
12.......	7.3	225			2790
13.......	7.6	212			2630
Average...	7.3	222	0.353	0.87	2700
21.......	7.0	220			2640
22.......	7.1	213			2865
23.......	7.5	210			2725
Average...	7.2	214	0.354	0.87	2725
31.......	7.4	215			2880
32.......	6.9	216			2525
33.......	6.4	213			2785
Average...	6.9	215	0.351	0.87	2730
41.......	6.2	214			2725
42.......	7.9	216			2640
43.......	7.1	198			2480
Average...	7.1	209	0.352	0.87	2615
51.......	6.0	222			2745
52.......	8.5	225			2890
53.......	6.3	212			2685
Average...	6.9	220	0.354	0.87	2775
131.......	6.7	205			2665
132.......	6.0	217			2525
133.......	6.9	222			2860
Average...	6.5	215	0.353	0.87	2685
Grand Average..	6.9	215	0.353	0.87	2645

*At age of 28 days.

Some differences were also found in the properties of the three larger sizes of mild steel, and hence the spirals used represented a considerable variation in quality. The high strength steel was heat-treated drawn wire of a special quality originally intended for use in suspension bridge cables, and it showed a high degree of uniformity in physical properties. Information concerning the spirals, including certain dimensions and physical properties described in Section 6, are given in Table 4.

TABLE 4

AVERAGE DIMENSIONS AND PROPERTIES OF SPIRAL REINFORCEMENT*

Group No.	Outside Diam. of Spirals, in.	Overall Section of Col., sq. in.	Spiral Wire		Pitch of Spiral, in.	Per cent of Spiral	Stress in Spiral, lb. per sq. in.		Ultimate Unit Elongation	Remarks
			Diam., in.	Section, sq. in.			When $\epsilon_s = 0.005$	At Ultimate		
1................	10.0	78.1	0.124	0.0120	0.98	0.50	67 000	70 500	0.010	Wire slightly elliptical
2................	10.0	78.1	0.187	0.0273	0.99	1.11	61 000	70 600	0.020	
3................	9.8	76.0	0.254	0.0503	0.99	2.07	38 000	53 500	0.162	Steel slightly crooked
4................	9.9	77.2	0.282	0.0624	0.96	2.64	40 000	53 400	0.156	
5................	9.9	77.2	0.376	0.1106	1.01	4.41	46 000	66 700	0.159	
13................	9.9	77.7	0.250	0.0486	1.00	1.96	106 000	197 400	0.039	

*Estimated maximum variation from average of each group: wire section, 0.2 to 2.6 per cent; pitch of spiral, 1 to 4 per cent; percentage of spiral, 1 to 4 per cent.

6. *Fabrication and Testing of Reinforcing Spirals.*—The spirals were all fabricated by the American System of Reinforcing, under the inspection of Mr. Brandtzaeg. The steel as received at the shop was in coils of large diameter. The spirals were coiled by machine and held to the desired pitch by means of spacers. Four spacers were used on each spiral, being made of a crimped No. 11 wire on the outside of the spiral attached to a ¼-in. round rod on the inside of the spiral. These spacers provided 0.3 per cent of longitudinal reinforcement which, however, was neglected in all calculations. The spirals were made one inch shorter than the finished columns and two extra turns were added at each end for anchorage.

The dimensions of all spirals are recorded in Table 4, together with information on the variability of the dimensions. The diameter of the spiral wire was measured with a micrometer at five points along the spiral, two measurements being taken at right angles to each other at each point. Measurements of the spacing were taken as one-fifth of the distance between every fifth winding, and seven of these measurements were taken along two sides of each spiral.

In order to utilize the strain-gage measurements that were taken in the tests of spirally reinforced members to determine the lateral stress in such a member, it was necessary first to have an accurate knowledge of the stress-strain relation for the spiral steel at all stages of loading. Hence a particular effort was made to determine the stress-strain properties of the spiral steel as it actually existed in a column. Since it was not feasible to test a curved member in tension, and since the processes of coiling and straightening had marked effects upon the properties of the steel, indirect methods were employed to determine the true stress-strain relation for each spiral.

During fabrication three test coupons were cut from the stock used in each spiral, two being cut from the extreme ends of the steel used in the spiral, but before it had been coiled. The first two, taken from the uncoiled wire, were nearly straight. The third, which had been bent to a 5-in. radius, was straightened in a uniform way before testing by pulling the wire through a vise.

The tests were made in a hand-operated Olsen wire-testing machine of 10 000-lb. capacity. Extensometer measurements were taken on an 8-in. gage length for the wire and on a 2-in. gage length for the heavier rolled sizes. Average stress-strain diagrams embodying the results of the tests are shown in Figs. 1 and 2. In general, each curve represents the average of three tests.

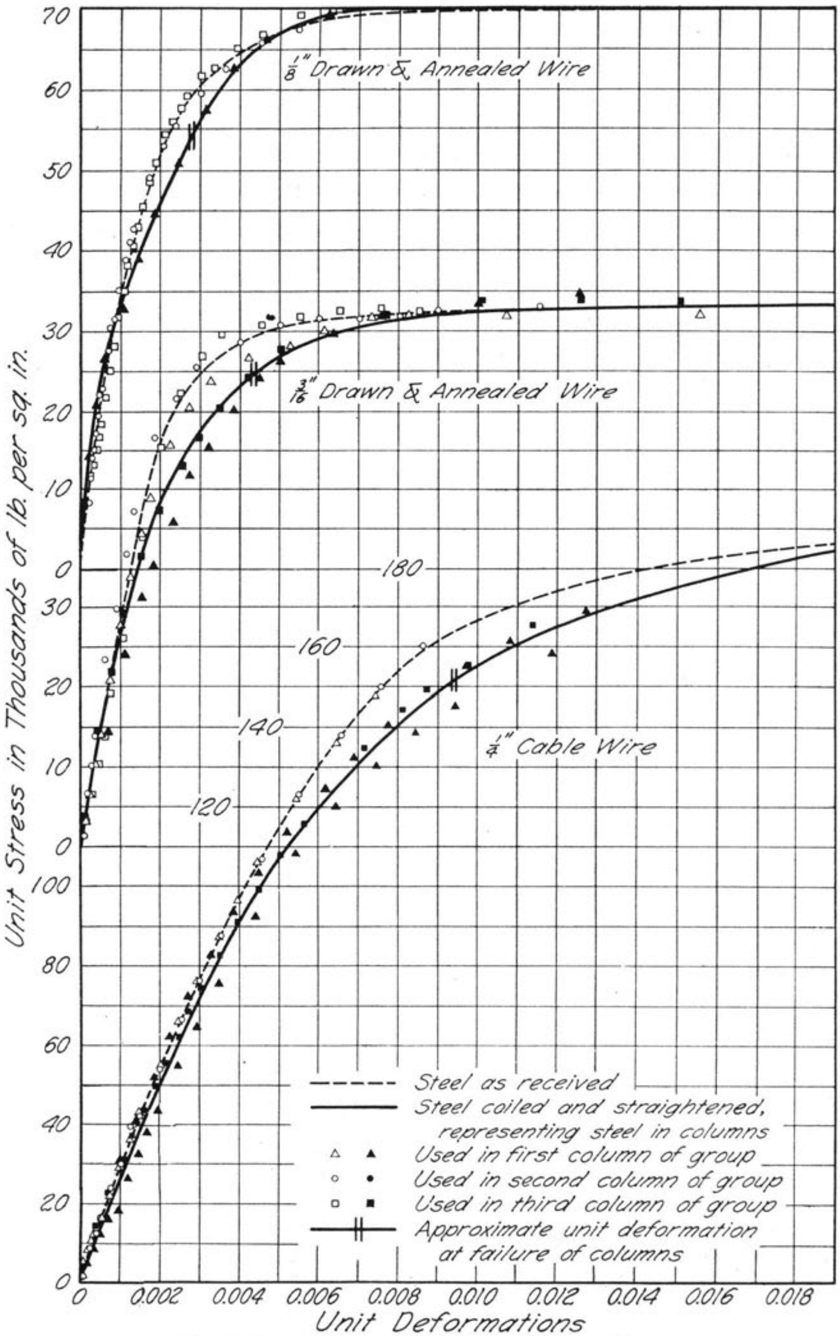

FIG. 1. STRESS-STRAIN CURVES FOR DRAWN WIRE

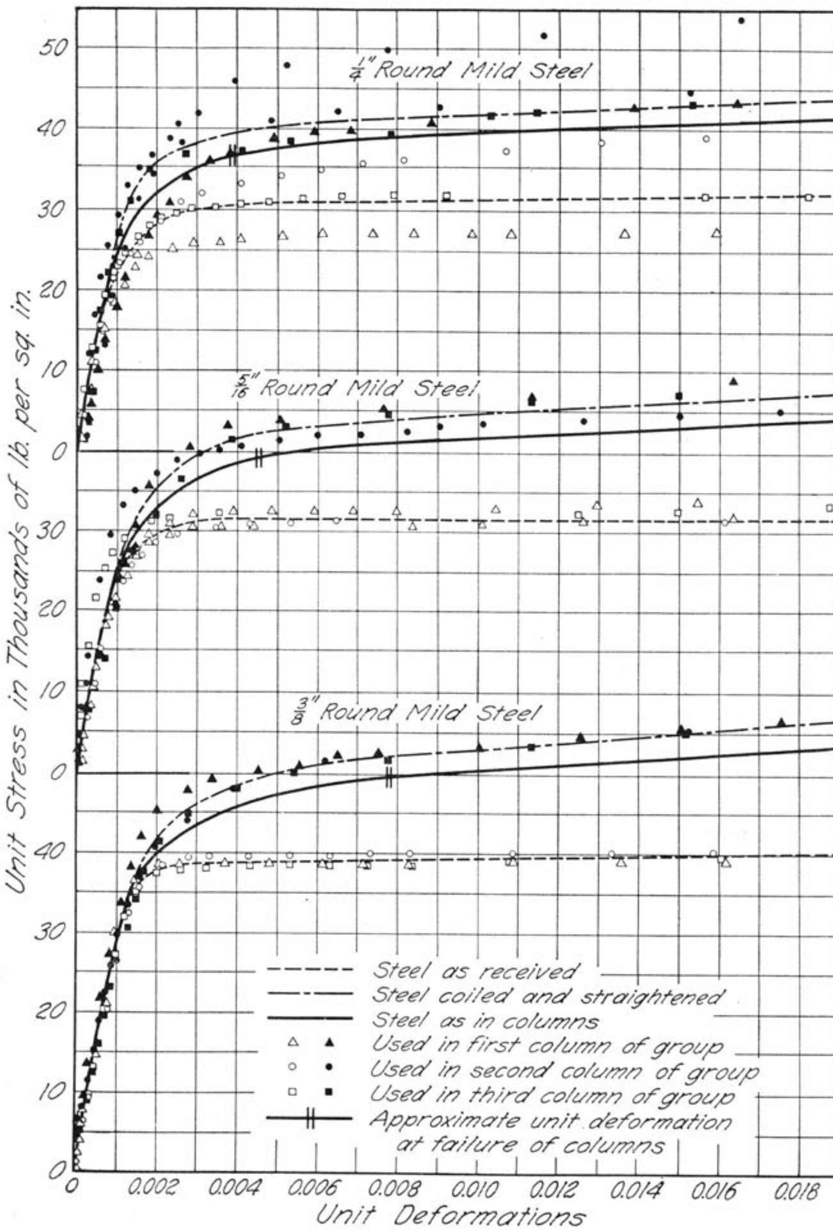

FIG. 2. STRESS-STRAIN CURVES FOR MILD STEEL

Figures 1 and 2 show a considerable difference between the properties of the material before it had been coiled into a spiral and after it had been coiled and straightened. It is evident that the amount of cold-working received by the spirals during coiling was somewhat less than that received by the test coupons that were coiled and straightened. The latter specimens had been allowed to rest a week between the straightening and testing. This rest gave somewhat higher elastic strength than that found when coupons were tested immediately after straightening, an effect noted many years ago by Bauschinger.* In order to agree with conditions in the spirals, only the tests of rested specimens are included in Figs. 1 and 2.

Two different effects of cold-working are seen in the different grades of steel. For the ⅛-in. and the 3/16-in. drawn wire and the ¼-in. cable steel of Fig. 1, the effect of the cold-working produced in coiling and straightening the coupons appears to be an appreciable lowering of the stress-strain curve up to and around the yield point, though the difference between the two curves is not very great. For the ¼-in., the 5/16-in. and the ⅜-in. mild steel of Fig. 2, however, a great difference is seen between the two curves, the effect of cold working apparently being to raise the whole upper part of the stress-strain curve a large amount. This effect is so great that it has an important bearing upon the interpretation of the column tests, and it was felt necessary to get further information regarding the part of the effect due to coiling and the part due to straightening of the coupons. With this object in view a number of auxiliary tests were made, the most satisfactory being a small series of cross-bending tests of pieces of the ⅜-in. spiral steel, loaded on a span of 4 inches, with a concentrated load at midspan. Tests were made on (A) the steel straight as received (B) the steel bent as in the spirals and (C) the steel cut from the spirals, straightened, and rested for eight days before testing. While the load-deflection relations obtained from these tests may not be analogous to the stress-strain relations to be found in tension tests, it was felt that the relative effects of different degrees of cold-working might be at least roughly indicated. In the cross-bending test it was possible not only to test the steel curved just as it was in the spiral, but also to study the effect of applying the load so as either to increase or to decrease the original curvature. Two specimens of type A, two of type B, with downward load applied to the inside of the spiral, two of type B, with load applied to the outside of the spiral, and two each for the corresponding cases of type C were tested.

*Bauschinger, J. See "Handbook of Testing Materials" by A. Martens, translated from the German by G. C. Henning, 1899.

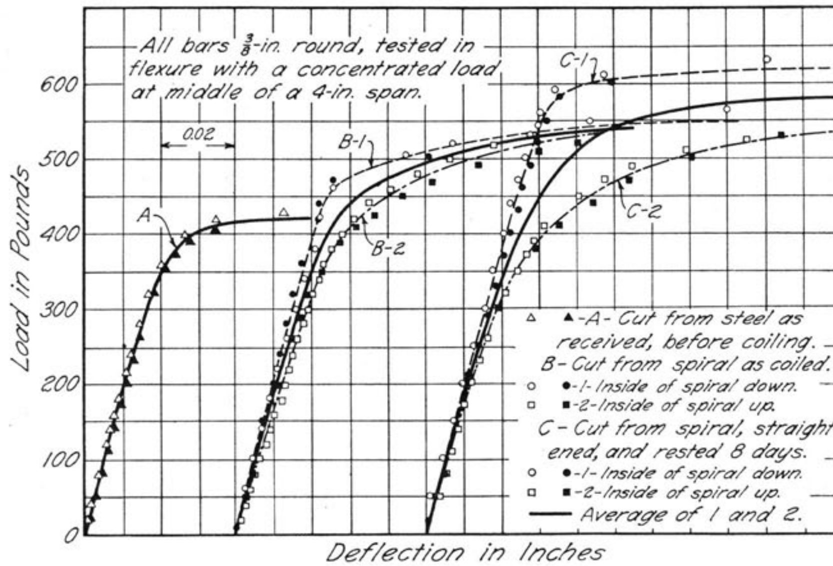

FIG. 3. LOAD-DEFLECTION CURVES FOR 3/8-IN. ROUND BARS

The load-deflection curves obtained from these tests are given in Fig. 3, which shows that there was a distinct difference between the curves for the three types of specimens as well as consistent differences for types B and C between the curves for those specimens tested with the center of curvature upward and those with it downward. Figure 3 shows that the average curve for type B falls between those for type A and type C, as would be expected. A study of the curves shows that near the yield point the effect of coiling the spiral was about half as great as that of the combined coiling and straightening; with greater deflections the effect of coiling was relatively much larger, reaching about 80 per cent of the combined effect at a deflection four times that found at the yield point. This information was used as a guide in locating curves for the 1/4-in., the 5/16-in., and the 3/8-in. mild steel rounds of Fig. 2, which are considered as representing the stress-strain curves of the spiral steel as it existed in the columns. For the other three groups of spirals, made of drawn wire and of cable steel, the difference between the curves of Fig. 1 for the steel as received and those obtained from the steel after coiling and straightening was so small that the latter were taken as representing the properties of the spiral steel used in the columns.

It is felt that this study of the properties of the spiral reinforcement has added greatly to the reliability of the column test data to follow; it should not be implied that the effect of cold-working can always be judged from the results of these tests, but these results should emphasize the fact that the usual tests of samples of spiral steel which have been subjected to an unknown amount of cold-working cannot be relied upon to furnish stress-strain curves that are applicable to the steel as it exists in a column.

7. *Making and Curing of Test Pieces.*—The concrete for all test pieces was mixed in a batch mixer of one-third cubic yard capacity, all materials being weighed and dumped into the mixer in the order: gravel, sand, cement, and water. Mixing was continued for four minutes, whereupon the batch was discharged into a large pan, further turned over a few times with shovels, and then deposited in the forms. The forms were made of 10-in. steel pipe split into four parts, and held in position by three heavy circular steel bands. In placing the concrete, it was puddled with a ¾-in. round steel rod, and the form was tapped with a heavy hammer during pouring.

Consistency tests and 6 by 12-in. test cylinders were made from a portion of each batch. Slump and flow tests were first made, then the same concrete was used in making the cylinders. Both the large test pieces and the cylinders were capped with neat cement soon after pouring. The test pieces were removed from the forms after two days and were stored in the laboratory under wet burlap for 24 days. During the curing period the temperature of the laboratory varied between 65 and 75 deg. F., but with no marked change in the mean temperature throughout the period. Two days before the date of testing, the burlap was removed from the columns and the gage holes for strain measurements were prepared. At this time the small steel plugs required for measurements of concrete deformations were inserted and firmly anchored by means of plaster of paris. Gage lines on the spiral steel, which was practically at the column surface, were also drilled, except in the case of the spirals of ⅛-in. wire, on which the gage holes were omitted because of the small diameter of the wire. The location of gage lines is shown diagrammatically in Fig. 4, in which the surface of the column is developed to indicate the lines which were laid out on the four sides of the column at three points along the length.

8. *Testing Apparatus and Procedure.*—The large specimens were tested in a 600 000-lb. Riehle testing machine; the 6 by 12-in. cylinders in a 300 000-lb. Olsen testing machine.

Measuring instruments used included Berry type strain gages of 4- and 8-in. gage lengths, Howard type gages, fitted with dial indicators,

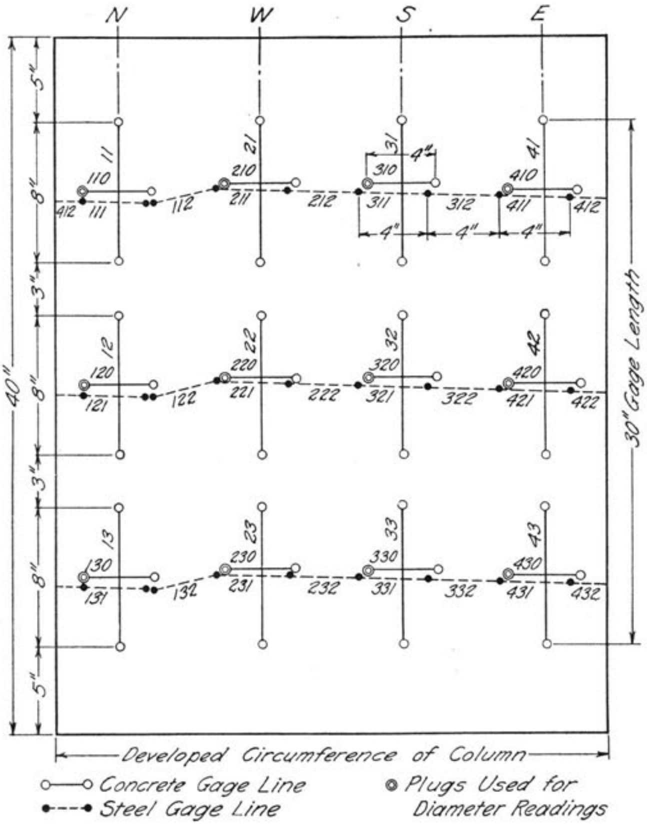

FIG. 4. DIAGRAM SHOWING LOCATION OF GAGE LINES ON COLUMNS

of 4-, 8-, and 30-in. gage lengths, and a special diameter gage designed for the tests by Mr. Brown. A view of these gages is given in Fig. 5. All gages were equipped with conical points. In one test the diameter gage was fitted with special contact points, consisting of conical sockets or cups which engaged a ball bearing embedded in the steel gage plugs of the test piece, a detail that has been used in other gages. This was intended to eliminate the effect of wear of the edges of gage holes, which affected the readings of this gage particularly, since the direction of deformation was parallel to the axes of the contact points and gage holes, but no appreciable advantage of the ball and socket contact was noted.

Ames micrometer dials reading 0.001 in. were employed in all of the strain gages, the Berry gages having a lever ratio of about 7.5 to 1,

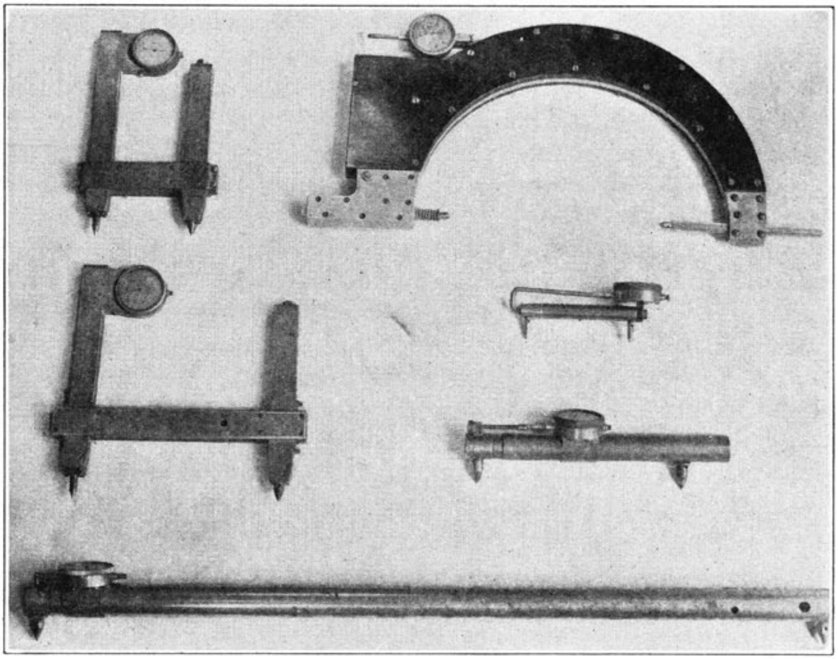

FIG. 5. VIEW OF STRAIN GAGES USED IN TESTS

the diameter gage 5 to 1, while the Howard gages indicated directly without multiplication. All were used on gage holes made with a No. 54 drill. In the diameter gage the points were held in the gage holes with a constant pressure produced by a strong spring at the lever end of the instrument. The probable errors in unit deformations obtained by the use of the different instruments, as judged from the consistency of the readings, were roughly as follows: 8-in. Berry gage, 0.000008; 4-in. Berry gage, 0.000025; diameter gage, 0.000010; 30-in. Howard type gage, 0.000033; 8-in. Howard type gage, 0.00005; and 4-in. Howard type gage, 0.0005. The latter two gages were used only after the deformations had exceeded the range of the Berry gages. It is felt that the gages used furnished a satisfactory degree of precision for most purposes of the tests.

After all gage holes had been prepared, the column was placed in the testing machine, plumbed and centered. At times it was found necessary to embed the base of the test piece in plaster of paris to secure uniform bearing. At the beginning of a test an initial or "zero" load of 1000 to 2000 pounds was applied to the column through a spherical bearing block, which was then prevented from further rotation by the use of wedges. The initial strain measurements were next taken, and the remainder of the test proceeded with the column loaded between two fixed and fairly parallel planes. Loads were applied in increments varying from 25 000 to 35 000 pounds in the early stages of the test, and gradually decreasing to 10 000 to 15 000 pounds during the later stages. The testing machine was always run at its nominal speed of 0.05 inch per minute; the load recorded was that indicated by the scale beam at the instant the machine was being stopped; and a record was kept of any decrease in the load during the period, usually 7 to 12 minutes, in which strain measurements were taken. Strain measurements were always taken up to the maximum load, and generally beyond it. When failure came through breaking of the spiral reinforcement the tests were discontinued, but in cases where the columns failed through a general yielding, the test was carried somewhat beyond the maximum load. A number of special tests were made at this final stage of loading, to which reference will be made later.

9. *Record of Tests.*—A brief description of the behavior of the test pieces under load may be useful. Certain terms used in the description will be defined at the outset. Thus, while the test pieces are not columns in the ordinary usage of the word, since the height is only four times the diameter, they will be so designated for lack of a better term. The maximum load carried will be defined as the greatest load reached, with the testing machine running continuously. Beyond this load further operation of the machine produced a recorded decrease in resistance. In the tests of spirally reinforced columns it was found that, after such a maximum load was passed and loading was stopped for a short time and then resumed, a second "maximum" was observed. This second maximum was usually slightly higher than the first. (See Table 6, Section 20.) Similarly a third or a fourth maximum was sometimes noted. To avoid uncertainty, the first maximum load observed will always be denoted as the maximum load carried by the column, since this is the load that would be observed in a continuous test to complete failure. The load indicated by the machine at any time a stop was made to take strain measurements will be considered as the load on the column, although it was observed that, particularly at the higher loads, a considerable decrease in the load occurred during the time the machine was stopped. The starting load, or load on the column when the machine was again started, was always recorded and the difference between stopping and starting loads will be discussed in Section 19.

A. Plain Concrete Columns

This lot of five plain columns, of 1:2.1:2.5 concrete, was designated as Group 0.

Group 0.—The maximum load carried by Column 02 was 2290 lb. per sq. in. A number of nearly vertical cracks were developed just as the maximum load was reached, and loading was discontinued.

A set of strain gage readings was taken on Column 03 at a load of 1940 lb. per sq. in. During the readings an increase in lateral deformation indicated that a large plastic flow was taking place. Cracks also developed during the period in which the readings were taken. When the machine was started again, the load had fallen off and the column failed before the load reached the amount previously applied. It is probable that the maximum load attained would have been greater than 1940 lb. per sq. in. if the column had been loaded continuously to failure.

In the test of Column 04, the special conical sockets mentioned in Section 8 were used on the diameter gage. The arrangement worked very satisfactorily, but not particularly better than the usual conical points when the gage holes for the latter were properly worn down, and on account of the greater simplicity the conical points and drilled gage holes were used in subsequent tests. Strain gage readings were taken on this column at a load of 2120 lb. per sq. in., which proved to be the maximum load. It was noted that in a period of about 7 minutes the unit deformations at the midheight of the column showed an average increase of 0.00021 on longitudinal lines and 0.00044 on radial lines. Cracks developed during the taking of readings, and the column failed to reach a higher load when testing was resumed.

The maximum load on Column 05 was 2080 lb. per sq. in. Complete strain readings were taken at this load. Loading on this column was continued until a complete failure of the shearing type occurred on planes inclined 50 to 60 deg. to the horizontal.

Early strain readings on Column 06 were probably affected by rapid temperature changes in the laboratory. The maximum load on this column was 2220 lb. per sq. in.

Figure 6 shows a view of Columns 02, 03, and 04 after testing.

B. Spirally Reinforced Concrete Columns

This lot of columns, of 1:2.1:2.5 concrete, had spiral reinforcement of varying amounts and qualities.

Group 1.—The columns of this group had 0.5 per cent of spiral reinforcement of ⅛-in. annealed drawn wire. Because of the small di-

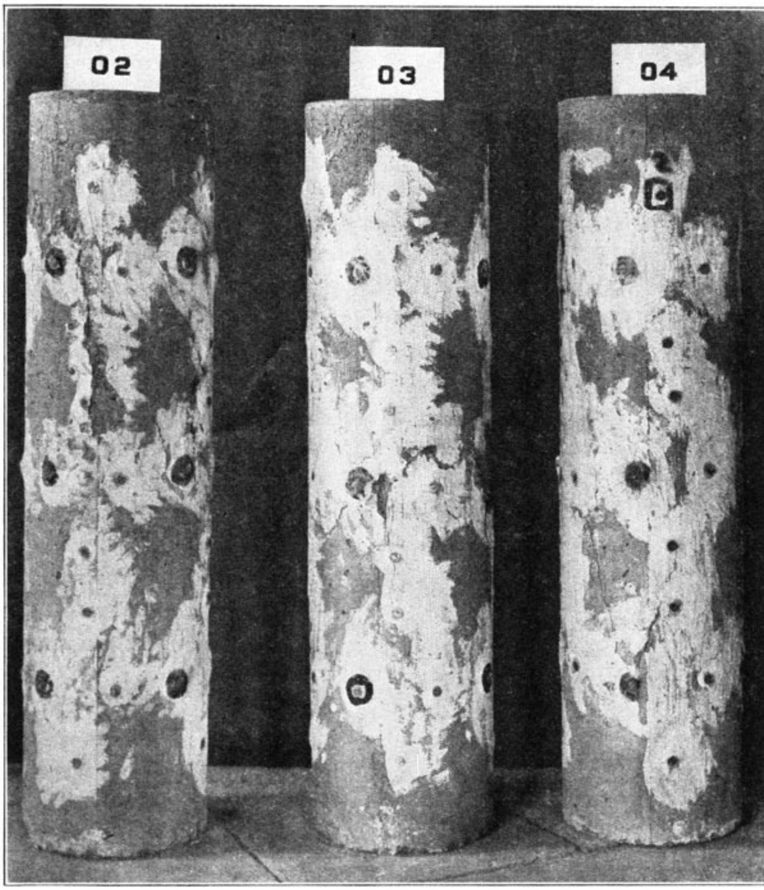

Fig. 6. View of Columns 02, 03, and 04, after Testing

ameter of the spiral wire no gage holes were drilled in the spirals of this group; instead, light punch marks were used. No breaks in the wire occurred at these marks.

In the test of Column 11 slight spalling of the plaster of paris surrounding the gage plugs was noted at a unit load of 2620 lb. per sq. in., and the maximum load reached was 2720 lb. per sq. in. After the machine had been stopped to permit strain measurements, a second and a third maximum were observed, at loads of 2740 and 2680 lb. per sq. in. Cracking and spalling of the concrete had become quite general when, with further motion of the testing machine head, the spiral broke near the upper gage lines. After this the column still withstood a load of 1900 lb. per sq. in.

Slight spalling was noted on Column 12 at a load of 2450 lb. per sq. in., and this action increased until the maximum load was reached at 2760 lb. per sq. in. At this point the cracking and spalling were not noticeable from a short distance away. With further intermittent loading, a second, third, and fourth maximum were noted at loads of 2800, 2640, and 2600 lb. per sq. in. Failure occurred through the the breaking of two loops of the spiral soon after the fourth maximum was reached.

At the maximum load on Column 13 there was slight cracking and bulging of the concrete between spirals. After strain readings had been taken a second maximum of 2610 lb. per sq. in. was reached, and shortly afterward the column failed through the breaking of a spiral.

Group 2.—The columns of this group had about 1.1 per cent of spiral reinforcement of $3/16$-in. annealed drawn wire. In the test of Column 21, slight spalling was observed at a load of 2360 lb. per sq. in. and cracking and spalling increased as the load rose slowly to the maximum of 3570 lb. per sq. in. As the machine was run beyond the maximum the load fell off to 3150 lb. per sq. in. where the spiral broke at two of the drilled gage holes.

The first perceptible spalling of Column 22 came at a load of 2930 lb. per sq. in., and the maximum load was reached at 3635 lb. per sq. in. After a set of readings had been taken the load was again raised to a maximum of 3640 lb. per sq. in., whereupon the test was discontinued without breaking the spiral. Gage holes in the spiral of this column were not drilled, but were light punch marks.

Slight spalling of the concrete of Column 23 was noted at a load of 2720 lb. per sq. in. Spalling and cracking continued but was not very pronounced at the maximum load of 3570 lb. per sq. in. With the machine running the load remained constant for some time, then the spiral broke at two places at gage holes in the wire. After the spiral broke, a load of 3410 lb. per sq. in. was again applied before the spirals started slipping and rapid decrease in load followed.

Group 3.—The columns of this group had 2.05 per cent of spiral reinforcement of $1/4$-in. rolled low carbon steel. In the test of Column 31, a vertical crack in the concrete was noted at a load of 2640 lb. per sq. in., and as the load increased spalling developed near this crack. At the load of 3530 lb. per sq. in. the column was taking load very slowly, and spalling was quite general in the upper part of the column at the maximum load of 3740 lb. per sq. in. After strain measurements were taken, loading was resumed and a second maximum of 3760 lb. per sq. in. was reached. No breaking of spirals occurred and the test was discontinued.

No surface spalling of Column 32 was seen until a load of 3390 lb. per sq. in. was reached. Thereafter the spalling increased and the rate of loading decreased until the maximum load of 3800 lb. per sq. in. was reached. After strain measurements had been taken, a systematic set of loading tests was made wherein the machine was run for four periods of 2 minutes each, with a one-half minute rest between periods. In each two-minute period a maximum load was reached, the consecutive values being 3900, 3870, 3850 and 3810 lb. per sq. in. The results of these tests are discussed in Section 20. The spiral was unbroken when the tests were discontiuned, though the column had shortened about 2 per cent.

One day after the foregoing tests the spiral reinforcement was stripped from the column, which was then retested as a plain concrete column. It took load quite rapidly up to the maximum of 1040 lb. per sq. in., which is 49 per cent of the strength of the plain columns of Group 0. Figure 7 shows the column with spirals removed, before and after retesting.

At a unit load of 2870 lb. per sq. in. on Column 33, there was slight cracking of the concrete and buckling over the spirals. At 3320 lb. per sq. in. spalling had become quite general and the load was increasing very slowly, requiring more than six minutes to reach the maximum load of 3450 lb. per sq. in. With the machine running the load stayed constant at 3440 lb. per sq. in. for 17 minutes, when the average longitudinal unit shortening was 0.02 and the lateral unit deformation was 0.01. A systematic loading test such as that of Column 32 was made, and after this the test was discontinued. The spiral reinforcement remained unbroken.

Group 4.—The columns of this group had 2.6 per cent of spiral reinforcement of $\frac{5}{16}$-in. low carbon rolled steel. In the test of Column 41 cracking sounds were heard at a load of 2620 lb. per sq. in., but no appreciable spalling occurred until a load of 3830 lb. per sq. in. was reached. The maximum load was 4260 lb. per sq. in. Spalling of the column was quite general and the average unit deformations were: longitudinal, 0.012; lateral, 0.003. When the testing machine was again started a second maximum of 4330 lb. per sq. in. was reached; the load was left on the column for 10 hours and gradually decreased to 3000 lb. per sq. in. With the machine started again the column took load quite rapidly and a third maximum load of 4570 lb. per sq. in. was reached, whereupon the test was discontinued.

Some spalling of Column 42 occurred at a load of 3400 lb. per sq. in. The maximum load was 4240 lb. per sq. in. The average deformations taken just beyond the maximum load were: longitudinal,

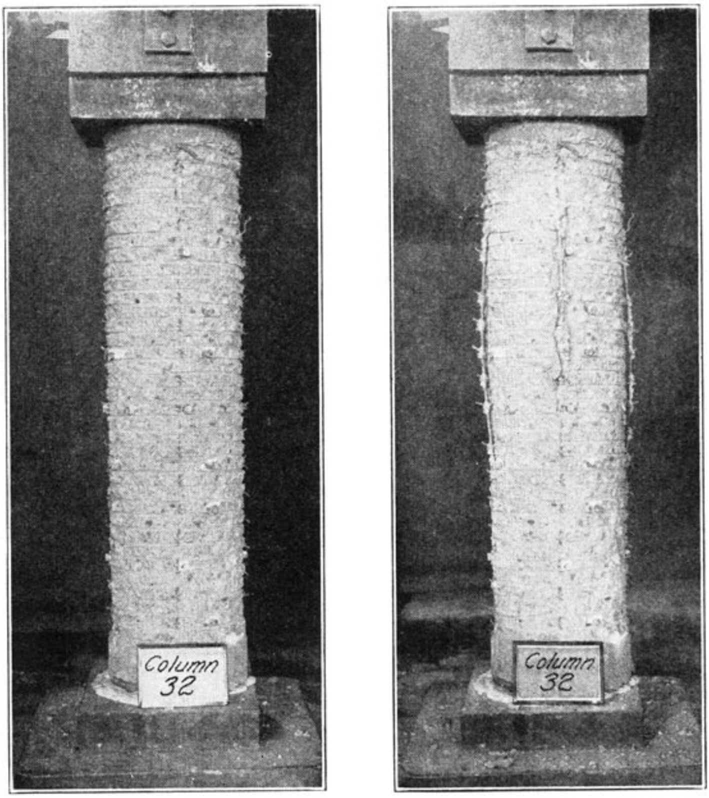

FIG. 7. VIEWS OF COLUMN 32 WITH SPIRALS REMOVED, BEFORE AND AFTER RETESTING

0.014; lateral, 0.007. Repeated loadings to a maximum, followed by periods of rest, similar to the tests of Columns 32 and 33, were then made on the column. After these tests the spiral reinforcement was still intact and loading was discontinued.

Column 43 developed a few small cracks at a load of 3540 lb. per sq. in. and some spalling at 4070 lb. per sq. in. The column took load very slowly near the maximum load, which was 4390 lb. per sq. in. Strain measurements taken soon after the maximum load showed an average shortening of the column of 1.9 per cent. After repeated loading similar to that of Column 42 the test was discontinued.

Group 5.—The columns of this group had 4.4 per cent of ⅜-in. low carbon rolled steel.

In the test of Column 51 spalling began at a load of 4760 lb. per sq. in. and was almost complete when the maximum load of 6460 lb.

per sq. in. was reached. After the machine had been stopped to take readings, a second maximum of 6530 lb. per sq. in. was applied, and a third still higher was reached after a two-hour rest. By this time the column was bending appreciably and the test was stopped.

Some spalling of Column 52 began at a load of 3520 lb. per sq. in., and had become quite general when the maximum load of 6220 lb. per sq. in. was reached. Failure occurred in the upper part of the column; the large deformations noted near the top of the column were evidently due to the fact that the spiral reinforcement was about 1½ in. below the upper bearing surface. Average unit deformations just after maximum load were: longitudinal, 0.021; radial, 0.007. With further loading, second, third, and fourth maxima of 6280, 6300, and 6220 lb. per sq. in., respectively, were reached. The test was discontinued as the column showed a slight amount of bending.

Column 53 showed spalling at a load of 3240 lb. per sq. in., and practically the entire surface spalled off before the maximum load of 6600 lb. per sq. in. was reached. The column took load very slowly as the maximum was approached, a 16-minute run being required to raise the load from 6400 to 6600 lb. per sq. in. The average unit deformations at this point were: longitudinal, 0.027; lateral, 0.009. With the machine running for 24 minutes more, the load decreased to 6590 lb. per sq. in., and the test was discontinued.

Group 13.—The columns of this group had 2 per cent of spiral reinforcement of ¼-in. high strength cable wire.

In the test of Column 131 slight spalling of the concrete was seen at a load of 3540 lb. per sq. in. and spalling was nearly complete at the load of 5470 lb. per sq. in. The column took load slowly up to the maximum of 8460 lb. per sq. in., when the spiral broke with a loud report at a drilled gage hole. The spiral uncoiled and removed the lateral restraint from a length of column of several inches, where violent crushing of the concrete took place. The average unit deformations of the column read just before the failure were: longitudinal, 0.044; lateral, 0.012.

Spalling of Column 132 began at a load of 3040 lb. per sq. in. Failure occurred in a way similar to that of Column 131, through breaking of the spiral at a load of 7370 lb. per sq. in. The average unit deformations shortly before failure occurred were: longitudinal, 0.035; radial, 0.011.

In the test of Column 133 buckling of the concrete outside the spirals was noted at a load of 3660 lb. per sq. in., and at a load of 5300 lb. per sq. in. spalling of the surface was almost complete. The column took load very slowly from a load of 6270 lb. per sq. in. up to failure, which was due to breaking of the spiral at a gage hole, at a load of 7800 lb. per sq. in. The average unit deformations shortly before failure were: longitudinal, 0.039; radial, 0.011.

In connection with the foregoing record of the tests it may be noted that for Columns 11, 21, 33, 42, 51, 131, and 132 load was released nearly to zero for an interval of about an hour and then reapplied; such release of load being made at loads generally less than one-half the maximum load. The release and reapplication of load apparently had little effect upon subsequent behavior of the columns.

Figure 8 shows views of the columns of Series 1 after they had been tested. It should be kept in mind that most of these columns were loaded for some time after the maximum load was reached. However, the figure shows to some extent the relative amounts of surface spalling and deformation to which the different columns were subjected.

10. *General Results of Tests.*—Table 5 gives a summary of the principal results of general interest. The table is self-explanatory, except perhaps with regard to the values of initial modulus of elasticity, Poisson's ratio, and bulk modulus. The method of obtaining these properties of the material will be described in Section 11.

The major portion of the test observations is shown in the form of stress-strain curves for the columns. The readings on individual gage lines at any section did not differ greatly, but there were frequently marked differences between the strains measured at different sections of the column. The stress-strain curves given in Figs. 9 and 10 represent the average of the readings taken at sections near the top, middle, and bottom of typical columns. The longitudinal strains represent the average of four readings taken on the four sides of the column. The radial strains represent the average of two diameter-gage readings at right angles to each other. The circumferential strains on concrete are the average of four readings, while those on steel are the average of the readings on eight gage lines, which together completely encircled the column.

In computing unit loads, the average cross-sectional area of the column was used; in the case of the spirally reinforced members the section from outside to outside of the spiral was used. The same section was considered to govern the gage length on diameter measurements, for while the gage plugs extended from 3/8 to 1/2 in. beneath the surface of the concrete, it was felt that the plaster of paris surrounding the plug moved more nearly with the surface of the column. In the spiral columns in which the outer surface spalled off, the gage length

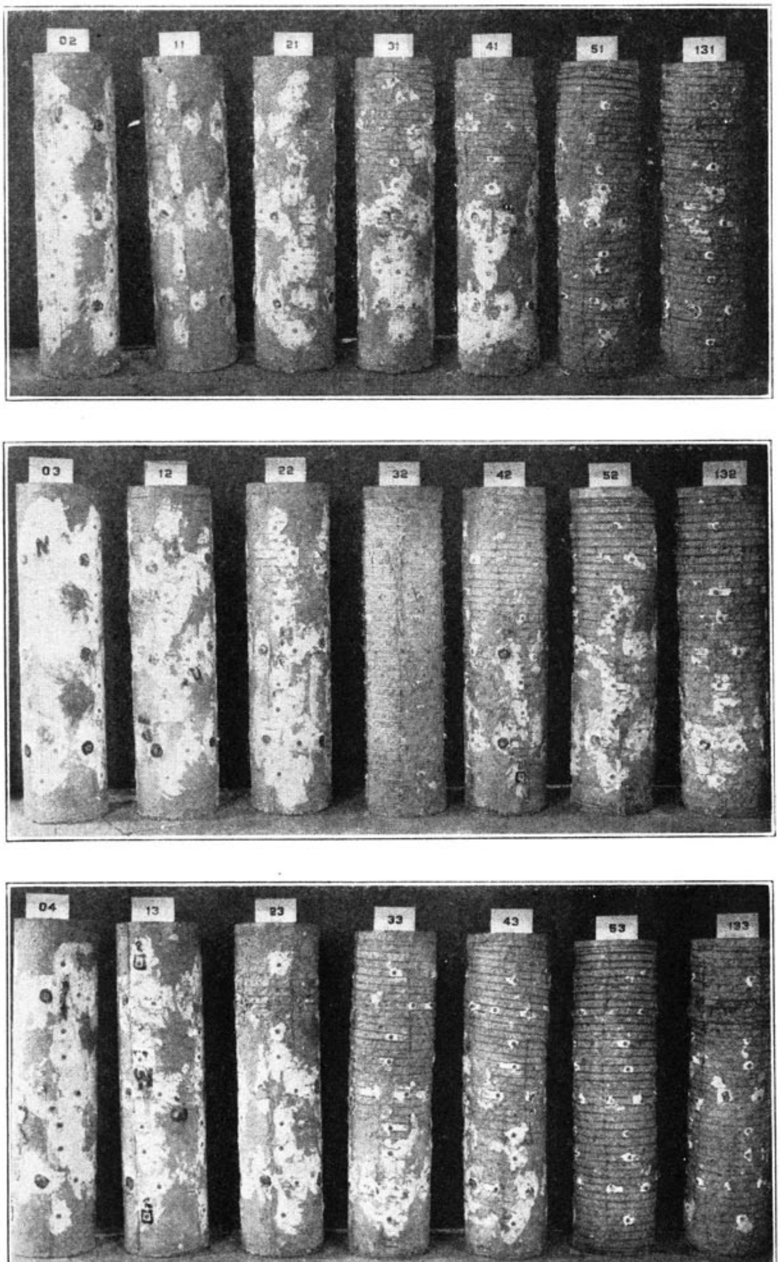

FIG. 8. VIEW OF ALL COLUMNS AFTER TESTING

TABLE 5

SUMMARY OF RESULTS OF COLUMN TESTS

Column No.	Percentage of Reinforcement	Maximum Unit Load—lb. per sq. in.	Unit Load at which Spalling Began—lb. per sq. in.	Comp. Strength of 6 by 12-in. Cylinders—lb. per sq. in.	Initial Modulus of Elasticity—thousands of lb. per sq. in.	Estimated Initial Values of	
						Poisson's Ratio	Bulk Modulus—thousands of lb. per sq. in.
02	0	2290	....	2530	3140	0.09	3900
03	0	1940	....	2550	3020	0.11	3900
04	0	2120	....	2290	3060	0.12	4000
05	0	2080	....	2280	2940	0.12	3950
06	0	2220	....	2475	3300	....	....
Average	0	2130	....	2425	3090	0.11	3940
11	0.49	2720	2620	2675	2800	0.13	3800
12	0.50	2760	2450	2790	2650	0.11	3400
13	0.50	2565	2565	2630	2900	0.11	3720
Average	0.50	2680	2545	2700	2780	0.12	3640
21	1.10	3570	2360	2640	2800	0.12	3680
22	1.11	3635	2930	2865	2100	0.13	2840
23	1.11	3570	2720	2725	2800	0.13	3800
Average	1.11	3590	2670	2725	2570	0.13	3440
31	2.09	3740	2640	2880	2300	0.13	3100
32	2.10	3800	3390	2525	2700	0.12	3550
33	2.03	3450	2870	2785	2550	0.12	3360
Average	2.07	3665	2965	2730	2520	0.12	3340
41	2.61	4260	3830	2725	2600	0.13	3500
42	2.64	4240	3400	2640	2700	0.12	3600
43	2.66	4390	3540	2480	2650	0.12	3500
Average	2.64	4295	3590	2615	2650	0.12	3530
51	4.41	6460	4290	2745	2750	....	....
52	4.40	6220	3520	2890	2650	0.11	3400
53	4.42	6600	3240	2685	2350	0.12	3100
Average	4.41	6430	3680	2775	2580	0.12	3250
131	1.99	8460	3540	2665	2900	0.11	3720
132	1.90	7370	3040	2525	2250	0.11	2890
133	1.99	7800	3660	2860	2500	0.11	3160
Average	1.96	7880	3410	2685	2550	0.11	3260

was taken as the average diameter from outside to outside of the spiral.

Figures 11, 12, and 13 show stress-strain curves plotted from the average of the readings for each column. Figures 11 and 12 show values of the longitudinal and radial deformations of the concrete, while Fig. 13 shows values of the radial deformations and the deformations measured on the spiral steel. The circumferential concrete deformations, which agreed closely with the radial ones, are not shown. It will be noted that in Figs. 11 and 12 the scale for radial deformations is twice that used for longitudinal ones. This was done so that

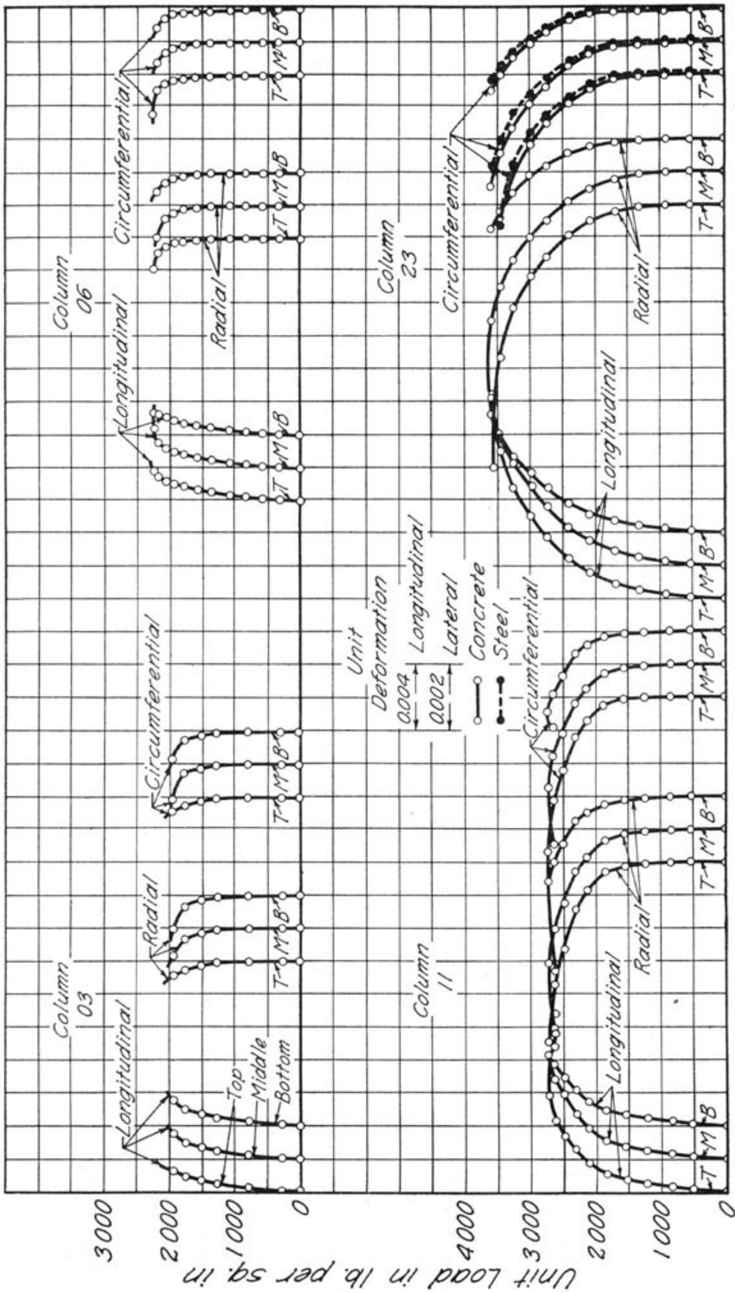

FIG. 9. TYPICAL STRESS-STRAIN CURVES FOR SECTIONS AT TOP, MIDDLE, AND BOTTOM OF COLUMNS

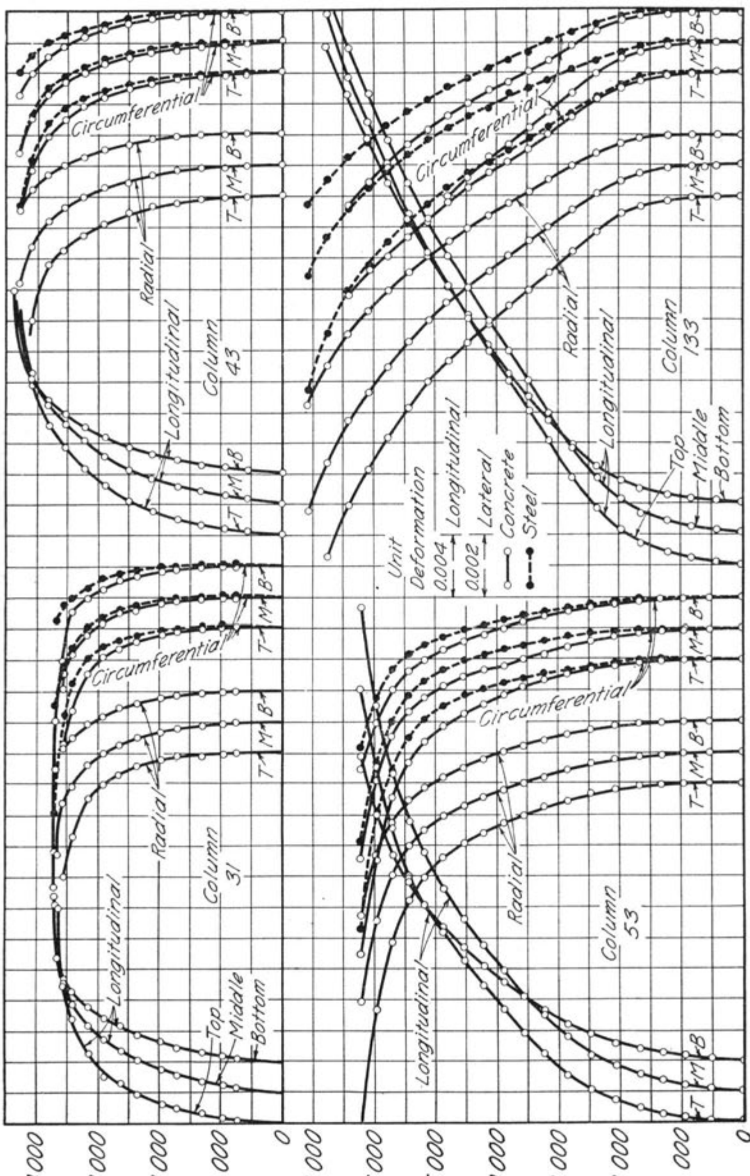

FIG. 10. TYPICAL STRESS-STRAIN CURVES FOR SECTIONS AT TOP, MIDDLE, AND BOTTOM OF COLUMNS

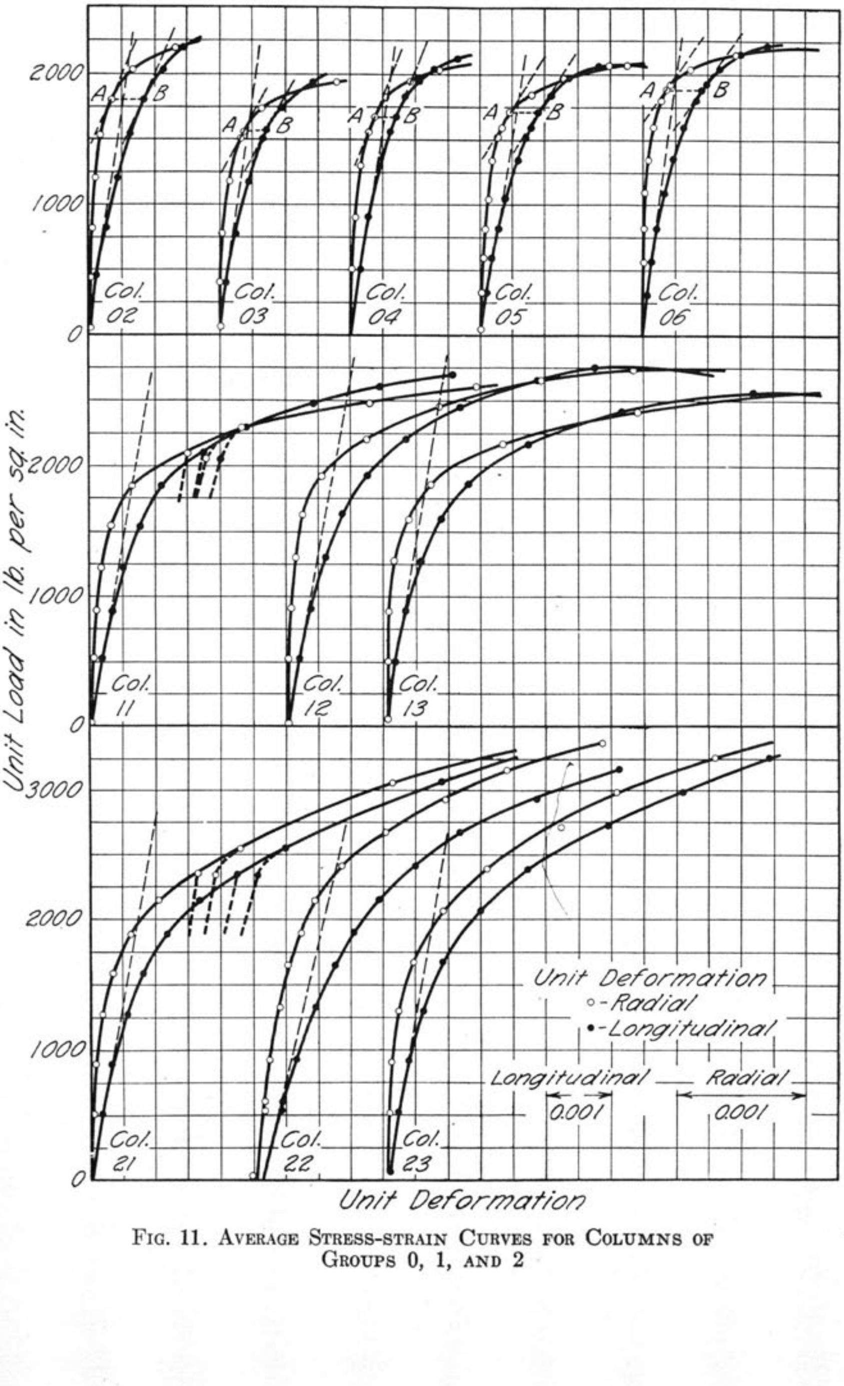

FIG. 11. AVERAGE STRESS-STRAIN CURVES FOR COLUMNS OF GROUPS 0, 1, AND 2

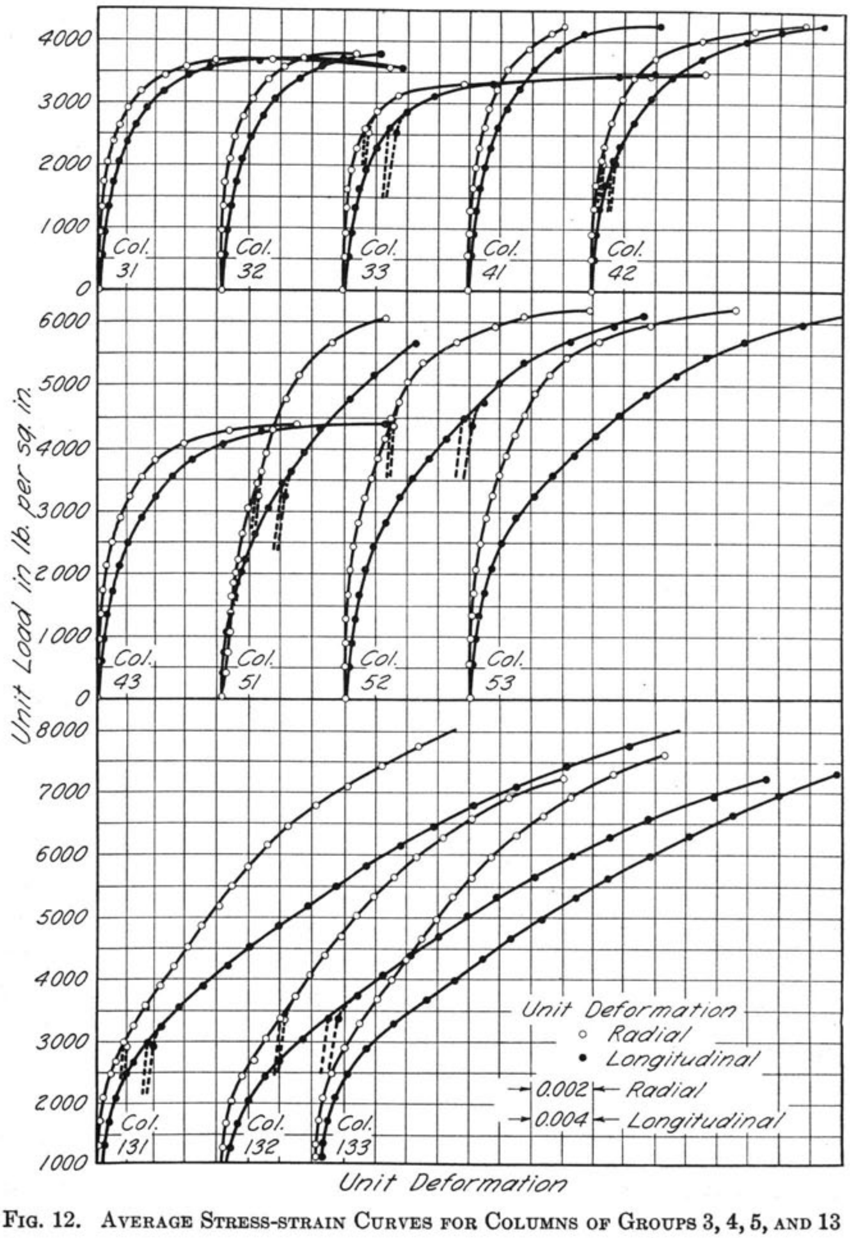

FIG. 12. AVERAGE STRESS-STRAIN CURVES FOR COLUMNS OF GROUPS 3, 4, 5, AND 13

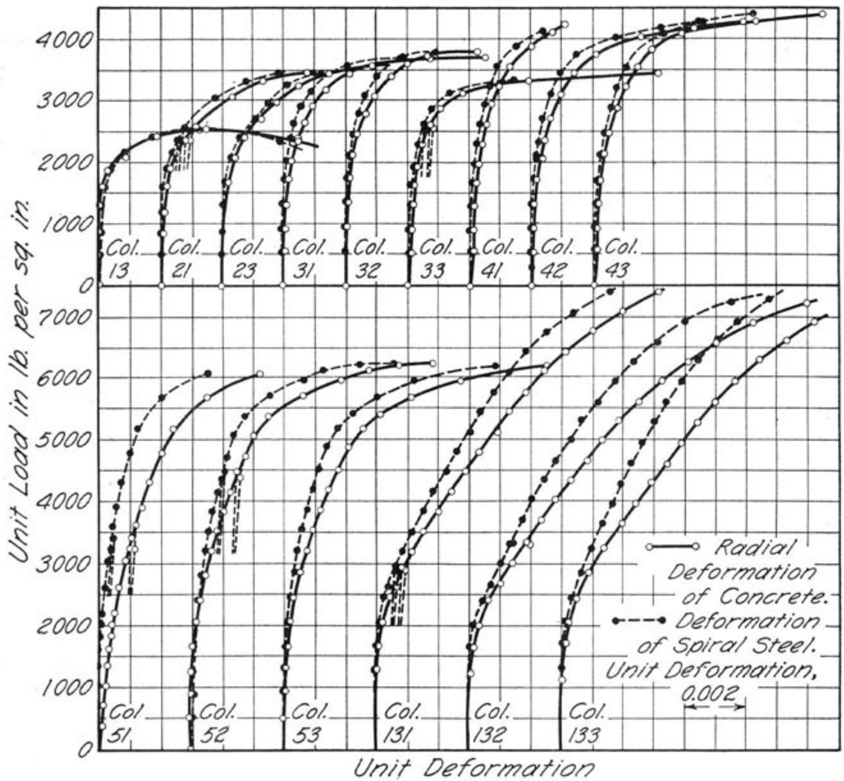

FIG. 13. AVERAGE STRESS-LATERAL-STRAIN CURVES FOR ALL COLUMNS

the horizontal distance between the curves would be proportional to the unit change in volume of the column, thus furnishing a graphical record of volume changes throughout the loading of each column.

The curves of Fig. 13 also furnish a specific comparison, since the horizontal distance between these curves represents a relative movement between the spiral steel and the enclosed concrete. A further discussion of these curves will be found in Section 21.

III. TESTS OF PLAIN CONCRETE COLUMNS

11. *Elastic Properties of the Material.*—While the main object of the tests was the study of the behavior of the material as it approached failure, some information was also secured regarding the deformations produced at low loads. Average values of the deformations of all plain columns except Column 06, which was omitted be-

cause of uncertainties due to rapid temperature variations during testing, have been used in the determination of the initial elastic properties of the material. Although the relative error of observations at low loads was large, it was evident that the stress-strain curves were practically straight lines up to one-fourth or one-third of the maximum load, and the initial slopes of the curves were used in deriving values of the modulus of elasticity, E, Poisson's ratio, μ, and from these the bulk modulus, K, which is equal to the quantity $\left(\frac{E}{1-2\mu}\right)$. These values, given in Table 5, may be termed elastic constants, and represent fairly definite properties of the material, although they were not determined with a high degree of precision.

12. *Longitudinal and Lateral Deformations.*—The average stress-strain curves of Fig. 11 show some interesting features of the deformations of plain concrete at medium and high stresses. These curves show the development of an increasing amount of curvature with increase in load for both the longitudinal and the lateral deformations. The longitudinal deformation curves show a gradual increase in curvature up to or very close to the maximum load and the shape of these curves agrees fairly well with that of a parabola, which has been used by Professor Talbot* to represent the stress-strain curve for concrete in compression. The curves for lateral deformations, however, are of a different shape, showing a region of greatest curvature at from 70 to 85 per cent of the maximum load. Beyond this region of greatest curvature the rate of increase in lateral deformation is more than one-half that for longitudinal deformations, as may be seen from the slopes of the curves in Fig. 11. The increase in lateral deformation is so rapid that before the maximum load is reached the lateral unit deformation becomes equal to one-half the longitudinal unit deformation (the two curves intersect) and in some cases at the maximum load the lateral unit deformation is equal to or greater than the longitudinal unit deformation. This relatively large value of the lateral strain is not consistent with the rational behavior of a homogeneous body and evidently indicates the development of internal discontinuity, presumably a splitting, of the material.

Another characteristic of the lateral deformation near the maximum load is seen in the fact that where a flow or continued deformation was noted under a stationary load, the amount noted was relatively much larger on lateral gage lines than on longitudinal ones.

*"Tests of Concrete and Reinforced Concrete Columns; Series of 1906," Univ. of Ill. Eng. Exp. Sta. Bul. 10, 1907.

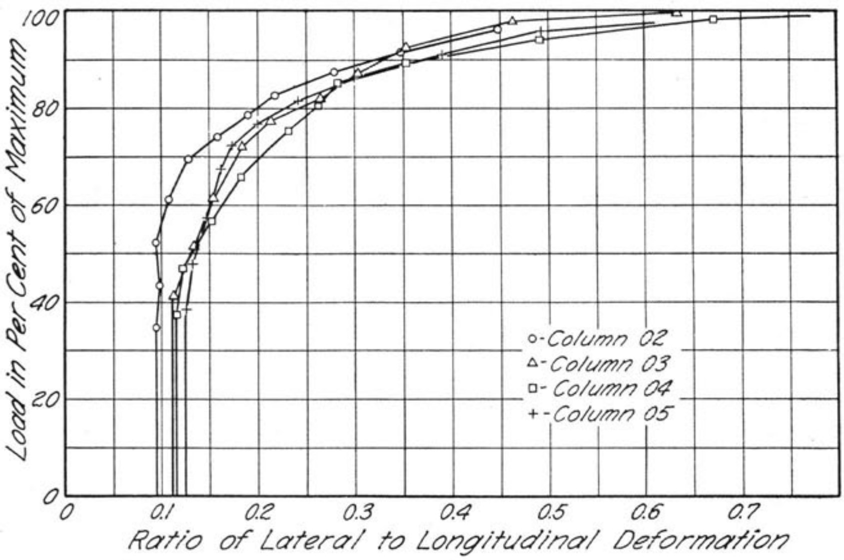

FIG. 14. RATIO OF LATERAL TO LONGITUDINAL DEFORMATIONS OF PLAIN COLUMNS

Thus, on Column 04 at the maximum load, the longitudinal unit deformation increased by 0.00021 in seven minutes, while the lateral unit deformation increased twice as much in the same period of time.

For the curved portion of the stress-strain curves, wherein plastic, or inelastic, deformations are developing, it is convenient to consider quantities similar in nature to the elastic constants mentioned in Section 11, but varying in magnitude with the load. Thus from the average stress-strain curves, values of the secant modulus of elasticity, the secant bulk modulus, and the ratio of longitudinal to lateral unit deformation have been determined at stresses ranging from 800 lb. per sq. in. to the ultimate strength. Figure 14 shows values of the ratio of lateral to longitudinal deformation, so found, plotted against unit load. The term "Poisson's ratio" applies to the initial values of this ratio, which appear to be nearly constant up to the load of 800 lb. per sq. in.; the term does not apply to the variable values found at the higher loads. The curve for Column 02 differs somewhat from the others, due to the fact that there was more wear on gage holes during the test, and hence less measured lateral swelling of the column. All of the curves, however, show a fairly regular increase in the ratio of lateral to longitudinal strain, this increase becoming more rapid at from 70 to 80 per cent of the maximum load. It is seen that the ratio

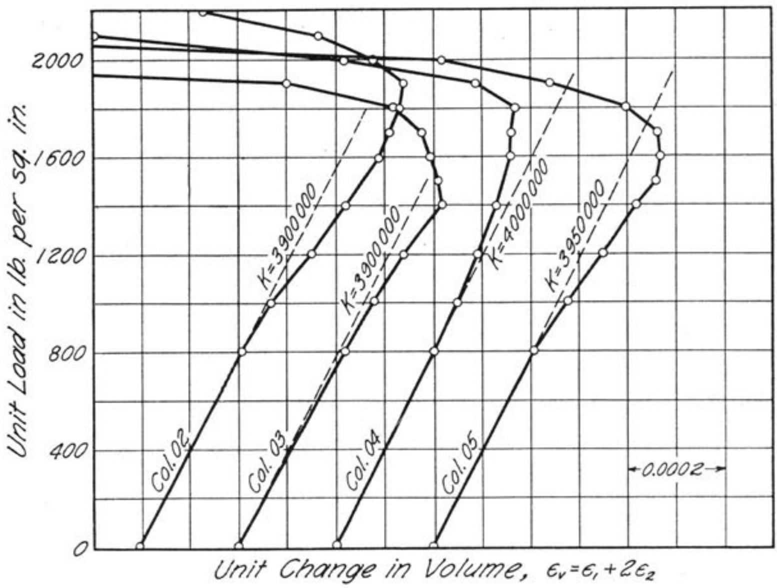

FIG. 15. VOLUME CHANGES IN PLAIN CONCRETE COLUMNS

was generally more than 0.5 at the maximum load, and on columns which were loaded beyond the maximum load the ratio reached values of 2.0 and more.

13. *Volume Changes of the Material.*—For a simple compression member subject to a longitudinal unit deformation ϵ_1 and lateral unit deformations ϵ_2, the volumetric unit deformation, or unit change in volume is $\epsilon_v = \epsilon_1 + 2\epsilon_2$. As the average stress-strain curves of Figs. 11 and 12 have been plotted, the horizontal distance between the two curves represents the unit change in volume at the corresponding load. Referring to Fig. 11 it is seen that during the application of load, up to perhaps two-thirds of the maximum load, the volume decreases roughly in proportion to the load applied; as the region of sharp curvature of the lateral stress-strain diagram is approached, the rate of change in volume becomes less. At the load indicated on the figure by the line A-B, at which tangents to the two curves become parallel, a small increase in load causes no corresponding change in volume. At higher loads the volume increases with increase in load, until at the point where the two curves intersect the volume is just what it was before any load was applied. At the maximum load the apparent volume of the test piece has actually been increased by the application of a compressive load.

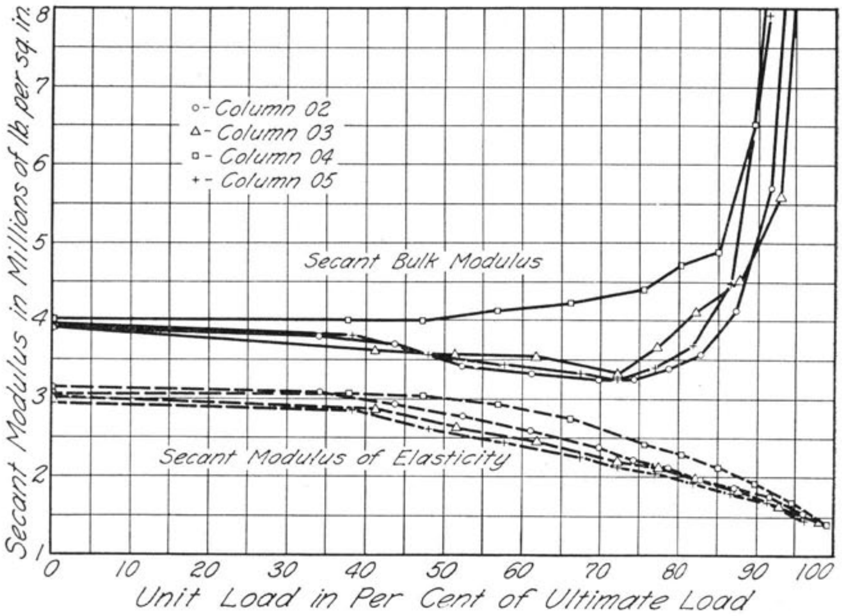

FIG. 16. SECANT MODULUS OF ELASTICITY AND BULK MODULUS OF PLAIN CONCRETE

For the five plain columns tested it has been found that the load indicated by the line A-B was 77 to 85 per cent of the maximum load, and that the slope of the tangent to the longitudinal stress-strain curve (or tangent modulus of elasticity) at this load averaged exactly one-third of the initial modulus of elasticity. These relations seem to be quite consistent, and will be used in making further comparisons.

To illustrate the changes in volume described above, the unit changes in volume have been computed from the average longitudinal and radial strains for each plain column and are plotted against the unit load in Fig. 15. Straight lines representing values of the bulk modulus, K, given in Table 5, are also shown in the figure. It is seen that at medium and low loads the deviation of the change in volume from the straight-line relation is small, much less in fact than the corresponding deviation for either the longitudinal or lateral deformations. The rapid changes in volume at loads of 77 to 85 per cent of the maximum and beyond, as previously described, are clearly shown in the figure.

The values of secant modulus of elasticity and secant bulk modulus described in Section 12 and given in Fig. 16 also throw light on the way in which volume changes of the material took place. These values have been plotted against unit load, the value at zero load being the initial value of the modulus listed in Table 5. It is seen that the values of the secant bulk modulus at low and medium loads are more nearly constant than are values of secant modulus of elasticity. Figure 16 also emphasizes the fact that at loads of from 75 to 85 per cent of the maximum, abrupt changes take place in the behavior of the material—changes that would not be apparent from a consideration of longitudinal deformations alone.

14. *Development of Cracks and Failure.*—With the fairly rapid loading used in these tests no cracks were observed on the surface of any plain column until the maximum load was reached. The cracks that developed at or somewhat beyond the maximum load are barely visible in the views of Fig. 8. The cracks were of two distinct types: (a) vertical cracks, extending for some distance along the column and not accompanied by spalling of the adjacent concrete surface; and (b) more or less horizontal cracks, usually rather short and not extending inward normal to the surface. These latter cracks were frequently accompanied by spalling of thin flakes of concrete, followed by general breaking up of the surface.

Two of the five plain columns were crushed down until complete failure occurred, while the others were loaded only until maximum load was reached. The appearance of the two columns at complete failure was characteristic of concrete of this grade. Rupture of the material followed more or less conical surfaces, inclined at from 30 to 35 deg. to the axis of the cylinder, a so-called shearing type of failure.

15. *Characteristics of Deformation and Failure of Concrete in Simple Compression.*—The following description of important features of the deformation and general behavior of concrete in simple compression is in accord with previous discussions* of the subject. It seems convenient to divide the action of the column into three distinct stages of loading, each having certain well-defined characteristics.

In the first stage the deformations are all very nearly proportional to the loads, and hence to each other. While it is not known just how far the proportionality of loads and strains obtains, the stress-strain curves do not show appreciable deviation from straight lines up to at least one-fourth of the maximum load. Similar results were obtained

*"A Study of the Failure of Concrete Under Combined Compressive Stresses," Univ. of Ill. Eng. Exp. Sta. Bul. 185, 1928.

by Johnson,* who used a mirror extensometer apparatus in making very careful measurements of longitudinal and lateral strains in concrete and mortar cylinders.

The second stage of loading is marked by the deviation of the stress-strain curves from the straight line followed in the first stage; in general, the ratio of lateral to longitudinal deformation increases during this stage, while the change in volume remains more nearly a linear function than do any of its component parts.

The third and final stage of loading is marked by the abrupt change in the curve for lateral deformation previously described. This change is not accompanied by a like increase in longitudinal deformations, and consequently both the volume and the ratio of lateral to longitudinal strain show rapid increases. The abrupt transition from the second to the third stage seems to be best explained as being due to an internal splitting or breaking of the continuity of the material, since this would make it possible to have an increase in compressive stress accompanied by an increase in bulk of the material. The fact that the lateral strains increase more rapidly than the longitudinal ones further indicates that the initial failures within the material are of a splitting rather than a shearing type. Since vertical cracks do not appear on the surface until near the maximum load, it may be argued that the initial splitting occurs on minute sections widely scattered through the mass, and only after a considerable amount of displacement has occurred do these cracks merge to form larger continuous planes of failure.

Brandtzaeg† has offered a theory of failure of a material like concrete in compression which agrees in many particulars with the phenomena observed in the tests. The first departure from elastic action of the material, corresponding to the beginning of the second stage, is considered to be due to the starting of plastic sliding on elementary planes at scattering points and in every direction within the specimen. The spreading and increase of this plastic deformation in turn sets up lateral pressures which are resisted by the tensile strength of the elements still intact. Tensile failure of the latter elements, the splitting mentioned above, marks the beginning of the third stage of loading. The loss of lateral restraint which is produced by the splitting action results in a rapid increase in plastic deformation or yielding and further loading soon leads to failure. While the final breakdown of

*Johnson, A. N. "Direct Measurement of Poisson's Ratio for Concrete," Proc. A.S.T.M., Vol. 24, Part 2, 1924.

†Brandtzaeg, A. "Failure of a Material Composed of Non-Isotropic Elements." Det Kgl. Norske Videnskabers Selskabs Skrifter 1927. Nr. 2 Trondhjem, Norway. Also "A Study of the Failure of Concrete under Combined Compressive Stresses," Univ. of Ill. Eng. Exp. Sta. Bul. 185, 1928.

the material may follow the directions of predominant elementary sliding planes, it is evident that failure is precipitated by the tensile or splitting failure of elements of the material. With very strong concrete the splitting failure is the most noticeable, whereas with weak concrete the general disorganization and very gradual plastic yielding are familiar phenomena of failure.

IV. Tests of Spirally Reinforced Concrete Columns

16. *Rate of Loading and Deformation of Columns.*—Since all tests were made with the machine running at the same nominal speed, the rate at which a column took load depended mainly upon the deformations accompanying a given load. As may be judged from the stress-strain diagrams, the rate of taking load gradually decreased with increase in load. The columns took load rather steadily until a load corresponding to the maximum for plain columns was reached and the spiral began to be effective; at this point the load came on rather irregularly and slowly, a condition which may have been due to the attendant large lateral deformation required to bring the spiral reinforcement to a bearing against the concrete. At loads near the ultimate, very large longitudinal deformations were produced with little increase in load, as described in Section 9. It is clear that in most cases not only was the concrete in a condition of plastic yielding but the steel also had reached a point far beyond its limit of proportionality.

Although all columns reached the "spiral stage" (the stage of loading at which the spiral reinforcement began to become effective) at about the same load, the first appearance of cracks on the surface was noted at somewhat higher loads for the columns having the heavier reinforcement. The first cracks appeared when the lateral unit deformation had reached values of 0.001 to 0.002. Spalling of the surface concrete developed rather slowly on the heavily reinforced columns, but, as shown in Fig. 8, the spalling was much more complete at the ultimate load for these columns than for those with smaller percentages of reinforcement.

17. *Properties of Columns at Low Loads.*—It is well known that the lateral stresses exerted by the spiral reinforcement are negligible below a unit load equal to the strength of the plain concrete, and therefore it is possible to determine "elastic" constants for spirally reinforced columns similar to those found in Section 11 for plain

columns. The initial slopes of the stress-strain curves of Figs. 11 and 12 have been determined, giving principal weight to the first three points on each curve. As noted before, the tests were made to study the action of the material at failure, and the information secured at low loads is not as complete as is desirable. Values of the initial modulus of elasticity are given in Table 5, together with average values of Poisson's ratio and bulk modulus. The values of these latter quantities, however, are estimated rather than directly observed, inasmuch as inconsistent or improbable test values were discarded in arriving at these averages. It is believed that these values represent a reasonably close estimate of the elastic properties of the material.

The values of the initial modulus of elasticity given in Table 5 are generally somewhat lower for the spirally reinforced columns than for the plain columns. Similar results have been noted in other tests in which the presence of the spiral apparently reduced the density and stiffness of the outer shell of concrete.

18. *Release and Reapplication of Load.*—In a few cases it was found necessary to discontinue testing for an hour or two after the column had been loaded well into the spiral range, and, to avoid the plastic deformation which might occur under this rather high stress, the load on the column was reduced to an amount just sufficient to hold all bearing surfaces in alignment. After one to two hours the original load was reapplied. The behavior of the columns (Nos. 11, 21, 33, 42, 51, 52, 131, and 132) is indicated by the stress-strain diagrams of Figs. 11 to 13. Generally, strain readings were taken before and after release of load and before and after reapplication, and the points so determined have been joined by straight lines in the stress-strain diagrams. These straight lines indicate the average rate of recovery of deformation with release of load, and the average rate of deformation with reapplication of load. It appears that the slope of the curve during release of load, sometimes termed the modulus of elasticity in regression, was but slightly less than the initial modulus of elasticity for the column, while the slope during reapplication of load was on the average about two-thirds as great as the initial modulus of elasticity.

There was nothing in the stress-strain relations or in other observations of the tests to indicate that this release and reapplication of load, at stresses of from 35 to 75 per cent of the ultimate strength of the column, had any effect upon the maximum load carried by the column. There may have been some effect on the deformations near

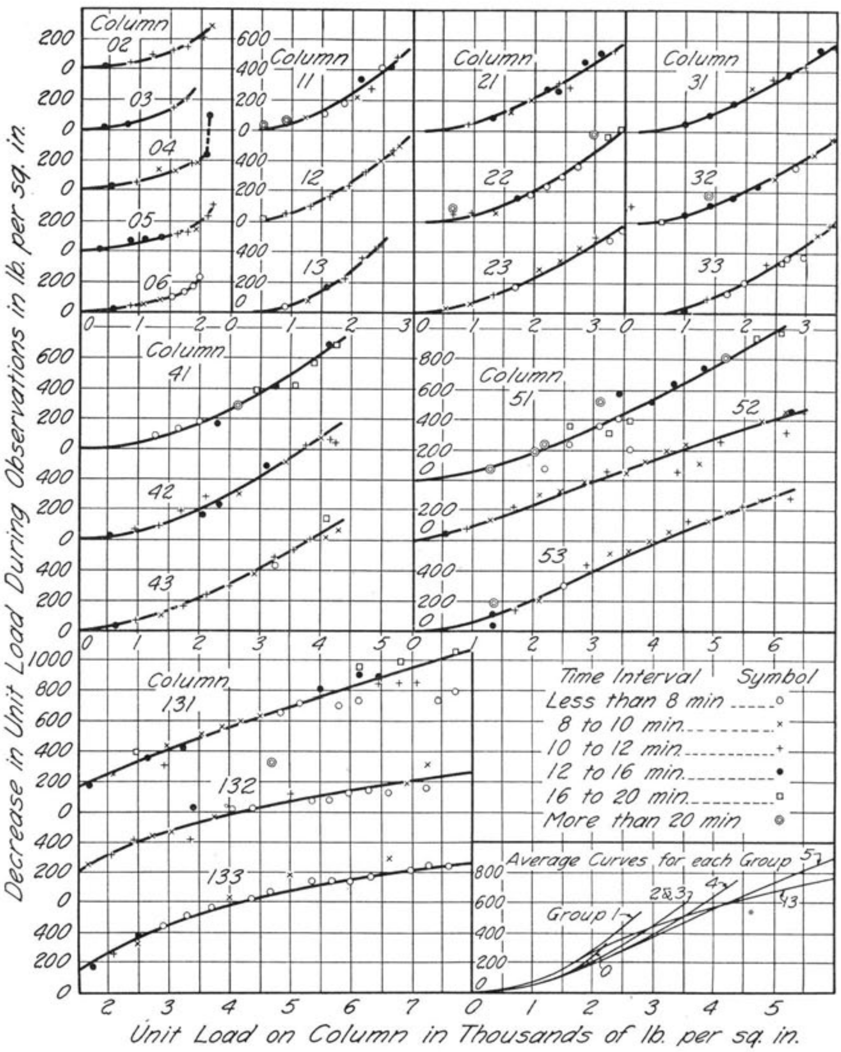

FIG. 17. SELF-RELEASE OF LOAD DURING STRAIN MEASUREMENTS ON COLUMNS

the point of reapplication of load, but the curves compare very well with those for columns in which the testing proceeded continuously.

19. *Release of Load Due to Yielding of Column.*—Figure 17 gives information on the amount by which the load decreased due to yielding of the concrete while the screw-power testing machine was stopped

during strain measurements. While some part of the decrease may have been due to plastic flow of grease in the screws and thrust bearings of the testing machine, tests made in which the concrete column was replaced by a rigid steel block indicate that this effect was not more than 3 to 5 per cent of the total decrease. The decrease in load is plotted in Fig. 17 against the load at which the machine was stopped, and different symbols have been used to indicate the time interval during which the machine was stopped. It is evident that the load carried by the column is the factor of principal importance in determining the amount of yielding and the resultant drop in load, though a slight effect of the time interval during which yielding occurs may be seen. The shortest interval for which the testing machine was stopped was 7 minutes, and the longest about 20 minutes. Previous tests with a similar grade of concrete have indicated that most of the decrease in load observed over a short period occurred in the first minute after the testing machine was stopped, the rate of the decrease varying roughly in inverse ratio to the square of the time interval. If this condition obtains even approximately for these tests, little difference can be expected between the amount of load released in 8 minutes and that released in 20 minutes.

The shape of the curves in Fig. 17 is much the same for all types of test pieces, except that the curves for the plain concrete columns show a rather rapid release of load as the maximum load is approached, while those for the spirally reinforced columns show a nearly uniform slope. As a means of comparing the action of columns of the different types, average curves for each type are shown in the lower part of Fig. 17, and, to afford numerical comparisons, the percentage of load released at a load somewhat below the ultimate load has been computed. At a load 90 per cent of the ultimate the average percentage of load released was found to be as follows: Group 0, plain concrete, 9 per cent; Groups 1, 2, 3, and 4, 14 to 15 per cent; Group 5, 13 per cent; and Group 13, 11 per cent. Spirally reinforced columns, in which the concrete is in a semi-plastic state throughout much of the "spiral range" of loading might be expected to show greater yielding than plain concrete; it is evident also that the yielding of the spiral reinforcement had progressed further in Groups 1 to 5 than in Group 13.

Plastic flow or yielding of concrete is usually associated with the idea of long continued loading. The yielding noted in these tests over periods of only a few minutes may be similar in character to that occurring over a period of months or years, but differing in degree.

These tests show the need for a comprehensive study of all phases of plastic yielding. The effect of such yielding upon the strength of concrete columns having longitudinal and spiral reinforcement is a matter of much concern to designing engineers.

20. *Behavior of Columns near Maximum Load.*—All of the reinforced columns except those of Group 13 passed the maximum load without fracture of the spirals. It was noted in Section 9 that, after a column passed the maximum load and the loading was discontinued to permit strain observations, when load was reapplied it rose to a new maximum and again fell off. This was repeated several times on some columns, the second maximum attained usually being the highest of all, and the difference between the maxima being relatively small. While the first maximum reached was always considered to be the maximum for the column, there is still a question as to what the maximum might have been had the testing machine been stopped just below the maximum load, since the period of rest and plastic yielding or compacting appeared to raise the strength slightly upon reapplication of load.

To secure some information upon the effect of the length of time elapsing between loadings to a maximum and upon the magnitude of the difference between maxima, systematic loading tests were made on Columns 32, 33, 42, and 43 after the maximum load for each column had been reached in the ordinary course of testing. The testing machine was alternately run and stopped for fixed periods of time, and a record was kept of the maximum load and the time required to attain it in each case. The results of these tests are given in Table 6. There seems to be no appreciable difference in effect when the period of rest was one-half minute from that when it was ten minutes. The period of rest between the first maximum and the repeated loading tests evidently had a slight effect. For Column 43, in which this period was about three hours, the second maximum was about 5 per cent greater than the first; in the other tests the differences in load sustained were less. These tests indicate that while the manner of applying the maximum load may have a measurable effect upon its magnitude, the amount of the variation that may be produced is relatively small.

It is of interest that in all but one of the spirals made of annealed drawn wire, fracture of the spiral occurred with further loading after the maximum load on the column was reached; none of the rolled mild steel spirals were broken in the tests, but each of the high carbon

TABLE 6

RESULTS OF REPEATED MAXIMUM LOADING TESTS

Column No.	"First Maximum" Unit Load, lb. per sq. in.	Unit Deformation, in Thousandths				Unit Load, lb. per sq. in.			Time, minutes and seconds		
		Beginning		End		Starting Machine	Maximum	Stopping Machine	Between Loadings	From Start to Stop	From Start to Maximum
		Axial	Steel	Axial	Steel						
32	3800	9.0	3.0			....	3900	3850	....	2:00	1:15
						3630	3870	3840	0:30	2:00	0:45
						3610	3850	3800	0:30	2:00	0:40
				19.6	9.5	3580	3810	3760	0:30	2:00	1:10
33	3450	19.8	12.0			....	3510	3470	....	2:00	1:10
						2830	3520	3440	10:00	2:00	1:00
						2820	3480	3420	10:00	2:00	0:45
				29.0	19.0	2930	3470	3390	10:00	2:00	0:45
42	4240	15.1	5.6			....	4330	4250	....	3:00	0:40
						4020	4250	4180	0:30	3:00	0:30
				35.0	....	3990	4200	4130	0:30	3:00	0:30
43	4390	20.0	10.0			....	4630	4450	....	4:00	2:45
						3830	4480	4270	10:00	4:00	2:00
				29.0	15.0	3720	4320	4150	10:00	4:00	0:45

spirals of Group 13 broke at gage holes in the steel, and caused failure at a time when the column was still taking load rapidly.

It has been stated that during the "spiral range" the column undergoes large deformations and is in a semi-plastic condition. This has led to the erroneous idea that the concrete within the spirals at the maximum load is a disintegrated, granular mass. To show that the core still had considerable strength and stability of its own,* the spiral reinforcement was removed from Column 32 after the completion of the repeated loadings indicated in Table 6. This core, which had suffered a longitudinal unit deformation of 0.0196 and a lateral unit deformation of 0.0105, was still able to carry a unit load of 1040 lb. per sq. in. or practically one-half of the strength of the plain columns of Group 0.

21. *Depression of the Spiral Steel into the Concrete.*—In Fig. 13 stress-strain curves are shown for lateral deformations on both concrete and steel gage lines. The concrete deformations were measured on 10-in. radial, or diametral, gage lines, while the steel deformations were on 4-in. circumferential lines. It is seen that the readings on the concrete were consistently greater than those on the steel, indicating a depression of the steel into the concrete as the bearing pressures increased, or more correctly a flow of the concrete through the spiral reinforcement. The relative amount of the depression and its effect upon unit deformations are greater than would be found in a column of larger diameter; in this case there is no question but that these factors must be considered in a study of stress-strain relations.

To show more clearly the amount of the depressions Fig. 18 has been prepared from the results of the various column tests. In this figure the difference between lateral deformations of concrete and of steel are plotted as ordinates, and unit stresses in the steel, calculated from the measured strains and the stress-strain curves of Figs. 1 and 2, are plotted as abscissas. Since the ordinates represent relatively small differences between two measured quantities, the irregularity of the plotted points does not seem unreasonable. It appears that the relation between depression and spiral stress is very nearly a linear one. The slopes of the straight lines of Fig. 18, which represent the rate of depression of the spiral steel into the concrete, are listed in Table 7. The slopes are expressed as the differences between concrete and steel unit deformations accompanying an increment of spiral stress of 10 000 lb. per sq. in.

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*For accounts of similar tests see Mörsch, E. "Der Eisenbetonbau," Fifth Edition, p. 214; also Univ. of Ill. Eng. Exp. Sta. Bul. 20 and 185.

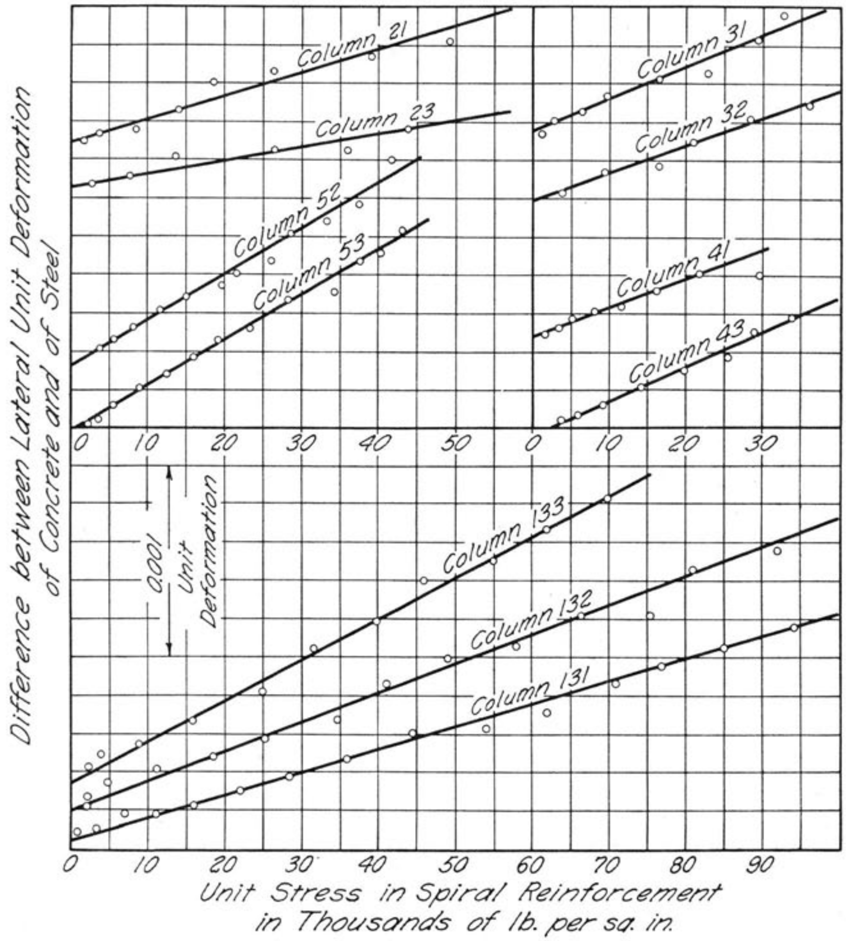

FIG. 18. DEPRESSION OF SPIRAL STEEL INTO CONCRETE CORE

The depression of the spiral into the concrete should be related to the bearing pressure between spiral and concrete. For a known spiral stress the bearing pressure may be computed, assuming the pressure to be uniformly distributed over a cylinder of concrete having a height equal to the diameter of the spiral wire and a diameter equal to the mean diameter of the spiral. Letting f_b denote the bearing unit stress in the spiral, d the diameter of the spiral wire and D the diameter of the spiral,

$$f_b = \frac{f_s \pi d}{2D} \tag{1}$$

TABLE 7

RATE OF DEPRESSION OF SPIRAL STEEL INTO CONCRETE

Column No.	Rate of Depression of Spiral Steel into the Concrete,*† K	Bearing Unit Pressure between Concrete and Steel, lb. per sq. in.,† f_b	Modulus of Depression, lb. per sq. in. $\frac{f_b}{K} = N$
21........	0.00012		
23........	0.00007		
Average....	0.00010	300	3 010 000
31........	0.00017		
33........	0.00016		
Average....	0.00016	410	2 550 000
41........	0.00015		
43........	0.00018		
Average....	0.00017	455	2 680 000
52........	0.00024		
53........	0.00024		
Average....	0.00024	605	2 540 000
131........	0.00012		
132........	0.00016		
133........	0.00021		
Average....	0.00016	400	2 520 000

*From slope of curves of Fig. 18, representing difference in measured lateral unit deformations of concrete and of spiral.

†Corresponding to an increase in spiral steel stress of 10 000 lb. per sq. in.

The values of the bearing stresses f_b, corresponding to an increment of steel stress of 10 000 lb. per sq. in., have been computed and are given in Table 7. The ratio, N, of the bearing stress to the rate of depression is given in the last column of the table. This ratio, which might be termed the "modulus of depression," evidently depends upon the quality of the concrete, the pitch of the spirals, the diameter of the column and other factors. It is interesting that the values of the ratio stay so nearly constant for the wide range of spiral sizes used in these tests.

An attempt was made to determine how much of the depression could be calculated as elastic deformation, and how much was plastic deformation. Although no exact method of analysis was found, an approximate calculation of the elastic depression of the spiral into the concrete showed this to be very small, and from this it appears that the relative movement between spiral and concrete was very largely a plastic flow of the concrete between the spiral wires. The fact that this flow occurred, even though the pitch of the spirals was only one inch, emphasizes the need of keeping the pitch of spirals to a minimum if the reinforcement is to be fully effective.

Curves similar to those of Fig. 18, but based upon circumferential rather than diametral strain measurements on the concrete, have also been plotted; in general the points are more irregular and the average curves have a slightly smaller slope. However, since the precision of measurement of the circumferential strains was much less than for those taken with the diameter gage, these readings have been considered as merely confirmatory check readings, and are omitted here.

22. *Maximum Loads.*—As an aid to the study of stress-strain conditions in the spirally reinforced columns near the maximum load, complete records of the average strains at a load just below the maximum and at one just beyond the maximum have been compiled, as given in Table 8. The table includes vertical and lateral unit stresses and longitudinal and lateral strains; from these sufficient information was available to afford a good estimate of the strains and the lateral stresses existing at the maximum load. The unit strains given are observed average values for each column. In a few cases steel deformations have been computed from the lateral concrete deformations by using the information given in Fig. 18. The spiral stresses have been computed from the strains by use of the average stress-strain curves of Figs. 1 and 2. The spiral stresses and the calculated lateral compressive stresses in the concrete at maximum load have been estimated from the readings at the loads just preceding and just following the maximum, the possible range in the estimated values being thus held within quite narrow limits.

The average maximum unit load and the average lateral unit stress at this load for each of the six groups of columns have been plotted in Fig. 19, together with the corresponding quantities for the plain columns of Group 0. With the exception of the points representing Groups 2 and 13, the points fall on a straight line, which may be represented by the equation

$$f_1 = f_c' + 4.1f_2 \qquad (2)$$

where f_1 is the axial unit load, f_2 the lateral unit stress, and f_c' the compressive strength of plain concrete of the quality used. This same equation applied very well to the results of tests of concrete in combined compression made in this same investigation and described in Bulletin 185. In those tests the lateral stress was produced by hydraulic pressure and was independent of the lateral deformation of the test piece.

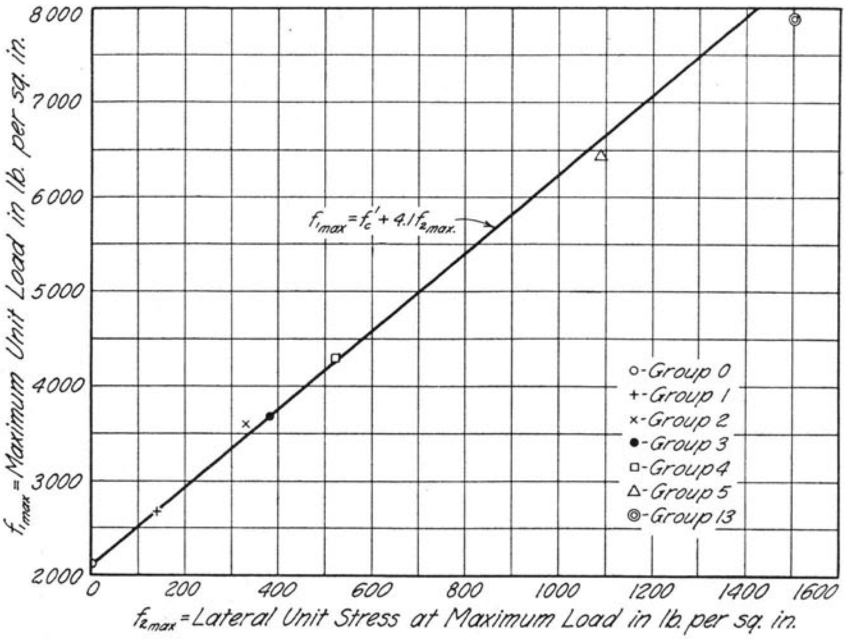

FIG. 19. MAXIMUM UNIT LOADS AND LATERAL STRESSES FOR ALL COLUMNS

The relation between the lateral pressure f_2, the spiral stress f_s, and the percentage of spiral reinforcement p, may be found by use of the conventional analysis of tension in thin-walled cylinders to be

$$f_2 = \frac{pf_s}{2} \tag{3}$$

This value of f_2 may be substituted in Equation (2) to give the following expression for the relation between the unit load and the stress in the spiral reinforcement:

$$f_1 = f_c' + 2.05pf_s \tag{4}$$

Equation (4) indicates that in the spiral range the unit load depends upon the strength of the plain concrete, the percentage of reinforcement, and the stress in the spiral. It is of interest to know whether this relation holds true at the time of failure, and whether failure is due to refusal of the spiral to take further stress. Table 9 gives estimated steel stresses at maximum load, together with the slope of the stress-strain curve, or tangent modulus of elasticity at these stresses. These steel stresses were determined by reference to the data of Table

TABLE 8

RECORD OF SPIRALLY REINFORCED COLUMNS NEAR MAXIMUM LOAD

Column No.	Per cent of Spiral	Maximum Unit Load, lb. per sq. in.	Situation just before or just after maximum load								Estimated Lateral Unit Stress at Max. Load, lb. per sq. in.	Action of Spiral at Failure
			At Unit Load			Av. Unit Strains in thousandths				Lateral Unit Stress, lb. per sq. in.		
			Relative to Max.	lb. per sq. in.	Per cent of Max.	Axial	Diametral	Steel	Volume¶			
11	0.49	2720	Before	2710	99.8	5.75	4.07	3.48*	−2.39	144	150	Yielded and broke
			After	2630	96.8	8.07	7.09	6.40*	−6.11	169		
12	0.50	2760	Before	2760	100.0	4.70	2.66	2.18*	−0.60	118	120	" "
			After	2640	95.6	7.10	5.00	4.35*	−2.90	161		
13	0.50	2565	Before	2565	100.0	5.61	3.54	2.99*	−1.47	137	140	" "
			After	2355†	91.9	8.94	6.40	5.72*	−3.86	171		
Average		2680									140	
21	1.10	3570	Before	3460	97.0	9.23	4.72	3.81	−0.21	311	340	Yielded and broke
22	1.11	3635	Before	3545	97.6	9.08	3.79	3.26*	1.50	296	320	Yielded
			After	3580	98.5	11.70	5.62	5.01*	0.46	342		
23	1.11	3570	Before	3435	96.4	7.58	3.47	3.18	−0.36	293	330	Yielded and broke
			After	3570†	100.0	10.00	5.42	3.49	−0.84	330		
Average		3590									330	
31	2.08	3740	Before	3670	98.2	10.27	4.42	3.77	1.43	380	390	Yielded
			After	3570	95.5	18.7	13.8	13.0*	−8.9	421		
32	2.10	3800	Before	3720	97.9	7.00	2.63	1.86	1.74	331	350	"
			After	3780	99.5	9.03	4.29	2.94	0.45	370		
33	2.03	3450	Before	3320	96.3	9.63	3.90	3.40	1.83	365	410	"
			After	3440	99.8	19.8	12.6	12.0	−5.4	410		
Average		3665									380	

*Computed from diametral strains, using Fig. 18.
†After spiral had broken.
‡By extrapolation in Fig. 13.
¶Reduction in volume recorded as positive.

TABLE 8 (Concluded)

RECORD OF SPIRALLY REINFORCED COLUMNS NEAR MAXIMUM LOAD

Column No.	Per cent of Spiral	Maximum Unit Load, lb. per sq. in.	Situation just before or just after maximum load								Estimated Lateral Unit Stress at Max. Load, lb. per sq. in.	Action of Spiral at Failure
			At Unit Load			Av. Unit Strains in thousandths				Lateral Unit Stress, lb. per sq. in.		
			Relative to Max.	lb. per sq. in.	Per cent of Max.	Axial	Diametral	Steel	Volume¶			
41	2.61	4260	Before	4120	95.5	7.42	2.67	2.36	2.08	447	480	Yielded
42	2.64	4240	Before	4140	97.8	12.6	5.23	4.20	2.14	514	540	"
			After	4230	99.7	15.1	6.9	5.57	1.3	540		
43	2.66	4390	Before	4260	97.0	10.5	4.25	3.33	2.00	497	530	"
			After	4380	99.8	18.5	10.3	9.8	−2.1	559		
Average		4295									520	
51	4.41	6460	Before	5660	87.5	12.5	3.53	2.08	4.44	868	1070	Yielded
52	4.40	6220	Before	6120	98.5	19.2	5.77	4.58	7.66	1020	1040	"
			After	6220	100.0	23.5	8.2	6.5	7.1	1070		
53	4.42	6600	Before	6220	94.3	26.6	8.54	6.93	9.52	1075	1170	"
Average		6430									1090	
131	1.99	8460	Before	8310	98.3	43.8	13.9	11.83	16.0	1635	1650‡	Broke
132	1.90	7370	Before	7220	97.8	35.1	11.5	8.56	12.1	1350	1410‡	"
133	1.99	7800	Before	7590	97.4	39.2	11.3	8.28	16.6	1400	1450‡	"
Average		7880									1500	

‡By extrapolation in Fig. 13.
¶Reduction in volume recorded as positive.

TABLE 9

STRESS SITUATION IN SPIRAL REINFORCEMENT AT MAXIMUM LOAD

Column No.	Percentage of Spiral Reinforcement	Estimated Stress in Steel at Maximum Load, lb. per sq. in.	Tangent Modulus of Elasticity of Steel at Maximum Load, millions of lb. per sq. in.	Calculated Increase in Axial Unit Load with Increase of 0.001 in the Unit Deformation of Spiral at Maximum Load, lb. per sq. in.
11		59 000	8.0	
12		47 000	10.0	
13		55 000	8.5	
Average	0.50	53 700	8.8	90.0
21		61 500	2.9	
22		57 500	4.7	
23		59 500	3.8	
Average	1.11	59 500	3.8	86.5
31		38 000	0.8	
32		33 500	3.2	
33		39 000	0.5	
Average	2.07	36 800	1.5	63.5
41		37 000	2.6	
42		40 800	0.4	
43		40 000	0.7	
Average	2.64	39 300	1.2	65.0
51		48 400	0.7	
52		47 400	0.9	
53		52 900	0.3	
Average	4.41	49 600	0.6	54.0
131		165 000	5.0	
132		148 000	8.0	
133		146 000	8.6	
Average	1.96	153 000	7.2	289.0

8 and of Figs. 1 and 2. While a certain degree of approximation is involved in the estimation of the stress at maximum load, and in assuming that the curves of Figs. 1 and 2 correctly represent the conditions existing in the spiral, the values seem to be quite consistent in showing that the spiral stresses were still increasing as the columns failed. The last column of Table 9 shows the calculated increase in unit load corresponding to an increase in unit deformation of the spiral of 0.001 at the time the maximum load was reached. Since failure occurred while the lateral stress was still increasing, it is evident that the rate of increase of f_1 indicated by Equation (4) does not hold true at the maximum load; the increase in unit load made possible by the lateral

support of the spiral is counteracted by some other effect, possibly a loss of cohesion and of interlocking of particles of the material of the concrete core which had hitherto contributed to the strength. It seems quite probable that failure may have been dependent upon the rate at which lateral pressure was supplied, so that some minimum rate of increase of lateral pressure with respect to deformation was needed in order to produce a corresponding increase in vertical unit load. The rate at which failure occurred, as given in Table 9, does not seem to vary greatly for the first five groups, though it seems to be appreciably higher for the columns having the least reinforcement. The foregoing reasoning does not apply to the columns of Group 13, which failed through the breaking of the spirals at gage holes while the column was still taking load rapidly and not by a gradual yielding process.

The stress-strain curves of Figs. 1 and 2 are marked to indicate the stress existing in the spirals when the columns failed. This stress does not seem to have any definite relation to the ultimate strength of the steel, nor to the slope of the stress-strain curve at the points indicated (as given in Table 9) so that it is difficult to make any generalization as to the value of f_s to be used in Equation (4) to give the ultimate strength of the spirally reinforced column. The values of the calculated rate of increase in vertical load at the time of failure give some indication that the ultimate load is reached when the product of the percentage of spiral reinforcement and the tangent modulus of elasticity of the steel reaches a definite amount. Whether this amount would differ with other materials and different loading conditions is, of course, unknown.

23. *Relation between Lateral and Longitudinal Stresses throughout Loading.*—It has been noted in previous tests* that in the "spiral range" of loading of a spirally reinforced column the increments of load were nearly proportional to the increments of spiral stress, which are in turn proportional to increments of lateral unit stress in the column. In Section 22 a linear relation was shown between maximum loads and lateral unit stresses for the columns of this bulletin. In order to study the relation existing at loads below the maximum the curves of Fig. 20 have been plotted. Average values of vertical unit load are plotted against values of lateral unit stress for all load increments at which readings were taken on each column, and average curves have been drawn through the points for each group of columns.

*"Tests of Concrete and Reinforced Concrete Columns, Series of 1907," Univ. of Ill. Eng. Exp. Sta. Bul. 20, 1907; also Jour. Am. Concrete Inst. July, 1915.

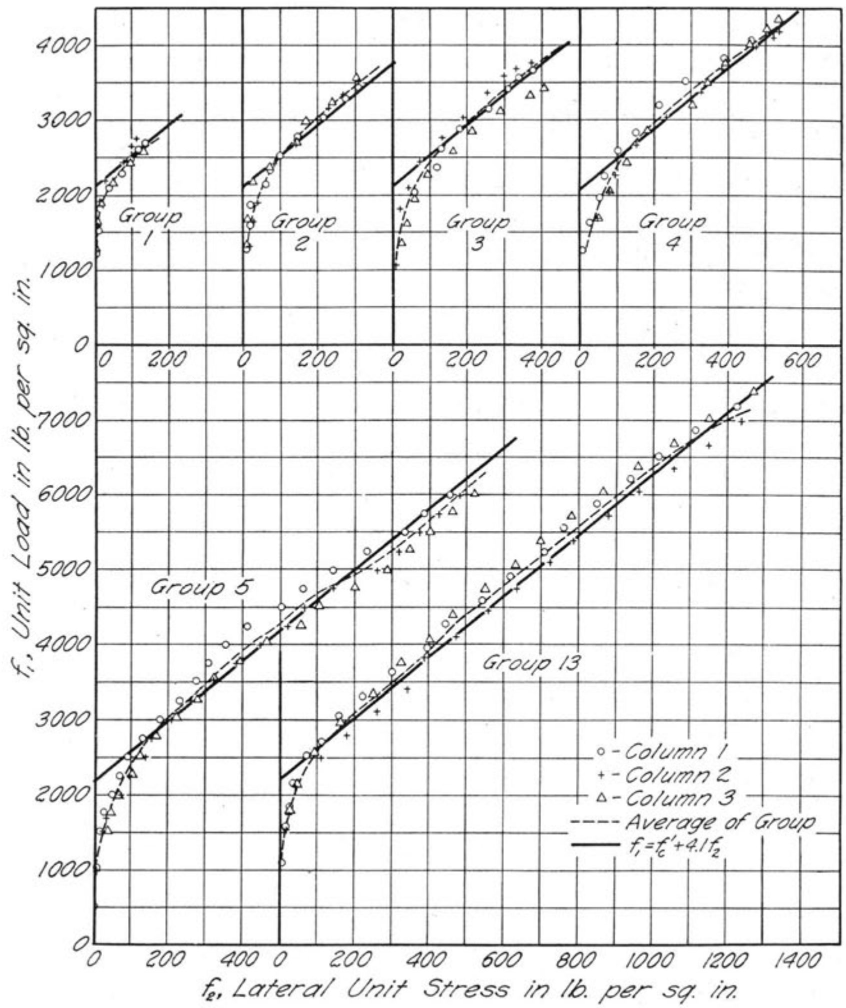

FIG. 20. LATERAL STRESSES IN COLUMNS AT VARIOUS UNIT LOADS

The lateral unit stresses were computed from the average of all measured strains in the spirals of each column by the method described in Section 22. For Columns 11, 12, 13, and 22, on which no spiral strains were measured, the lateral stresses have been calculated from the lateral concrete strains, allowing for the depression of the spiral into the concrete, as noted in Section 21.

Besides the average curves of Fig. 20, a straight line has been drawn through the points for each group of columns. This line is identical with the straight line of Fig. 19 and is represented by Equation (2)

$$f_1 = f_c' + 4.1f_2$$

wherein f_c' is the ultimate strength of the plain concrete columns, 2130 lb. per sq. in. At the unit load of 2130 lb. per sq. in. the column is beginning to deform rapidly and the spiral reinforcement is just beginning to become effective, so that there is appreciable deviation between the experimental curve and the straight line. At the load of 2500 lb. per sq. in. the deviation has become negligible and beyond this load the two curves are practically coincident.

That one equation represents so well the relation between unit load and lateral stresses for these six groups of columns having a great variation in amount and quality of reinforcement, with the accompanying differences in the amount of lateral deformation existing at a given load, is of interest, and seems to indicate an important law of behavior of the material, particularly since the same relation holds at the maximum loads for the columns, and at the ultimate loads of the cylinders tested in three-dimensional compression in Series 3A and 3B, Bulletin 185. In so far as numerical values are concerned, it should be noted that the columns for these tests were made of concrete of the same quality while the tests of Bulletin 185 were made on concretes of different proportions, but the same kind of cement and aggregates. In contrast with the value of 4.1 noted above, Talbot found values of the ratio $\frac{f_1 - f_c'}{f_2}$ to vary from 2.8 to 4.0 for concretes of varying mixtures and materials. Considère, reasoning from the principles of internal friction in granular materials, estimated the ratio to be 4.8 for spirally reinforced concrete columns; later he revised this value to 4.2.

It is important to note that Equation (2) was obtained from tests of short members, four diameters in length. Spirally reinforced columns in the plastic stage are very unstable as regards lateral deflection, and an increase in slenderness may be expected to reduce the effectiveness of the spiral very rapidly. This may account for values of the ratio $\frac{f_1 - f_c'}{f_2}$ much lower than 4.1 which have been found in other tests.

24. *Volume Changes of the Material.*—In Section 13 it was noted that radical changes in the volume of the plain concrete columns occurred as the maximum load was approached; hence it was felt desir-

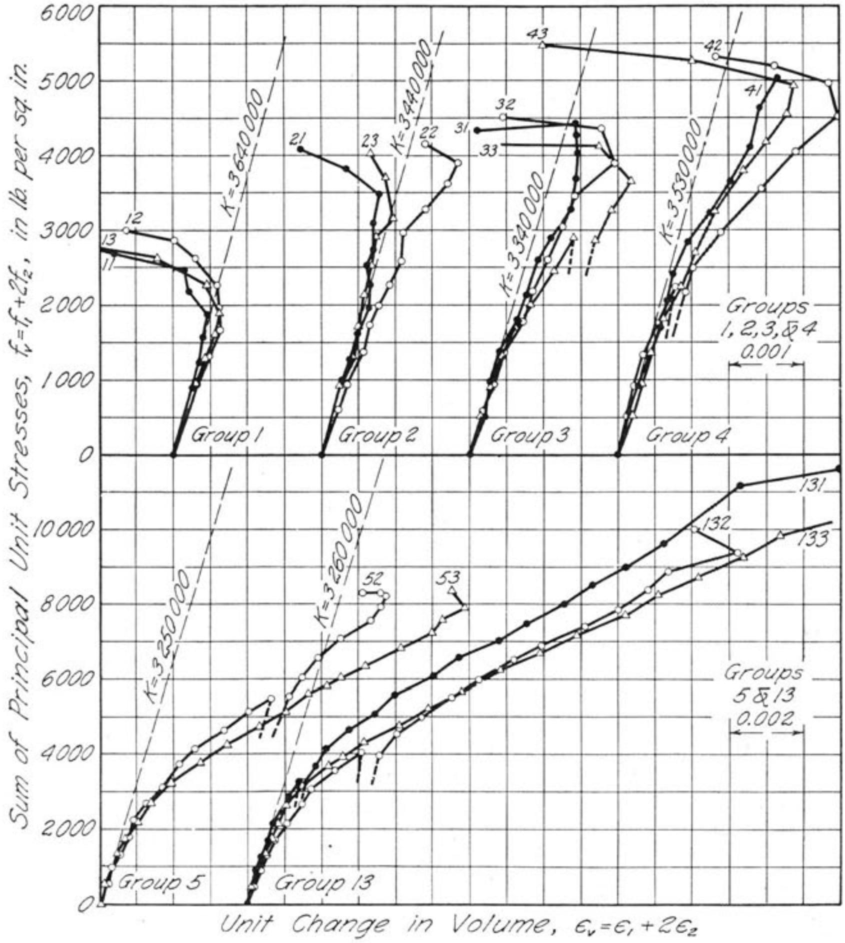

FIG. 21. VOLUME CHANGES IN SPIRALLY REINFORCED CONCRETE COLUMNS

able to make a study of the corresponding volume changes in the spirally reinforced columns. The unit change in volume ϵ_v is equal to the algebraic sum of the three principal unit deformations, or $\epsilon_1 + 2\epsilon_2$, where ϵ_1 represents the axial unit deformation and ϵ_2 the lateral unit deformation. In an elastic cylinder subjected to an axial unit stress f_1 and lateral unit stresses f_2, the unit change in volume is

$$\epsilon_v = \frac{1 - 2\mu}{E}(f_1 + 2f_2) = \frac{f_1 + 2f_2}{K} = \frac{f_v}{K} \tag{5}$$

where μ is Poisson's ratio, E is the initial modulus of elasticity, K is the initial bulk modulus, and f_v is the sum of the principal stresses, or the "volume stress." The term "volume stress" is derived from the fact that in an elastic cylinder the quantity f_v is proportional to the change in volume.

For the purpose of studying the volume changes, values of ϵ_v and f_v have been computed for all loads at which readings were taken and are plotted in Fig. 21. The deformations used are the average values for each column, except that the only lateral deformations used are those measured with the diameter gage. Each curve represents the average relation between volume stress and volume change for a column. For each group of columns an initial tangent is also drawn; the slope of the tangent K is the average initial bulk modulus for the group, as given in Table 5.

The curves do not differ greatly from those of Fig. 16 for plain columns at loads below 1500 lb. per sq. in., the tendency being toward a slightly greater change in volume than that indicated by the initial bulk modulus K. The curves of Group 1 follow the straight line up to a load of about 2000 lb. per sq. in.; then an abrupt change occurs, and the volume increases rapidly until failure is reached. As in the case of the plain columns, the net change in volume of these columns at failure is a considerable increase. The curves for the columns of Group 2 differ somewhat, Columns 21 and 23 following the initial tangent fairly closely until near the maximum load, when an increase in volume begins. The curve for Column 22 is similar to the other two, but shows a greater decrease in volume. The curves of Group 3 and 4 are similar to those of Column 22, but show a more pronounced deviation to the right of the initial tangent in the upper part of the diagram, followed by the characteristic sudden change in direction and subsequent increase in volume with further loading. The marked deviation of these curves to the right of the initial tangent suggests that the material within this range of loading is undergoing a considerable amount of inelastic compacting, while the final rapid increase in volume indicates the development of splitting or internal discontinuity within the material, similar to the behavior of plain concrete near failure.

The curves for Groups 5 and 13 in Fig. 21 appear somewhat different from those just described; however, this is due in part to the large proportion of the curves representing the action of these columns within the "spiral range." The curves of Group 5 are like those of Group 4 in that the greatest curvature is seen at a value of f_v of 2500 to 3000 lb. per sq. in.; beyond this the curve becomes nearly straight, though the rate of change of volume is six or seven times as great as at the initial load. Like the preceding curves, these curves show a reversal in direction near the maximum load, indicating an increase in volume with further loading. Measurements on Column 53, which held the maximum load practically unchanged during 24 minutes of continuous running of the machine, showed a net increase in volume (not shown on the curve) at the end of this loading period. The curves for Group 13 are similar to those of Group 5, except that the loads and changes in volume are much greater, and only one of the columns showed any tendency toward an increase in volume at failure. As previously stated, failure occurred suddenly due to breaking of the spiral wire. It is of interest that the decrease in volume of Column 133 was more than 1.5 per cent.

The curves of Fig. 21 for Columns 33, 42, 52, 131, and 132 show breaks where the load was released for a short time and reapplied. When the load was released the recovery in volume was slow. The slope of the curve representing release of load was about twice as great as the slope of the initial tangent at first application of load, while the slope of the curve representing the reapplication of load was about the same as that of the initial tangent.

Although the curves of Fig. 21 appear somewhat unlike, it is evident that they represent the same kinds of phenomena. The reduction in volume at low loads represents nearly elastic action of the material. As soon as the spiral range of the column is reached, the reduction in volume becomes greater than can be explained by elastic deformations, and the inference is that inelastic compacting of the material is produced by the applied stresses. The inelastic compacting is of greatest magnitude for those columns in which the "spiral range" is relatively greatest. Within the spiral range the rate of reduction is nearly constant though six or seven times as great as the initial rate shown by the tangents drawn at zero stress. For the columns which failed gradually through lack of the needed lateral support from the spiral, the compacting was counteracted, and finally far exceeded by an internal splitting action, which produced an increase rather that a decrease in volume as the maximum load was approached.

The similarity of the curves for Groups 5 and 13 suggests that a quantitative relation might be found between ϵ_v and f_v for all groups. By superimposing all of the curves it is found that, neglecting the portion near the maximum load where the volume is increasing, all of the curves follow the same general trend shown by the curves of Group 13. While it does not seem desirable to express this trend by plotting an average curve and deriving its empirical equation, it is well to know that such a relation might be found. It is rather surprising that the curves for the change in volume should be independent of the amount of spiral reinforcement. However, this may be explained by the fact that within the spiral range there is a definite relation between f_1 and f_2, as given by Equation (2), so that any given value of f_v represents certain fixed values of f_1 or f_2 as follows:

$$f_v = f_1 + 2f_2 = 1.49f_1 - 0.49f_c' = 6.1f_2 + f_c'$$

Since f_v always represents a definite stress situation in the material, it is reasonable to conclude that the important factor is the lateral stress f_2 and not the amount of reinforcement that produces the stress, although it is evident that in the more lightly reinforced columns in which the lateral stresses were small the compacting of the material did not occur to as great an extent as it did in the other columns at the same loads. This is the principal item of difference between the various curves of Fig. 21.

25. *Variations in Properties along Length of Column.*—While the study of stress-strain relations has been confined in the foregoing sections principally to the average of a large number of observations in the attempt to define general relations or well-defined trends, still it is desirable to note the extent of variation of the individual observations from the average values used. It was characteristic of the strains measured on the four sides of any column at a given section that the values were quite consistent but that there was considerable variation between strains at the top, middle, and bottom of the column. Column 52 may be taken as representative of the group in having a very noticeable difference between the measured strains at the top and bottom.

Figure 22 shows the relation between axial unit loads and lateral unit stresses determined at sections at the top, middle, and bottom of Column 52. The procedure followed in determining the values used was similar to that employed in Section 22. The values for the three sections of the column are identified by different symbols, and average values are also shown by the dotted line, while the solid straight line represents Equation (2). It is seen that the points of Fig. 22 are slightly more scattered than are the points shown in Fig. 20, which is reasonable, since the latter represent the average of a larger number of observations. It is evident, however, that, despite the individual differences in deformations, the latter have varied in reasonable conformity with the general law expressed by Equation (2). The points

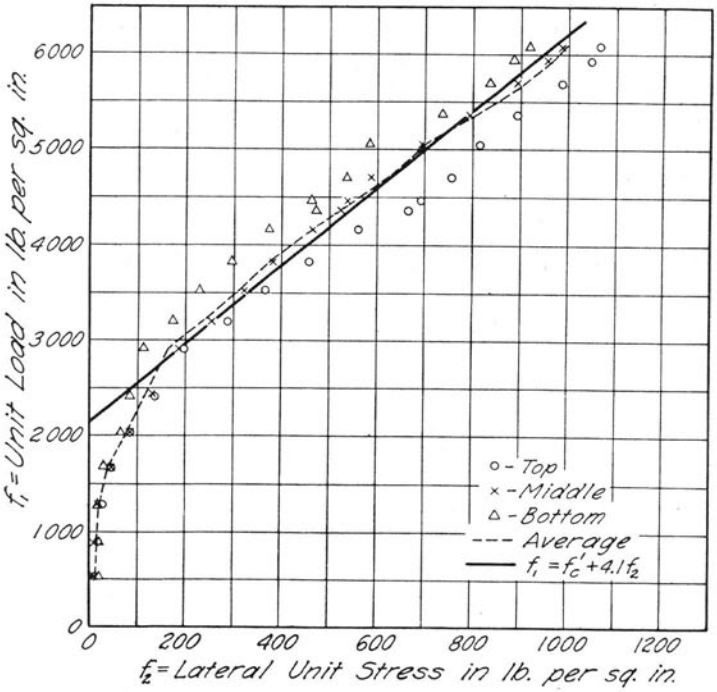

FIG. 22. UNIT LOADS AND LATERAL STRESSES IN COLUMN 52

indicate a rather consistent difference in trend for the top, middle, and bottom sections, indicating the probability of a difference in strength and stiffness between the three sections; hence, if the appropriate values of f_c' could be inserted in Equation (2) for the three sections, the agreement between the observed values and the line representing the general equation would undoubtedly be still better.

As a second study of variations along the length of the column, values of the change in volume, ϵ_v, at the top, middle, and bottom of Column 52 have been computed, and are plotted in Fig. 23 against values of the volume stress, f_v. As might be expected the change in volume varies with the measured strains, being fairly large at the top and middle sections, and departing only slightly from the initial tangent at the bottom section. This is another indication that the concrete at the top of this column was weaker and less dense than that at the bottom, since the amount of compacting occurring might be expected to be an inverse function of the density.

The studies of individual strain variations are of interest in showing that in spite of incidental variations in the material, the relation

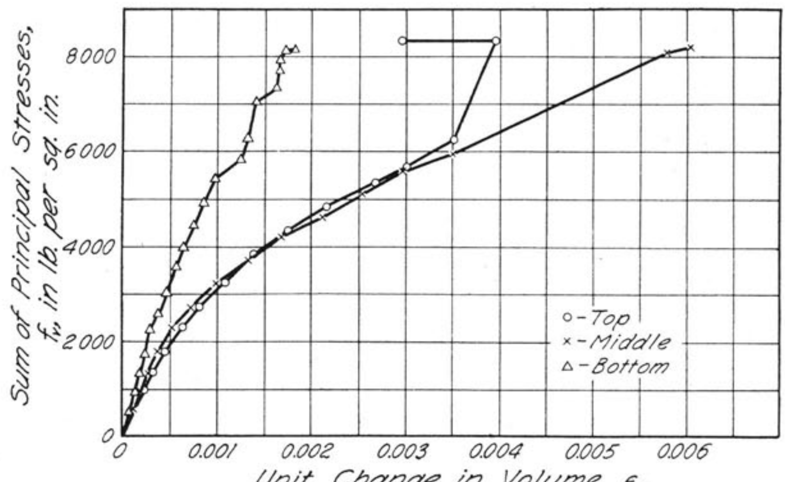

FIG. 23. VOLUME CHANGES IN COLUMN 52

expressed by Equation (2) evidently governs the behavior of all parts of the column. Volume change, on the other hand, is seen to be an extremely variable quantity. Considering that much of the volume change consists of an inelastic compacting of the concrete, it is noteworthy that the average values given in Fig. 21 show the consistent agreement in general trend that they do.

26. *Stress-strain Relations within the Spiral Range.*—In the foregoing sections three important relations between stresses and strains within the spiral range of action of the spirally reinforced columns have been noted; the effect of the depression of the spiral steel into the concrete, the relation between longitudinal and lateral unit stresses at any load, and the relation between volume changes and the accompanying stress situation. With this information available it is possible to derive expressions for the relations between the longitudinal and lateral strains and the unit stresses in the material which apply fairly accurately for the range of conditions covered by these tests. Since such expressions may not be of general application, and since they are represented by rather complicated empirical equations, no attempt will be made to give them here. However, it is important to know that such relations can be derived, not so much for their quantitative use as for their value in indicating the relative importance of the various factors involved. Thus it can be shown that the lateral strain varies directly with the excess of longitudinal stress over

the plain concrete strength, and inversely with the percentage of spiral reinforcement; for a given load within the spiral range, the lateral deformation was about nine times as great for the columns of Group 1 as for those of Group 5. The relation between longitudinal strain and stress is not governed by any simple linear function, but it is evident that the strain decreases as the percentage of spiral reinforcement increases. Other variables which enter into the stress-strain relation are the diameter of the spiral wire, the modulus of elasticity and bulk modulus of the material and the "modulus of depression" described in Section 21.

27. *A Conception of the Action of Spirally Reinforced Columns.*— The theory of failure of concrete under combined stresses given in Bulletin 185 is evidently applicable to the spirally reinforced compression member. During the early stages of loading the action of the reinforced member differs little from that of the plain concrete one. After the limit of proportionality of stress and strain has been reached, marking the beginning of plastic deformation of the concrete, the increase in the rate of lateral deformation serves to produce a very small stress in the spirals. At the stage at which splitting and failure of plain concrete begins, the action of the spiral column is quite different. There is undoubtedly a tendency to splitting, but since such action would in turn produce a rapid increase in the lateral deformation and a consequent increase in the lateral compression exerted by the spirals, the splitting is restrained or retarded. Stated in another way, considering that small elements of the material have begun to deform plastically, these elements must be supported laterally if they are to carry load. The lateral support is afforded by the tensile strength of surrounding elastic elements and by the lateral pressure developed as the concrete bulges outward against the spirals. As lateral deformation progresses it is evident that some splitting takes place, the support of some elastic elements thus vanishing and an increase in plasticity following. This loss of support allows further lateral deformation and the needed lateral restraint is secured from the spiral reinforcement. The materal at this stage may be considered as "disorganized," the elements being in a highly plastic stage, held in equilibrium between the external loads and the lateral pressure of the spiral reinforcement. This equilibrium may be called a plastic equilibrium; it is essentially independent of the amount of lateral deformation.

It is evident that the maximum load on a spirally reinforced member is reached when the increase in lateral compression produced by the spirals fails to keep up with the loss of cohesion between particles. There seems to be a minimum rate of development of spiral stress required, since failure of columns occurred before the spiral steel had reached its ultimate strength. The material of columns highly reinforced should reach a more completely disorganized or plastic condition than that of a lightly reinforced one, in which the splitting effect should be more noticeable. This is in accord with the very gradual yielding noted in columns of Group 5 as compared to the more rapid failures of those of Group 1.

It is admitted that the foregoing statements are not all based on test observations; rather it is intended to give a visualization of the probable internal action causing the phenomena observed, an explanation consistent with a conception of failure that seems in accord with other sets of experiments.

V. Conclusions

28. *Summary of Results.*—In the foregoing discussion an effort has been made to correlate the results of the tests with the conception of failure of concrete in combined compression given in Bulletin 185, an attempt which has in some cases involved re-statement of well-known principles simply for the sake of a new viewpoint.

The following summary is intended to emphasize the principal findings of the bulletin; the first two paragraphs refer to the action of plain concrete members and the remainder to those containing reinforcement.

(1) The behavior of plain concrete in simple compression may be considered for three stages of loading, each having certain special characteristics. In the first stage the material acts like an elastic material, stresses and strains being proportional. The extent of this stage depends upon the quality and condition of the concrete. The second stage is marked by appreciable deviations, particularly of the lateral strains, from the linear stress-strain curves of the first stage, and a steady increase in the ratio of lateral to longitudinal strains. This stage evidently indicates the beginning of plastic deformation, perhaps a yielding of the bond at the surfaces of aggregate particles, within the material. As this deformation spreads it tends to set up lateral tensile stresses in the portions of the material still intact. The beginning of the third stage is marked by an abrupt increase in the ratio of lateral to longitudinal strains; as a consequence the volume of the material, which had been decreasing under increasing loads, changes its behavior radically and increases with further loading. In

these tests the third stage generally began at from 75 to 85 per cent of the maximum load.

(2) As failure was approached in the third stage of loading the ratio of lateral to longitudinal strain exceeded one-half, the apparent volume of the material showed a net increase, and small cracks, parallel to the direction of loading, began to appear on the surface. This condition was evidently produced by internal tension failure or splitting on minute surfaces, followed by extensive plastic deformation, or disorganization of the material, and final failure or collapse of the unstable mass. From the measured bulging of the material it is apparent that initial failure was due to a splitting action rather than to a sliding along continuous inclined planes, since the bulging was not accompanied by a like axial shortening, which would be an essential feature of a sliding action. The fact that final fracture frequently followed conical surfaces inclined at 55 to 60 deg. to the horizontal probably has no significance as regards initial failure.

(3) The action of spirally reinforced columns at the early stages of loading is essentially the same as that described for the first and second stages of loading of plain columns. During the second stage plastic deformation of the material begins and the lateral deformations become large enough to produce a small stress in the spiral, which in turn exerts a slight lateral pressure on the concrete core. The third stage, which has been denoted as the "spiral range" of action, begins at a load corresponding to that at which the splitting of plain concrete begins. Considering that certain small elements of the material have begun to deform plastically, these elements must be supported laterally if they are to carry load. This lateral support must be afforded by the tensile strength of surrounding elastic elements, and by the lateral pressure developed by the spiral. The large lateral deformations accompanying high tensile stress in the concrete develop pressure against the spiral and as the support of elastic elements is lost through a splitting action, the requisite support is gained from the spiral reinforcement. As this action increases, the concrete becomes more of a plastic mass, carrying further load only as fast as the lateral support of the steel can be developed. When loading is applied more rapidly than lateral support can be furnished, failure begins.

(4) An important result of the tests was the determination of a fairly definite relationship between longitudinal and lateral stresses within the spiral range. The relation $f_1 = f_c' + 4.1 f_2$ was found to apply at the maximum as well as at lower loads; and to individual columns as well as to the average of each group of columns. The fact

that this relation also applied quite well to the tests of Bulletin 185 adds to its usefulness. Two limitations to the applicability of the relation must be noted, however: (a) it was derived from tests of concrete made from a particular lot of aggregates and cement, and it is likely that concrete made from other materials might furnish different numerical constants, particularly as regards the coefficient 4.1; (b) the tests were made on short cylinders having a height equal to four diameters, a type of specimen in which the effectiveness of the spiral should be greater than in the more slender members generally used in practice.

(5) The foregoing relation between longitudinal and lateral stresses in the spiral range evidently represents plastic equilibrium of the material, and is independent of the lateral deformations taking place. The lateral deformations are governed by the deformations of the spiral required to develop the lateral pressure demanded for this equilibrium.

(6) The reliability of all properties dependent upon the observed spiral stresses is greatly enhanced by the careful studies made of the true stress-strain relations for the spiral steel as it existed in the columns. The results of these studies indicate that the true properties of the steel may not have been determined in earlier investigations.

(7) In all the tests except those of Group 13 the maximum load on a column was passed without breaking the spiral reinforcement, or even reaching the maximum strength of the steel. It appears that there may be a certain minimum rate of increase of lateral stress with deformation of the column necessary to allow the vertical load to increase proportionately; when this minimum rate is not maintained, the maximum load is passed. While the tests do not give a very clear definition of this minimum rate, it is evidently lower for the more highly reinforced members. The limiting rate of increase in lateral stress with respect to lateral deformation is a function of both the percentage of spiral steel and the tangent modulus of elasticity of the steel. The limiting value of the latter increases more rapidly than in an inverse ratio to the percentage of spiral as the percentage is decreased. This limiting value of the tangent modulus of elasticity fixes the maximum steel stress that can be utilized in giving strength to the column. The foregoing statements do not apply to the columns of Group 13, which failed prematurely through stress-concentration at gage holes in the spiral steel.

(8) In spite of the small pitch (1 inch) of the spirals used, there was in all cases a considerable depression of the spirals into the concrete core, or more correctly, a flow of the concrete between the spiral

wires. The amount of the depression was roughly proportional to the bearing pressure between the spiral and the concrete, being greatest for the columns having the largest percentage of spiral reinforcement.

(9) Volume changes of a column throughout loading gave valuable information regarding the general behavior of the column. At low loads, the volume change was closely proportional to the applied loads. In the spiral range the action differed from that preceding. Those columns having a large amount of reinforcement suffered a correspondingly large decrease in volume, the rate of decrease reaching 6 or 7 times the initial rate. This decrease was evidently due to inelastic compacting of the concrete. In the columns having very small amounts of reinforcement the inelastic deformation was relatively small, and at high loads the amount of increase in volume was relatively great, approaching that of the plain concrete. This increase in volume, which began well below the maximum load, was evidently due to the failure of the spiral to furnish sufficient lateral restraint.

(10) The action of the spirally reinforced column departed radically from the laws of behavior of elastic solids, particularly as to the independence of load-carrying capacity and lateral deformation. Volume changes, which in the spiral range were essentially inelastic in character, were quite irregular, though all followed the same general trend as represented by an average curve.

(11) The foregoing tests have provided three related sets of observations: (a) a relation between longitudinal and lateral stresses; (b) a general rule regarding the depression of the spiral steel into the concrete; and (c) a relation for the variations in volume during loading. Together these relations, if sufficiently general and accurate, provide complete information regarding load-deformation relations in spirally reinforced members. Of the three, the third is least reliable and general in application, while the first is the most applicable and useful. It alone furnishes a rational basis for an analysis of the strength properties of this type of member.

(12) It has long been recognized that much of the strength of spirally reinforced columns cannot be utilized because of the large attendant deformations; furthermore, the foregoing load-deformation relations for plain and spirally reinforced columns do not apply directly to the commercial column with both longitudinal and spiral reinforcement. However, the information presented should lead to a better understanding of the behavior of and to more rational rules of design for such members. There is particular need for a better knowledge of the deformations, as well as the stresses, in such members.

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