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A NEW APPROACH TO THE SENSITIVITY PROBLEM IN MULTIVARIABLE FEEDBACK SYSTEM DESIGN

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SUMMARY

A new point of view for the parameter variation problem in linear multivariable systems is proposed. The output deviations due to parameter variations for an open loop realization are related by a sensitivity matrix to the output deviations due to parameter variations for a closed loop (feedback) realization. By means of a performance index involving this sensitivity matrix, conditions are obtained for insuring that the feedback realization is less affected by parameter variations than an open loop realization having the same nominal transfer characteristics. A procedure based on this new sensitivity formulation is given for designing a multivariable feedback system.

1. INTRODUCTION

The use of feedback in control systems is often costly, complex, and may lead to stability problems. Any system transfer characteristic that can be obtained using feedback can be realized by an open-loop configuration also. However, feedback is used for two primary reasons:

a) Feedback <u>may</u> decrease the effects of parameter variations upon the system behavior.

b) Feedback <u>may</u> improve rejection of disturbance signals.

In this article we shall be concerned with the first of these reasons, which will be treated analytically using the concept of sensitivity.

The concept of sensitivity[1] has been useful in the analysis and synthesis of linear time-invariant feedback systems[1-9]. We consider a dynamic process to be controlled, called the plant, and we suppose that the characteristics of this process are not known exactly. For example, the plant at hand may be a sample out of an ensemble. Since a primary reason for the use of feedback is to reduce the effects of parameter variations, it is important to have design procedures which are intimately related to this primary goal. In the case of single-input single-output feedback systems, the "percentage change" sensitivity function defined as

$$S = \frac{dT(s,x)}{dx}\,\frac{x}{T(s,x)} = \frac{d\,\ell n\,T(s,x)}{d\,\ell n\,x} \tag{1}$$

plays an important role in the design procedures. Here, T is a transfer function depending on the complex frequency variable s and a quantity x, such as amplifier gain K or plant transfer function P. A good feedback design insures that $|S(j\omega)| \ll 1$ in the frequency band of interest. For large parameter variations, the sensitivity function may be defined as

$$S = \frac{\Delta T(s,x)}{T(s,x)}\,\frac{x}{\Delta x} \tag{2}$$

where $\Delta T(s,x)$ is the change in $T(s,x)$ due to a change Δx in x. In Equation (2), x and T represent the unchanged or nominal values. Horowitz[6] uses a similar expression except that he normalizes with respect to the changed values of x and T.

In the case of multivariable linear time-invariant systems, T is a matrix of transfer functions. Previous attempts to extend Equation (2) to the matrix case have not proved satisfactory as no suitable interpretation of the elements of the sensitivity matrix has been made. Horowitz[4] developed a design procedure which involves the elements of the matrix ΔT. It is not clear that given a ΔT specification, it is necessary to use a feedback structure rather than an open-loop structure.

The purpose of this paper is to develop a method for directly comparing an open-loop design with a closed-loop or feedback design for a multivariable, linear, time-invariant system. We present a matrix analog of Equation (2) together with its interpretation. Using this sensitivity matrix it will be possible to develop a design procedure for multivariable systems. Moreover, new insight into the role of sensitivity will be gained through this new formulation.

II. THE SENSITIVITY MATRIX AND ITS INTERPRETATION

Figure 1 shows an open-loop structure whereas Figure 2 shows the general feedback structure. We wish to insure that the designed feedback structure is better than the open-loop structure in reducing the effects of plant parameter variations. These variations may be due to tolerance in manufacturing or ignorance of the process. We assume that ranges in which parameter values may lie are known. We shall measure these effects quantitatively using a sensitivity matrix.

Let T represent the nominal or desired transmission matrix, P the nominal plant transfer matrix, $P' = P+\Delta P$ the actual plant transfer matrix, $T_o' = T+\Delta T_o$ the actual transmission matrix using an open-loop design, and $T_c' = T+\Delta T_c$ the actual transmission matrix using a closed-loop design. Then the Laplace transform of the error due to plant parameter variation between the nominal output and the actual output in the open-loop case is

$$\underline{E}_o(s) = [T - T_o'] \, \underline{R}(s) \tag{3}$$

Using a closed-loop design the corresponding error between the nominal output and the actual output is

$$\underline{E}_c(s) = [T - T_c'] \, \underline{R}(s) \tag{4}$$

If there are as many inputs as there are outputs, then $T - T_o'$ is square. Furthermore, if $T - T_o'$ is nonsingular[10] then

$$\underline{R}(s) = (T - T_o')^{-1} \, \underline{E}_o(s) \tag{5}$$

and Equation (4) can be rewritten as

$$\underline{E}_c(s) = (T - T_c')(T - T_o')^{-1} \, \underline{E}_o(s) = S \, \underline{E}_o \, (s) \tag{6}$$

The matrix S in Equation (6) which we will call the <u>sensitivity matrix</u> relates the error in the open-loop design to the error in the closed-loop design.

Let us examine the matrix S in Equation (6) in more detail. First, we note that from Figure 1,

$$T = PG_1 \tag{7}$$

and

$$T_o' = P'G_1 = (P+\Delta P)G_1 \tag{8}$$

Hence

$$(T - T_o') = PG_1 - P'G_1 = -(\Delta P)G_1 \tag{9}$$

Solving[11] for G_1 in Equation (7) and substituting in Equation (9),

$$(T - T_o') = -(\Delta P)P^{-1} \, T \, . \tag{10}$$

For the feedback case we have

$$T - T'_c = - \Delta T_c \tag{11}$$

Hence

$$S = (\Delta T_c)T^{-1} P (\Delta P)^{-1} \tag{12}$$

We are of course assuming that all indicated inverses exist.[11] Note the similarity between the expression for S in Equation (12) and that for the single-input, single-output case in Equation (2) where x is taken as P. Thus the sensitivity matrix we have defined in Equation (6) for the multivariable case is a generalization of the scalar sensitivity for the single variable case. If the form of Equation (2) is taken as a starting point for <u>defining</u> S as in Equation (12), then S is <u>interpreted</u> as the matrix which relates open-loop error to closed-loop error as in Equation (6).

For the two-degree-of-freedom configuration[3] in Figure 3, the expression for S can be written in terms of a generalized return difference. Substituting in Equation (6)

$$T = (I+PGH)^{-1} PG \tag{13}$$

$$T'_c = (I+P'GH)^{-1} P'G \tag{14}$$

and

$$T'_o = P'G_1 \tag{15}$$

where $G_1 = P^{-1}T$, we obtain

$$S = (I+L')^{-1} \tag{16}$$

where

$$L' = P'GH. \tag{17}$$

The quantity L' may be interpreted as a matrix return ratio for the actual system and $(I+L')$ as a generalized return difference. Observe that for incremental variations, $L' \approx L$, and hence S is the inverse of the matrix form of the classical return difference for this case. For large parameter variations, L may differ significantly from L'. Thus, in this case, L' and hence S must be evaluated for the entire range of ΔP.

III. SPECIFIC COMPARISON CRITERIA

We have defined a sensitivity matrix which relates the transforms of the instantaneous errors due to plant parameter variations for open-loop and closed-loop configurations. It is useful to have a scalar performance index for comparing these configurations. In this section we shall develop some scalar indices based on the sensitivity matrix S.

Suppose we choose the integrated square error as a performance index. To insure a closed-loop design yielding lower performance index than that corresponding to open-loop design we impose the condition

$$\int_0^\infty \underline{e}_c^T(t)\underline{e}_c(t)\,dt < \int_0^\infty \underline{e}_o^T(t)\underline{e}_o(t)\,dt \tag{18}$$

From Parseval's theorem, we can replace the integrals by corresponding ones involving frequency domain quantities. Thus

$$\int_{-\infty}^{\infty} \underline{E}_c^T(-j\omega)\underline{E}_c(j\omega)\,d\omega < \int_{-\infty}^{\infty} \underline{E}_o^T(-j\omega)\underline{E}_o(j\omega)\,d\omega \tag{19}$$

Replacing $\underline{E}_c$ by S $\underline{E}_o$ and transposing, we have

$$\int_{-\infty}^{\infty} \underline{E}_o^T(-j\omega)[S^T(-j\omega)S(j\omega) - I]\underline{E}_o(j\omega)\,d\omega < 0 \tag{20}$$

where I is the identity matrix. Thus, if $[S^T(-j\omega)S(j\omega)-I]$ is positive definite for all frequencies, Equation (20) can never be satisfied, and any closed-loop design is worse than the open-loop design in this case. If $[S^T(-j\omega)S(j\omega)-I]$ is negative definite for all frequencies, then Equation (20) will surely be satisfied. Therefore, a sufficient condition for insuring that feedback design is better than open-loop design in the sense of Equation (18) is that

$$S^T(-j\omega)S(j\omega)-I < 0 \text{ (negative definite)} \tag{21}$$

for all frequencies and for the entire range of ΔP. The relation of Equation (21) is a generalization of the condition for the single variable case. That is, for a single variable system, a good feedback design requires that the magnitude of the sensitivity function be less than one.

Instead of Equation (18) we may specify a more restrictive condition on the sensitivities such as

$$\int_0^\infty \underline{e}_c^T(t)\underline{e}_c(t)\,dt < \lambda \int_0^\infty \underline{e}_o^T(t)\underline{e}_o(t)\,dt \tag{22}$$

where λ is a positive number less than 1. For a given range of plant parameter variation, it may be desirable to design a feedback structure which satisfies Equation (22) with the least value for λ. Analogous to Equation (21), a sufficient condition for Equation (22) to be satisfied is

$$S^T(-j\omega)S(j\omega) - \lambda I < 0 \tag{23}$$

It is emphasized that a design corresponding to a smaller λ in Equation (23) does not necessarily yield a smaller integral of the squared closed-loop error. A smaller λ simply means a smaller lower bound on the integral of the closed-loop error, and the integral may have any value less than the lower bound.

It should be noted that if the above sufficiency criteria are satisfied, closed-loop design is guaranteed to be better than open-loop design for <u>any input</u>. This is to be contrasted with Mazer's approach[5] wherein the mean square closed-loop error is minimized for a given <u>specific input</u>.

IV. APPLICATION TO MULTIVARIABLE SYSTEM DESIGN

In this section we discuss the application of the above results to the design of multivariable feedback systems. We consider here "classical" design in terms of frequency response, root locus techniques, etc. The new formulation of sensitivity presented above is applicable also to systems described by state variable differential equations. Sensitivity considerations for systems described by state variables is the subject of a forthcoming companion paper.

Equations (16) and (21) (or (23)) are particularly useful in the design of multivariable systems. (Application to the single variable case yields the conventional result

$$|S(j\omega)| < 1 \tag{24}$$

as has been noted above). The basic idea in the use of Equations (16) and (21) (or (23)) for design is to obtain from these equations a set of inequalities involving the elements L'_{mn} of the matrix L'. We then shape these transmissions $L'_{mn}(j\omega)$ (or alter the poles and zeros of $L'_{mn}(s)$ in the s plane) to satisfy the inequalities, as well as any "filter" specifications (bandwidth, damping ratio, M_p, etc.). Moreover, the design must be checked for stability. In such a design procedure there is an inevitable conflict between the effort involved in the design and the savings in gain-bandwidth requirements of the loop transmissions. For simplicity, we may begin the design by considering $L' \cong L$ (small variations). Furthermore, the design labor is reduced materially by requiring the L matrix to have considerable symmetry.

For example, if $L \cong L'$ is taken to be diagonal, then the sensitivity matrix S will be diagonal. Thus $S^T(-j\omega)\ S(j\omega)$ will be diagonal, and Equations (16) and (21) (or (23)) require

$$|1 + L_{nn}(j\omega)|^2 > \lambda \tag{25}$$

for n = 1, 2, ..., m, where m is the number of input or output variables. Notice, however, that the matrices P, G, H, and T are not diagonal, in general, so each element L_{nn} depends on many entries of the P, G, and H matrices. Further economy in design labor may be obtained by requiring certain symmetries in G and H. Such symmetries are obtained usually at the cost of increased gain-bandwidth requirements.

Moreover, in the case of large parameter variations, $L' \neq L$. So even though the preliminary stages of the design were carried out using L, the actual matrix L' should be computed for the range of ΔP, and the design should be examined to insure that the sensitivity specifications are truly met.

Finally, the actual system must be checked for stability. Horowitz[6] shows that since

$$T' \equiv [I+P'GH]^{-1}P'G = [L+P\,P'^{-1}]^{-1}(I+L)T \tag{26}$$

we may check

$$\det\,(L(s) + P(s)\,P'(s)^{-1}) = 0 \tag{27}$$

for zeros in the right half s-plane. This must be done for all P' resulting from the range of parameter variation.

The trial-and-error design procedure may be summarized as follows:

1. Take $L' \cong L$, and also assume L is diagonal (or possesses some other convenient symmetry).
2. Obtain the sensitivity requirements using Equations (16) and (21) (or (23)). Express these in terms of the elements of L.
3. Using conventional frequency response or s-plane design techniques, adjust the compensation G and H to satisfy both the filter and sensitivity specifications. In carrying out the actual design of this compensation, the methods presented in Chapter 10 of Horowitz[6] are useful. Strictly speaking, the inequalities of Equation (25) must hold for all real frequencies ω. However, in many practical applications we may obtain adequate performance by requiring Equation (25) to hold only over a finite band of significant frequencies.
4. Evaluate L' and check all specifications. (Some redesign may be required here in the case of large parameter variations).
5. Check stability by examining $\det\,(L+P\,P'^{-1})$ for right half plane zeros.

V. CONCLUSIONS

A basic reason for the use of feedback is to render a closed-loop system less susceptible to the effects of plant parameter variations than an open-loop system having the same nominal input-output characteristics. In this paper we have presented an analytical measure of this susceptibility in terms of a sensitivity matrix. This direct comparison of open-loop and closed-loop performance is a new point of view for the parameter variation problem. For the single variable case this new formulation yields the "percentage change" expression employed heretofore. In addition, however, the new formulation yields a meaningful measure of the effects of parameter variations for the multivariable problem as well. By comparing the integrals of the sums of the squares of the open- and closed-loop errors, we obtain a condition on the sensitivity matrix that <u>guarantees</u> the superiority of the closed-loop design. By expressing the sensitivity matrix in terms of a matrix return difference for a specific configuration, we obtain a procedure for the design of multivariable feedback systems.

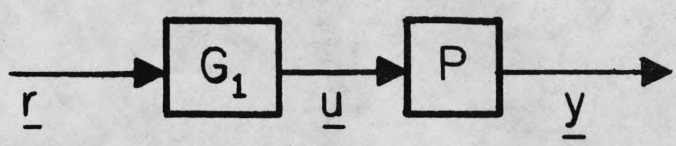

Figure 1. A Multivariable Open-loop Control System.

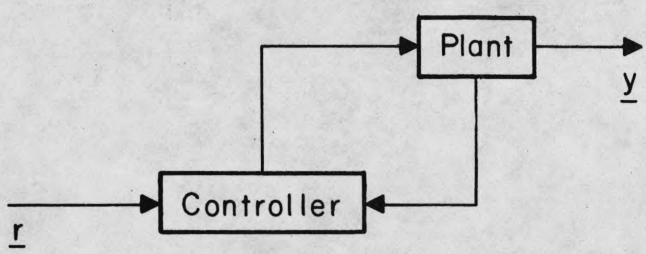

Figure 2. A General Multivariable Feedback Control System.

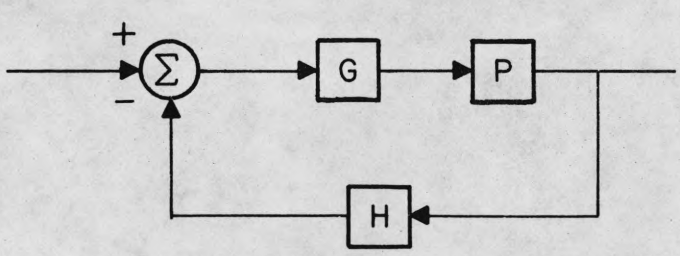

Figure 3. A Two-degree-of-freedom Multivariable Feedback Control System.

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10. The case where $(T-T_o')$ is singular or even when $(T-T_o')$ is not square can be handled using Penrose's generalized inverse which always exists. $(T-T_o')^{-1}$ in Equations (5) and (6) is then replaced by the generalized inverse. For simplicity, we assume that all matrices in this paper are square and nonsingular. For the generalized inverse of a matrix see Penrose, R., "A Generalized Inverse for Matrices," Proc. Cambridge Phil. Soc., 51, pp. 406-413, 1955.

11. See comment in 10.

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