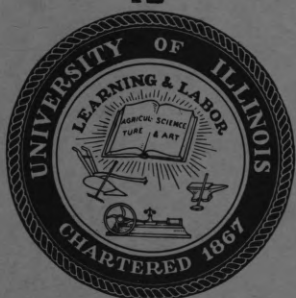


# Coordinated Science Laboratory



UNIVERSITY OF ILLINOIS – URBANA, ILLINOIS

**DIELECTRIC PROPERTIES OF A  
WEAKLY TURBULENT PLASMA**

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**R-251**

**April, 1965**

This work was supported wholly by the Joint Services  
Electronics Programs (U. S. Army, U. S. Navy, and  
U. S. Air Force) under Contract No. DA 28 043 AMC 00073(E).

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#### ACKNOWLEDGMENT

The author is much obliged to Professor M. Raether for many valuable suggestions and criticisms.



# DIELECTRIC PROPERTIES OF A WEAKLY TURBULENT PLASMA

Yoshi H. Ichikawa<sup>\*</sup>

## Abstract

Generalization of the Herring-Yoshikawa theory has been developed to take into account the dynamical effect of turbulent fluctuation on the dielectric properties of the weakly turbulent plasma. According to the hydrodynamic description of electron plasma, the turbulent fluctuation gives rise to appreciable modification of the dielectric properties in the low frequency region and at the frequency of twice of the electron plasma frequency. An expression of the static dielectric constant shows that the characteristic correlation distance of the weakly turbulent plasma becomes longer than the Debye distance of corresponding homogeneous plasma in thermal equilibrium state. Contrary to the hydrodynamic scheme, the integral equation has been derived for the induced fluctuation in the kinetic description of the plasma. This integral equation suggests that the turbulent fluctuation may modify the dielectric properties of the turbulent plasma also at the electron plasma frequency.

## 1. Introduction

Plasmas encountered in the nature are most likely in turbulent states in the sense that various types of noise and oscillations are sustained in the plasmas. Though such turbulent states of plasmas may resemble that of the ordinary fluids in some cases, we find frequently the plasma in a state which can be described as an assembly of many weakly interacting modes of oscillation. Such a state can be defined as the weakly turbulent state. When the density of plasma is low and the energy of these oscillation is sufficiently smaller than the thermal energy of plasma particles, the wave-particle interaction dominates over the wave-wave interaction and the particle-particle interaction at least for the case of electron plasma oscillation. The behaviour of a weakly unstable plasma under the above conditions has been analyzed by Drummond and Pines<sup>1)</sup> and Vedenov and his collaborators<sup>2)</sup> in terms of the quasi-linear theory of the Vlasov equation. Their theory is developed by taking advantage of the dynamical properties of the system. Kadomtsev and Petviashvili<sup>3)</sup>, on the other hand, have proposed a formal theory of the weakly turbulent plasma by employing a statistical approach to construct a hierarchy equation of correlation functions.

Now, in recent years, the anomalous transport phenomena in plasmas have been one of the main concerns of studies of plasmas. Among various attempts of giving theoretical account for the anomalous transport phenomena in plasmas, the work of Yoshikawa<sup>4)</sup> attracts our particular interest. Following the method of Herring<sup>5)</sup>, which is essentially a quasi-linear approximation for the effect of spatial inhomogeneity, he has illustrated

that density fluctuation existing in plasmas reduces the d-c electrical conductivity.

Since the dynamical nature of density fluctuation in plasma has not been taken into consideration in the work of Yoshikawa, we have undertaken the investigation of the dielectric response properties of a weakly turbulent plasma based on the kinetic description of the plasma. In order to investigate the effect of turbulent fluctuation on the frequency and wave number dependent dielectric properties, it is necessary to generalize the quasi-linear approximation to some extent. The generalization has been accomplished by using explicitly the statistical average over the ensemble of stationary turbulent fluctuations. An integral equation has been derived for the induced density fluctuation in phase space, yet we have not succeeded to solve it explicitly. The derivation of the integral equation will be described in the appendix of the present paper.

In the main part of this paper, therefore, we will discuss the problem on the basis of a hydrodynamic description of the electron plasma. A basic formulation of the problem is presented in Section 2. After calculating the induced density fluctuation of the weakly turbulent plasma, we give an expression of the dielectric constant of the weakly turbulent plasma in Section 3. In Section 4, we carry out explicit calculations of the dielectric constant for the case of a cold plasma. The results will be discussed in some detail in the following sections.



## 2. Basic Formulation

For simplicity, let us consider an electron plasma immersed in a uniform background distribution of positive ions of density  $n_0$ . We assume that the state of plasma can be determined by the number density  $n(\vec{x}, t)$  and the flux  $\vec{\Gamma}(\vec{x}, t)$ , the space-time variation of which is described in terms of the following set of equations,

$$\frac{\partial}{\partial t} n(\vec{x}, t) + \text{div } \vec{\Gamma}(\vec{x}, t) = 0 \quad (1.a)$$

$$\frac{\partial}{\partial t} \vec{\Gamma}(\vec{x}, t) = - \frac{1}{\tau} (\vec{\Gamma}(\vec{x}, t) + D \nabla n(\vec{x}, t) + \mu n(\vec{x}, t) \vec{E}(\vec{x}, t)) \quad (1.b)$$

$$\text{div } \vec{E}(\vec{x}, t) = 4\pi e(n(\vec{x}, t) - n_0) \quad (1.c)$$

where  $D$  and  $\mu$  are the diffusion coefficients and mobility of electrons in the plasma. The relaxation time  $\tau$  satisfies a relation  $\mu = e\tau/m$ .  $\mu$  and  $D$  are connected by the Einstein relation,  $\mu/D = e/kT_e$ .

Eliminating the flux  $\vec{\Gamma}(\vec{x}, t)$  from Eqs. (1.a) and (1.b), we obtain

$$\frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} + \frac{1}{\tau} \right) n - \frac{D}{\tau} \nabla^2 n + \frac{\mu}{\tau} \text{div } (n\vec{E}) = 0 \quad (2)$$

Eqs. (1.c) and (2) are the set of fundamental equations of the present problem. We assume a statistical ensemble of systems of plasma composed of a random distribution of the phase of rapid fluctuation. The number density  $n(\vec{x}, t)$  can be decomposed into two parts as,

$$n(\vec{x}, t) = n_0 + v(\vec{x}, t) \quad (3)$$

where  $n_0$  is the ensemble average of the number density defined as

$$n_0 = \langle n(\vec{x}, t) \rangle \quad (4)$$

and  $v$  is the component of the rapid density fluctuation. Then Eqs. (2) and (1.c) can be reduced to

$$\frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} + \frac{1}{\tau} \right) v - \frac{D}{\tau} \nabla^2 v + \frac{\mu}{\tau} n_0 \operatorname{div} \vec{E} + \frac{\mu}{\tau} \operatorname{div} (v \vec{E}) = 0 \quad (5.a)$$

and

$$\operatorname{div} \vec{E} = 4\pi e v \quad (5.b)$$

In the case of a weakly turbulent plasma, we may neglect the non-linear term of Eq. (5.a). Hence, the density fluctuation can be determined by the following equation,

$$\frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} + \frac{1}{\tau} \right) v - \frac{kT_e}{m} \nabla^2 v + \omega_p^2 v = 0 \quad (6)$$

where  $\omega_p^2$  is the well-known Langmuir frequency. Introducing the Fourier expansion of the density fluctuation

$$v(\vec{x}, t) = \frac{1}{V} \sum_{\vec{k}} v(\vec{k}, t) e^{i\vec{k}\vec{x}}, \quad (7)$$

we obtain a solution of Eq. (6) for the specified initial values,  $v(\vec{k}, t=0)$  and  $\dot{v}(\vec{k}, t=0)$ . Then, in the limit of  $\omega \gg 1/\tau$ , we can calculate the spectral density of the correlation of density fluctuation as

$$\frac{1}{VT} \langle |v(\vec{k}, \omega)|^2 \rangle = \frac{n_0 k^2}{m} \frac{n\Theta(k)}{\omega(k)^2} \cdot \frac{\pi}{2} \{ \delta(\omega - \omega(k)) + \delta(\omega + \omega(k)) \} \quad (8)$$

with the abbreviation  $\omega(k)^2 = \omega_p^2 + (kT_e/m)k^2$ . In Eq. (8), the turbulent temperature  $\Theta(k)$  is defined by the following relation,

$$\langle \frac{1}{2} |\dot{v}(\vec{k}, t=0)|^2 + \frac{1}{2} \omega(k)^2 |v(\vec{k}, t=0)|^2 \rangle = \nabla \cdot \frac{n_0 k^2}{m} n\Theta(k) \quad (9)$$

Although the major problem of a theory of turbulence is to determine the energy density spectrum or the equivalent turbulent temperature of the system, we take the turbulent temperature  $\Theta(k)$  defined in Eq. (9) as a given quantity to characterize the stationary state of the weakly turbulent plasma, since we are concerned primarily with the response properties of the weakly turbulent plasma.

Thus, our problem is well defined as follows: examine the response properties of the weakly turbulent plasma, the stationary state of which is characterized by the average number density  $n_0$ , the kinetic temperature  $T_e$ , and by the spectral density of the density correlation as given by Eq. (8). In order to investigate the dielectric response properties, let us introduce adiabatically a test charge  $eq(\vec{x}, t)$  into the system, and examine behaviour of the induced density fluctuation. Linearizing Eq. (2) with respect to the induced fluctuation, we obtain

$$\frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} + \frac{1}{\tau} \right) n^{(1)} - \frac{kT_e}{m} \nabla^2 n^{(1)} + \frac{e}{m} \text{div} (n^{(0)} \vec{E}^{(1)} + n^{(1)} \vec{E}^{(0)}) = 0 \quad (10)$$

where the unperturbed density  $n^{(0)}(\vec{x}, t)$  is defined by Eqs. (3) and (5.b) respectively, while the induced electric field  $\vec{E}^{(1)}$  satisfies the equation

$$\text{div} \vec{E}^{(1)} = 4\pi e n^{(1)} + 4\pi eq. \quad (11)$$

The Fourier expansions of Eqs. (10), (11) and (5.b) give rise to the following equations,

$$\{\omega(k)^2 - \omega^2 - i\omega/\tau\} n^{(1)}(\vec{k}, \omega) + \omega_p^2 q(\vec{k}, \omega)$$



$$\begin{aligned}
& - \frac{4\pi e^2}{m} \frac{1}{VT} \sum_{\vec{l}, \omega'} \frac{\vec{k} \cdot (\vec{k} - \vec{l})}{|\vec{k} - \vec{l}|^2} v(\vec{l}, \omega') q(\vec{k} - \vec{l}, \omega - \omega') \\
& - \frac{4\pi e^2}{m} \frac{1}{VT} \sum_{\vec{l}, \omega'} \left( \frac{\vec{k} \cdot \vec{l}}{l^2} + \frac{\vec{k} \cdot (\vec{k} - \vec{l})}{|\vec{k} - \vec{l}|^2} \right) v(\vec{l}, \omega') n^{(1)}(\vec{k} - \vec{l}, \omega - \omega') = 0
\end{aligned} \tag{12}$$

Solving Eq. (12) for the induced density fluctuation  $n^{(1)}(\vec{k}, \omega)$ , we can determine the longitudinal dielectric constant  $\epsilon(\vec{k}, \omega)$  from the following expression,

$$\frac{1}{\epsilon(\vec{k}, \omega)} = 1 + \frac{\langle n^{(1)}(\vec{k}, \omega) \rangle}{q(\vec{k}, \omega)} \tag{13}$$

### 3. Dielectric constant of the weakly turbulent plasma

Eq. (12) has a complicated structure arising from the fact that the inhomogeneous fluctuation existing in the unperturbed plasma a mode of the induced fluctuation of a wave number  $\vec{k}$  and frequency  $\omega$  couples with those of different wave number  $\vec{k} - \vec{l}$  and frequency  $\omega - \omega'$ . Since we are interested in the rapidly oscillating induced fluctuation, we can not apply the quasi-linear approximation in its original form as used by Herring and Yoshikawa. Instead of applying the quasi-linear approximation to the mode coupling effect, however, we may take full advantage of the fact that all we need to know is the quantity  $\langle n^{(1)}(\vec{k}, \omega) \rangle$  averaged over the ensemble of the turbulent fluctuation.

Now, Eq. (12) gives rise to the following formal expression for  $\langle n^{(1)}(\vec{k}, \omega) \rangle$ ,

$$\begin{aligned}
\langle n^{(1)}(\vec{k}, \omega) \rangle = & - D(\vec{k}, \omega) [q(\vec{k}, \omega) - \frac{1}{n_0} \frac{1}{VT} \sum_{\vec{\ell}, \omega'} \frac{\vec{k} \cdot (\vec{k} - \vec{\ell})}{|\vec{k} - \vec{\ell}|^2} \\
& + \langle v(\vec{\ell}, \omega') q(\vec{k} - \vec{\ell}, \omega - \omega') \rangle - \frac{1}{n_0} \frac{1}{VT} \sum_{\vec{\ell}, \omega'} \left( \frac{\vec{k} \cdot \vec{\ell}}{\ell^2} + \frac{\vec{k} \cdot (\vec{k} - \vec{\ell})}{|\vec{k} - \vec{\ell}|^2} \right) \\
& + \langle v(\vec{\ell}, \omega') n^{(1)}(\vec{k} - \vec{\ell}, \omega - \omega') \rangle]
\end{aligned} \tag{14}$$

where  $D(\vec{k}, \omega)$  is defined as

$$D(\vec{k}, \omega) = \frac{\omega_p^2}{\omega(k)^2 - \omega^2 - i\omega/\tau} \tag{15}$$

Referring to the case of a quiescent plasma in thermal equilibrium, we may express the quantity  $D(\vec{k}, \omega)$  as follows

$$D(\vec{k}, \omega) = 4\pi \cdot \frac{\alpha^{(0)}(\vec{k}, \omega)}{\epsilon^{(0)}(\vec{k}, \omega)} \tag{16.a}$$

by introducing the polarizability  $\alpha^{(0)}(\vec{k}, \omega)$  defined as

$$\epsilon^{(0)}(\vec{k}, \omega) = 1 + 4\pi\alpha^{(0)}(\vec{k}, \omega) \tag{16.b}$$

$$\alpha^{(0)}(\vec{k}, \omega) = \frac{ne^2}{m} \cdot \left[ \frac{\kappa T_e}{m} k^2 - \omega^2 - i\omega/\tau \right]^{-1} \tag{16.c}$$

The second term of the bracket of Eq. (14) can be set equal to zero, while the ensemble average of the third term can be calculated as

$$\begin{aligned}
\langle v(\vec{\ell}, \omega') n^{(1)}(\vec{k} - \vec{\ell}, \omega - \omega') \rangle = & D(\vec{k} - \vec{\ell}, \omega - \omega') \frac{1}{n_0} \frac{1}{VT} \times \\
& \left[ \sum_{\vec{j}, \omega''} \frac{(\vec{k} - \vec{\ell}) \cdot (\vec{k} - \vec{\ell} - \vec{j})}{|\vec{k} - \vec{\ell} - \vec{j}|^2} \langle v(\vec{\ell}, \omega') v(\vec{j}, \omega'') q(\vec{k} - \vec{\ell} - \vec{j}, \omega - \omega' - \omega'') \rangle \right]
\end{aligned}$$

$$+ \sum_{\vec{j}, \omega''} \left( \frac{(\vec{k}-\vec{\ell}) \cdot \vec{j}}{j^2} + \frac{(\vec{k}-\vec{\ell}) \cdot (\vec{k}-\vec{\ell}-\vec{j})}{|\vec{k}-\vec{\ell}-\vec{j}|^2} \right) \times \langle v(\vec{\ell}, \omega') v(\vec{j}, \omega'') n^{(1)}(\vec{k}-\vec{\ell}-\vec{j}, \omega-\omega'-\omega'') \rangle \quad (17)$$

Here, we can simplify the second term by noticing that

$$\begin{aligned} & \langle v(\vec{\ell}, \omega') v(\vec{j}, \omega'') q(\vec{k}-\vec{\ell}-\vec{j}, \omega-\omega'-\omega'') \rangle \\ &= \langle v(\vec{\ell}, \omega') v(\vec{j}, \omega'') \rangle q(\vec{k}-\vec{\ell}-\vec{j}, \omega-\omega'-\omega'') \\ &= \delta_{\vec{\ell}, -\vec{j}} \delta_{\omega', -\omega''} \langle |v(\vec{\ell}, \omega')|^2 \rangle q(\vec{k}, \omega) \end{aligned} \quad (18)$$

As far as the third term of Eq. (17) is concerned, we may truncate the ensemble average of the product of three factors,

$$\begin{aligned} & \langle v(\vec{\ell}, \omega') v(\vec{j}, \omega'') n^{(1)}(\vec{k}-\vec{\ell}-\vec{j}, \omega-\omega'-\omega'') \rangle \\ & \approx \langle v(\vec{\ell}, \omega') v(\vec{j}, \omega'') \rangle \langle n^{(1)}(\vec{k}-\vec{\ell}-\vec{j}, \omega-\omega'-\omega'') \rangle \\ &= \delta_{\vec{j}, -\vec{\ell}} \delta_{\omega'', -\omega'} \langle |v(\vec{\ell}, \omega')|^2 \rangle \langle n^{(1)}(\vec{k}, \omega) \rangle \end{aligned} \quad (19)$$

Substituting Eq. (17) with Eqs. (18) and (19) into Eq. (14), we obtain the ensemble average of the induced charge density fluctuation as

$$\langle n^{(1)}(\vec{k}, \omega) \rangle = - \frac{1 + \Gamma(\vec{k}, \omega)}{\epsilon^{(0)}(\vec{k}, \omega) + 4\pi\alpha^{(0)}(\vec{k}, \omega) \cdot \Delta(\vec{k}, \omega)} \cdot 4\pi\alpha^{(0)}(\vec{k}, \omega) q(\vec{k}, \omega) \quad (20)$$

where  $\Gamma(\vec{k}, \omega)$  and  $\Delta(\vec{k}, \omega)$  are defined as follows,

$$\Gamma(\vec{k}, \omega) = - \frac{1}{n_0^2} \frac{1}{VT} \sum_{\vec{\ell}, \omega'} \left( \frac{\vec{k} \cdot \vec{\ell}}{\ell^2} + \frac{\vec{k} \cdot (\vec{k}-\vec{\ell})}{|\vec{k}-\vec{\ell}|^2} \right) \frac{\vec{k} \cdot (\vec{k}-\vec{\ell})}{k^2} \times$$



$$D(\vec{k}-\vec{\ell}, \omega-\omega') \times \frac{1}{VT} \langle |\nu(\vec{\ell}, \omega')|^2 \rangle \quad (21.a)$$

and

$$\begin{aligned} \Delta(\vec{k}, \omega) = & -\frac{1}{n_0} \frac{1}{2} \frac{1}{VT} \sum_{\vec{\ell}, \omega'} \left( \frac{\vec{k} \cdot \vec{\ell}}{\ell^2} + \frac{\vec{k}(\vec{k}-\vec{\ell})}{|\vec{k}-\vec{\ell}|^2} \right) \left( 2 - \frac{\vec{k} \cdot \vec{\ell}}{\ell^2} - \frac{\vec{k} \cdot \vec{\ell}}{k^2} \right) \\ & \cdot D(\vec{k}-\vec{\ell}, \omega-\omega') \cdot \frac{1}{VT} \langle |\nu(\vec{\ell}, \omega')|^2 \rangle \end{aligned} \quad (21.b)$$

Finally, the longitudinal dielectric constant of the weakly turbulent plasma is obtained from Eqs. (13) and (20) as follows,

$$\epsilon(\vec{k}, \omega) = 1 + 4\pi\alpha(\vec{k}, \omega) \quad (22)$$

with

$$\alpha(\vec{k}, \omega) = \alpha^{(0)}(\vec{k}, \omega) \cdot \frac{1 + \Gamma(\vec{k}, \omega)}{1 - 4\pi\alpha^{(0)}(\vec{k}, \omega)(\Gamma(\vec{k}, \omega) - \Delta(\vec{k}, \omega))} \quad (23)$$

Since the contributions of the turbulent fluctuation,  $\Gamma(\vec{k}, \omega)$  and  $\Delta(\vec{k}, \omega)$ , decrease as  $\omega^{-2}$  in the high frequency region, the high frequency asymptotic form of  $\epsilon(\vec{k}, \omega)$  is the same as that of the homogeneous plasma in the thermal equilibrium state, and hence the longitudinal sum rule

$$\int_{-\infty}^{+\infty} \omega \operatorname{Im} \epsilon(\vec{k}, \omega) d\omega = \pi \omega_p^2 \quad (24)$$

holds even if the plasma is in the weakly turbulent inhomogeneous state.

#### 4. Dielectric properties of a weakly turbulent electron plasma

Now, we may examine in some detail the dielectric properties of the weakly turbulent electron plasma by analyzing the formal expression obtained as Eq. (22) in the preceding section. First, let us consider the static dielectric property. In the static limit, it is evident from Eqs. (16.a), (21.a) and (21.b) that the quantities  $\Gamma(\vec{k}, \omega)$  and  $\Delta(\vec{k}, \omega)$  are

both real. Substituting the expression of the spectral density of turbulent fluctuation, Eq. (8), into Eqs. (21.a) and (21.b), we obtain

$$\Gamma(\vec{k}, 0) = -\frac{1}{n_0} \frac{\Theta_0}{T_e} \cdot \frac{k_d^2}{8\pi^2} \int_0^\infty d\ell \ell^2 \frac{\ell^2}{\ell^2 + k_d^2} f(\ell) \int_0^\pi d\theta \sin \theta \frac{L_1(\vec{k}, \vec{\ell})}{k^2 - 2\vec{k} \cdot \vec{\ell}} \quad (25.a)$$

with the following abbreviation of

$$L_1(\vec{k}, \vec{\ell}) = \left( \frac{\vec{k} \cdot \vec{\ell}}{\ell^2} + \frac{k^2 - \vec{k} \cdot \vec{\ell}}{|\vec{k} - \vec{\ell}|^2} \right) \left( 1 - \frac{\vec{k} \cdot \vec{\ell}}{k^2} \right) \quad (25.b)$$

and a similar expression for the quantity  $\Gamma - \Delta$  by replacing the function  $L_1(\vec{k}, \vec{\ell})$  with the following function,

$$L_2(\vec{k}, \vec{\ell}) = \left( \frac{\vec{k} \cdot \vec{\ell}}{\ell^2} + \frac{k^2 - \vec{k} \cdot \vec{\ell}}{|\vec{k} - \vec{\ell}|^2} \right) \left( 1 - \frac{\vec{k} \cdot \vec{\ell}}{\ell^2} \right) \quad (25.c)$$

In writing down Eq. (25.a), we have expressed the turbulent temperature  $\Theta(k)$  as

$$\Theta(k) = \Theta_0 f(k). \quad (26)$$

$k_d^2 = (4\pi e^2 n / k T_e)$  is the Debye wave number, and  $\theta$  is the angle between the wave number  $\vec{k}$  and  $\vec{\ell}$ . Taking the limit of  $k \rightarrow 0$ , we can obtain simply the following expressions,

$$\Gamma(\vec{k}, 0) = -\frac{1}{6\pi} \cdot \epsilon \cdot \frac{\Theta_0}{T_e} \int_0^\infty dx \frac{x^2}{1+x^2} f(x) \quad (27.a)$$

and

$$\Gamma(\vec{k}, 0) - \Delta(k, 0) = -\frac{3}{4\pi} \cdot \epsilon \cdot \frac{\Theta_0}{T_e} \int_0^\infty dx \frac{1}{1+x^2} f(x) \cdot \left( \frac{k}{k_d} \right)^2 \quad (27.b)$$

where  $\epsilon$  is the standard plasma parameter ( $= (4\pi n \lambda_D^3)^{-1}$ ), and  $x$  is a

variable defined as  $x = \ell/k_d$ . Therefore, the static dielectric constant of the weakly turbulent electron plasma is determined as

$$\epsilon(\vec{k}, 0) = 1 + \frac{k_*^2}{k^2} \quad (28)$$

with the following definition of  $k_*$ ,

$$k_*^2 = \frac{1 - \frac{1}{6\pi} \cdot \epsilon \cdot \frac{\Theta}{T_e} \cdot \int_0^\infty dx \frac{x^2}{1+x^2} f(x)}{1 + \frac{3}{4\pi} \cdot \epsilon \cdot \frac{\Theta}{T_e} \cdot \int_0^\infty dx \frac{1}{1+x^2} f(x)} \cdot k_d^2 \quad (29)$$

The wave number  $k_*$  may be regarded as a fundamental quantity of the weakly turbulent electron plasma, which corresponds to the Debye wave number of the homogeneous plasma in thermal equilibrium state. Eq. (29) confirms that the wave number  $k_*$  is smaller than the Debye wave number  $k_d$  for the arbitrary spectrum of the turbulent fluctuation. Hence, we can conclude that the characteristic correlation length in the turbulent plasma is longer than the Debye length of the thermal equilibrium plasma.

Next let us examine the high frequency dielectric properties of the turbulent electron plasma. In order to illustrate how the dielectric constant of the turbulent electron plasma differs from that of the homogeneous plasma in thermal equilibrium state in the high frequency region, we may simplify the following calculation by considering the case of cold plasma. For the case of cold plasma, the function  $D(\vec{k}, \omega)$  becomes independent on the wave number  $\vec{k}$ . Then Eqs. (21.a) and (21.b) are reduced to the following expression,



$$\Gamma(\vec{k}, \omega) = \frac{k^{\Theta}_0}{4\pi e^2 n_0} \cdot \frac{1}{4n_0} \cdot D(\omega) \cdot \frac{1}{8\pi} \int d\vec{l} \, l f(l) L_1(\vec{k}, \vec{l}) \quad (30)$$

and a similar expression for the quantity  $\Gamma(\vec{k}, \omega) - \Delta(\vec{k}, \omega)$  by replacing  $L_1(\vec{k}, \vec{l})$  by  $L_2(\vec{k}, \vec{l})$ . The function  $D(\omega)$  is given by

$$D(\omega) = \frac{\omega_p^2}{\omega(\omega - 2\omega_p) + i(\omega - \omega_p)/\tau} - \frac{\omega_p^2}{\omega(\omega + 2\omega_p) + i(\omega + \omega_p)/\tau} \quad (31)$$

In the long wave length limit, we get

$$\Gamma(\vec{k}, \omega) = \frac{1}{30} \cdot \frac{k^{\Theta}}{4\pi e^2 n_0} \cdot k^2 \cdot D(\omega) \quad (32.a)$$

and

$$\Delta(\vec{k}, \omega) = - (3/2) \Gamma(\vec{k}, \omega) \quad (32.b)$$

where  $\Theta$  is defined as

$$\Theta = \Theta_0 \int f(l) l^2 dl / \int l^2 dl \quad (33)$$

and the use has been made of the following normalization,

$$\frac{1}{8\pi} \int d\vec{l} = n_0 \quad (34)$$

Examining Eq. (31), we can see that in the high frequency region the turbulent fluctuation in the cold electron plasma has a large effect primarily on the imaginary part of the dielectric constant for  $\omega \sim 2\omega_p$ .

Substituting Eqs. (32.a) and (32.b) with Eq. (31) into Eq. (22), we obtain the high frequency dielectric constant of the cold turbulent plasma as follows,

$$\epsilon(\vec{k}, \omega) = \epsilon_r(\vec{k}, \omega) + i\epsilon_i(\vec{k}, \omega) \quad (35.a)$$

$$\epsilon_r(\vec{k}, \omega) = 1 - \frac{(1+\Gamma_r)(\omega^2 - 2.5\omega_p^2\Gamma_r) + (\omega/\tau - 2.5\omega_p^2\Gamma_i)\Gamma_i}{(\omega^2 - 2.5\omega_p^2\Gamma_r)^2 + (\omega/\tau - 2.5\omega_p^2\Gamma_i)^2} \omega_p^2 \quad (35.b)$$

$$\epsilon_i(\vec{k}, \omega) = \frac{(1+\Gamma_r)(\omega/\tau - 2.5\omega_p^2\Gamma_i) - (\omega^2 - 2.5\omega_p^2\Gamma_i)\Gamma_i}{(\omega^2 - 2.5\omega_p^2\Gamma_r)^2 + (\omega/\tau - 2.5\omega_p^2\Gamma_i)^2} \omega_p^2 \quad (35.c)$$

where  $\Gamma_r$  and  $\Gamma_i$  are the real part and the imaginary part of  $\Gamma(\vec{k}, \omega)$  given by Eq. (32.a).

We shall illustrate this by calculating numerically the characteristic energy loss function, which is defined as

$$F(\vec{k}, \omega) = - \text{Im} \frac{1}{\epsilon(\vec{k}, \omega)} \quad (36)$$

for an arbitrary chosen value of  $\tau^{-1} = 0.1\omega_p$ . In Fig. 1, the broken line represents the result for a value of  $k = 0$ , which is nothing but the result for the homogeneous cold plasma. The real line represents the result for the value of  $k = \{4\pi e^2 n_0 / k\theta\}^{1/2}$ . There appears a small peak at the frequency of  $2\omega_p$ . This peak is due to the sharp maximum of the imaginary part of the dielectric constant. Another striking feature of the calculated energy loss function is apparent in the very low frequency region where it is seen that the imaginary part becomes negative because of the turbulent fluctuation. However, we are not going to explore this kind of instability, because the dielectric properties of plasmas at the low frequency region depend critically on effects of finite temperature of plasmas and also on effects of ion motion.

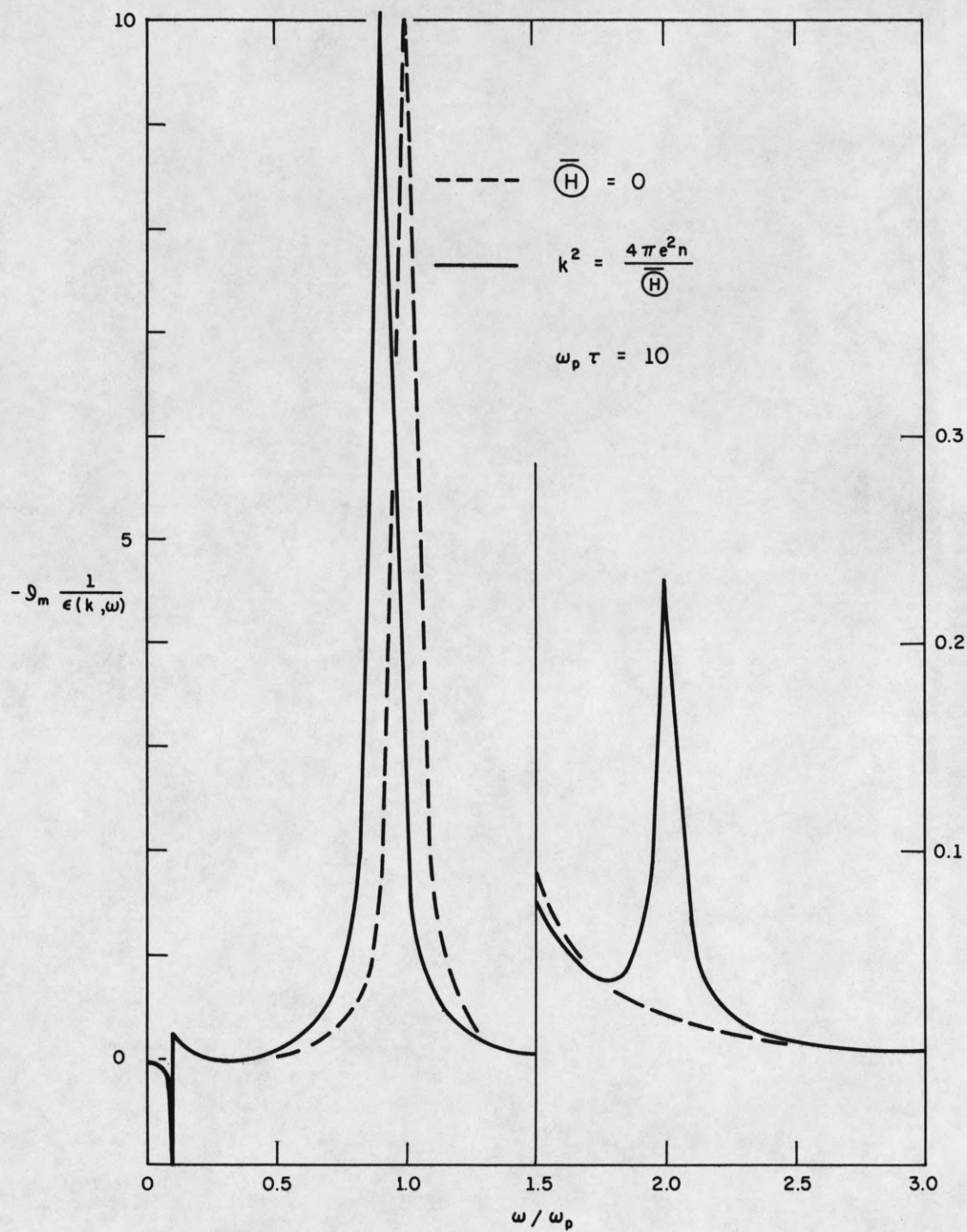


Fig. 1 The characteristic energy loss function  $-\text{Im}\{1/\epsilon(k, \omega)\}$ .



## 5. Concluding Discussions

The dynamical effect of the turbulent fluctuation on the dielectric properties of the weakly turbulent electron plasma has been examined by applying the truncation method of the ensemble average of products of fluctuations.

When the turbulent spectrum has a sharp maximum at the electron plasma frequency, we have found that the dielectric properties of the weakly turbulent differ very markedly from that of the homogeneous plasma in the thermal equilibrium state in the low frequency region and at twice of the electron plasma frequency. Investigating the static dielectric properties, we have obtained an explicit formula for the quantity  $k_*$  which characterizes the correlation length in the weakly turbulent electron plasma. The structure of Eq. (29) suggests that the contribution which appeared in the numerator of Eq. (29) can be attributed to a modification of the effective number density due to the turbulent fluctuation, while the denominator of Eq. (29) can be understood as the effective temperature of the turbulent electron plasma, which is the sum of the electron kinetic temperature and the turbulent energy density. We have not explored the dielectric properties of the turbulent electron plasma in the low frequency region, because there the contribution of the ions will become important.

The same method has been applied to the kinetic description of the low density and high temperature plasma in the Appendix. Derivation of the integral equation for the induced fluctuation of distribution function is just straightforward as in the case of a hydrodynamic description of the plasma. Although we are not able to write down an explicit solution of

the integral equation, we can analyze contributions of the various terms of the integral equation in comparison with the result obtained in the hydrodynamic description. It should be noticed that, according to the kinetic description presented in the Appendix, we have found the terms which represent appreciable effect of the turbulent fluctuation at the electron plasma frequency. This difference may be due to the use of the diffusion equation, Eq. (1.b), in the hydrodynamic description of the plasma where the non-linear drift term has been neglected.

## APPENDIX

In this Appendix, we will discuss a case of the low density and high temperature electron plasma. We may assume that kinetic properties of the electron plasma can be described by the non-linear Vlasov equation,

$$\frac{\partial F}{\partial t} + \vec{v} \cdot \vec{\nabla} F + \frac{e}{m} \vec{E} \cdot \frac{\partial}{\partial \vec{v}} F = -\nu F \quad (\nu \rightarrow 0) \quad (\text{A.1.a})$$

$$\text{div } \vec{E} = 4\pi e \left( \int F d\vec{v} - n \right) \quad (\text{A.1.b})$$

where the function  $F(\vec{x}, \vec{v}, t)$  is a one particle distribution function of electrons. A weakly turbulent state of the electron plasma can be specified by decomposing the distribution function  $F$  into two parts,

$$F(\vec{x}, \vec{v}, t) = F^{(0)}(\vec{v}, t) + f^{(0)}(\vec{x}, \vec{v}, t) \quad (\text{A.2})$$

where  $F^{(0)}(\vec{v}, t)$  is a spatially uniform slowly varying background distribution of the electrons, while  $f^{(0)}(\vec{x}, \vec{v}, t)$  represents a rapidly fluctuating component of the electron distribution in the system. We assume a statistical ensemble of plasmas having a random distribution of phases of the fluctuation  $f^{(0)}(\vec{x}, \vec{v}, t)$ . Corresponding to the decomposition of Eq. (A.2), we can reduce Eqs. (A.1.a) and (A.1.b) to the following set of equations,

$$\frac{\partial F^{(0)}}{\partial t} = -\frac{e}{m} \langle \vec{E}^{(0)} \cdot \frac{\partial}{\partial \vec{v}} f^{(0)} \rangle \quad (\text{A.3.a})$$

$$\frac{\partial f^{(0)}}{\partial t} + \vec{v} \cdot \vec{\nabla} f^{(0)} + \frac{e}{m} \vec{E}^{(0)} \cdot \frac{\partial F^{(0)}}{\partial \vec{v}} = -\frac{e}{m} \left( \vec{E}^{(0)} \cdot \frac{\partial f^{(0)}}{\partial \vec{v}} - \langle \vec{E}^{(0)} \cdot \frac{\partial f^{(0)}}{\partial \vec{v}} \rangle \right) \quad (\text{A.3.b})$$

and



$$\text{div } \vec{E}^{(0)} = 4\pi e \int f^{(0)} d\vec{v} \quad (\text{A.3.c})$$

where the bracket  $\langle \rangle$  stands for the ensemble average over the random distribution of the phase of  $f^{(0)}(\vec{x}, \vec{v}, t)$ .

Now we are interested in the linear response properties of the weakly turbulent plasma, of which the unperturbed state is determined by the above set of equations (A.3.a), (A.3.b) and (A.3.c). Let us introduce adiabatically an oscillating external charge  $eq(\vec{x}, t)$  into the systems and investigate the disturbance induced in the weakly turbulent plasma. The linear disturbance  $f^{(1)}(\vec{x}, \vec{v}, t)$  induced by the external charge is determined by the following set of equations,

$$\begin{aligned} \frac{\partial f^{(1)}}{\partial t} + \vec{v} \cdot \vec{\nabla} f^{(1)} + \frac{e}{m} \vec{E}^{(1)} \cdot \frac{\partial F^{(0)}}{\partial \vec{v}} + \frac{e}{m} \vec{E}^{(0)} \cdot \frac{\partial f^{(1)}}{\partial \vec{v}} + \\ \frac{e}{m} \vec{E}^{(1)} \cdot \frac{\partial f^{(0)}}{\partial \vec{v}} = -\nu f^{(1)} \end{aligned} \quad (\text{A.4.a})$$

and

$$\text{div } \vec{E}^{(1)} = 4\pi e \int f^{(1)} d\vec{v} + 4\pi eq \quad (\text{A.4.b})$$

Introducing the Fourier expansions,

$$\begin{pmatrix} f^{(1)}(\vec{x}, \vec{v}, t) \\ \vec{E}^{(1)}(\vec{x}, t) \end{pmatrix} = \frac{1}{VT} \sum_{\vec{k}, \omega} \begin{pmatrix} f^{(1)}(\vec{k}, \vec{v}, \omega) \\ \vec{E}^{(1)}(\vec{k}, \omega) \end{pmatrix} e^{i(\vec{k} \cdot \vec{x} - \omega t)} \quad (\text{A.5})$$

we can reduce Eqs. (A.4.a) and (A.4.b) as follows,

$$-i(\omega - \vec{k} \cdot \vec{v} + i\nu) f^{(1)}(\vec{k}, \vec{v}, \omega) + \frac{e}{m} E_{\alpha}^{(1)}(\vec{k}, \omega) \frac{\partial F^{(0)}}{\partial v_{\alpha}} +$$

$$\begin{aligned} & \frac{1}{VT} \sum_{\vec{\ell}, \omega'} \frac{e}{m} E_{\alpha}^{(0)}(\vec{\ell}, \omega') \frac{\partial}{\partial v_{\alpha}} f^{(1)}(\vec{k}-\vec{\ell}, \vec{v}, \omega-\omega') + \\ & \frac{1}{VT} \sum_{\vec{\ell}, \omega'} \frac{e}{m} E_{\alpha}^{(1)}(\vec{\ell}, \omega') \frac{\partial}{\partial v_{\alpha}} f^{(0)}(\vec{k}-\vec{\ell}, \vec{v}, \omega-\omega') = 0 \end{aligned} \quad (\text{A.6.a})$$

and

$$E_{\alpha}^{(1)}(\vec{k}, \omega) = \frac{4\pi e}{k^2} i k_{\alpha} \overline{f^{(1)}(\vec{k}, \omega)} + \frac{4\pi e}{k^2} i k_{\alpha} q(\vec{k}, \omega) \quad (\text{A.6.b})$$

with the abbreviation

$$\overline{f^{(1)}(\vec{k}, \omega)} = \int f^{(1)}(\vec{k}, \vec{v}, \omega) d\vec{v} \quad (\text{A.6.c})$$

Since the above set of equations corresponds to Eq. (12) of Section 2, we may proceed just in the same way as in Section 3. Applying a similar truncation approximation as in Eq. (18), we obtain the following integral equation for the function  $\langle f^{(1)}(\vec{k}, \vec{v}, \omega) \rangle$ ,

$$\begin{aligned} & \{ \epsilon^{(0)}(\vec{k}, \omega) + 4\pi(\beta(\vec{k}, \omega) + \gamma(\vec{k}, \omega)) \} \langle f^{(1)}(\vec{k}, \omega) \rangle + \\ & \left( \frac{e}{m} \right)^2 \frac{1}{(VT)^2} \sum_{\vec{\ell}, \omega'} \langle E_{\alpha}^{(0)}(\vec{\ell}, \omega') E_{\beta}^{(0)}(\vec{\ell}, \omega')^* \rangle \cdot \\ & \int \frac{1}{\omega - \vec{k} \cdot \vec{v} + i\nu} \cdot \frac{\partial}{\partial v_{\alpha}} \left( \frac{1}{\omega - \omega' - (\vec{k} - \vec{\ell}) \cdot \vec{v} + i\nu} \frac{\partial}{\partial v_{\beta}} \langle f^{(1)}(\vec{k}, \vec{v}, \omega) \rangle \right) d\vec{v} \\ & = - 4\pi(\alpha^{(0)}(\vec{k}, \omega) + \beta(\vec{k}, \omega) + \gamma(\vec{k}, \omega) q(\vec{k}, \omega)) \end{aligned} \quad (\text{A.7})$$

with the abbreviations

$$\beta(\vec{k}, \omega) = e \left( \frac{e}{m} \right)^3 \frac{k_\beta}{k^2} \frac{1}{(VT)^2} \sum_{\vec{l}, \omega'} \langle E_\alpha^{(0)}(\vec{l}, \omega') E_\gamma^{(0)}(\vec{l}, \omega')^* \rangle \cdot \int \frac{1}{\omega - \vec{k} \cdot \vec{v} + i\nu} \cdot \frac{\partial}{\partial v_\alpha} \left[ \frac{1}{\omega - \omega' - (\vec{k} - \vec{l}) \cdot \vec{v} + i\nu} \frac{\partial}{\partial v_\beta} \left( \frac{\partial F^{(0)} / \partial v_\gamma}{-\omega' + \vec{l} \cdot \vec{v} + i\nu} \right) \right] d\vec{v} \quad (\text{A.8.a})$$

and

$$\begin{aligned} \gamma(\vec{k}, \omega) = & 4\pi e \left( \frac{e}{m} \right)^4 \frac{k_\epsilon}{k^2} \frac{1}{(VT)^2} \sum_{\vec{l}, \omega'} \frac{k_\alpha - l_\alpha}{|\vec{k} - \vec{l}|^2} \cdot \frac{1}{\epsilon^{(0)}(\vec{k} - \vec{l}, \omega - \omega')} \\ & \langle E_\beta^{(0)}(\vec{l}, \omega') E_\delta^{(0)}(\vec{l}, \omega')^* \rangle \cdot \int \frac{1}{\omega - \vec{k} \cdot \vec{v} + i\nu} \left\{ \frac{\partial}{\partial v_\beta} \left( \frac{\partial F^{(0)} / \partial v_\alpha}{\omega - \omega' - (\vec{k} - \vec{l}) \cdot \vec{v} + i\nu} \right) \right. \\ & + \left. \frac{\partial}{\partial v_\alpha} \left( \frac{\partial F^{(0)} / \partial v_\beta}{\omega' - \vec{l} \cdot \vec{v} + i\nu} \right) \right\} d\vec{v} \cdot \int \frac{1}{\omega - \omega' - (\vec{k} - \vec{l}) \cdot \vec{v} + i\nu} \left\{ \frac{\partial}{\partial v_\delta} \left( \frac{\partial F^{(0)} / \partial v_\epsilon}{\omega - \vec{k} \cdot \vec{v} + i\nu} \right) + \right. \\ & \left. \frac{\partial}{\partial v_\epsilon} \left( \frac{\partial F^{(0)} / \partial v_\delta}{\omega' - \vec{l} \cdot \vec{v} + i\nu} \right) \right\} d\vec{v} \quad (\text{A.8.b}) \end{aligned}$$

The quantities  $\epsilon^{(0)}(\vec{k}, \omega)$  and  $\alpha^{(0)}(\vec{k}, \omega)$  are defined as

$$\epsilon^{(0)}(\vec{k}, \omega) = 1 + 4\pi\alpha^{(0)}(\vec{k}, \omega) \quad (\text{A.9.a})$$

and

$$\alpha^{(0)}(\vec{k}, \omega) = - \frac{e^2}{mk^2} \int \frac{k_\alpha}{\omega - \vec{k} \cdot \vec{v} + i\nu} \frac{\partial F^{(0)}}{\partial v_\alpha} d\vec{v} \quad (\text{A.9.b})$$

Comparing Eq. (A.7) with Eq. (20), we find that both equations have similar structure, especially, the quantity  $\gamma(\vec{k}, \omega)$  corresponds to the quantities  $\Gamma(\vec{k}, \omega)$  and  $\Delta(\vec{k}, \omega)$  given as Eqs. (21.a) and (21.b). These quantities represent the modification of the dielectric properties of the turbulent plasma in the low frequency region and at the frequency region of twice of



the electron plasma frequency. However, the quantity  $\beta(\vec{k}, \omega)$  and the second term of the left hand side of Eq. (A.7) have no counter parts in Eq. (20). The contributions of these factors become very appreciable at the electron plasma frequency. Thus, it appears to be very interesting to examine the dielectric properties of the turbulent electron plasma on the basis of Eq. (A.7). Yet, we have not been able to solve the integral equation, (A.7) so far.

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1. ORIGINATING ACTIVITY (Corporate author)		2a. REPORT SECURITY CLASSIFICATION	
Coordinated Science Laboratory University of Illinois, Urbana, Illinois		Unclassified	
3. REPORT TITLE		2b. GROUP	
DIELECTRIC PROPERTIES OF A WEAKLY TURBULENT PLASMA			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)			
5. AUTHOR(S) (Last name, first name, initial)			
Ichikawa, Yoshi H.			
6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS.	
April, 1965	24	5	
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)		
Da 28 043 AMC 00073(E)	R-251		
b. PROJECT NO.			
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)		
d.			
10. AVAILABILITY/ LIMITATION NOTICES			
Qualified requesters may obtain copies of this report from DDC. DDC release to OTS is NOT authorized.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY	
		U. S. Army Electronics Laboratories. Fort Monmouth, New Jersey 07703	
13. ABSTRACT			
<p>Generalization of the Herring-Yoshikawa theory has been developed to take into account the dynamical effect of turbulent fluctuation on the dielectric properties of weakly turbulent plasma. According to the hydrodynamic description of electron plasma, the turbulent fluctuation gives rise to appreciable modification of the dielectric properties in the low frequency region and at the frequency of twice of the electron plasma frequency. An expression of the static dielectric constant shows that the characteristic correlation distance of the weakly turbulent plasma becomes longer than the Debye distance of corresponding homogeneous plasma in thermal equilibrium state. Contrary to the hydrodynamic scheme, the integral equation has been derived for the induced fluctuation in the kinetic description of the plasma. This integral equation suggests that the turbulent fluctuation may modify the dielectric properties of the turbulent plasma also at the electron plasma frequency.</p>			

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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